Sliding Modes Control of the Asynchronous Motor with an Observer of a Special Class of the Non-linear Systems Interconnected to an Estimator

AHMED CHOUYA
Department of Engineering Electrical
University of Djilali Bounâama
Khemis-Miliana City, 44 225
ALGERIA

Abstract: - The present paper deals sliding modes control of the asynchronous motor, where the sensor mechanical speed, it replaced by a software sensor. It is an observer of a special class of the non-linear systems interconnected to an estimator. The performances of the sliding modes control of the asynchronous motor with an observer of a special class of the non-linear systems interconnected to an estimator proposed are shown by a simulation treating the evaluation of the rotor flux and rotating speed an asynchronous motor.

Key-Words: - Adaptive observer, high gain observer, sliding mode observer, induction machine, sensor-less speed.

Received: August 25, 2022. Revised: April 19, 2023. Accepted: May 15, 2023. Published: July 5, 2023.

1 Introduction

The new industrial applications require variable speed transmissions having high dynamic performances. These last years several techniques were developed to allow the asynchronous variators to reach these performances, (cf.[1], [2], [3], [4], [5], [6], [7], [8], [9]).

However vectorial control and linearising input-output, which allows a decoupling between the variables of control, remain the most used considering the high dynamic performances which it offers for a broad range applications [10].

With the aim of improve the dynamic performances of the regulation speed of the asynchronous motor, we considered it interesting to call upon an observer of state to rebuild the state (flux and speed) starting from the size and manipulated variable (tension) to control [11]. In the following section a short recall of the asynchronous model used, section 3 presents the non-linear observer; sliding modes control is in section 4; where stability (motor+observer) is checked; results of simulations are given in section 5 by a simulation with the software MATLAB/SIMULINK.

2 Dynamic Model of Asynchronous Motor

In this study, the model of the motor rests on the following hypothesis, [12], [13]:

- The fluxes and the currents are proportional by the intermediary of inductances and the mutual.
- The losses iron are neglected .
- The air-gap is constant (squirrel-cage rotor).
- The homo-polar components are null.

It results from these assumptions that the various mutual between rotor and stator can be expressed like functions sinusoidal of the rotor position.
Its vector state is composed by the stator currents and rotor fluxes, as follows:

$$\begin{aligned}
\begin{bmatrix} \frac{di_{s\alpha}}{dt} \\ \frac{di_{s\beta}}{dt} \\ \frac{d\psi_{r\alpha}}{dt} \\ \frac{d\psi_{r\beta}}{dt} \end{bmatrix} &= \begin{bmatrix} -\gamma & 0 & \frac{K}{T_r} & pK\Omega \\ 0 & -\gamma & -pK\Omega & \frac{K}{T_r} \\ \frac{M}{T_r} & 0 & -\frac{1}{T_r} & -p\Omega \\ 0 & \frac{M}{T_r} & p\Omega & -\frac{1}{T_r} \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix} \\
&+ \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \end{bmatrix} \\
\frac{d\Omega}{dt} &= \frac{p}{J}\frac{M}{L_r}\left(i_{s\beta}\psi_{r\alpha} - i_{s\alpha}\psi_{r\beta}\right) - \frac{C_{res}}{J} \\
\frac{dC_{res}}{dt} &= 0
\end{aligned} \tag{1}$$

with:

$$T_r = \frac{L_r}{R_r} \quad ; \quad \sigma = 1 - \frac{M^2}{L_s L_r}$$
$$K = \frac{M}{\sigma L_s L_r} \quad ; \quad \gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r}$$

L_s, L_r and M are inductances cyclic stator, rotor and cyclic mutual inductance between stator and rotor respectively; R_s and R_r are stator and rotor resistances ; σ is the scattering coefficient; T_r is the time constant of the rotor dynamics; J is the rotor inertia; C_{res} is the resistive torque; p is the pole pair induction; the model (1) is for a reference frame binds to the fields stator.

3 Synthesis of Observer of a Special Class of the Nonlinear Systems Interconnected to an Estimator

The model of the asynchronous machine (1) can be rewritten in the shape of interconnected two subsystems:

$$\begin{bmatrix} \frac{di_{s\alpha}}{dt} \\ \frac{d\Omega}{dt} \end{bmatrix} = \begin{bmatrix} 0 & pK\psi_{r\beta} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ \Omega \end{bmatrix} + \begin{bmatrix} -\gamma i_{s\alpha} + \dfrac{K}{T_r}\psi_{r\alpha} + \dfrac{1}{\sigma L_s} v_{s\alpha} \\ \dfrac{p}{J}\dfrac{M}{L_r}\left(i_{s\beta}\psi_{r\alpha} - i_{s\alpha}\psi_{r\beta}\right) - \dfrac{C_{res}}{J} \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} \dfrac{di_{s\beta}}{dt} \\ \dfrac{d\psi_{r\alpha}}{dt} \\ \dfrac{d\psi_{r\beta}}{dt} \end{bmatrix} = \begin{bmatrix} -\gamma & 0 & \dfrac{K}{T_r} \\ 0 & -\dfrac{1}{T_r} & 0 \\ \dfrac{M}{T_r} & 0 & -\dfrac{1}{T_r} \end{bmatrix} \begin{bmatrix} i_{s\beta} \\ \psi_{r\alpha} \\ \psi_{r\beta} \end{bmatrix} + \begin{bmatrix} -p\Omega K\psi_{r\alpha} + \dfrac{1}{\sigma L_s} v_{s\beta} \\ \dfrac{M}{T_r} i_{s\alpha} - p\Omega\psi_{s\beta} \\ p\Omega\psi_{r\alpha} \end{bmatrix} \quad (3)$$

The interconnected two subsystems (2) and (3) can be represented in a more compact interconnected form as follows:

$$\begin{cases} \dot{X}_1 = F_1(X_2)X_1 + G_1(U_1, X_1, X_2) \\ Y_1 = C_1 X_1 \end{cases} \quad (4)$$

$$\begin{cases} \dot{X}_2 = A_2 X_2 + G_2(U_2, X_1, X_2) \\ Y_2 = C_2 X_2 \end{cases} \quad (5)$$

Where
$X_1 = [i_{s\alpha}, \Omega]^T$, $X_2 = [i_{s\beta}, \psi_{s\alpha}, \psi_{s\beta}]^T$, $C_1 = [1, 0]$, $C_2 = [1, 0, 0]$, $F_1(X_2) = \begin{bmatrix} 0 & pK\psi_{r\beta} \\ 0 & 0 \end{bmatrix}$,

$$A_2 = \begin{bmatrix} -\gamma & 0 & \frac{K}{T_r} \\ 0 & -\frac{1}{T_r} & 0 \\ \frac{M}{T_r} & 0 & -\frac{1}{T_r} \end{bmatrix}, \quad U = [v_{s\alpha}, v_{s\beta}]^T,$$

$$G_1(U_1, X_1, X_2) = \begin{bmatrix} -\gamma i_{s\alpha} + \dfrac{K}{T_r}\psi_{r\alpha} + \dfrac{1}{\sigma L_s} v_{s\alpha} \\ \dfrac{p}{J}\dfrac{M}{L_r}\left(i_{s\beta}\psi_{r\alpha} - i_{s\alpha}\psi_{r\beta}\right) - \dfrac{C_{res}}{J} \end{bmatrix},$$

$$G_2(U_2, X_1, X_2) = \begin{bmatrix} -p\Omega K\psi_{r\alpha} + \frac{1}{\sigma L_s} v_{s\beta} \\ \frac{M}{T_r} i_{s\alpha} - p\Omega\psi_{s\beta} \\ p\Omega\psi_{r\alpha} \end{bmatrix}$$

and $U_1 = [v_{s\alpha}, 0]^T$, $U_2 = [v_{s\beta}, 0, 0]^T$, $Y = [i_{s\alpha}, i_{s\beta}]^T$.

Before giving the hypothesis, we will make the remarks and one presents notations used hereafter.

1. Set $\Lambda_1(X_2)$ diagonal matrix defined by:

$$\Lambda_1(X_2) = \begin{bmatrix} 1 & 0 \\ 0 & pK\psi_{r\beta} \end{bmatrix} \quad (6)$$

2. Set S the single solution of the algebraic equation of LYAPUNOV:

$$S + A^T S + SA - C_1^T C_1 = 0 \quad (7)$$

where A identity matrix.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

One can show that the explicit solution (7) is given by:

$$S(i, j) = (-1)^{i+j} C_{i+j-2}^{j-1} \text{ where } C_j^i = \frac{j!}{i!(j-i)!}$$

3. Set $\bar{S} = \Lambda_1(X_2)^T S \Lambda_1(X_2)$ and $\bar{C}_1 = C_1 \Lambda_1(X_2)$

With an aim of designing an observer cascades for the subsystem (4), we pose the following hypothesis:

Hypothisis 1 *:*

H_1 *:($v_{s\alpha}$, $\psi_{r\alpha}$, $\psi_{r\beta}$, $i_{r\alpha}$) and ($v_{s\beta}$, Ω, $i_{r\beta}$) are limited and supposed to be regularly persistent to guarantee the property of observability of the hypothesis (4) and (5) respectively.*

H_2 *: F_1 is global LIPSCHITZ compared to X_2, i.e.:*

$$\left\| \Delta_\theta^{-1} \left(F_1(X_2) X_1 - F_1\left(\hat{X}_2\right) X_1 \right) \right\| \le \xi_1 \|\bar{e}_1\| ;$$

$\xi_1 > 0$: *Constant of LIPSCHITZ*

H_3 *: G_1 is global LIPSCHITZ compared to X_2, i.e.:*

$$\left\| \Delta_\theta^{-1} \left(G_1(U_1, X_1, X_2) - G_1\left(U_1, \hat{X}_1, \hat{X}_2\right) \right) \right\| \le \xi_2 \|\bar{e}_1\|$$

$\xi_2 > 0$: *Constant of LIPSCHITZ*

H_4 *: G_2 is global* LIPSCHITZ *compared to* X_1*, i.e.:*

$$\left\|G_2\left(U_2, X_1, X_2\right)-G_2\left(U_2, \hat{X}_1, \hat{X}_2\right)\right\| \leq \xi_3\left\|e_2\right\|$$

$\xi_3>0$: *Constant of* LIPSCHITZ

H_5 *: There are two positive constants α_1 et β_1 such as:*

$$0<\alpha_1^2<F_1^T\left(\hat{X}_2\right) F_1\left(\hat{X}_2\right)<\beta_1^2 \tag{8}$$

According to the contribution of [16] the observer of the subsystem (4) is given by:

$$\left\{\begin{array}{rcl}\dot{\hat{X}}_1 & = & F_1(\hat{X}_2)\hat{X}_1+G_1(U_1,\hat{X}_1,\hat{X}_2) \\ & & +\theta\Delta_\theta\bar{S}^{-1}\bar{C}_1^T\left(Y_1-\hat{Y}_1\right) \\ \hat{Y}_1 & = & \bar{C}_1\hat{X}_1\end{array}\right. \tag{9}$$

withe $\Delta_\theta=\left[\begin{array}{cc}1 & 0 \\ 0 & \theta\end{array}\right]$ for $\theta>0$.

The estimator of the system (5) is given by the following equations:

$$\left\{\begin{array}{rcl}\dot{\hat{X}}_2 & = & A_2\hat{X}_2+G_2(U_2,\hat{X}_1,\hat{X}_2) \\ \hat{Y}_2 & = & C_2\hat{X}_2\end{array}\right. \tag{10}$$

Let us note that the observer (9) is composed of a term which is the copy of the dynamics of the subsystem (4) and of another term which represents the part of correction. This correction is a function of the error in estimation between the measured currents and their considered multiplied by the gains of the observer. When with the estimator (10), it represents only the copy of the dynamics of the subsystem (4).

Now we present the analysis of total stability of the observer of a special class of the systems nonlinear interconnected with an estimator. This analysis of stability is made by THE LYAPUNOV THEORY. We will see that the convergence dynamics of the observer is fixed by that of the estimator and thus cannot be arbitrarily selected.

3.1 Analyze Stability of the Observer and the Estimator

To study the stability of the observer, we defines the equations of the errors in estimation as follows:

$$e_1 = X_1-\hat{X}_1 \tag{11}$$

$$e_2 = X_2-\hat{X}_2 \tag{12}$$

we can easily check that:

$$\Lambda_1(\hat{X}_2)F_1(\hat{X}_2)=A\Lambda_1(\hat{X}_2)$$

Thus, by multiplying the left and right side of the equation (7) by $\Lambda_1^T(\hat{X}_2)$ and $\Lambda_1(\hat{X}_2)$ respectively, to obtain to us:

$$\theta\bar{S}+F_1^T(\hat{X}_2)\bar{S}+\bar{S}F_1(\hat{X}_2)-\bar{C}_1^T\bar{C}_1=0 \tag{13}$$

The dynamic ones of these errors are given by:

$$\begin{aligned}\dot{e}_1 = & \left(F_1(\hat{X}_2)-\theta\Delta_\theta\bar{S}^{-1}\bar{C}_1^T\bar{C}_1\right)e_1+F_1(X_2)X_1 \\ & -F_1(\hat{X}_2)X_1+G_1(U_1,X_1,X_2) \\ & -G_1(U_1,\hat{X}_1,\hat{X}_2)\end{aligned} \tag{14}$$

Set $\bar{e}_1=\Delta_\theta^{-1}e_1$, then $\Delta_\theta^{-1}F_1(\hat{X}_2)\Delta_\theta=\theta F_1(\hat{X}_2)$.

$$\begin{aligned}\dot{\bar{e}}_1 = & \theta\left(F_1(\hat{X}_2)-\bar{S}^{-1}\bar{C}_1^T\bar{C}_1\right)\bar{e} \\ & +\Delta_\theta^{-1}\left\{F_1(X_2)X_1-F_1(\hat{X}_2)X_1\right\} \\ & +\Delta_\theta^{-1}\left\{G_1(U_1,X_1,X_2)-G_1(U_1,\hat{X}_1,\hat{X}_2)\right\}\end{aligned} \tag{15}$$

$$\dot{e}_2 = A_2e_2+G_2(U_2,X_1,X_2)-G_2(U_2,\hat{X}_1,\hat{X}_2) \tag{16}$$

Lemma 1 *:*
If the hypothesis 1 is satisfied, then the system (9)-(10) is an observer asymptotic of the system (4)-(5). Moreover, the speed of convergence of the error in estimation $e=[\bar{e}_1;e_2]^T$ can be selected as fast as that imposed by the estimator (10) of the subsystem (5).

3.2 Proof of the Lemma 1

To prove convergence, let us consider the following equation of LYAPUNOV:

$$V = V_1+V_2 \tag{17}$$

where $V_1=\bar{e}_1^TS\bar{e}_1$ and $V_2=e_2^Te_2$. By calculating the derivative of V along the trajectories of $\bar{e}_1$ and e_2, we obtains:

$$\begin{aligned}\dot{V} = & 2\bar{e}_1^T\bar{S}\dot{\bar{e}}_1+2\bar{e}_1^T\Lambda_1^T\bar{S}\dot{\Lambda}_1\bar{e}_1+\dot{e}_2^Te_2+e_2^T\dot{e}_2 \\ = & \theta\left(2\bar{e}_1^T\bar{S}F_1(\hat{X}_2)\bar{e}_1-2\bar{e}_1^T\bar{C}_1^T\bar{C}_1\bar{e}_1\right) \\ & +2\bar{e}_1^T\bar{S}\Delta_\theta^{-1}\left\{F_1(X_2)X_1-F_1(\hat{X}_2)X_1\right\} \\ & +2\bar{e}_1^T\bar{S}\Delta_\theta^{-1}\left\{G_1(U_1,X_1,X_2)-G_1(U_1,\hat{X}_1,\hat{X}_2)\right\} \\ & +2\bar{e}_1^T\Lambda_1^T\bar{S}\dot{\Lambda}_1\bar{e}_1+e_2^T\left\{F_2(\hat{X}_1)+F_2(\hat{X}_1)^T\right\}e_2 \\ & +2e_2^T\left\{G_2(U_2,X_1,X_2)-G_2(U_2,\hat{X}_1,\hat{X}_2)\right\}\end{aligned} \tag{18}$$

By taking account of the hypothesis 1, and by introducing the norms, then:

$$\begin{aligned}\dot{V} \leq & -\theta V-\theta\left\|\bar{C}\bar{e}\right\|^2 \\ & +2\left\|\bar{S}\bar{e}_1\right\|\left\|\Delta_\theta^{-1}\left\{F_1(X_2)X_1-F_1(\hat{X}_2)X_1\right\}\right\|\end{aligned}$$

$$\begin{aligned}&+2\left\|\bar{S}\bar{e}_1\right\|\left\|\Delta_\theta^{-1}\left\{G_1(U_1,X_1,X_2)\right.\right.\\&\left.\left.-G_1(U_1,\hat{X}_1,\hat{X}_2)\right\}\right\|\\&+2\bar{e}_1^T\Lambda_1^T\bar{S}\dot{\Lambda}_1\bar{e}_1+e_2^T\left\|F_2(\hat{X}_1)+F_2(\hat{X}_1)^T\right\|e_2\\&+2e_2^T\left\|G_2(U_2,X_1,X_2)-G_2(U_2,\hat{X}_1,\hat{X}_2)\right\| \quad (19)\end{aligned}$$

$$\begin{aligned}\dot{V} &\leq -\theta V+2\breve{\kappa}^2\lambda_{\max}(S)\left\|\bar{e}_1\right\|^2\{\xi_1+\xi_2\}\\&\quad+2\eta\kappa\lambda_{\max}(S)\left\|\bar{e}_1\right\|^2+\left(\mu+\widetilde{\xi}_3\right)\left\|e_2\right\|^2 \quad (20)\end{aligned}$$

where

* $\eta=\sup\left\|\dot{\Lambda}\right\|$ and $\widetilde{\xi}_3=2\xi_3$
* μ is selected a positive constant to satisfy the *condition of* LIPSCHITZ.

$$\mu=\max\left(2\gamma,\frac{2}{T_r}\right) \quad (21)$$

* $\lambda_{\min}(S)$ and $\lambda_{\max}(S)$ are the eigenvalues minimal and maximum of S.
* $\breve{\kappa}=\max(1,\beta_1)$
* $\|e_2\|^2=e_2^Te_2$

By rewriting the preceding expression of V (17) according to V_1 and V_2, we obtains:

$$\dot{V}\leq-(\theta-\widetilde{\xi}_1)V_1-(\mu-\widetilde{\xi}_3)V_2 \quad (22)$$

Where

$$\widetilde{\xi}_1=2\sigma(S)\frac{\breve{\kappa}^2}{\kappa^2}\left(\xi_1+\xi_2+\kappa\eta\right);$$

$$\kappa=\min(1,\alpha_1)\quad,\quad\sigma(S)=\sqrt{\frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}}$$

If we take:

$$\left\{\begin{array}{l}\pi_1=\theta-\widetilde{\xi}_1>0\\\text{and}\\\pi_2=\mu-\widetilde{\xi}_3>0\end{array}\right.$$

Finally, while taking:

$$\zeta=\min(\pi_1,\pi_2) \quad (23)$$

it follows that:

$$\dot{V}\quad\leq\quad-\zeta\left(V_1+V_2\right)\leq-\zeta V \quad (24)$$

This finishes the proof of convergence of the observer. Thus V is a LYAPUNOV FUNCTION and the speed of convergence of the error in estimation is fixed by dynamics of the estimator (10) of the subsystem (5).

4 Synthesis of the Control by Sliding Modes

For a more general case, let us consider the system described by the following equation:

$$\left\{\begin{array}{lcl}\dot{x}&=&f(x,u)\\y&=&h(x)\end{array}\right.$$

where $x\in\Re^n$ is the vector of state, $u\in\Re^m$ the control, $y\in\Re^r$ the vector of out-put. Let us suppose that our system is commandable and observable. The objective of the sliding mode control is, firstly, to synthesize a variety (surface) $S(x,t)\in\Re^m$ such as all the trajectories of the system obey one desired behavior of continuation, regulation and stability. Secondly, to determine a law of control (commutation), $u(x,t)$, who is able to attract all the trajectories of state towards the sliding surface and to maintain them on this surface. One will study the applicability of this method in the case of the asynchronous motor. The advantages of this approach are:

- The process of sliding is of a order reduced in comparison to the original system.
- The sliding mode presents properties of robustness with respect to the variation of certain types of parameters.

However, a question arises:*how to synthesize sliding surfaces for various classes of system?*

It is supposed that all the states are measured. Our objective is to build a law of control $U=\left[\begin{array}{cc}u_a & u_b\end{array}\right]^T$ to force the states of the motor, who are speed and rotor flux, to join the sliding surface $S=\left[\begin{array}{cc}S_1 & S_2\end{array}\right]^T$. This surface is defined by:

$$\left\{\begin{array}{lcl}S_1&=&\frac{k_1}{\eta}(\hat{\Omega}-\Omega_{ref})+(\hat{i}_{s\beta}\hat{\psi}_{r\alpha}-\hat{i}_{s\alpha}\hat{\psi}_{r\beta})\\&&-\frac{\dot{\Omega}_{ref}}{\eta}\\S_2&=&\frac{T_r}{2}k_2(\hat{\phi}-\phi_{ref})+[M(\hat{i}_{s\alpha}\hat{\psi}_{r\alpha}\\&&+\hat{i}_{s\beta}\hat{\psi}_{r\beta})-\hat{\phi}]-\dot{\phi}_{ref}\frac{T_r}{2}\end{array}\right. \quad (25)$$

where $\eta=\frac{p}{J}\frac{M}{L_r}$ and $k_1,k_2>0$, $\dot{\Omega}_{ref}$ and $\dot{\phi}_{ref}$ the derivative compared to the time of Ω_{ref} and ϕ_{ref} (speed desired and norm of desired flux), respectively. On $S\equiv0$, we have

$$\left\{\begin{array}{lcl}\eta(\hat{i}_{s\beta}\hat{\psi}_{r\alpha}-\hat{i}_{s\alpha}\hat{\psi}_{r\beta})&=&\\-k_1(\hat{\Omega}-\Omega_{ref})+\dot{\Omega}_{ref}&&\\\frac{2}{T_r}[M(\hat{i}_{s\alpha}\hat{\psi}_{r\alpha}+\hat{i}_{s\beta}\hat{\psi}_{r\beta})-\hat{\phi}]&=&\\-k_2(\hat{\phi}-\phi_{ref})+\dot{\phi}_{ref}&&\end{array}\right. \quad (26)$$

knowing that

$$\left\{\begin{array}{lcl}\dot{\Omega} & = & \eta(\hat{i}_{s\beta}\hat{\psi}_{r\alpha}-\hat{i}_{s\alpha}\hat{\psi}_{r\beta}) \\ \dot{\phi} & = & \dfrac{2}{T_r}[M(\hat{i}_{s\alpha}\hat{\psi}_{r\alpha}+\hat{i}_{s\beta}\hat{\psi}_{r\beta})-\hat{\phi}]\end{array}\right. \tag{27}$$

one obtains

$$\left\{\begin{array}{lcl}\dot{\Omega} & = & -k_1(\hat{\Omega}-\Omega_{ref})+\dot{\Omega}_{ref} \\ \dot{\phi} & = & -k_2(\hat{\phi}-\phi_{ref})+\dot{\phi}_{ref}\end{array}\right. \tag{28}$$

Then

$$\left\{\begin{array}{lcl}\dfrac{d}{dt}(\hat{\Omega}-\Omega_{ref}) & = & -k_1(\hat{\Omega}-\Omega_{ref}) \\ \dfrac{d}{dt}(\hat{\phi}-\phi_{ref}) & = & -k_2(\hat{\phi}-\phi_{ref})\end{array}\right. \tag{29}$$

Consequently, on $S\equiv 0$, the speed of the rotor and the square of rotor flux must converge exponentially towards their references. However, to continue Ω_{ref} and ϕ_{ref}, it is sufficient to make the sliding surface gravitational and invariant. That is to say the following proposal:

Proposal:
Let us consider the sliding surface $= [\begin{array}{cc} S_1 & S_2 \end{array}]^T$ defined in (25) and the control law by sliding mode $U=U_i+U_e$, with

$$\left\{\begin{array}{lcl}U_i & = & -D^{-1}\left[\begin{array}{cc} u_{01} & 0 \\ 0 & u_{02}\end{array}\right]\left[\begin{array}{c} sign(S_1) \\ sign(S_2)\end{array}\right] \\ U_e & = & -D^{-1}F\end{array}\right. \tag{30}$$

and

$$\left\{\begin{array}{lcl}u_{01} & > & |A| \\ u_{02} & > & |B|\end{array}\right. \tag{31}$$

where

$$F=\left[\begin{array}{c} A \\ B\end{array}\right], \qquad D=\left[\begin{array}{cc} -\dfrac{1}{\sigma L_s}\hat{\psi}_{r\beta} & \dfrac{1}{\sigma L_s}\hat{\psi}_{r\alpha} \\ \dfrac{M}{\sigma L_s}\hat{\psi}_{r\alpha} & \dfrac{M}{\sigma L_s}\hat{\psi}_{r\beta}\end{array}\right]$$

and

$$\begin{aligned} A \;=\; & \left(k_1-\frac{1}{T_r}-\gamma\right)f_2-p\hat{\Omega}\left(f_1+K\hat{\phi}\right) \\ & -\frac{k_1}{\eta}\dot{\Omega}_{ref}-\frac{1}{\eta}\ddot{\Omega}_{ref} \qquad (32) \\ B \;=\; & \left(\frac{T_r k_2}{2}-1\right)\dot{\hat{\phi}}+M\left[\frac{M}{T_r}m_i-\left(\frac{1}{T_r}+\gamma\right)f_1\right. \\ & \left.+\frac{K}{T_r}\hat{\phi}+p\hat{\Omega}f_2\right]-\frac{T_r}{2}k_2\dot{\phi}_{ref}-\frac{T_r}{2}\ddot{\phi}_{ref} \qquad (33)\end{aligned}$$

with

$f_1=\hat{i}_{s\alpha}\hat{\psi}_{r\alpha}+\hat{i}_{s\beta}\hat{\psi}_{r\beta}$, $\quad f_2=\hat{i}_{s\beta}\hat{\psi}_{r\alpha}-\hat{i}_{s\alpha}\hat{\psi}_{r\beta}$, $m_i=\hat{i}_{s\alpha}^2+\hat{i}_{s\beta}^2$ then, S is gravitational and invariant.

Proof:
That is to say the following function of LYAPUNOV $V=\dfrac{1}{2}S^TS$; then, its derivative compared to time is $\dot{V}=S^t\dot{S}$ with

$$\dot{S}=\left[\begin{array}{c}\dot{S}_1 \\ \dot{S}_2\end{array}\right]=F+DU_i$$

where

$$U_i=\left[\begin{array}{c}U_{i1} \\ U_{i2}\end{array}\right]=-D^{-1}\left[\begin{array}{cc} u_{01} & 0 \\ 0 & u_{02}\end{array}\right]\left[\begin{array}{c} Sign(S_1) \\ Sign(S_2)\end{array}\right]$$

We can rewrite $\dot{S}$ in the following form:

$$\dot{S}=F-\left[\begin{array}{cc} u_{01} & 0 \\ 0 & u_{02}\end{array}\right]Sign(S)$$

The S variety is gravitational if $S^T\dot{S}<0$ i.e.

$$S^T\left(F-\left[\begin{array}{cc} u_{01} & 0 \\ 0 & u_{02}\end{array}\right]Sign(S)\right)<0$$

then

$$\left\{\begin{array}{lcl}u_{01} & > & |A| \\ u_{02} & > & |B|\end{array}\right.$$

We can choose:

$$\begin{aligned} u_{01} \;=\; & \left|\left(k_1-\frac{1}{T_r}-\gamma\right)f_2-p\hat{\Omega}\left(f_1+K\hat{\phi}\right)\right. \\ & \left.-\frac{k_1}{\eta}\dot{\Omega}_{ref}-\frac{1}{\eta}\ddot{\Omega}_{ref}\right| \qquad (34) \\ u_{02} \;=\; & \left|\left(\frac{T_r k_2}{2}-1\right)\dot{\hat{\phi}}+M\left[\frac{M}{T_r}m_i-\left(\frac{1}{T_r}+\gamma\right)f_1\right.\right. \\ & \left.\left.+\frac{K}{T_r}\hat{\phi}+p\hat{\Omega}f_2\right]-\frac{T_r}{2}k_2\dot{\phi}_{ref}-\frac{T_r}{2}\ddot{\phi}_{ref}\right| \qquad (35)\end{aligned}$$

Then, the condition of existence of the sliding requires only the knowledge of the maximum value of the couple of load which the engine can support. However, $S=0$ is invariant if $\dot{S}=0$, i.e.

$$\left(\begin{array}{c}0 \\ 0\end{array}\right)=F+DU_e, \quad \text{or} \quad U_e=-D^{-1}F$$

It should be noted that this law of control is different from that proposed by [15], in the latter the author gives the basic concepts for the synthesis of a control to variable structure for the electric actuators.
In addition, the force of the sliding mode is its robustness with respect to the variation of parameters. It is easy to show that this law of control

is robust compared to the errors of modeling and the variation of certain parameters. This is possible by taking the gains of the regulator u_{01} and u_{02} sufficiently large.
It is as very known as the technique of the sliding mode gives the undesirable problem of chattering, but one can cure that by replacing the function *Sign* by a continuous function in the vicinity of the origin.

$$Sign(S_i) = \begin{cases} 1 & \text{if} \quad S_i \ > \ 0 \\ -1 & \text{if} \quad S_i \ < \ 0 \end{cases} \tag{36}$$

In the design of the control, we supposed that only measurements of the current and the tension are available, we will need to consider flux rotor and rotating speed seen of an application in real time.

5 Results and Simulations

The vector of state of the motor is initialized with the stopped state $\begin{bmatrix} i_{s\alpha} & i_{s\beta} & \psi_{r\alpha} & \psi_{r\beta} & \Omega \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$; whereas that of the observer in a functional state $\begin{bmatrix} \hat{i}_{s\alpha} & \hat{i}_{s\beta} & \hat{\psi}_{r\alpha} & \hat{\psi}_{r\beta} & \hat{\Omega} \end{bmatrix}^T = \begin{bmatrix} 0.2 & 0.2 & 1 & 1 & 10 \end{bmatrix}^T$, and the results are given for the motor of which a direct starting, i.e. a resistive torque null ($C_{res} = 0$) and these characteristics are given in table (Tab.1):

Parameters	Notation	Value	Unit
Poles pairs	p	2	
Stator resistance	R_s	9.65	Ω
Rotor resistance	R_r	4.3047	Ω
Stator inductance	L_s	0.4718	H
Rotor inductance	L_r	0.4718	H
Mutual Inductance	M	0.4475	H
Rotor inertia	J	0.0293	$Kg.m^2$
Resistive torque	C_{res}	0	$N.m$

Table 1: Parameters of the induction motor.[17]

We conceived simulation by carrying out the diagram general in blocks as the figure shows it (Fig.1).

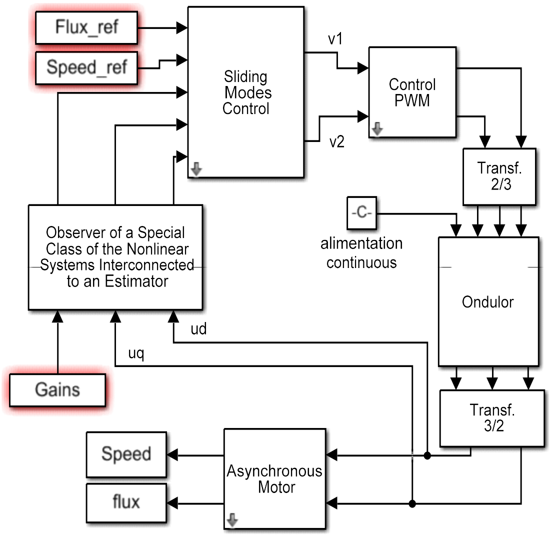

Figure 1: Diagram general of the sliding modes control of the asynchronous motor with an observer of a special class of the nonlinear systems interconnected to an estimator.

Simulation was made by MATLAB-SIMULINK. The simulation results are given by the curves obtained law of sliding modes control which coefficients $k_1 = 8500000, k_2 = 600000$ and an observer of a special class of the non-linear systems interconnected to an estimator which gain of observation $\theta = 41$.

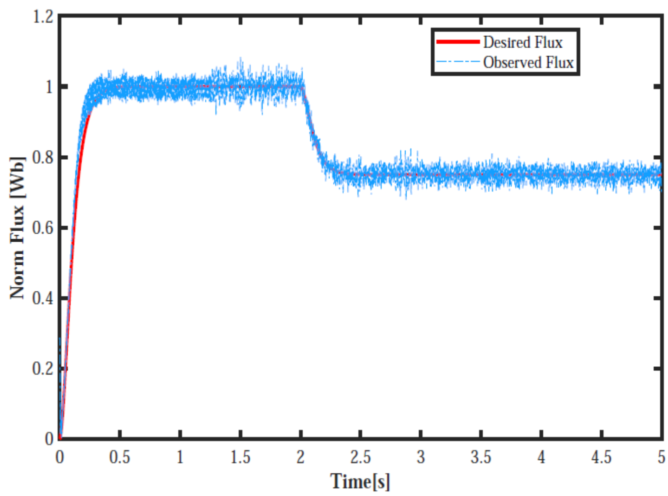

Figure 2: Curves of continuation of flux of sliding modes control of the asynchronous motor with an observer of a special class of the non-linear systems interconnected to an estimator and desired flux.

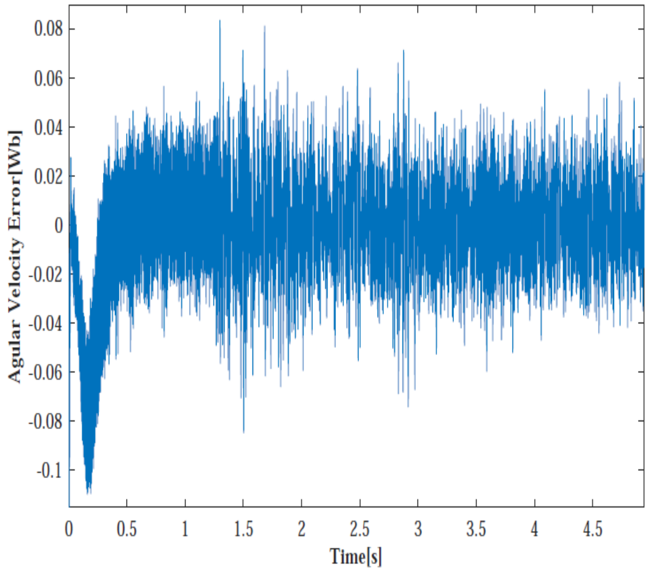

Figure 3: Curve error of continuation of flux of sliding modes control of the asynchronous motor with an observer of a special class of the non-linear systems interconnected to an estimator.

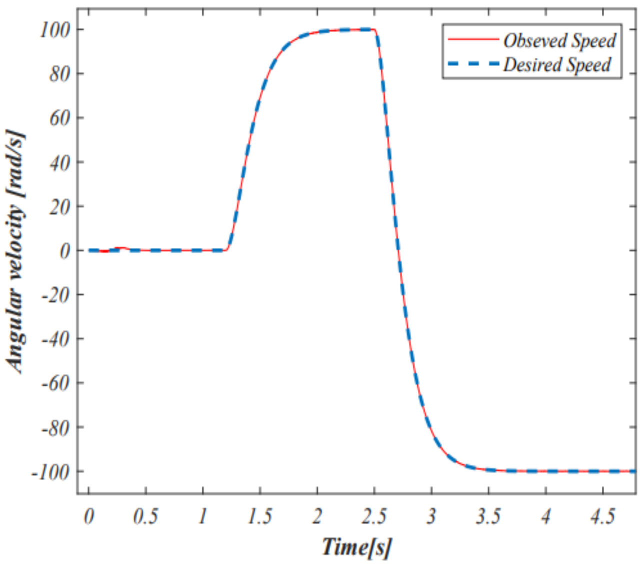

Figure 4: Curves of continuation speed of sliding modes control of the asynchronous motor with an observer of a special class of the non-linear systems interconnected to an estimator and desired speed.

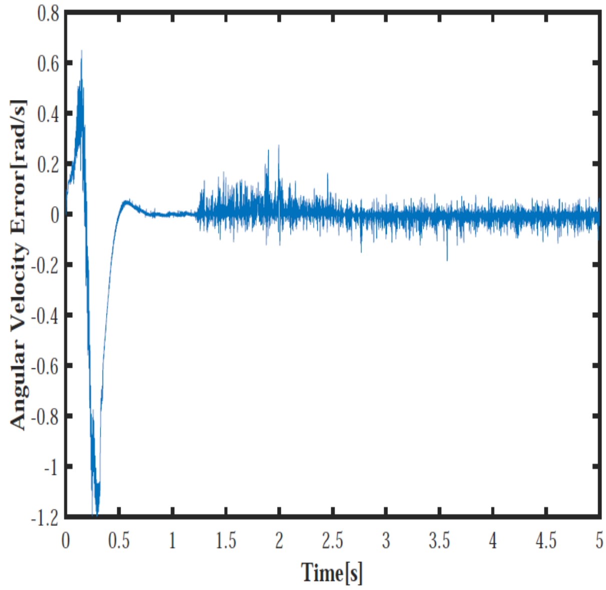

Figure 5: Curve error of continuation speed of sliding modes control of the asynchronous motor with an observer of a special class of the non-linear systems interconnected to an estimator.

6 Conclusion

Current simulations made it possible to validate the operation of the control in the majority of the dynamic modes (starting, inversion of direction of rotation). The empirical determination of the gains is thus specific to each motor, which constitutes a serious constraint. The global convergence of the controlled outputs is reached. The stability of the whole of the system (motor, observer,control) carried out. One notices an error of convergence of the parameters observed due to the difference of the dynamics of the process and observer. It is noted that the results are satisfactory.

References:

[1] A. Chouya, A. Mansouri, M. Chenafa and A. Bouhena, "Influence de la zone morte pour l'observation par mode de glissement du moteur asynchrone", *Third International Conference of Electrotechnics*, ICEL05, 13-14 Nouvember 2005, ref:RE-ME2-5,pp.25. USTO. Oran-Algeria.

[2] A. Chouya, A. Mansouri, M. Chenafa and A .Bouhena, "Etude et mise en oeuvre d'un observateur à grand gain de la machine asynchrone", *Journées Nationales sur l'Electrotechnique et l'Automatique*,

JNEA'2006, 18-19 décembre 2006, ref: EPC31, ENSET, Oran-Algérie.

[3] A. Chouya, A. Mansouri, M. Chenafa and A. Bouhena, "Observateur à Grand Gain pour des Systèmes Uniformément Localement Observables : Application au Moteur Asynchrone", *4th International Conference on Computer Integrated Manufacturing*, CIP'2007, 03-04 November 2007, réf:F13, pp.200, Ferhat ABBAS University, Laboratory of Automatic, Setif, Algeria.

[4] A. Chouya, "Observation by Sliding Mode in Triangular Form in Output Injection of an Asynchronous Motor", *9th International Conference on Modeling and Simulation of Electric Machines, Converters and Systems*, ELECTRIMACS2008, Université Laval, ref:33-Ld5S-1112, 08-11 June 2008, Québec-CANADA.

[5] M. Farza, "Modélisation et synthèse d'observateurs pour des classes de systèmes non-linéaires application aux proces des Biochimiques", *Habilitation a diriger des recherches*, 21 décembre 2000, Docteur en automatique de l'INPG - France.

[6] A. Chouya, "Study and Implementation of a High Gain Observer of the Asynchronous Machine", *International Conference on Modelling, Identification and Control*, ICMIC'2008, ref:S2-17 , 29 Juin -2 Juillet 2008, Shanghai Jiao Tong University, Shanghai, CHINA.

[7] A. Chouya, "High Gain Observer Interconnected with an Estimator: Application to the Induction Motor without Mechanical Sensor", *International Conference on Modelling, Identification and Control*, ICMIC'2008, ref:S2-16 , 29Juin-2Juillet 2008, Shanghai Jiao Tong University, Shanghai, CHINA.

[8] A. Chouya, " High Gain Observers Dedicated to the Systems Uniformly Locally Observable: Application to the Induction Motor", *4th WSEAS/IASME International Conference on DYNAMICAL SYSTEMS and CONTROL* (CONTROL'08), 26-28 October 2008, ref:593-137, Island of CORFU ("Kerkyra"), GREECE

[9] A. Chouya, "A Nonlinear Observer of Estimating Rotor Flux in Induction Motor", *The 7th WSEAS International Conference on CIRCUITS, SYSTEMS, ELECTRONICS, CONTROL & SIGNAL PROCESSING, CSECS 2008*, ref:604-103, 15-17 December 2008, Canary Islands, SPAIN.

[10] A. Chouya, "Cascade Observer Interconnected with an Estimator: Application to the Asynchronous Motor Without Mechanical Sensor", *9th International Conference on Modelling and Simulation of Electric Machines, Converters and Systems*, ELECTRIMACS2008, Université Laval, ref:33-fLfq-1112, 08-11June 2008, Québec-CANADA.

[11] Malek Ghanes, "Observation et commande de la machine asynchrone sans capteur mécanique", *Thèse de Doctorat, Université de NANTES école centrale de NANTES*, 03 Novembre 2005.

[12] D.Lubineau,"Commande non linéaire de moteur asynchrone avec observateur", *Laboratoire d'automatique de Grenoble*, Institut National Polytechnique de Gronoble, Avril 1999.

[13] Caron, J.P. and Htier, J.P., "Modélisation et commande de la machine asynchrone", *Technip-Paris, series Méthodes et pratiques de l'ingénieur*, Octobre, volume 7, 1995.

[14] A. Chouya, "Observabilité et observateurs linéaires et non-linéaires de la machine asynchrone", *mémoire magister en Automatique*, 16 Mars 2006, IMSI-université d'Oran-Algerie.

[15] V. I. Utkin, " Sliding mode control design principles" , *IEEE, Transactions on Industrial Electronics*, vol(40), pp:23-36, 1993.

[16] L. Rossignol, M. Farza, M.M'Saad, J.F. Massieu and R. Alvarez Salas, "Observer design for a class of non-linear systems : application to an induction motor", *15th Triennial World Congress, Barcelona,* Spain, 2002 IFAC.

[17] A. Mansouri, M. Chenafa, A. Bouhena, and E. Etien, "Powerful non-linear observer associated with field oriented control of an induction motor", *Int. J. Appl. Math. Comput. Sci.(amcs)*, 2004, vol. 14, n. 2, pp. 209-220.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The author contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The author has no conflict of interest to declare that is relevant to the content of this article.