Investigation of magneto-convection in viscoelastic fluid saturated anisotropic porous layer under local thermal non-equilibrium condition

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Abstract: - The problem of magneto-convection in viscoelastic fluid saturated anisotropic porous layer under local thermal non-equilibrium (LTNE) effect is investigated. Extended Darcy model with time derivative term for viscoelastic fluid of the Oldroyd type with an externally imposed vertical magnetic field is used to model the momentum equation. The entire investigation has been split into two parts: (i) linear stability analysis (ii) weakly non-linear stability analysis. We perform normal mode technique to examine linear stability analysis while truncated representation of Fourier series method is used for weakly non-linear stability analysis. The onset of convection is set in through oscillatory rather than stationary mode due to competition between the processes of thermal, magnetic effect and viscoelasticity. A comparative study between anisotropic and isotropic porous medium is made as a function of Q (Chandrasekhar number), Γ (non dimensional inter phase heat transfer coefficient), λ_1 (Relaxation time) and λ_2 (Retardation time). Apart from this, Q, Γ and λ_2 stabilize the system in oscillatory case while λ_1 destabilize the system. Furthermore ξ (mechanical anisotropic parameter), η_s (thermal anisotropic parameter for solid phase), destabilizes the system and η_f (thermal anisotropic parameter for fluid phase) stabilizes the system. The effect of Q, λ_1 , λ_2 , Γ , ξ , η_f and η_s on heat transfer is also examined.

Key-Words: - Extended Darcy model, An external vertical magnetic field, Rayleigh Number, Viscoelastic fluid, LTNE, Porous medium.

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1. Introduction

Thermal convection in porous medium is still a captivating area of research due to various applications in geophysics ([1], [2]). Classical Rayleigh Bernard convection is often examined due to its clarity and well defined controlling non-dimensional parameters [3]. Abundance of work related to thermal convection of Newtonian fluid is published and documented.

But limited work is reported on viscoelastic fluids (a non-Newtonian fluid). Possessing the potential to acquire both viscous and elastic properties simultaneously, viscoelastic fluids surrounded a wide variety of physical systems. It can be found in a wide range of liquids, colloids, polymers, organic and polymer alloys, and biological materials. Regardless of the considered specific chemical composition, intriguing and original dynamics generally originates from the property of these fluids to retain stresses even in the absence of a gradient of velocity and the ensuing ability to produce highly non-linear behaviour; while an initial flow can produce long-chain molecules stretching, the deformation of the molecules (evolving with a characteristic time that does not match that of the main flow) can cause secondary flows which further stretch them, thereby allowing the amplification of an initial small disturbance through an iterative cause and effect coupling mechanism. Thermal instability in viscoelastic fluid was introduced by [4]. Rudraiah et al. [5] developed a linear stability theory for a viscoelastic fluid in a porous media. According to them, the effect of elasticity of the fluid causes the system to become unstable. Kim et al. [6] have studied thermal instability of a viscoelastic fluid saturated porous layer. They discovered that over stability is a favoured mode for a specific parameter range using linear stability analysis. They also discovered that the onset of convection takes the form of a supercritical and stable bifurcation, regardless of the elastic parameter values. Malashetty et al. [7], is investigated the linear stability of a viscoelastic fluid saturated densely packed horizontal porous layer heated from below and cooled from above with LTNE model. According to their analysis, the processes of viscous relaxation and thermal diffusion compete, causing the first convective instability to be oscillatory rather than stationary. In a viscoelastic fluid saturated rotating porous layer heated from below, kang et al. [8] investigated linear and weakly nonlinear stability analysis of thermal convection. They found that rotation reduces the heat transfer capacity for both stationary and over stable convection modes.

Most of the work related to thermal convection in viscoelastic fluid saturated porous medium is examined under local thermal equilibrium (LTE) condition. But in a situation where hot fluid flow into a cold porous matrix there will exist a difference in the average local temperature of the two phases, then the phases are not in LTE. Then single energy equation is replaced by two equations, one for each phase. Initially, Combarnous and Bories [9] studied the effect of LTNE on nonlinear Darcy-Bernard convection. A detailed review of the effect of LTNE on natural convection in porous media is given by [10]. In (2016), Srivastava and Bhadauria [11] investigated the LTNE effect on fingering instability in a porous medium with cross diffusion effect under magnetic field. They concluded that the inter-phase heat transfer coefficient stabilizes the system that delay the onset of convection. Recently most of the work related to LTNE model is done with nano-fluid saturated porous medium and well documented in [12].

Not much attention is devoted the combined effects of magneto-convection and LTNE effects of fluid saturated porous medium. The study of magnetoconvection has great importance in petroleum reservoir [13]. In (2011), Srivastava et al. [14], investigated the onset of magneto convection in an electrically conducting fluid saturated anisotropic porous medium under LTNE effect. They showed that increasing heat transfer coefficient and magnetic effect parameter stabilizes the system.

For including many practical situations, porous medium is considered to be anisotropic rather than isotropic for their mechanical and thermal properties. Asymmetric geometric of porous matrix leads to anisotropic characteristic and it seems in numerous systems in industry and in nature. Castinel and Combarnous [15] was the first to explore the onset of thermal convection in a horizontal porous layer with anisotropic permeability. Later on Epherre (1975) [16], extended the stability analysis to porous media with anisotropic thermal diffusivity. Neild and Bejan [17] has written a comprehensive review on convective flow across anisotropic porous media.

While there is no doubt that each of the abovementioned studies on the subject enlightened that field but recognized challenge in pushing the current knowledge to investigate the combined effect of these parameters for linear as well as non-linear stability analysis. Here we focus the linear and nonlinear stability analysis of a viscoelastic fluid saturated anisotropic porous medium under LTNE effect. The following is a summary of the paper. In Sect. 2, a brief mathematical formulation of the physical problem is presented. Linear stability analysis in oscillatory convection and a weakly nonlinear theory with heat and mass transfer for F/F case are reported in Sect. 3. Sect. 4. contains the results and discussion. Finally, in Sect. 5., some key aspects of the analysis are summarised.

2. Mathematical model

2.1. The physical domain

We consider a non-Newtonian fluid saturating anisotropic porous layer of depth d, which is heated from below, and confined between two parallel horizontal planes, z = 0 and z = d, as shown in Fig. 1. A constant magnetic field $H_b = H_b \hat{k}$ is maintained externally in vertically upward direction. Taking a cartesian coordinates with origin at bottom of the porous medium, horizontal coordinate x and vertical coordinate z increases upwards. The surfaces are extended infinitely in x and y directions and constant temperature gradient ΔT is maintained across the porous layer. We are assuming that temperature of porous matrix and fluid is different that is LTNE is maintained through the porous matrix.



Fig. 1: Schematic of the problem considered.

2.2. Governing equation

The modified Darcy law for the viscoelastic fluid of the Oldroyd type is used to model momentum equation, [7]. The balance equations for mass, momentum and two energy equations, can be cast in dimensional form a

$$\begin{pmatrix} 1 + \lambda_1 \frac{\partial}{\partial t} \end{pmatrix} \begin{pmatrix} \frac{\rho_0}{\varepsilon} \frac{\partial \boldsymbol{q}}{\partial t} - \mu_m(\boldsymbol{H}.\nabla)\boldsymbol{H} \end{pmatrix} + \begin{pmatrix} 1 + \lambda_2 \frac{\partial}{\partial t} \end{pmatrix} \mu K^{-1}. \boldsymbol{q} = \begin{pmatrix} 1 + \lambda_1 \frac{\partial}{\partial t} \end{pmatrix} (-\nabla P + \rho \boldsymbol{g}), \quad (1)$$

following [4], heat flow in fluid saturated porous matrix is described by two-phase heat equations (first for fluid and second for solid)

$$\varepsilon(\rho_0 c)_f \frac{\partial T_f}{\partial t} + (\rho_0 c)_f (\boldsymbol{q}.\nabla) T_f = \varepsilon \kappa_f \nabla^2 T_f + h(T_s - T_f),$$
(2)

$$(1-\varepsilon)(\rho_0 c)_s \frac{\partial T_s}{\partial t} = (1-\varepsilon)\kappa_s \nabla^2 T_s - h(T_s - T_f),$$
(3)

$$\frac{\partial H}{\partial t} + \boldsymbol{q} \cdot \nabla \boldsymbol{H} - \boldsymbol{H} \cdot \nabla \boldsymbol{q} = \Lambda \nabla^2 \boldsymbol{H}, \qquad (4)$$

$$\nabla \boldsymbol{.} \boldsymbol{q} = \boldsymbol{0}, \tag{5}$$

$$\nabla \cdot \boldsymbol{H} = \boldsymbol{0}, \tag{6}$$

here q = (u, v, w) is velocity of the fluid, $H = (H_x, H_y, H_z)$ is magnetic field, μ_m is magnetic permeability, λ_1 is relaxation time, λ_2 is retardation time, P is pressure, ρ is fluid density, ε is porosity, T_f is temperature of fluid, T_s is temperature of solid, η_f is thermal diffusivity of fluid and η_s is thermal diffusivity of solid respectively. The relation

between the reference density and temperature is assumed to be,

$$\rho = \rho_0 [1 - \beta (T_f - T_0)].$$
⁽⁷⁾

The appropriate boundary conditions for temperature and magnetic field are: -

$$T_f = T_s = T_0 + \Delta T \text{ at } z = 0 \text{ and } T_f = T_s = T_0 \text{ at } z = d , \qquad (8)$$

$$\boldsymbol{H} \times \hat{\boldsymbol{k}} = 0 \quad at \ z = 0, d. \tag{9}$$

2.3. Basic state

The basic state of the fluid is assumed to be quiescent, and is given by,

$$q_{b} = (0,0,0), P = P_{b}(z), T_{f} = T_{fb}(z), T_{s} = T_{sb}(z), \rho = \rho_{b(z)}, H = H_{b}\hat{k}.$$
 (10)

Using (10) in Eqs. (1) - (7) yield

$$\frac{dp_b}{dz} = -\rho_b g, \ \frac{d^2 T_{fb}}{dz^2} = 0, \ \ \frac{d^2 T_{sb}}{dz^2} = 0.$$
(11)

The basic state solution for temperature and solute are given by: -

$$T_{fb}(z) = T_l - \Delta T \frac{z}{d}, T_{sb}(z) = T_l - \Delta T \frac{z}{d}.$$
 (12)

2.4. Perturbed state

On the basic state, we superpose infinitesimal perturbation in the form,

$$\begin{aligned} \boldsymbol{q} &= \boldsymbol{q}_{\boldsymbol{b}}(z) + \boldsymbol{q}'(x, y, z, t), \ T_{f} &= T_{fb}(z) + \\ T_{f}'(x, y, z, t), T_{s} &= T_{sb}(z) + T_{s}'(x, y, z, t) \\ P &= P_{b}(z) + P'(x, y, z, t), \rho = \rho_{b}(z) + \\ \rho'(x, y, z, t), \boldsymbol{H} &= \boldsymbol{H}_{\boldsymbol{b}}(z) + \boldsymbol{H}'(x, y, z, t). \end{aligned}$$
(13)

Where primes indicate perturbations. Introducing (13) in Eqs. (1)-(6), and using basic state from Eq. (11), we obtain,

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left(\frac{\rho_0}{\varepsilon} \frac{\partial q'}{\partial t} - \mu_m H_b \frac{\partial H'}{\partial z}\right) + \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \mu K^{-1} \cdot \mathbf{q}' = \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left(-\nabla P' + \beta \rho_0 \mathbf{g} T_f'\right),$$

$$(14)$$

$$\varepsilon(\rho_0 c)_f \frac{\partial T'_f}{\partial t} + (\rho_0 c)_f (q' \cdot \nabla) T'_f + (\rho_0 c)_f w' \left(\frac{dT_{fb}}{dz}\right) = \varepsilon \kappa_f \nabla^2 T'_f + h \left(T'_s - T'_f\right), \quad (15)$$

$$(1-\varepsilon)(\rho_0 c)_s \frac{\partial T'_s}{\partial t} = (1-\varepsilon)\kappa_s \nabla^2 T'_s - h(T'_s - T'_f),$$

$$\frac{\partial H'}{\partial t} + (\mathbf{q}' \cdot \nabla)H' - (H' \cdot \nabla)\mathbf{q}' - H_b \frac{\partial \mathbf{q}'}{\partial z} = \Lambda \nabla^2 H,$$
(17)

$$\nabla \boldsymbol{.} \, \boldsymbol{q}' = \boldsymbol{0}, \tag{18}$$

$$\nabla \cdot \boldsymbol{H}' = \boldsymbol{0},\tag{19}$$

$$\rho' = -\rho_0 \beta (T'_f). \tag{20}$$

We non-dimensionalized the Eqs. (14)-(20) using following transformations,

$$(x, y, z) = d(x^*, y^*, z^*), t = \frac{(\rho_0 c)_f d^2}{K_f} t^*, (u', v', w') = \frac{\varepsilon K_f}{(\rho_0 c)_f d} (u^*, v^*, w^*), P' = \frac{\varepsilon \mu K_f}{(\rho_0 c)_f K} P^*, \lambda_1 = \frac{(p_0 c)_f d^2}{K_f} \lambda_1^*, \lambda_2 = \frac{(p_0 c)_f d^2}{K_f} \lambda_2^*, \mathbf{H}' = H_b \mathbf{H}^*, T'_f = (\Delta T) T_f^*, T'_s = (\Delta T) T_s^*.$$
(21)

We obtained the non-dimensional governing equations (after dropping the asterisks for simplicity) as

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left[\frac{1}{Va} \frac{\partial q}{\partial t} - QP_m \frac{\partial H}{\partial z} + \nabla P - Ra_T T_f \hat{\kappa}\right] + \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \boldsymbol{q}_a = 0,$$
(22)

$$\left(\frac{\partial}{\partial t} - \eta_f \nabla^2_1 - \frac{d^2}{dz^2} \right) T_f + (\boldsymbol{q}.\nabla) T_f = w + \Gamma(T_s - T_f),$$
(23)

$$\alpha \frac{\partial T_s}{\partial t} = \left(\eta_s \nabla_1^2 + \frac{d^2}{dz^2}\right) T_s - \gamma \Gamma \left(T_s - T_f\right), \qquad (24)$$

$$\frac{1}{\varepsilon}\frac{\partial H}{\partial t} + (\boldsymbol{q}.\nabla)\boldsymbol{H} - (\boldsymbol{H}.\nabla)\boldsymbol{q} - \frac{\partial \boldsymbol{q}}{\partial z} = p_m \nabla^2 \boldsymbol{H}.$$
 (25)

Where the non-dimensional parameters, $Va = \frac{\varepsilon \mu d^2}{\rho_0 \kappa_f K_z}$ is Vadasz number, $Ra_T = \frac{\beta g(\rho_0 c)_f K_z d\Delta T}{\varepsilon \nu \kappa_f}$ is Rayleigh number, $\eta_f = \frac{k_{fz}}{k_{fx}}$ is fluid thermal conductivity ratio, $\eta_s = \frac{k_{sz}}{k_{sx}}$ is solid thermal conductivity ratio, $Q = \frac{\mu_m H_b^2 K_z}{\rho_{0\nu A}}$ is Chandershekhar number, $P_m = \frac{\Lambda(\rho_0 c)_f}{\varepsilon \kappa_f}$ is magnetic Prandtl number, $\gamma = \frac{\varepsilon k_{fz}}{(1-\varepsilon)k_{sz}}$ is specific heat ratio and $\boldsymbol{q}_a = \left(\frac{u}{\xi}, \frac{v}{\xi}, w\right)$ is the anisotropic modified velocity vector. Eliminating the pressure term from Eq. (22) by taking the curl twice, and get

$$\begin{pmatrix} 1 + \lambda_1 \frac{\partial}{\partial t} \end{pmatrix} \begin{bmatrix} \frac{1}{Va} \frac{\partial}{\partial t} (\nabla^2 \boldsymbol{q}) - Q P_m \nabla^2 \left(\frac{\partial H}{\partial z} \right) - \\ \hat{\boldsymbol{i}} Ra \frac{\partial^2 T_f}{\partial x \partial z} - \hat{\boldsymbol{j}} Ra \frac{\partial^2 T_f}{\partial y \partial z} + \hat{\boldsymbol{k}} Ra \nabla_1^2 T_f \end{bmatrix} + \begin{pmatrix} 1 + \\ \lambda_2 \frac{\partial}{\partial t} \end{pmatrix} C = 0.$$
 (26)

Where,

$$C = (C_1, C_2, C_3), C_1 = \frac{1}{\xi} \frac{\partial^2 v}{\partial y \partial x} + \frac{\partial^2 w}{\partial x \partial z} - \frac{1}{\xi} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), C_2 = \frac{1}{\xi} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial y \partial z} - \frac{1}{\xi} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right), C_3 = - \left(\nabla_1^2 + \frac{1}{\xi} \frac{\partial^2}{\partial z^2} \right) w.$$

The boundaries are considered to be impermeable, isothermal and perfect electrically conducting; therefore we have the following conditions,

$$w = \frac{\partial^2 w}{\partial z^2} = T_f = T_s = \frac{\partial H_z}{\partial z} = 0 \text{ at } z = 0, 1.$$
 (27)

3. Linear Stability Analysis

Taking vertical component and neglecting nonlinear terms from Eqs. (23)-(26), we get,

$$\begin{pmatrix} 1 + \lambda_1 \frac{\partial}{\partial t} \end{pmatrix} \left[\frac{1}{Va} \frac{\partial (\nabla^2 w)}{\partial t} - Q P_m \nabla^2 \left(\frac{\partial H_z}{\partial z} \right) - Ra \nabla_1^2 T_f \right] + \left(1 + \lambda_2 \frac{\partial}{\partial t} \right) \left(\nabla_1^2 + \frac{1}{\xi} \frac{\partial^2}{\partial z^2} \right) w = 0, \quad (28)$$

$$\left(\frac{\partial}{\partial t} - \eta_f \nabla_1^2 - \frac{d^2}{dz^2}\right) T_f - w - \Gamma \left(T_s - T_f\right) = 0, \quad (29)$$

$$\left(\alpha \frac{\partial}{\partial t} - \eta_s \nabla_1^2 - \frac{d^2}{dz^2}\right) T_s + \gamma \Gamma \left(T_s - T_f\right) = 0, \quad (30)$$

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t} - P_m \nabla^2\right) H_z = \frac{\partial w}{\partial z}.$$
(31)

Combining Eqs. (28) and (31)

$$\begin{pmatrix} 1 + \lambda_1 \frac{\partial}{\partial t} \end{pmatrix} \begin{bmatrix} \frac{1}{Va} \left(\frac{1}{\varepsilon} \frac{\partial}{\partial t} - P_m \nabla^2 \right) \frac{\partial (\nabla^2 w)}{\partial t} - \\ QP_m \nabla^2 \frac{\partial^2 w}{\partial z^2} \end{bmatrix} - \left(\frac{1}{\varepsilon} \frac{\partial}{\partial t} - P_m \nabla^2 \right) Ra \nabla_1^2 T_f + \\ \left(1 + \lambda_2 \frac{\partial}{\partial t} \right) \left(\frac{1}{\varepsilon} \frac{\partial}{\partial t} - P_m \nabla^2 \right) \left(\nabla_1^2 + \frac{1}{\xi} \frac{\partial^2}{\partial z^2} \right) w = 0.$$
(32)

For predicting the threshold of both marginal and oscillatory, applying linear stability theory. Assuming the solutions to be periodic waves of the forms

$$\begin{pmatrix} W \\ T_f \\ T_s \end{pmatrix} = \begin{pmatrix} W(z) \\ \ominus(z) \\ \phi(z) \end{pmatrix} e^{i(lx+my)+\sigma t}.$$
 (33)

Where *l* and *m* are horizontal wave number and σ (a complex quantity) is growth rate. W, θ and Φ are the amplitudes of velocity and temperature field respectively. Substituting (33) in the Eqs. (29), (30) and (32), we get,

$$\left[\frac{\sigma(D^2-a^2)}{Va} - \frac{QP_m D^2(D^2-a^2)}{\left(\frac{\sigma}{\varepsilon} - P_m(D^2-a^2)\right)} + \frac{(1+\lambda_2\sigma)}{(1+\lambda_1\sigma)} \left(-a^2 + \frac{D^2}{\xi}\right)\right] W(z) + R_a a^2 \ominus (z) = 0, \qquad (34)$$

 $-W(z) + \left(\sigma + \eta_f a^2 - D^2 + \Gamma\right) \ominus (z) - \Gamma \Phi(z) = 0, \tag{35}$

$$-\gamma\Gamma \ominus (z) + (\alpha\sigma + \eta_s a^2 - D^2 + \gamma\Gamma)\Phi(z) =$$
(36)

Where $D = \frac{d}{dz}$ and $a^2 = l^2 + m^2$. The corresponding boundary condition (27) will be of the form: -

$$W = D^2 W = \Theta = \Phi = 0 \quad at = 0,1. \tag{37}$$

We take solution of Eqs. (34) - (36) satisfying the boundary condition for free-free case:

$$[W(z), \ominus (z), \Phi(z)] = [W_0, \ominus_0, \Phi_0] \sin(n\pi z), (n = 1, 2, 3,).$$
(38)

Substituting (38) into (34) – (36), we obtain a matrix equation considering n=1

$$\begin{bmatrix} \frac{\sigma\delta^2}{va} + \frac{QP_m\delta^2\pi^2}{\frac{\sigma}{\varepsilon} + P_m\delta^2} + \frac{(1+\lambda_2\sigma)}{(1+\lambda_1\sigma}\delta_1^2 & -a^2Ra & 0\\ & -1 & \sigma + \delta_2^2 + \Gamma & -\Gamma\\ & 0 & -\gamma\Gamma & \alpha\sigma + \delta_3^2 + \gamma\Gamma \end{bmatrix} \begin{bmatrix} W_0\\ \Theta_0\\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix}.$$

Where,

$$\delta^2 = a^2 + \pi^2$$
, $\delta_1^2 = a^2 + \frac{\pi^2}{\xi}$, $\delta_2^2 = \eta_f a^2 + \pi^2$ and $\delta_3^2 = \eta_s a^2 + \pi^2$.

For non-trivial solution of $W_0, \bigoplus_0 and \Phi_0$, we need to make the determinant of the above matrix as zero, we get,

$$Ra_{T} = \frac{1}{a^{2}(\sigma\alpha + \delta_{3}^{2} + \gamma\Gamma)} [(\sigma + \delta_{2}^{2} + \Gamma)(\sigma\alpha + \delta_{3}^{2} + \gamma\Gamma)] \left[\frac{\sigma\delta^{2}}{Va} + \frac{QP_{m}\delta^{2}\pi^{2}}{\frac{\sigma}{\varepsilon} + P_{m}\delta^{2}} + \frac{(1+\lambda_{2}\sigma)}{(1+\lambda_{1}\sigma)}\delta_{1}^{2}\right].$$
(40)

3.1 Stationary state

For the direct bifurcation (i.e., steady onset), we have $\sigma = 0$ at the margin of stability. Then, the Rayleigh number at which marginally stable steady mode exists, becomes,

$$Ra_{T}^{st} = \frac{1}{a^{2}(\delta_{3}^{2} + \gamma\Gamma)} [(\delta_{2}^{2} + \Gamma)(\delta_{3}^{2} + \gamma\Gamma) - \gamma\Gamma^{2}] [Q\pi^{2} + \delta_{1}^{2}], \qquad (41)$$

in the absence of $\Gamma = 0$ the Eq. (41) reduces to

$$Ra_T^{st} = \frac{(\eta_f a^2 + \pi^2)(a^2 + \frac{\pi^2}{\xi} + Q\pi^2)}{a^2},$$
(42)

Further if porous media is isotropic in mechanical and thermal properties, Eq.(42) reduces to

$$Ra_T^{st} = \frac{(a^2 + \pi^2)(a^2 + \pi^2 + Q\pi^2)}{a^2},$$
(43)

the above result is same as result of [18]. If the magnetic field is absent

$$Ra_T^{st} = \frac{(a^2 + \pi^2)^2}{a^2}.$$
(44)

Which has the critical value $Ra_T^{st} = 4\pi^2$ for $a_c = \pi$, as obtained by [19] and [20].

The above calculations clearly states that the stationary Rayleigh number for the magnetoconvection for viscoelastic fluid saturated anisotropic porous medium under LTNE effect is same as for magneto-convection in Newtonian fluid saturated porous medium under LTNE effect.

3.2 Oscillatory state

The growth rate σ is in general a complex quantity such that $\sigma = \sigma_r + i\omega$. The system with $\sigma_r < 0$ is always stable, while for $\sigma_r > 0$ it will become

(39)

unstable. For natural stability state $\sigma=0$. We put $\sigma_r = i\omega(\omega \text{ is real})$ in Eq. (40) and obtain,

 $Ra_T = \Pi_1 + (i\omega)\Pi_2,$ (45) the expression for Π_1 is given by

$$\begin{split} &\Pi_{1} = M_{1} \times \left[M_{2} + M_{3} + M_{4} + M_{5}\right].\\ &\text{Where } M_{1} = \frac{1}{a^{2}(L_{1}^{2} + \alpha^{2}\omega^{2})}, M_{2} = T_{7}T_{8}\omega^{2} + \\ &\frac{T_{7}T_{10}}{(T_{3} + \omega^{2})} - \frac{T_{7}T_{13}}{\varepsilon(T_{3} + \omega^{2})}\omega^{2} + \frac{T_{7}T_{14}}{1 + \lambda_{1}^{2}\omega^{2}} + \\ &\frac{T_{7}T_{16}}{(1 + \lambda_{1}^{2}\omega^{2})}\omega^{2}, M_{3} = \frac{T_{7}T_{18}}{(1 + \lambda_{1}^{2}\omega^{2})}\omega^{2} - \alpha T_{9}\omega^{4} - \\ &\frac{\alpha T_{10}}{(T_{3} + \omega^{2})}\omega^{2} + \frac{\alpha T_{13}}{\varepsilon(T_{3} + \omega^{2})}\omega^{4} - \frac{\alpha T_{14}}{(1 + \lambda_{1}^{2}\omega^{2})}\omega^{2} - \\ &\frac{\alpha T_{16}}{(1 + \lambda_{1}^{2}\omega^{2})}\omega^{4}, M_{4} = \frac{\alpha T_{18}}{(1 + \lambda_{1}^{2})}\omega^{2} - T_{7}T_{8}\omega^{2} + \\ &\frac{T_{7}T_{11}}{(T_{3} + \omega^{2})}\omega^{2} + \frac{T_{7}T_{12}}{\varepsilon(T_{3} + \omega^{2})}\omega^{2}, and M_{5} = \frac{T_{7}T_{19}}{(1 + \lambda_{1}^{2}\omega^{2})}\omega^{2} - \\ &\frac{T_{15}T_{7}}{(1 + \lambda_{1}^{2}\omega^{2})}\omega^{2} + \frac{T_{7}T_{17}}{(1 + \lambda_{1}^{2}\omega^{2})}\omega^{2}. \end{split}$$

Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (45) it follows that either $\omega = 0$ (steady onset) or $\Pi_2 = 0$ ($\omega \neq 0$, oscillatory onset). For oscillatory onset $\Pi_2 = 0$ ($\omega \neq 0$) and this gives a dispersion relation of the form

$$N_1(\omega^2)^3 + N_2(\omega^2)^2 + N_3\omega^2 + N_4 = 0.$$
 (46)

Where the constants $N_1 = \varepsilon T_7 T_9 \lambda_1^2 + \alpha \varepsilon T_{19} - \alpha \varepsilon T_8 \lambda_1^2, N_2 = \varepsilon T_7 T_8 \lambda_1^2 - \varepsilon T_7 T_{19} - \alpha \varepsilon T_8 (1 + \lambda_1^2 T_3) + \alpha \varepsilon T_{11} \lambda_1^2 + \alpha T_{12} \lambda_1^2 - \alpha \varepsilon T_{15} + \varepsilon \alpha T_{17} + \alpha \varepsilon T_3 T_{19} + \varepsilon T_7 T_9 (1 + T_3 \lambda_1^2) - T_7 T_{13} \lambda_1^2 + \varepsilon T_7 T_{16}, N_3 = \varepsilon T_7 T_8 (1 + \lambda_1^2 T_3) - \varepsilon T_7 T_{11} \lambda_1^2 - T_7 T_{12} \lambda_1^2 + \varepsilon T_7 T_{15} - \varepsilon T_7 T_{17} - \varepsilon T_3 T_7 T_{19} - \varepsilon \alpha T_3 T_8 + \alpha \varepsilon T_{11} + \alpha T_{12} - \varepsilon \alpha T_3 T_{15} + \varepsilon \alpha T_3 T_{17} + \varepsilon T_3 T_7 T_{16} and N_4 = \varepsilon T_3 T_7 T_8 - \varepsilon T_7 T_{11} - T_7 T_{12} + \varepsilon T_3 T_7 T_{15} - \varepsilon T_3 T_7 T_{17} + \varepsilon T_3 T_7 T_{15} - \varepsilon T_3 T_7 T_{17} + \varepsilon T_3 T_7 T_{15} - \varepsilon T_3 T_7 T_{17} + \varepsilon T_7 T_{14} + T_3 T_7 T_{15} - \varepsilon T_3 T_7 T_{17} + \varepsilon T_7 T_{14} + T_3.$

Now Eq. (45) with $\Pi_2 = 0$, gives oscillatory Rayleigh number Ra_T^{osc} at the margin of stability as

$$Ra_T^{osc} = \Pi_1. \tag{47}$$

Also, for the oscillatory convection to occur, ω^2 must be positive. The symbols $L_1 - L_6$, $T_1 - T_{17}$ and Π_2 are defined in **Appendix-I.**

4. Weakly non-linear analysis

In this section, we consider the non-linear analysis using a truncated representation of Fourier series considering two terms. Although the linear stability analysis is sufficient for obtaining the stability condition of the motionless solution and the corresponding eigen functions describing qualitatively the convective flow, it cannot provide information about the values of the convection amplitudes, nor regarding the rate of heat transfer. In order to obtain this additional information, we perform the non-linear analysis, which is useful to understand the physical mechanism with minimum amount of mathematical analysis and is a step forward toward understanding full non-linear problem.

Introducing stream function $\psi as u = \frac{\partial \psi}{\partial z}, w = -\frac{\partial \psi}{\partial x}$ and $H_x = \frac{\partial \phi}{\partial z}, H_z = -\frac{\partial \phi}{\partial x}$ into vertical component of Eq. (26) and Eqs. (23-25), we obtain,

$$\begin{pmatrix} 1 + \lambda_1 \frac{\partial}{\partial t} \end{pmatrix} \begin{bmatrix} \frac{1}{Va} \frac{\partial}{\partial t} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \psi - QP_m \frac{\partial}{\partial z} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \phi \\ \frac{\partial^2}{\partial z^2} \phi + Ra \frac{\partial T_f}{\partial x} \end{bmatrix} + \left(1 + \lambda_2 \frac{\partial}{\partial t} \right) \left(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi} \frac{\partial^2}{\partial z^2} \right) \psi = 0,$$

$$(48)$$

$$\frac{\partial \psi}{\partial x} + \left(\frac{\partial}{\partial t} - \eta_f \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2}\right) T_f - \frac{\partial(\psi, T_f)}{\partial(x, z)} - \Gamma(T_s - T_f) = 0,$$
(49)

$$\alpha \frac{\partial T_s}{\partial t} - \left(\eta_s \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) T_s + \gamma \Gamma \left(T_s - T_f\right) = 0, \quad (50)$$

$$\frac{1}{\varepsilon}\frac{\partial\phi}{\partial t} - \frac{\partial(\psi,\phi)}{\partial(x,z)} - \frac{\partial\psi}{\partial z} - P_m\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\phi = 0.$$
(51)

A minimal double Fourier series which describes the finite amplitude steady-state convection is given by,

$\psi =$	$A_1(t)$ sin	n(πa	(x) s	in(π	z),			(52)
m	\mathbf{D}		``		· · ·	D (.)		(50)

 $T_f = B_1(t)\cos(\pi ax)\sin(\pi z) + B_2(t)\sin(2\pi z)$ (53) $T_s = C_1(t)\cos(\pi ax)\sin(\pi z) + C_2(t)\sin(2\pi z),$ (54)

$$\phi = D_1(t) \sin(\pi ax) \cos(\pi z) + D_2(t) \sin(2\pi ax).$$
(51)

Where the amplitude $A_1(t)$, $B_1(t)$, $B_2(t)$, $C_1(t)$, $C_2(t)$, $D_1(t)$ and $D_2(t)$ are to be determined from the dynamics of the system. Substituting Eqs. (52) - (55) into Eqs. (48)-(51) and equating the coefficients of like terms, we obtain the following non-linear autonomous system of differential equations,

$$\frac{dA_1}{dt} = E_1, \tag{56}$$

$$\frac{aB_1}{dt} = -(\pi a)A_1 - (\eta_f \pi^2 a^2 + \pi^2 + \Gamma)B_1 + \Gamma C_1 - \pi^2 a A_1 B_2,$$
(57)

. ...

$$\frac{dB_2}{dt} = -(4\pi^2 + \Gamma)B_2 + \Gamma C_2 + \frac{\pi^2 a}{2}A_1B_1, \qquad (58)$$

$$\frac{dC_1}{dt} = \frac{\gamma\Gamma}{\alpha}B_1 - \frac{(2\pi^2 + a^2 + \gamma\Gamma)}{\alpha}C_1, \tag{59}$$

$$\frac{dC_2}{dt} = \frac{\gamma\Gamma}{\alpha}B_2 - \left(\frac{4\pi^2 + \gamma\Gamma}{\alpha}\right)C_2, \tag{60}$$

$$\frac{dD_1}{dt} = -\varepsilon Pm(\pi^2 a^2 + \pi^2)D_1 + \varepsilon \pi^2 a A_1 D_2, \quad (61)$$

$$\frac{dD_1}{dt} = -\varepsilon Pm\pi^2 a^2 D_2 - \varepsilon \frac{\pi^2 a}{2} A_1 D_2, \qquad (62)$$

$$\frac{dE_1}{dt} =$$

$$\frac{\left[\frac{\pi^{2}a^{2}Ra_{T}\lambda_{1}-\left(\pi^{2}a^{2}+\frac{\pi^{2}}{\xi}\right)-\lambda_{1}\pi^{2}\varepsilon QPm(\pi^{2}a^{2}+\pi^{2})\right]}{\left(\frac{\pi^{2}a^{2}+\pi^{2}}{Va}\right)\lambda_{1}}A_{1} + \frac{\pi aRa_{T}\lambda_{1}(\eta_{f}\pi^{2}a^{2}+\pi^{2}+\Gamma)-\pi aRa_{T}}{\left(\frac{\pi^{2}a^{2}+\pi^{2}}{Va}\right)\lambda_{1}}B_{1} - \frac{\pi aRa_{T}\lambda_{1}}{\left(\frac{\pi^{2}a^{2}+\pi^{2}}{Va}\right)\lambda_{1}}C_{1} - \frac{\left[QPm^{2}\pi\lambda_{1}\varepsilon(\pi^{2}a^{2}+\pi^{2})^{2}-QPm\pi(\pi^{2}a^{2}+\pi^{2})\right]}{\left(\frac{\pi^{2}a^{2}+\pi^{2}}{Va}\right)\lambda_{1}}D_{1} - \frac{\left(\frac{\pi^{2}a^{2}+\pi^{2}}{Va}\right)\lambda_{1}}{\left(\frac{\pi^{2}a^{2}+\pi^{2}}{Va}\right)\lambda_{1}}E_{1} - \frac{\left(\frac{\pi^{2}a^{2}+\pi^{2}}{Va}\right)\lambda_{1}}{\left(\frac{\pi^{2}a^{2}+\pi^{2}}{Va}\right)\lambda_{1}}A_{1}D_{2} + \frac{\pi^{3}a^{2}Ra_{T}\lambda_{1}}{\left(\frac{\pi^{2}a^{2}+\pi^{2}}{Va}\right)\lambda_{1}}A_{1}B_{2}. (63)$$

Qualitative predictions of above autonomous differential equations are discussed and stated as. Eqs. (56)-(63) represent a system which is uniformly bounded in time and possesses many properties of the full problem. They are same as the original Eqs. (14)-(20), and therefore Eqs. (56)-(63) must be dissipative. This shows that volume in the phase space must contract. For proving this statement that, the volume contraction, it is necessary to show that velocity field has a constant negative divergence. Indeed,

Here over dot represent a time derivative.

All physical parameter used in above expression inside the square bracket is non-negative, therefore overall right-hand quantity is negative and therefore system is bounded and dissipative. So, the trajectories are attracted to a set of measure zero in the phase space, or anyone say that they may be attracted to a fixed point, a limit cycle or, perhaps, a strange attractor.

4.1. Heat transport

It is known fact that for higher value of Rayleigh number the onset of convection is generally governed by heat transfer within system. Consequently, here we are defining the Nusselt number (following Srivastava and Bera [21]) as below. The Nusselt number is defined by,

$$Nu = \frac{H'}{D_Z \frac{\Delta T}{d}} = (1 - 2\pi B_2).$$
(64)

Where, B_2 are found in terms of A_1 . Calculating B_2 , in the steady case, which is independent of viscoelastic parameters, so complete calculation is not given in this paper.

5. Result and Discussion

We selected a range of parameters from several published papers and experimental data presented in Neild and Bejan [17] book to explore the sensitivity of the system regarding various regulating parameters. We have attempted to construct a picture of LTNE (dimensionless inter-phase heat transfer coefficient) using linear and weakly nonlinear stability theories affects the onset of magnetoconvection in viscoelastic fluid saturated anisotropic porous medium. Because the critical Rayleigh number characterises the stability of a system, we computed the equations for stationary and oscillatory critical Rayleigh numbers analytically. In the stationary case, the critical Rayleigh number is found to be independent of the viscoelastic parameters. It is because of absence of base flow in present case. The critical Rayleigh number for the oscillatory mode is calculated as a function of Chandrasekhar number, viscoelastic parameters, dimensionless inter phase heat transfer coefficient, mechanical and thermal (fluid and solid both) anisotropic parameters. It is clearly noted that the over stable motions are possible prior to the steady motion. The oscillatory neutral stability curves in (a, Ra_T)-plane for different values of parameters is shown in Figs. 2-8. In these figures, we have presented the variation of Ra_T as a function for different values of parameters when other parameters are fixed. These neutral curves are topologically connected which consent to the linear stability criteria expressed in terms of critical Rayleigh number (Ra_T^C). By this, one can conclude that below critical Rayleigh number system is stable and above critical Rayleigh number it is unstable.

The characteristic curves for different value of O have been presented in Fig. 2. We can see from this graph that when the Chandrashekhar number (magnetic field) rises, the lowest value of Rayleigh number rises as well, indicating that the effect of Q is to stabilize the system. This can be explained by recalling the definition of Q. When the magnetic the field strength permeating medium is considerably strong, it induces viscosity into the fluid, and the magnetic lines are distorted by convection. Then these magnetic lines hinder the growth of disturbances, leading to the delay in the onset of instability. Same patterns have been shown in isotropic case. We also find that curves corresponding to the isotropic lies above the anisotropic case for same values of O.



Fig. 2: Oscillatory neutral stability curves for different values of Chandrasekhar number (Q).

The influence of the relaxation parameter (λ_1) on the neutral stability curve is seen in Fig. 3. We can see that increasing λ_1 reduces the critical value of the Rayleigh number, implying that λ_1 has destabilizing effect on magneto-convection in viscoelastic fluid saturated porous medium.



Fig. 3: Oscillatory neutral stability curves for different values of stress relaxation parameter (λ_1).

The effect of retardation parameter (λ_2) on the neutral curves is shown in Fig. 4. We discovered that when the value of λ_2 increases, the minimum of the Rayleigh number increases, enhancing stability of the system. It is also note that the minimum Rayleigh number shift toward the larger values of the wave number with increase in the value of λ_2 , indicating that the cell width decreases with increasing λ_2 . The behaviour of viscoelastic parameters is obvious and similar to as reported by [8].



Fig. 4: Oscillatory neutral stability curves for different values of retardation parameter (λ_2).

It can be pointed out from Fig. 5 that increasing the value of dimensionless inter phase heat transfer coefficient (Γ), increases the value of critical Rayleigh number thus stabilize the system. For explaining this effect, we recall the definition of dimensionless inter-phase heat transfer coefficient $\left(\Gamma = \frac{hd^2}{\varepsilon \kappa_{fz}}\right)$. Increasing the value of Γ means only increasing the values of inter-phase heat transfer coefficient as d, κ_{fz} and ε are fixed parameters.

Increasing inter-phase heat transfer coefficient means heat transfer between solid and fluid increases that is heat involving in transferring one phase to other phase not in onset of convection, delay the convection that is stabilizes the system.



Fig. 5: Oscillatory neutral stability curves for different values of dimensionless inter-phase heat transfer coefficient (Γ).

Fig. 6, show the effect of mechanical anisotropic parameter on onset of convection in oscillatory mode. We discovered that when the value of ξ increases, the minimum of the Rayleigh number drops, implying that increasing the value of ξ destabilises the system. This effect of ξ can be understood by the fact that it is ratio of horizontal permeability (K_x) to the vertical permeability (K_z) (which is also involve in another controlling parameter so fixed). So, increasing the value of ξ means increasing the value of horizontal permeability, enhances the fluid mobility in the vertical direction and hence reduce the critical Rayleigh number and destabilize the system.



Fig. 6: Oscillatory neutral stability curves for different values of mechanical anisotropic parameter (ξ) .

Fig. 7 depicts the influence of the thermal anisotropic parameter η_f . It has been discovered that increasing the value of η_f raises the critical Rayleigh number, reduces the onset of convection, hence stabilizes the system. It can be understand by the fact that $\eta_f = \frac{\kappa_{fx}}{\kappa_{fz}}$, so increasing the value of η_f depends on increasing thermal permeability in x direction that is κ_{fx} , as κ_{fz} is fixed using in the other controlling parameters. As κ_{fx} increases, heat defuse horizontally and delay the onset of convection.



Fig. 7: Oscillatory neutral stability curves for different values of thermal anisotropic parameter for fluid (η_f) .

In Fig. 8, the effect of thermal anisotropic parameter (η_s) for solid matrix is shown on onset of convection. It is observed that increasing value of η_s , decreases the minimum value of Rayleigh number, indicating that increasing the value of η_s destabilizes the system. It can be explained as: $\eta_s = \frac{\kappa_{sx}}{\kappa_{sz}}$ which clearly show that increasing the value of η_s , increasing the value of thermal permeability in solid matrix in x direction, so conduction situation is increasing, and it enhance the onset of convection.



Fig. 8: Oscillatory neutral stability curves for different values of thermal anisotropic parameter for solid (η_s) .

The transient behaviour of heat transfer is investigated using the Runge-Kutta fourth order approach with appropriate initial conditions to solve an independent system of differential equations numerically. The Nusselt number (Nu) is a time dependent quantity. The variation of Nu with time (t) for various parameters is shown by Figs. 9(a)-9(i). It can be easily visualized that although Nu oscillates initially but become steady with time. The Chandrashekhar number (Q), retardation parameter (λ_2) , dimensionless heat transfer coefficient (Γ) , mechanical anisotropic parameter (ξ) , thermal anisotropic parameter for fluid (η_f) , magnetic Prandtl number Pm, is to suppress heat transfer. Whereas the effect of increasing the stress relaxation parameter (λ_1) , thermal anisotropic parameter for solid (η_s) , ratio of heat capacities on heat transfer (γ) , increases the heat transfer.









Fig. 9: Variation of Nusselt number with time t for different values of (a) Q (b) $\lambda_1(c)\lambda_2$ (d) Γ (e) ξ (f) η_f (g) η_s (h) γ (i) Pm.

Finally, streamlines, isotherms for fluid and solid, magnetic streamlines are drawn for different value of Chandrashekhar number (Q = 20, 201), and for other fixed parameters are depicted in Fig. 10a1-10d2. We see that streamlines are equally divided. The effect of increasing the Chandrasekhar number O is to decrease the wavelength of the cells, thereby contracting the cells. We observed that isotherms for fluid are almost horizontal at the boundaries and oscillatory in the middle of the porous layer, thus showing conductive nature at the boundaries and convective behaviour in the middle of the system. The isotherms become more oscillatory in nature on increasing the value of Q. We also noted that isotherms for solid are horizontal for whole domain indicates that heat transfer in solid matrix is move through conduction mode. Here also we observed that the effect of increase in the magnitude of the magnetic field is to contract the cells, thereby reducing the wavelength of the cells.









Fig. 10: Comparison of streamlines a_1 , a_2 , isotherms for fluid b_1 , b_2 , and solid c_1 , c_2 , magnetic streamlines d_1 , d_2 for Q = 20 and Q = 201 receptively.

Figs. 11-14 illustrate how the streamline, isotherm, and magnetic streamline pattern varies for unsteady cases over a range of small times (0.01, 0.009,0.005, 0.001).

In Fig. 11a-a3, the streamlines pattern is depicted for various times. The figure demonstrates how streamlines initially lack outlines until they gradually evolve with the passage of time. Convection is improving as evidenced streamlines spread over time.



Fig. 11: Unsteady streamlines for different small time (a) t = 0.01, (a1) t = 0.009, (a2)t = 0.005, (a3) t = 0.001.

Figs. 12a-a3 and 13a-a3 respectively show the isotherms for fluid and solid for various times. These graphs make it quite clear that convection state develops as a contour over time.



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Fig. 12: Unsteady isotherms for fluid for different small time (a) t = 0.01, (a1) t = 0.009, (a2) t = 0.005, (a3) t = 0.001.







Fig. 13: Unsteady isotherms for solid for different small time (a) t = 0.01, (a1) t = 0.009, (a2) t = 0.005, (a3) t = 0.001.

Figs. 14a-a3 shows the magnetic streamlines for the unsteady case at various times.





Fig. 14: Unsteady magnetic streamlines for different small time (a) t = 0.01, (a1) t = 0.009, (a2) t = 0.005, (a3) t = 0.001.

6. Conclusion

We have attempted to understand the effect of magnetic field on natural convective flow in a horizontal porous layer, saturated with viscoelastic fluid under LTNE condition. The porous medium is assumed to be thermally and mechanically anisotropic. To this end, we adopted two phase model for heat equations. In this article, linear and non-linear stability analysis is performed. Linear stability analysis is performed using normal mode technique, while for non-linear analysis, minimal representation of Fourier series including only two terms have been considered. The following conclusions are drawn:

1. It is clear that $Ra_T^{osc} < Ra_T^{st}$

2. Effect of increasing Q, λ_2 , Γ and η_f is found to delay the onset of oscillatory convection.

3. On increasing the value of λ_1 , ξ and η_s , the value of Rayleigh number corresponding to oscillatory convection decrease, thus it advances the onset of convection.

4. The effect of different controlling parameters on heat transfer is also shown.

5. We also discuss the effect of Chandrashekhar number (Q) on streamlines, isotherms (fluid and solid), and magnetic streamlines as well as unsteady streamlines, isotherms (fluid and solid) and magnetic streamlines for different small time.

Appendix-I

$$\begin{aligned} \pi_2 &= M_1 \times \left[M_6 + M_7 + M_8 + M_9 \right], \text{Where,} \\ M_6 &= T_7 T_8 - \frac{T_7 T_{11}}{(T_3 + \omega^2)} - \frac{T_7 T_{12}}{\varepsilon (T_3 + \omega^2)} + \frac{T_7 T_{15}}{(1 + \lambda_1^2 \omega^2)} - \frac{T_7 T_{17}}{(1 + \lambda_1^2 \omega^2)} - \frac{T_7 T_{19}}{(1 + \lambda_1^2 \omega^2)} \omega^2, M_7 &= -\alpha T_8 \omega^2 + \frac{\alpha T_{11}}{(T_3 + \omega^2)} \omega^2 + \frac{\alpha T_{12}}{\varepsilon (T_3 + \omega^2)} \omega^2 - \frac{\alpha T_{15}}{(1 + \lambda_1^2 \omega^2)} \omega^2 + \end{aligned}$$

$$\begin{split} &\frac{\alpha T_{17}}{(1+\lambda_1^2\omega^2)}\omega^2, M_8 = \frac{\alpha T_{14}}{(1+\lambda_1^2\omega^2)}\omega^4 + T_7 T_9 \omega^2 + \\ &\frac{T_7 T_{10}}{(T_3+\omega^2)} - \frac{T_7 T_{13}}{\varepsilon(T_3+\omega^2)}\omega^2, M_9 = \frac{T_7 T_{14}}{(1+\lambda_1^2\omega^2)} + \\ &\frac{T_7 T_{16}}{(1+\lambda_1^2\omega^2)}\omega^2 + \frac{T_7 T_{18}}{(1+\lambda_1^2\omega^2)}\omega^2. \end{split}$$

And other symbols are defined as:

$$\begin{split} L_1 &= \delta_3^2 + \gamma \Gamma, \qquad L_2 = \frac{\delta^2}{Va}, \qquad L_3 = \delta^2 Pm, \\ L_4 &= Q\pi^2, \qquad L_5 = \delta_2^2 + \Gamma, \\ L_6 &= \gamma \Gamma^2, \qquad T_1 = L_3^2 L_4 \varepsilon^2, \\ T_2 &= L_3 L_4 \varepsilon^2, \qquad T_3 = \varepsilon^2 L_3^2, \\ T_4 &= \lambda_2 - \lambda_1, \qquad T_5 = \lambda_1 \lambda_2, \\ T_6 &= L_1 L_5 - L_6, T_7 = L_1 + \alpha L_5, \\ T_8 &= L_1 L_2, \qquad T_9 = \alpha L_2, \\ T_{10} &= T_1 L_1, \qquad T_{11} = \alpha T_1, \\ T_{12} &= T_2 L_2, \qquad T_{13} = \alpha T_2, \\ T_{14} &= \delta_1^2 L_1, \qquad T_{15} = L_1 T_4 \delta_1^2, \\ T_{16} &= L_1 T_5 \delta_1^2, \qquad T_{19} = \alpha T_5 \delta_1^2. \end{split}$$

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References: -

[1] Stevens, B., Atmospheric moist convection, *Annual Review of Earth and Planetary Sciences*, 33(1), 2005, 605-643.

[2] Roberts, P.H., *Theory of the geodynamo, In Treatise on Geophysics (Second Edition) (ed. Gerald Schubert)*, Oxford: Elsevier, 2015, pp. 57-90.

[3] Bodenschatz, E., Pesch, W. and Ahlers, G., Recent developments in Rayleigh-Benard convection, *Annual Review of Fluid Mechanics*, 32 (1), 2000, pp. 709-778.

[4] Herbert, D.M., On the stability of viscoelastic liquids in heated plane Couette flow, *Journal of Fluid Mechanics*, 17, 1963, pp. 353-359.

[5] Rudraiah, N., Kaloni, P.N. and Radhadevi, P.V., Oscillatory convection in a viscoelastic fluid through a porous layer heated from below, *Rheologica Acta*, 28, 1989, pp. 48-53.

[6] Kim, M.C., Lee, S.B., Kim, S. and Chung, B.J., Thermal instability of viscoelastic fluids in porous media, *International Journal of Heat and Mass Transfer*, 46, 2003, pp. 5065-5072.

[7] Malashetty, M.S., Shivakumara, I.S., Kulkarni, S. and Swamy, M., Convective Instability of

Oldroyd-B Fluid Saturated Porous Layer Heated from Below using a Thermal Non-equilibrium Model, *Transport in Porous Media*, 64, 2006, pp. 123-139.

[8] Kang, J., Fu, C. and Tan, W., Thermal convective instability of viscoelastic fluids in a rotating porous layer heated from below, *Journal of Non-Newtonian Fluid Mechanics*, 166, 2011, pp. 93-101.

[9] Combarnous, M. and Bories, S.A., Modelisation de la convection naturelle au sein d'une couche poreuse horizontal `a l'aide d'un coefficient de transfert solide-fluide, *International Journal of Heat and Mass Transfer*, 17, 1974, pp. 505-515.

[10] Rees, D.A.S. and Pop, I., Local thermal nonequilibrium in porous medium convection, *In Transport Phenomena in Porous Media III (ed. D. B. Ingham & I. Pop)*, Pergamon, 2005, pp. 147-173.

[11] Srivastava, A.K. and Bhadauria, B.S., Influence of Magnetic Field on Fingering Instability in a Porous Medium with Cross-Diffusion Effect: a Thermal Non-equilibrium Approach, *Journal of Applied Fluid Mechanics*, 9(6), 2016, pp. 2845-2853.

[12] Kuznetsov, A.V. and Nield, D.A., Effect of local thermal non-equilibrium on the onset of convection in porous medium layer saturated by a nanofluid, *Transport Porous Media*, 83(1), 2010, pp. 425-436.

[13] Wallace, W.E., Pierce, C.I. and Sawyer, W.K., *Experiments on the flow of mercury in porous media in a transverse magnetic field, Report RI-7259 (PB-184327)*, Washington DC: Bureau of Mines, 1969.

[14] Srivastava, A.K., Bhadauria, B.S. and Kumar, J., Magneto convection in an anisotropic porous layer using thermal non-equilibrium model, *Special Topics & Reviews in Porous Media-An International Journal*, 2(1), 2011, pp. 1-10.

[15] Castinel, G. and Combarnous, M., Ceitere d' apparition de la convection naturelle dans une couche poreuse anisotropic horizontal, *Comptes rendus de l'Académie des Sciences*, 278, 1974, pp. 701-704.

[16] Epherre, J.F., Critere d' apparition de la convection naturelle dans une couche poreuse anisotrope, *Revue Générale de Thermique*, 168, 1975, pp. 949-950.

[17] Neild, D.A. and Bejan, A., *Convection in Porous Media*, Springer-Verlag, New York, 2017.

[18] Bhadauria, B.S. and Sherani, A., Onset of Darcy-convection in a magnetic-fluid saturated porous medium subject to temperature modulation of the boundaries, *Transport in porous Media*, 73, 2008, pp. 107-127.

[19] Horton, C.W. and Rogers, F.T., Convection currents in a porous medium, *Journal of Applied Physics*, 16, 1945, pp. 367-370.

[20] Lapwood, E.R., Convection of a fluid in a porous medium, *Proceedings of the Cambridge Philological Society*, 44, 1948, pp. 508-521.

[21] Srivastava, A.K. and Bera, P., Influence of Chemical Reaction on Stability of Thermo-Solutal Convection of Couple-Stress Fluid in a Horizontal Porous Layer, *Transport in porous Media*, 97, 2013, pp. 161-184.

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