Fast processing without processors: The Conductance Matrices

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Abstract: - The proposed work, starting from some basic principles of In Memory Computing, aims to describe how some relatively complex calculus, such as equation solving, can have a fast answer with a simple HW. be performed in a fast and accurate way through a very simple hardware structure, avoiding complex nonlinear operators, without involving processors and providing a huge reduction of computation time and consumption. This fact opens the way to an alternative way to solve distributed tasks e.g. in microrobotics or other real time applications.

Key-Words: - In Memory Computing, Phase Changing Memory, Polynomial Equations, Matrix Algebra, Field Oriented Control, Linear Observer

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1 Introduction

Computation time is a topic that is continuously investigated in order to increase the processor's speed and optimize the overall performance. This is also strictly related to power consumption, meaning that, according to studies in [1][2][9], 95% of energy consumption is in data movement among processors and memory (the "memory wall" problem). In Memory Computing (IMC) [10] aims to perform some operations still inside the memory without involving the processor and limiting the crossing of the "memory wall" (see Figure 1)

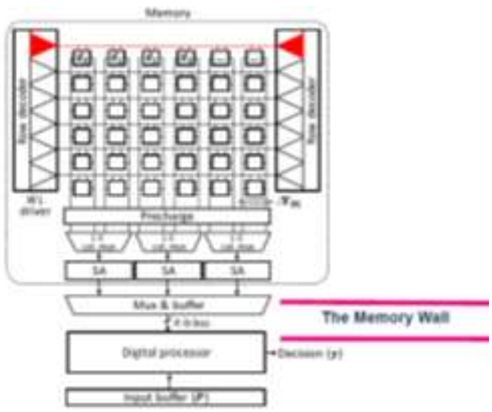

Figure 1. Von Neumann Architecture limitation: Memory Wall

A Phase Changing Memory (PCM) could be a good technological basis for IMC, allowing matrix products of data while they are stored [3]. This fact may open the way to the solution of some nontrivial problems, if they can be expressed through a suitable matrix product chain.

2 In Memory Computing with PCM

Phase-change materials are typically compounds of Ge or Sb that can be switched reversibly between amorphous and crystalline phases of different electrical resistivity. The amorphous phase tends to have high electrical resistivity, while the crystalline phase exhibits a low resistivity, sometimes three or four orders of magnitude lower. A PCM device [3] [11] consists of a certain volume of this phase change material sandwiched between two electrodes. An access device such as a field effect transistor (FET) is typically placed in series with a PCM device to constitute a complete PCM cell, modulating the crystallization in the material. This allows it to store an analogue resistive value until the next rewriting. By putting PCM cells in a suitable grid structure (see Fig. 2) and assuming as input an array of voltages, it can be calculated the output currents through the matrix product **GV=I**.

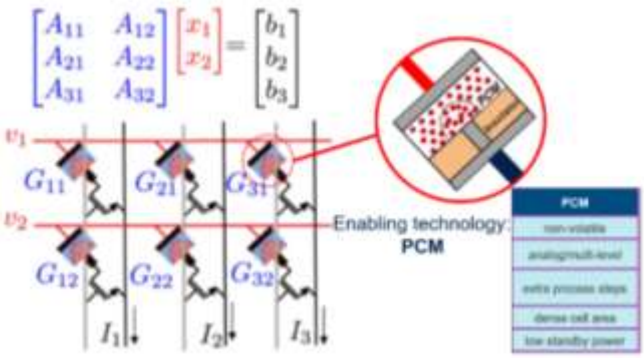

Figure 2. PCM-based In Memory Computing matrix product. Colors of parameters identify the matrices involved in the product

The conductance matrix **G** allows the calculation of multiple products and sums, if voltages are taken as inputs and currents as outputs:

$$I_1 = G_{11}v_1 + G_{12}v_2 \quad I_2 = G_{21}v_1 + G_{22}v_2 \quad I_3 = G_{31}v_1 + G_{32}v_2 \tag{1}$$

Inverting this assumption, divisions can also be implemented. By forcing unitary values to currents, it is possible to have as output voltages the inverse values of each conductance:

$$v_1 = 1/G_{11}, \quad v_2 = 1/G_{22}, \quad v_3 = 1/G_{33} \tag{2}$$

An example is given in Figure 3.

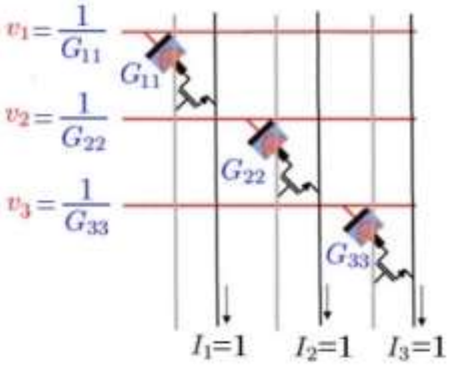

Figure 3. Circuit implementing simple inversion of values stored in the Conductance Matrix. Here $I_1=1$, $I_2=1$ $I_3=1$ are forced to obtain $v_1=1/G_{11}$, $v_2=1/G_{22}$ $v_3=1/G_{33}$

Cascade connections of structure as in Figure 2 and 3 can simultaneously perform all these operations at the same time in which the related parameters are stored, guaranteeing immediate results for a wide class of linear problems. One of this is the solution of linear algebraic systems:

$$AX=B \quad \rightarrow \quad X=A^{-1}B \tag{3}$$

The evaluation of inverse matrix A^{-1} can be reduced to a routine performing few matrix sums and products and one scalar divisions, e.g. by using the Cayley-Hamilton formula:

$$A^{-1} = (-a_1 I - a_2 A - \ldots - a_{n-1}A^{n-2} - A^{n-1})/a_0 \tag{4}$$

$$(n = \dim(A))$$

The values $a_0, a_1, \ldots a_{n-1}$ are the coefficient of characteristic polynomial of A, easily derivable with well-known sums and products of its coefficients (e.g. $a_{n-1} = -Tr(A)$, $a_0 = det(A) \ldots$).

In the next section we will see how this approach can be able to solve other nontrivial problems that would usually require more complex operators and the use of a processor.

3 How can polynomial equations be solved only via matrix products?

Polynomial Equations are fundamental in a wide variety of problems, being also the basis of differential equations solutions and very often it is a nontrivial task for processors due to the several iterations that are needed in some cases when required solutions are more than two. If we focus our attention to polynomial equations whose degree is less than four, these ones, according to Galois experience, can be solved directly through radicals without trial-and-error routines and could be good candidates to be implemented via In Memory Computing. However, radicals seem to be a huge task to perform if dealing only with sums and products. Let us consider for example the second order equation:

$$\lambda^2 + b\lambda + c = 0 \tag{5}$$

Using methods explained in [4] and [5] and assuming for the moment that $\Delta>0$, factorizing $\Delta/4$ as $\Delta/4 = g \cdot h$, a good approximation of the square root is an intermediate value between the positive numbers g and h . This could be an average among arithmetic mean and harmonic mean:

$$\sqrt{gh} \approx \left(\frac{g+h}{2} + \frac{2gh}{g+h}\right)/2 \tag{6}$$

It must be noted that formally the same could be applied also if $\Delta<0$. In fact it can be written:

$$\sqrt{\Delta/4} = i\sqrt{-\Delta/4} = i\sqrt{gh} \approx i\left(\frac{g+h}{2} + \frac{2gh}{g+h}\right)/2 \tag{7}$$

Let us consider for simplicity the case $\Delta>0$: one simple candidate for g is a perfect square sufficiently close to $\Delta/4 = b^2/4 - c$. If we add the positive quantity c^2/b^2 ($b \neq 0$), it holds:

$$\frac{\Delta}{4} < \frac{b^2}{4} - c + \frac{c^2}{b^2} = \left(\frac{b}{2} - \frac{c}{b}\right)^2 \tag{8}$$

Noting that (8) is never zero if $\Delta>0$, it is possible to assume:

$$g = \left|\frac{b}{2} - \frac{c}{b}\right|; \qquad h = \frac{\frac{b^2}{4} - c}{\left|\frac{b}{2} - \frac{c}{b}\right|} \tag{9}$$

Considering the general solution of (5), absolute value can be omitted due to ± operator needed for the two solutions:

$$\lambda_{1,2} = \frac{-b}{2} \pm \left(\frac{g}{4} + \frac{h}{4} + \frac{gh}{g+h}\right) \tag{10}$$

In matrix form, g and h can be given by:

$$\begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & -1 - \frac{2c}{b^2-2c} \end{bmatrix} \begin{bmatrix} b \\ c/b \end{bmatrix} = \begin{bmatrix} g \\ h \end{bmatrix} \tag{11}$$

And from g and h, solutions are:

$$\begin{bmatrix} 1 & 1/(g+h) & -1 \\ -1 & -1/(g+h) & -1 \end{bmatrix} \begin{bmatrix} 1/4 & h/4 \\ h & 0 \\ 1 & c/b \end{bmatrix} \begin{bmatrix} g \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{12}$$

It can be proven that, in order to cover also the cases in which Δ<0 (complex roots), the choice of g and h should be the following:

$$g = \frac{b}{2} - \frac{sign(\Delta)c}{b} \qquad \text{and}$$

$$h = \frac{b}{2} - \frac{sign(\Delta)c}{b} - \frac{2(1-sign(\Delta))b^4 + 2c^2}{b^3 - sign(\Delta)2bc} \tag{13}$$

It must be noted that (13) becomes (9) if sign(Δ)=1 (Δ>0). General solution depends also on sign(Δ) as follows:

$$\begin{bmatrix} i^{\frac{1-sign(\Delta)}{2}} & \frac{i^{\frac{1-sign(\Delta)}{2}}}{g+h} & -1 \\ -i^{\frac{1-sign(\Delta)}{2}} & -\frac{i^{\frac{1-sign(\Delta)}{2}}}{g+h} & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{h}{4} \\ h & 0 \\ 1 & \frac{sign(\Delta)c}{b} \end{bmatrix} \begin{bmatrix} g \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{14}$$

4 Hardware implementation

In this section it will be described how to implement the previous section method. Using the same approach seen in section 2, Figure 2, let us consider the quantities b, g and h as conductance. This means that *g+h* and *gh/(g+h)* are respectively parallel and series connections of g with h. In order to also cover the complex roots case, four current values should be the circuit output, indicating the real and imaginary part of each of the two solutions, and two switches driven by sign(Δ) should be included. Input voltages should be v1 and - v1 due to the sign change between first and second solution. A possible dedicated implementation is in Fig. 4.

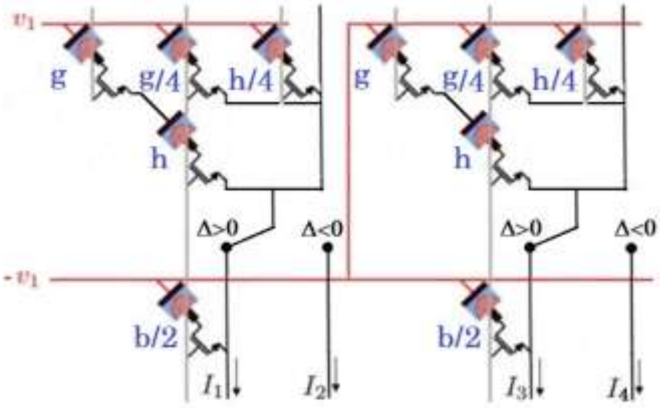

Figure 4. PCM-based HW structure implementing all the solutions of a second grade equation, knowing *b*,*g* and *h*. Here I_1=Re(λ_1) , I_2=Im(λ_1) , I_3=Re(λ_2) and I_4=Im(λ_2).

However, it could be better in some case to evaluate first the quantities b/2, g/4, h/4 and gh/(g+h) (e.g. using (11)) and adopting a more general matrix approach with constant parameters stored in the final matrix like in Fig. 5. This can be useful in particular when all the quantities in (10) can be easily calculated by a simple network (e.g when c<<b, meaning g+h≅b, gh/(g+h)≅b/4-c/b).

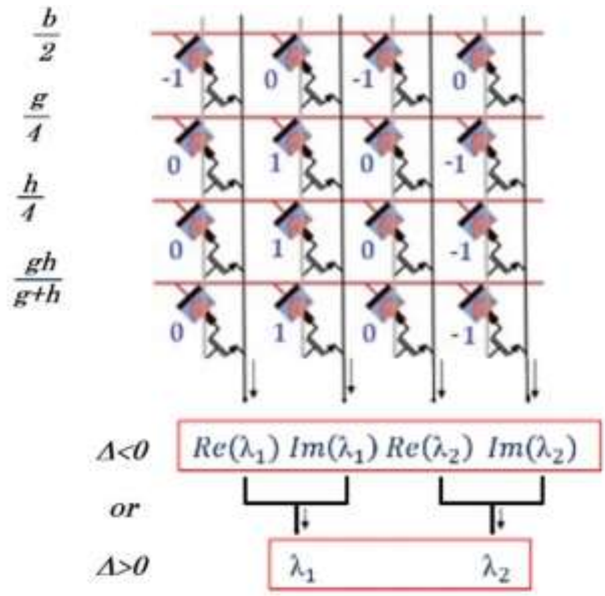

Figure 5. Alternative approach started from the separate evaluation of *b/2, g/4, h/4* and *gh/(g+h)*

In both cases, equation solutions are obtained via elementary operations, involving the evaluation of auxiliary parameters g and h, according to the flow (b,c)→(g,h)→(λ_2,λ_3). This approach will be followed also to extend the method to higher grade equations.

5 Extension to third and fourth grade polynomial equations

If in the second grade formula the only "critical" operator is square root, further ones must be considered when complexity increases. For example, cubic root and sin(arcsin(.)/3) are also involved in the third (and fourth) grade formula, making the approach previously described no more sufficient. However, due to parameter constraints and formula format, simple and efficient approximations will be used, like the following ones coming from classical literature [6]:

$$\sqrt[3]{a^3+b} \approx a + \frac{b}{3a^2}$$

$$\sqrt[3]{a-x} + \sqrt[3]{a+x} \approx \sqrt[3]{a}\left(2 - \frac{2x^2}{9a^2} - \frac{20x^4}{243a^4}\right)$$

$$\sin\left(\frac{\sin^{-1}(x)}{3}\right) \approx \frac{x}{3}\left(1 + \frac{13}{81}x^2 - \frac{1}{324}x^4\right) \quad (15)$$

Let us briefly describe how the routine can be extended to these more complex cases, keeping, when possible, a modular structure to algorithms and implementation.

5.1 The 3rd degree equation solution

The general form of the equation is in this case the following:

$$\lambda^3 + b\,\lambda^2 + c\,\lambda + d = 0 \quad (16)$$

with one additional parameter and one more solution with respect to the previous case. To solve it, the auxiliary variables p, q and of course Δ should be found. These are fundamental to find the three solutions (one real, the other two real or complex, depending on the sign of Δ). Examples of matrix chains able to evaluate them can be:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & b & 1 \end{bmatrix} \begin{bmatrix} -3b & 9 & 0 \\ 2b & -9 & 0 \\ 0 & 0 & 27 \end{bmatrix} \begin{bmatrix} b \\ c \\ d \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} p & 1 \end{bmatrix} \begin{bmatrix} p/27 & 0 \\ 0 & q/4 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \Delta \quad (18)$$

Finding one of the solutions reduces the problem to the previous section second grade case. To this aim, it could be structured in a modular structure according to the flow:

$$(b,c,d) \rightarrow (p,q) \rightarrow (\lambda_1, B(\lambda_1), C(\lambda_1)) \rightarrow (g,h) \rightarrow (\lambda_2, \lambda_3)$$

meaning that p and q help to calculate the first (real) solution λ_1 and the coefficients $B(\lambda_1)$ and $C(\lambda_1)$ of a new second grade equation, to be solved with methods described in Section 3. It can be easily demonstrated from polynomial division that

$$B(\lambda_1) = b + \lambda_1 \;\; ; \;\; C(\lambda_1) = \lambda_1^2 + b\,\lambda_1 + c \quad (19)$$

The solution λ_1 depends even in this case from the sign of Δ

$$\lambda_1 = \frac{1}{3}\sqrt[3]{-\frac{q}{2} - \sqrt{\Delta}} + \frac{1}{3}\sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} - \frac{b}{3} \quad (\Delta \geq 0)$$

$$\lambda_1 = -\frac{2}{3}\sqrt{\frac{-p}{3}}\sin\left(\frac{1}{3}\sin^{-1}\frac{3q\sqrt{3}}{2p\sqrt{-p}}\right) - \frac{b}{3} \quad (\Delta < 0)$$

(20)

Using approximation formulas in (15), (20) can be written as:

$$\lambda_1 = \left(\frac{c}{2b} - \frac{3d}{2b^2} - \frac{b}{3}\right)\left(2 - \frac{8\Delta}{9q^2} - \frac{320\Delta^2}{243q^4}\right) - \frac{b}{3} \quad (\Delta \geq 0)$$

$$\lambda_1 = -\frac{q}{3p}\left(1 - \frac{13q^2}{12p^3} - \frac{9q^4}{64p^6}\right) - \frac{b}{3} \quad (\Delta < 0)$$

(21)

Many different matrix representations (and therefore hardware implementations) can be used to calculate (21). An example for Δ>0 could be:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{c}{b} & -\frac{4c}{9b} & -\frac{160c}{243b} \\ -\frac{3d}{b^2} & \frac{4d}{3b^2} & \frac{160d}{81b^2} \\ -\frac{2b}{3} & \frac{8b}{27} & \frac{320b}{729} \\ -\frac{b}{3} & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \Delta/q^2 \\ \Delta^2/q^4 \end{bmatrix} = \lambda_1$$

(22)

This can be implemented in hardware in a very similar way to the one described in fig. 5.

A representation for Δ<0 is simpler and given by:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -\frac{q}{3p} & 0 & 0 \\ 0 & \frac{13q}{36p} & 0 \\ 0 & 0 & \frac{3q}{64p} \\ -\frac{b}{3} & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ q^2/p^3 \\ q^4/p^6 \end{bmatrix} = \lambda_1 \tag{23}$$

with the same procedure for hardware implementation.

5.2 The 4th degree equation solution

Let now us consider the equation:

$$\lambda^4 + b\lambda^3 + c\,\lambda^2 + d\,\lambda + e = 0 \tag{24}$$

This is the highest degree equation that is solvable by formulas and is very common in a wide variety of applications, such as the observer problem in Field Oriented Control of motors and kinematic description of robotic arm motion. The solution method can be summarized in five steps, using partially the previously described routines.

Step 1: Matrix calculation of auxiliary variables p, q, r. A compact formulation for them can be the following:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & b & 2 \end{bmatrix} \begin{bmatrix} -3b & 8 & 0 \\ 8b & -32 & 0 \\ 0 & 0 & 32 \end{bmatrix} \begin{bmatrix} b \\ c \\ d \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

$$\left(\begin{bmatrix} b & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3b & 0 & 0 & 0 \\ 0 & 16b & 0 & 0 \\ 0 & 0 & -64 & 0 \\ 0 & 0 & 0 & 256 \end{bmatrix} \begin{bmatrix} b \\ b \\ b \\ 1 \end{bmatrix} \right)^T \begin{bmatrix} b \\ c \\ d \\ e \end{bmatrix} = r \tag{25}$$

Step 2: Solve the auxiliary third degree equation $s^3 + 2ps^2 + (p^2 - 4r)s - q^2 = 0$ with method described in section 5.1.

Step 3: Using p, q and the positive solution s, calculate the following auxiliary values:

$$\alpha = \sqrt{s}\ , \quad \beta = \frac{s}{2} + \frac{p}{2} - \frac{q}{2\alpha}, \quad \gamma = \frac{s}{2} + \frac{p}{2} + \frac{q}{2\alpha} \tag{26}$$

(note: α can be evaluated using (6) with suitable values)

Step 4: Solve the two second order equations $x^2 + \alpha x + \beta = 0$ and $x^2 - \alpha x + \gamma = 0$ with method described in section 3. Here we have two parallel flows:

$(\alpha, \beta) \rightarrow (g_1, h_1) \rightarrow (x_1, x_2)$ & $(-\alpha, \gamma) \rightarrow (g_2, h_2) \rightarrow (x_3, x_4)$

Step 4: Final solutions are given by the linear relation:

$$\lambda_i = (x_i - b)/4 \tag{27}$$

6 A case study: Field Oriented Control motor observer

Many applicative areas may benefit from an immediate solution of algebraic equations, but one the most consistent boost can be given to the observer problem in the mainframe of the electrical motor control. A linear observer [7] is an artificial system able to estimate as fast as possible the state variables of the main system (here, the motor) in order to arrange an optimal control of it. This task is performed to mimic the behavior of the system by choosing the suitable values of variable gain vector **H** of the observer. Taking into account that the FOC-based state variables [8][12] of a motor lead to a fourth order dynamical system, a block diagram of the whole controlled system is depicted in Figure 6

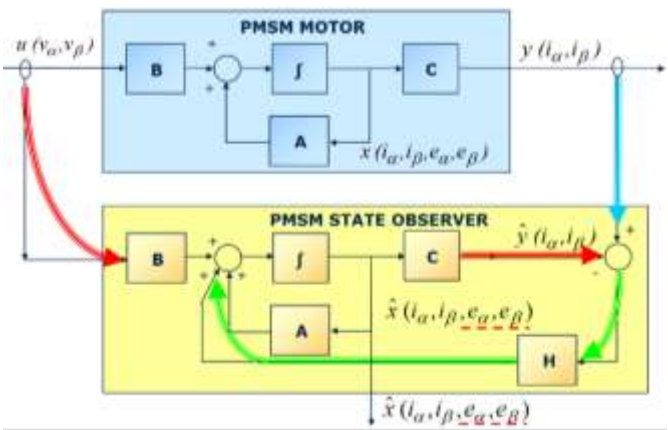

Figure 6. State observer of a PMSM motor.

From the theory, the observer matrix is defined as follows:

$$A\text{-}HC=\begin{bmatrix}-r_s/L_s & 0 & -1/L_s & 0\\ 0 & -r_s/L_s & 0 & -1/L_s\\ 0 & 0 & 0 & p\omega_r\\ 0 & 0 & -p\omega_r & 0\end{bmatrix}-\begin{bmatrix}h_1 & 0\\ 0 & h_1\\ h_2 & 0\\ 0 & h_2\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}=$$

$$=\begin{bmatrix}-r_s/L_s-h_1 & 0 & -1/L_s & 0\\ 0 & -r_s/L_s-h_1 & 0 & -1/L_s\\ -h_2 & 0 & 0 & p\omega_r\\ 0 & -h_2 & -p\omega_r & 0\end{bmatrix} \quad (28)$$

where r_s, L_s, p and ω_r are the parameters of the motor. The observer dynamics is established by the solutions of characteristic polynomial of (28), which has exactly the same form of (24). The coefficients can be obtained by the product

$$\begin{bmatrix}b\\ c\\ d\\ e\end{bmatrix}=\begin{bmatrix}2 & 0 & 2 & 0\\ \frac{r_s}{L_s}+2h_1 & p\omega_r & h_1 & -\frac{2}{L_s}\\ 2p^2{\omega_r}^2-\frac{2}{L_s}h_2 & 0 & 2p^2{\omega_r}^2 & -\frac{2}{L_s}h_1\\ \frac{r_s}{L_s}p^2{\omega_r}^2 & \frac{2r_s}{L_s}p\omega_r h_1 & h_1p^2{\omega_r}^2 & \frac{h_2}{L_s}\end{bmatrix}\begin{bmatrix}r_s/L_s\\ p\omega_r\\ h_1\\ h_2\end{bmatrix}$$

(29)

and the related equation can be solved as described in the previous section. It must be noted that these coefficients depend on the gains h_1 and h_2 of the observer: this means that it is possible to see the effect of their changes immediately on the eigenvalues, allowing a sort of real-time tuning of the gains.

6.1 Numerical simulation

Let us consider a motor with the following parameters: $r_s=2\Omega$ $L_s=0.1mH$, $p=3$ and $\omega_r=16rad/s$. Formula (29) becomes:

$$\begin{bmatrix}b\\ c\\ d\\ e\end{bmatrix}=\begin{bmatrix}2 & 0 & 2 & 0\\ 2\cdot10^4+2h_1 & 48 & h_1 & -2\cdot10^4\\ 4608-2\cdot10^4h_2 & 0 & 4608 & -2\cdot10^4h_1\\ 4608\cdot10^4 & 19\cdot10^5h_1 & 2304h_1 & 10^4h_2\end{bmatrix}\begin{bmatrix}2\cdot10^4\\ 48\\ h_1\\ h_2\end{bmatrix}$$

$$=\begin{bmatrix}4\cdot10^4+2h_1\\ {h_1}^2+4\cdot10^4h_1-2\cdot10^4h_2+4\cdot10^8\\ 4608h_1-2\cdot10^8h_2+9216\cdot10^4\\ 2304{h_1}^2+912\cdot10^5h_1+10^4{h_2}^2+9216\cdot10^8\end{bmatrix}$$

(30)

Stability of the observer ($Re(\lambda_i)<0$) requires that all terms in (30) should be positive. It can be easily shown that if $d>0$, this is always true. The search space could be restricted to all h_1 and h_2 satisfying $h_2<0.46+2.3\cdot10^{-4}h_1$ (roughly, $h_1>0$ and $h_2<0$) Moreover, from matrix A in (28), it can be easily seen that the fastest dynamics of the motor is given by the (double) eigenvalue $\lambda_1=\lambda_2=-r_s/L_s=-2\cdot10^4$. This means that the observer eigenvalues must be five times this quantity to have a fast and accurate estimation of state variables [8][12]. Solutions of the equation represented by (30) can be immediately calculated, focusing on the couples h_1, and h_2 giving solutions with real part $<-10^5$ ($Re(-\lambda_{max})>10^5$). From Fig. 7, it can be seen that a good choice of the gains of the observer can be $h_1=2\cdot10^5$ and $h_2=-5\cdot10^6$ (top area in the graphics)

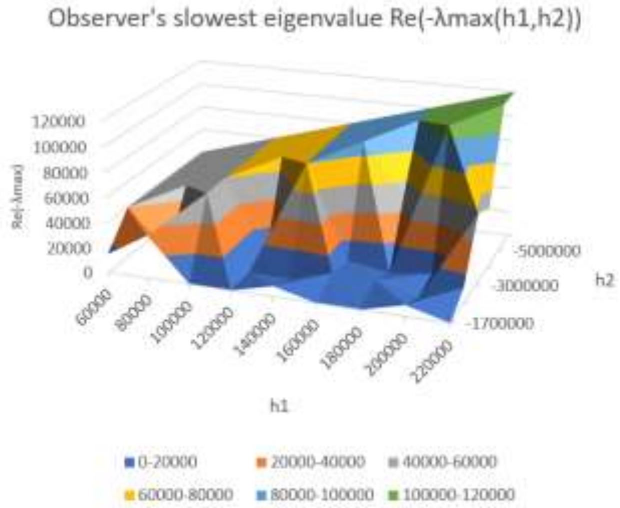

Figure 7. Evolution of the slowest eigenvalue of the observer for different values of h_1 and h_2 . Absolute values >100000 (on the top) identify the area of good choice of the gains.

7 Conclusions

In this paper, it has been described how the same mechanism of In Memory Computing can be used in order to build very simple hardware structures, able to perform nontrivial tasks such as polynomial equation solving. It has been also shown how can be removed the obstacles of having complex operators in the resolution formulas by using suitable approximations that are more than acceptable in most cases. Moreover, the presented approach is modular, meaning that the problem is decomposed in subproblems needing lower-level routines. A case study in which it is required the solution of a fourth order equation continuously updated has been also presented.

The advantage of avoiding some nontrivial computations to the processor may offer fast and low-consumption solutions to some real-time tasks, for example those dealing with complex control problems. Further investigations will involve an additional module to solve differential equations for robotic application, in order to provide a cheap and at the same time powerful computation unit locally to every link of a multi-armed robot or to a distributed environment of robots cooperating for a target task.

Even if HW implementation will benefit for sure of future technological evolution of PCM cells (i.e. increased storage capability), the Conductance matrix approach seems to be promising, in a shorter time frame, also with traditional resistive networks that could be highly performant for fixed-structure problems solving, like PID controllers tuning or parametric system identification [13].

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

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Conflict of Interest

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