Development of an Adjustable Frequency Device for a Test Vehicle Based on Curved Beam Theory

[1]J. D. YAU, [2]S. URUSHADZE
[1]Department of Civil Engineering of Tamkang University in Taiwan, Tamkang, TAIWAN
[2]Theoretical and Applied Mechanics, CAS, Prague, CZECH REPUBLIC

Abstract— In this article, an adjustable frequency device based on curved beam theory is designed to control vertical stiffness of an instrumented vehicle that it can detect dynamic data when moving on a test beam for frequency measurement. The adjustable frequency device consists of a set of two-layer cantilever semi-circular thin-beams to support a lumped mass for vibrations, in which a rotatable U-frame is used to change its subtended angle for adjustment of the supporting stiffness and corresponding vertical frequencies of the vehicle. Based on curved beam theory, an analytical frequency equation of the single-degree-of-freedom test vehicle was derived and applied to mobile frequency measurement of a simple beam. To determine the sectional rigidity of the semi-circular thin-beams, both theoretical and experimental studies were be carried out in the ITAM laboratory of the Academy of Science in Czech. The analytical and experimental results indicated that the present semi-circular beam model with guided ends is applicable to prediction of natural frequencies of the test vehicle considering different supporting stiffness.

Keywords—Curved beam, experiment, frequency, vibration.

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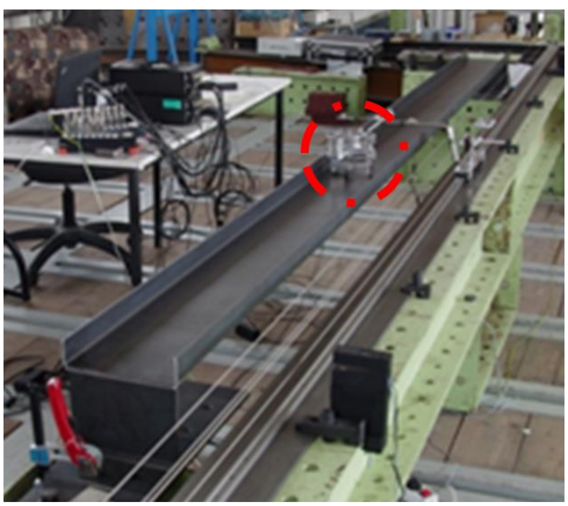

Fig. 1 An instrumented vehicle staying on a test beam.

1. Introduction

THE use of a moving test vehicle to measure the frequencies of a bridge has attracted a lot of attention from engineering researchers. Different from a large number of fixed-sensors being deployed on a bridge for structural healthy monitoring (SHM), the passing test vehicle over a bridge is regarded as a vibration message receiver to collect the dynamic data for bridge frequency measurement, as indicated in Fig. 1. Such an alternative monitoring approach for bridge structure health monitoring possesses the following competitions: (1) Mobility in sensor deployment; (2) Economy in sensor maintenance and monitoring workers; and (3) Efficiency and portability through repetitive experiments by changing vehicle's speeds [1, 2]. Such a monitoring approach through vehicle's response to detect bridge frequencies is called vehicle-scanning method (VSM) [1]. However, the vibration frequency of the instrumented vehicle should be known before applying the VSM to bridge frequency measurement due to possible coupling phenomena of vehicle-bridge system [3]. To illustrate the applicability of the VSM to bridge SHM in laboratory, a single-degree-of-freedom (SDF) instrumented vehicle that its vertical frequencies is adjustable by changing the supporting stiffness of a set of curved thin-beam system was designed and fabricated to offer more feasible measurements of bridge frequencies.

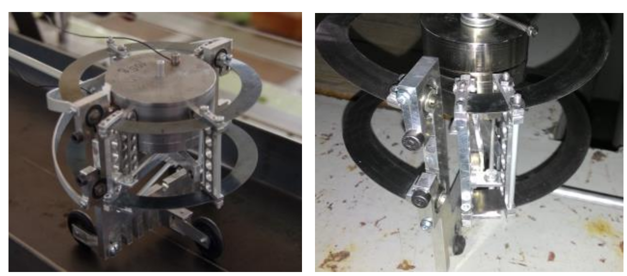

Fig. 2 Details of the supporting semi-circular thin-beam system

As shown in Fig.2, the fabricated test vehicle consists of a double two-layer cantilever semi-circular beam system that its arc-spring stiffness (or vertical frequency) can be adjusted through the change of effective arc-length (or subtended angle) of the double curved beam unit. In this study, a theoretical frequency equation as well as experimental measurement of free vibration for the test vehicle will be carried out and verified. The results shows the present curved-beam based model with adjustable subtended angle is feasible to predict the natural frequencies of the test vehicle considering different supporting stiffness.

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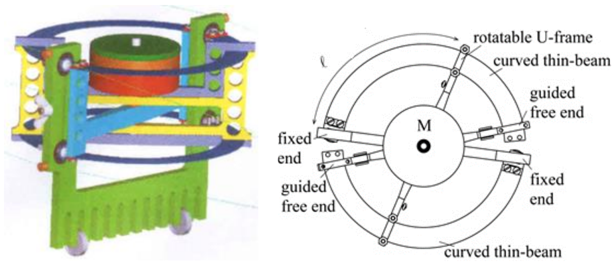

Fig. 3 Schematics of a test vehicle with curved thin-beam carrying a lumped mass (M)

2. Problem Formulation

Figure 3 depicts a schematic representation of an SDF test vehicle model that can measure beam frequency using the vehicle-scanning method (VSM) [1]. The test vehicle consists of a set of semi-circular curved thin-beams to support the lumped mass M, in which a rotatable U-frame is designed to adjust the supporting stiffness for different vehicle's frequencies. To investigate the dynamic properties of the SDF test vehicle, a theoretical model based on curved-beam theory will be presented in the following.

2.1 Governing equations and basic assumptions

With reference to Figs. 2 and 3, the following considerations are adopted:

1. The semi-circular beam is regarded as a *massless* curved-thin beam of Bernoulli-Euler type;
2. The U-frames equipped with the semi-circular beams supporting the lumped mass M are regarded as a rigid frame with mass m;
3. The vertical displacements at the guided free ends of the U frames are identical;
4. The guided free end is restraint in both torsional and bending rotations but free at vertical displacement. Please submit your manuscript electronically for review as e-mail attachments.

With above consideration, the coupled equations of the curved thin-beam in out-of-plane reads [4]

$$m_b u + EI\left(u'' - \frac{\theta_x}{R}\right)'' - \frac{GJ}{R}\left(\frac{u'}{R} + \theta_x\right) = 0$$
$$\rho I_b \theta_x - EI\left(u'' - \frac{\theta_x}{R}\right) - GJ\left(\frac{u'}{R} + \theta_x\right) = 0 \qquad (1a,b)$$

Here (m_b, ρI_b) represent the mass and polar inertia of moment of the thin-beam, and (u, θ_x) the flexural and torsional deformations, respectively, (EI, GJ) as flexural and torsional rigidities on the section of the curved beam element, and (R, L) the radius and arc length of the curved beam element with subtend angle of ϕ (= L/R) respectively. The coupled equations in (1a) and (1b) reveal the torsional and bending coupling nature of curved beams in out of plane. To simplify the stiffness matrix formulation of the supporting cantilever curved thin-beam system, their inertia would be neglected for its negligible mass much smaller than the lumped-mass M in the following theoretical formulation.

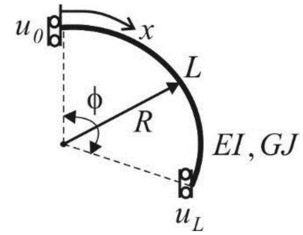

Fig. 4 Guided curved-beam element with a subtended angle ϕ

2.2 Guided curved-beam element

As depicted in Fig. 4, a curved beam element with rotational-restraint guided ends, in which (u_0, u_L) represent the vertical displacements at x = 0 and L, respectively. Here, the boundary conditions of the curved beam element with rotational restraints can be expressed as

$$u(0,t) = u_0, u(L,t) = u_L$$
$$\theta_x(0,t) = u'(0,t) = \theta_x(L,t) = u'(L,t) = 0 \qquad (2a,b)$$

where (u_0, u_L) denote the nodal displacements of the guided curved-beam element. Let us consider the *linear* generalized strains relating to flexure and constant strain of torsional deformation in the out of plane of the curved beam element depicted in Fig. 4, that is, [4]

$$u'' - \frac{\theta_x}{R} = \frac{b_1 x/R + b_2}{R^2}, \quad \theta_x' + \frac{u'}{R} = \frac{b_3}{R^2} \qquad (3a,b)$$

and the corresponding displacement and torsional functions are expressed as [4]

$$u(x) = \frac{x}{R}b_1 + b_2 + \frac{x}{R}b_3 + \cos\frac{x}{R}b_4 + \sin\frac{x}{R}b_5 + b_6$$
$$\theta_x(x) = \frac{-1}{R}\left(\frac{x}{R}b_1 + b_2 + \cos\frac{x}{R}b_4 + \sin\frac{x}{R}b_5\right) \qquad (4a,b)$$

Here, $b_1 \sim b_6$ are the undetermined coefficients. These coefficients can be determined by substituting the displacement and torsional expressions of (4) into the boundary conditions of guided ends, as given in (2) to yield the following shape functions for a guided curved-beam element, as shown in Fig. 4,

$$u(x,t) = f_u(L-x)u_0 + f_u(x)u_L$$
$$\theta_x(x,t) = f_\theta(x)(u_0 - u_L) \qquad (5a,b)$$

where the shape functions (f_u, f_θ) derived are written as

$$f_u(x) = \frac{\frac{x}{R}\cos\frac{L}{2R} + \sin\left(\frac{L}{2R} - \frac{x}{R}\right) - \sin\frac{L}{2R}}{2\left(\frac{L}{2R}\cos\frac{L}{2R} - \sin\frac{L}{2R}\right)} \qquad (6a)$$

$$f_\theta(x) == \frac{\sin\left(\frac{L}{2R} - \frac{x}{R}\right) - (1 - \frac{2x}{L})\sin\frac{L}{2R}}{2R\left(\frac{L}{2R}\cos\frac{L}{2R} - \sin\frac{L}{2R}\right)} \qquad (6b)$$

On the other hand, the out of plane strain energy U of a curved beam element with length *L* (= Rϕ) is equal to [4]

$$U = \frac{1}{2}\int_0^L \left[EI\left(u'' - \frac{\theta_x}{R}\right)^2 + GJ\left(\theta_x' + \frac{u'}{R}\right)^2\right]dx \qquad (7)$$

The bracketed terms in (7) represent the strain energy caused by the coupled flexural and torsional deformations of (u, θ_x). By substituting the shape functions of (5) into the strain energy equation of (7) and applying the variational principle, the virtual strain energy (δU) can be expressed in terms of nodal displacements (u_0, u_L) as follows

$$\delta U = \langle \delta u_0 \quad \delta u_L \rangle \begin{bmatrix} k(L,\phi) & -k(L,\phi) \\ -k(L,\phi) & k(L,\phi) \end{bmatrix} \begin{Bmatrix} u_0 \\ u_L \end{Bmatrix} \tag{8}$$

where the stiffness coefficient of $k(L, \phi)$ is given as

$$k(L,\phi) = \frac{EI}{R^3\phi}\left[\frac{1}{3}\left(\frac{(\phi/2)\times\tan(\phi/2)}{(\phi/2)-\tan(\phi/2)}\right)^2 + \frac{GJ}{EI}\right]_{\phi=L/R} \tag{9}$$

Let us consider the degenerated case that the subtended angle ϕ is reduced to zero for a straight beam element. It is expected that the stiffness coefficient of $k(L, \phi)_{\phi\to 0}$ becomes $12EI/L^3$ of a guided straight beam element with two end-rotational restraints.

3. Natural frequency

As shown in Fig. 3, the SDF dynamic system supporting a lumped mass M is composed by two layers of cantilever semi-circular thin-beams with a rotatable U-frame. By assembling these curved beam elements, the structural matrix equation of the SDF supporting system carrying a lumped mass M can be expressed as

$$\begin{bmatrix} M+m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{u}_L \\ \ddot{u}_{\pi R-L} \end{Bmatrix} + 4k(L,\phi) \begin{bmatrix} 1+\kappa & -\kappa \\ -\kappa & \kappa \end{bmatrix} \begin{Bmatrix} u_L \\ u_{\pi R-L} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{10a}$$

with the stiffness ratio

$$\kappa = \frac{k(\pi R - L, \pi - \phi)}{k(L,\phi)} \tag{10b}$$

Here, m is the mass of the supporting U-frame guiding the cantilever semi-circular thin-beam unit. Since the vertical displacements of (u_L, $u_{\pi R-L}$) at the guided ends are rigidly locked by the rotatable U-frame, they can be assumed to be identical, that is, $u_L = u_{\pi R-L}$. Thus the governing equation in vertical vibration for the test vehicle equipped with the semi-circular thin-beam system to support the lumped mass M in (1) reads

$$(M+2m)\ddot{u}_L + 4k(L,\phi)u_L = 0 \tag{11}$$

Solving the eigenvalue problem of free vibration yields the following characteristic equation for natural frequency of the test vehicle as

$$\frac{4EI}{R^3\phi}\left[\frac{1}{3}\left(\frac{(\phi/2)\times\tan(\phi/2)}{(\phi/2)-\tan(\phi/2)}\right)^2 + \frac{GJ}{EI}\right] - (M+2m)\omega^2 = 0 \tag{12}$$

By expressing the angular frequency as $\omega = 2\pi f$, the characteristic equation of (12) can be rewritten as

$$\frac{1}{3}\left(\frac{(\phi/2)\times\tan(\phi/2)}{(\phi/2)-\tan(\phi/2)}\right)^2 = -\frac{GJ}{EI} + \frac{(M+2m)R^3\pi^2}{EI}\phi f^2 \tag{13}$$

The analytical expression of (13) provides an exact relationship between the subtended angle ($\phi = L/R$) and the vertical frequency (f), from which we can identify the sectional rigidities of (EI, GJ) of the supporting semi-circular thin-beam systems through a series of experimental measurements.

For simplicity to illustrate (13), let us adopt the following symbols for mathematical curve-fitting using the least square method [5].

$$X = \phi f^2,\ Y = \frac{1}{3}\left(\frac{(\phi/2)\times\tan(\phi/2)}{(\phi/2)-\tan(\phi/2)}\right)^2 \tag{14a,b}$$

$$a = -\frac{GJ}{EI} = -\frac{2}{1+\nu},\ b = \frac{(M+2m)R^3\pi^2}{EI} \tag{14c,d}$$

Here the section properties of $J = wt^3/3$ and $I = wt^3/12$ for a thin rectangular section (w x t) are used in (14c). So the characteristic equation of free vibration shown in (13) can be rewritten in an alternative linear equation as

$$Y = a + bX \tag{15}$$

Then one can determine the coefficients of (a, b) by measuring the vertical frequency f incorporating with the subtend angle ϕ through a series of frequency measurements of free vibration test by changing the subtended angle.

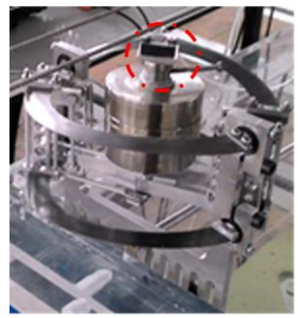

Fig. 5 A Brüel-Kjaer piezoelectric accelerometer mounted on the lumped mass for response measurement.

4. Experimental setup

As described in the previous sections, the test vehicle has the feature of adjustable frequencies by rotating the U-frame of the double semi-circular curved beam system to control the vertical stiffness. To measure the vertical free vibration of the test vehicle, a piezoelectric accelerometer, Brüel-Kjaer, type 4374 with sampling rate of 1000 Hz, is mounted on the lumped mass M of the instrumented test vehicle, as shown in Fig. 5. The vibration signals recorded from accelerometers were transmitted via charge amplifiers into a computer DEWE–43.

5. Experimental results

The material and geometrical properties of the semi-circular thin-beam are listed in the following Table 1. The materials used for the curved thin-beam is of steel and the rotatable U-frame of aluminum. By changing the subtended angle ϕ, one can measure the corresponding vertical frequencies of the test vehicle through free vibration test. Table 2 lists the theoretical and experimental results through a series of vibration tests by considering different subtended angles. As indicated, they exist some difference from the measured frequencies with respect to different subtended angles.

Table 1 Properties of the supporting curved-beam system

E (kN/mm^2)	ν	t (mm)	w (mm)	R (mm)	M (kg)	m (kg)	I_0 (mm^4)	J_0 (mm^4)
2.0E5	0.3	6	12.8	56.4	1.0	0.057	230	918

Table 2 Theoretical and Measured frequencies

ϕ (deg.)	75	90	105	120	135
f_0 (Hz) Analyt. Measured	11.0 (11.2)*	8.5	7.3	6.7	6.1 (5.5)*
f (Hz) Fitted	11.1	8.9	7.4	6.3	5.5

As mention previously, by using least square method [5], the coefficients of (a, b) in the linear equation of Eq. (15) are fitted from the measure data shown in Table 2 and listed in Table 3. Fig. 6 plotted the fitting relationship of the measured frequencies (f) with respect to the subtended angle (ϕ). Then the corresponding flexural (EI) and torsional (GJ) rigidities can be calculated from the fitted equation. According to the fitted curve plotted in Fig. 6, the relationship between the subtended angle ϕ and the measured frequency f is of nonlinearity even (15) is a linear equation in terms of X (= ϕf^2). However, the analytical expression of (15) helps us extract the key parameters of the adjustable frequency device based on curved beam theory.

Table 3 Fitted sectional parameters of the curved thin-beam

Y	a	b	$EI_0=Ewt^3/12$ (kN-m^2)	EI fitted	$GJ_0=Gwt^3/3$ (kN-m^2)	GJ fitted
$a+bX$	-1.57	0.043	45.9	48.9	70.6	76.8

(I_0, J_0) are obtained from the definition of bending and torsional constants.

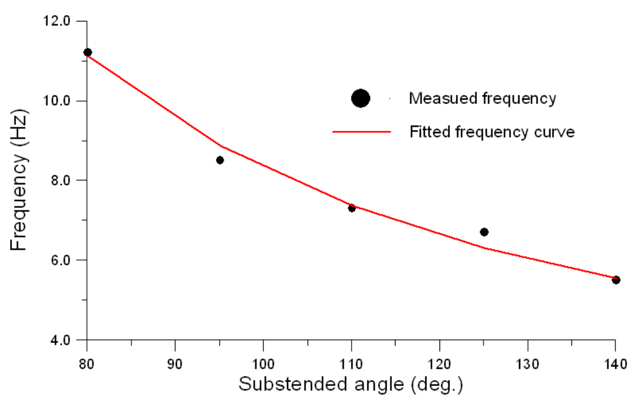

Fig. 6 Frequency fitting method

6. Conclusion

The present work is in part of structural health monitoring of bridge structures using a passing vehicle. The natural frequency of the test vehicle should be known before using a passing vehicle to collect the dynamic data of a bridge for frequency measurement. For this, an SDF test vehicle that its vertical frequency can be adjusted by changing the supporting stiffness of the semi-circular thin-beam system was designed and fabricated to offer more feasible frequency measurements in laboratory.

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J.D. Yau got his Ph.D. in the field of Civil Engineering from National Taiwan University (NTU) in 1996 for vibration problems of vehicle-bridge interaction of high speed rails. He joined the faculty at TamKang University (TKU) in 1999 where he has served as Assistant Professor (1999-2003), Associate Professor (2003-09), and Chair (2004-2007) in the Department of Architecture and Building Technology. In 2010, Dr. Yau became a Professor in TKU. Since 2013, Dr. Yau was a Jointly Appointed-Professor of Department of Architecture and Department of Civil Engineering in TKU. Dr. Yau has published over 60 referred journal papers and articles. His major research area is centered on moving load problems, dynamics of high speed rails, maglev dynamics, and nonlinear analysis of framed structures.

S. Urushadze, Head of Laboratory for Experimental Mechanics has been involved in long term monitoring programmes of wind induced deformation measurements on TV towers. His research interests include dynamic identification of structural degradation, testing and vibration of bridges, vibration damage assessment of historic masonry, earthquake problems. He is a local coordinator of a new EC Research Fund for Steel and Coal project on fatigue of bridges.