

# Adaptive Linearizing Control with MRAC Regulator for DC-DC Boost Converter

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**Abstract:** In this paper; we treat the converter boost DC-DC by an adaptive linearizing controller. Where regulators located at the feed forward and feedback. Small signal model is used as a linearizing technique. Massachusetts Institute of Technology (MIT) rule is applied as an adaptive mechanism to determine the optimal control parameters in some conditions. The used adaptive control technique is Model Reference Adaptive Control (MRAC), this method as able to control system in various output voltage. The proposed method has a stable response and able to reach the model reference smoothly. However, the response of the system has instantaneously overshoot and follows the response back of model reference.

**Keywords:** Automatic Control, Power Electronics, Feedback, Converters

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## 1. Introduction

Systems theory has seen significant progress over the years. Most analytical and synthetic techniques are based on linear models of the controlled processes. Nevertheless, the nonlinear nature of physical systems and due to the increasingly increasing performance demanded in industrial applications, so the use of advanced control techniques (adaptive control, optimal control, sliding mode control,... ) becomes essential.

Nowadays, advanced control techniques are becoming one of the most active areas of research. At the same time, we have powerful calculators and a variety of software tools. This facilitates the synthesis of advanced control laws and their execution, without difficulties in real time.

It takes electronic components to achieve the desired voltage. The recent components of the power converter are of high quality and high efficiency. One of them is the DC-DC step-up converter which allows to increase the output voltage [1]. In order to create rapid changes in response of the DC-DC boost converter, it must operate at high frequency [2]. In this condition, the DC-DC boost converter requires a controller to handle the desired value.

Adaptive controls are widely used by researchers to solve dynamic problems [3] and some of them use Proportional-Integral-Derive as its control structure. Conventional Proportional-Integral-Derive is based on a mathematical model, such that it has stability, reliability and control capabilities. Conventional Proportional-Integral-Derive controllers are effective in linear systems, but it is not suitable for non-linear systems and high-order systems. Determination of Proportional-Integral-Derive parameters has been used in many ways [6]. Several methods have their own advantages and disadvantages for determining the baseline adaptive control parameters of the model to achieve a stable system. The fixed parameter in Proportional-Integral-Derive controller is not quite robust or not able to adapt and therefore the adaptive controller techniques is required improve system response [7]. Several adaptive control techniques are used to solve this problem and one of them uses Direct Model Reference Adaptive Control (DMRAC) [8]. Model Reference Adaptive

Control performances are provided by model as reference, this means that the plant's response must follow the model's response. The following parameter adjustment mechanism is calculated by using Massachusetts Institute of Technology (MIT) rule [9].

## 2. DC-DC Boost Converter Model

### 2.1 Basic Modeling

DC-DC boost converter is commonly used in DC systems and also known as DC boost converter. The output voltage demand must be greater than the input voltage and continuous. It uses two semiconductors such as a controlled power device and an uncontrolled device. They basically consist of a series inductor and a parallel capacitor. The electric circuit of the Boost converter is presented by the figure 1.

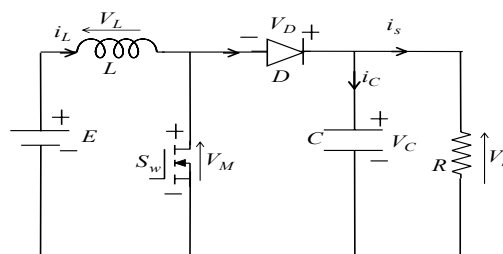


Fig. 1. Diagram of the Boost converter.

The equivalent circuit of the Boost converter presented by the figure 1 is :

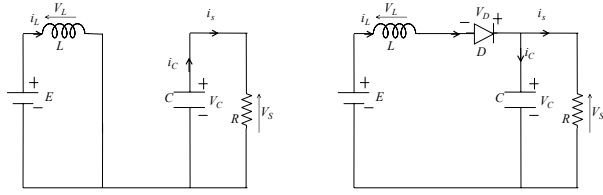


Fig. 2. Diagram of the Boost converter with  $S_w$  closed (left) and  $S_w$  opened (right)

On the interval,  $t_0 \leq t \leq t_0 + \alpha T$ , the switch  $S_w$  is closed and the diode  $D$  is blocked. The linear model which represents the left configuration of the circuit describes in figure 2 is given by :

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dV_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{E}{L} \\ 0 \end{bmatrix} \quad (1)$$

On the interval,  $t_0 + \alpha T \leq t \leq t_0 + T$ ,  $S_w$  is opened and the diode  $D$  is busy. The linear model which represents the right configuration of the circuit describes in figure 2 is given by :

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dV_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{E}{L} \\ 0 \end{bmatrix} \quad (2)$$

The general equation of the model of instantaneous state which governs the operation of the Boost converter is (see [14]):

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dV_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1-\alpha}{L} \\ \frac{1-\alpha}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{E}{L} \\ 0 \end{bmatrix} \quad (3)$$

## 2.2 Small-Signal Modeling of the Boost Converter

In order to define the small signal model (SSM) of the boost converter, it is necessary to substitute each variable. Each parameter is presented in steady state and in small signal variation as follows (see [12])

$$\begin{cases} \alpha = \alpha_e + \tilde{\alpha} \\ x = x_e + \tilde{x} \end{cases}$$

One first of all will study the equilibrium state of the system. One thus has (see [16]):

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1-\alpha}{L} \\ \frac{1-\alpha}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_{Le} \\ V_{Ce} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} E \quad \Leftrightarrow \quad \begin{bmatrix} i_{Le} \\ V_{Ce} \end{bmatrix} = \begin{bmatrix} \frac{E}{R(1-\alpha_e)^2} \\ \frac{E}{1-\alpha_e} \end{bmatrix} \quad (4)$$

To obtain the small-signals model of the Boost, we will linearize the model of average state around the equilibrium state  $(x_e, \alpha_e)$ . One then uses a development limited of TAYLOR

to order 1. After one immediate calculation, the system of linearized state is written :

$$\begin{bmatrix} \frac{d\tilde{i}_L}{dt} \\ \frac{d\tilde{V}_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1-\alpha_e}{L} \\ \frac{1-\alpha_e}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \tilde{i}_L \\ \tilde{V}_C \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_{Le} \\ V_{Ce} \end{bmatrix} \tilde{\alpha} \quad (5)$$

$$\Leftrightarrow \begin{bmatrix} s & \frac{1-\alpha_e}{L} \\ -\frac{1-\alpha_e}{C} & s + \frac{1}{RC} \end{bmatrix} \begin{bmatrix} \tilde{i}_L \\ \tilde{V}_C \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_{Le} \\ V_{Ce} \end{bmatrix} \tilde{\alpha} \quad (6)$$

Since  $V_C = V_s$ ; we finds the first transfer function binding the output voltage  $V_s$  with the duty cyclic  $\alpha$ :

$$\frac{\tilde{V}_s}{\tilde{\alpha}} = -\frac{E}{RC(1-\alpha_e)^2} \cdot \frac{s - \frac{R(1-\alpha_e)^2}{L}}{s^2 + \frac{1}{RC}s + \frac{(1-\alpha_e)^2}{LC}} \quad (7)$$

We deduces the transfer function binding the inductor current  $\tilde{i}_L$  with the duty cyclic  $\tilde{\alpha}$ :

$$\frac{\tilde{i}_L}{\tilde{\alpha}} = \frac{E}{L(1-\alpha_e)} \cdot \frac{s + \frac{2}{RC}}{s^2 + \frac{1}{RC}s + \frac{(1-\alpha_e)^2}{LC}} \quad (8)$$

the transfer function binding the output voltage  $\tilde{V}_s$  with the inductor current  $\tilde{i}_L$ :

$$\frac{\tilde{V}_s}{\tilde{i}_L} = -\frac{1}{RLC(1-\alpha_e)} \cdot \frac{s - \frac{R(1-\alpha_e)^2}{L}}{s + \frac{2}{RC}} \quad (9)$$

## 3. Synthesis of Adaptive Control with MRAC Regulator

From equation (5), we can deduce the law control according to:

$$\tilde{\alpha} = \frac{1}{\left(\frac{V_{Ce}}{L}\right)^2 + \left(\frac{i_{Le}}{C}\right)^2} \left( \frac{V_{Ce}}{L} \left( s\tilde{i}_L + \frac{1-\alpha_e}{L}\tilde{V}_s \right) - \frac{i_{Le}}{C} \left( -\frac{1-\alpha_e}{C}\tilde{i}_L + \left( s + \frac{1}{RC} \right) \tilde{V}_s \right) \right) \quad (10)$$

In Massachusetts Institute of Technology (MIT) rule is :

$$\mathcal{L} \left\{ \frac{d\theta}{dt} \right\} = -\gamma \frac{\partial J(e)}{\partial e} \frac{\partial e}{\partial \theta} \quad (11)$$

Where  $e$  represents the error between the plant and model output. The  $\theta$  is adjustable parameter and it is set in such a way such that  $J$  is minimized to zero.

We will present the synthesis of each closely related regulator separately to clarify the synthesis methodology of each of them.

We want to obtain in closed loop a response of the first order type. To achieve this objective, we take a MRAC of the type:

$$\tilde{I}_L(s) = \rho_1 V_{ref} - \rho_2 V_s \quad (12)$$

The closed loop system can be represented by the figure 3.

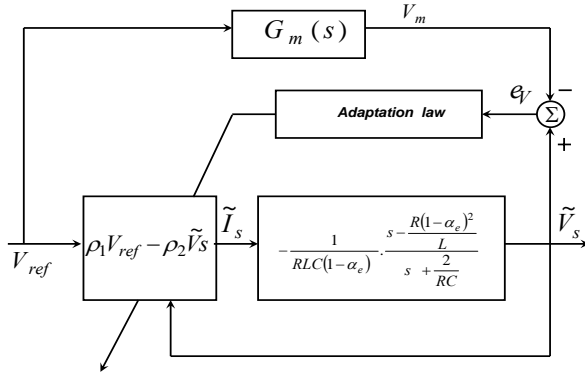


Fig. 3. Diagram block in closed loop output voltage of MRAC regulator.

The reference model of the closed loop system is selected with a first order transfer function :

$$G_m(s) = \frac{s + b_{0m}}{a_{1m}s + a_{0m}}$$

That is to say the optimality criterion  $J(e)$  of the adjustment loop is expressed by the absolute value in [15] and [10]:

$$J(e) = |e| \quad (13)$$

Its derivative is :

$$\frac{\partial J(e)}{\partial e} = \text{sign}(e) \quad (14)$$

The out-put is written:

$$\tilde{V}_s(s) = \frac{-\rho_1(s - K_V)}{(K_{1V} - \rho_2)s + K_{2V} + \rho_2 K_V} V_{ref}(s) \quad (15)$$

With  $K_V = \frac{R(1-\alpha_e)^2}{L}$ ,  $K_{1V} = RL(1 - \alpha_e)$  and  $K_{2V} = \frac{2R}{C}(1 - \alpha_e)$ .

The error  $e = \tilde{V}_s - V_m$ , its derivative compared to the parameters gives :

$$\frac{\partial e}{\partial \rho_1} = \frac{-(s - K_V)}{(K_{1V} - \rho_2)s + K_{2V} + \rho_2 K_V} V_{ref}(s) \quad (16)$$

$$\frac{\partial e}{\partial \rho_2} = -\frac{\rho_1(s - K_V)^2}{((K_{1V} - \rho_2)s + K_{2V} + \rho_2 K_V)^2} V_{ref}(s) \quad (17)$$

For  $e = 0 \Rightarrow \tilde{V}_s = V_m$  then  $-K_V = b_{0m}$ ,  $-\rho_2 + K_{1V} = a_{1m}$  and  $K_{2V} + \rho_2 K_V = a_{0m}$ .

$$\begin{aligned} \frac{\partial e}{\partial \rho_1} &= -\frac{1}{\rho_1} \cdot \frac{s + b_{0m}}{a_{1m}s + a_{0m}} V_{ref}(s) \\ &= -\frac{1}{\rho_1} V_m \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\partial e}{\partial \rho_2} &= -\frac{1}{\rho_1} \cdot \frac{s + b_{0m}}{a_{1m}s + a_{0m}} \cdot \frac{s + b_{0m}}{a_{1m}s + a_{0m}} V_{ref}(s) \\ &= -\frac{1}{\rho_1} \cdot \frac{s + b_{0m}}{a_{1m}s + a_{0m}} V_m(s) \end{aligned} \quad (19)$$

Taking into account (14), (18) and (19), one can write the equation of gradient  $\rho_1$  and  $\rho_2$ :

$$\mathcal{L} \left\{ \frac{d\rho_1}{dt} \right\} = -\kappa_1 \frac{\partial J(e)}{\partial e} \frac{\partial e}{\partial \rho_1} \quad \text{with } \kappa_1 > 0 \quad (20)$$

$$\rho_1 = \frac{\kappa_1}{s} \text{sign}(e) \cdot \frac{1}{\rho_1} V_m(s) \quad (21)$$

And

$$\mathcal{L} \left\{ \frac{d\rho_2}{dt} \right\} = -\kappa_2 \frac{\partial J(e)}{\partial e} \frac{\partial e}{\partial \rho_2} \quad \text{with } \kappa_2 > 0 \quad (22)$$

$$\rho_2 = \frac{\kappa_2}{s} \text{sign}(e) \cdot \frac{1}{\rho_1} \cdot \frac{s + b_{0m}}{a_{1m}s + a_{0m}} V_m(s) \quad (23)$$

## 4. Results and Simulations

The search is based on the output voltage generated by the step-up converter which has not been properly regulated. This problem occurs when there are changes in the reference voltage. This research which will be carried out in a boost converter using an adaptive controller when the regulator located in feed-forward and feedback and a boost converter using MRAC.

To examine practical utility, the proposed regulator has been simulated for a boost (see [13]), whose parameters are shown in the table I.

TABLE I  
DC-DC BOOST CONVERTER PARAMETERS.([13])

Parameters	Notation	Value	Unit
Input Supply Voltage	$E$	10	V
Inductor	$L$	100	$\mu H$
Resistor Load	$R$	10	$\Omega$
Capacitor	$C$	10	$\mu F$
Normal switching frequency	$f$	20	KHz
Switch off	$Sw$	$\alpha = 0$	
Switch on	$Sw$	$\alpha = 1$	
Duty cycle	$\alpha_e$	0.5	
Desire Output Voltage	$V_{ce}$	20	V
Inductor steady-state current	$i_{Le}$	4	A

By using these parameters, the model of DC-DC boost converter (3) is utilized as a plant of the system. The derivation MRAC based on **MIT** rule for inductor output voltage Regulator obtain (21) and (23). The value of  $\kappa$  is specified to achieve the appropriate response.

We show a detailed scheme general of the adaptive control with MRAC regulator in figure 4. The performance of boost

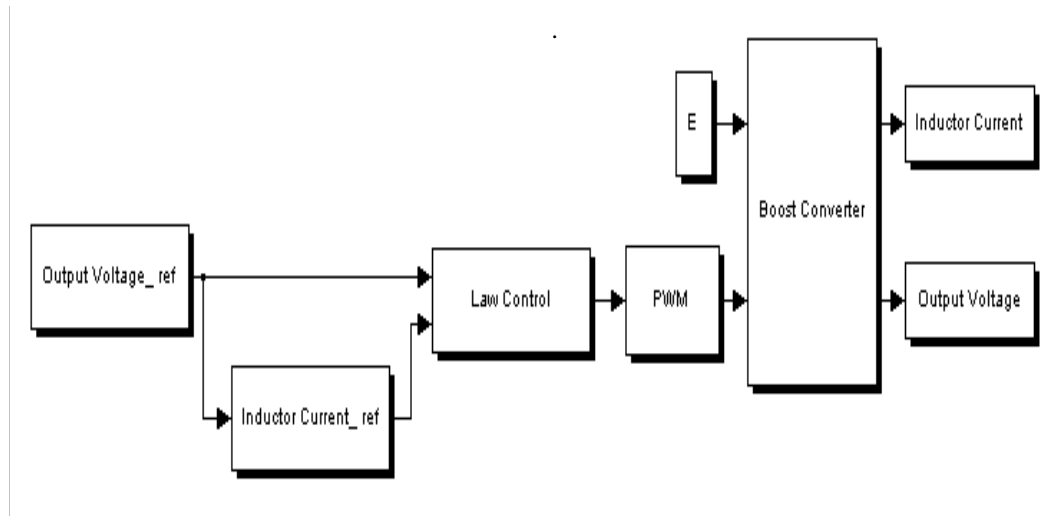


Fig. 4. Diagram general of adaptive control with MRAC regulator for DC-DC boost converter.

converter in proposed controller is proven in simulation such that any changed responses are able to be observed. The input voltage and resistor load of the boost converter are  $10V$  and  $10\Omega$ , respectively. Reference voltage is set to be  $20V$  and of  $\kappa_1 = 2 \times 10^3$  and  $\kappa_2 = 7 \times 10^4$ . As regulation parameters  $\rho_1$  and  $\rho_2$  are initialized at 10 and 35 respectively.

Figure (Fig.5) shows the evolution of the voltages (desired and output), where after the transient state; the output voltage follows the desired voltage  $20V$  which is double the supply voltage  $E = 10V$ . Because of the duty cycle equal to  $\alpha = 0.5$ , The voltage error is shown in figure (Fig.6(a)). It cancels out at  $1ms$ , with a voltage mean error  $1.0067V$ . The histogram (figure Fig.6(b)) shows more than 1000 samples centralizing at  $0V$ .

The forms of induction current and desired current appear in Figure (Fig.7) where after the transient state the induction current follows the desired current of  $4A$ . The current error is shown in figure (Fig.8(a)) with a current mean error  $0.1529A$ . It cancels out at  $1ms$ . Over 700 samples has  $0A$  error and the rest of the samples are around  $0A$  (see Fig. 8(b)).

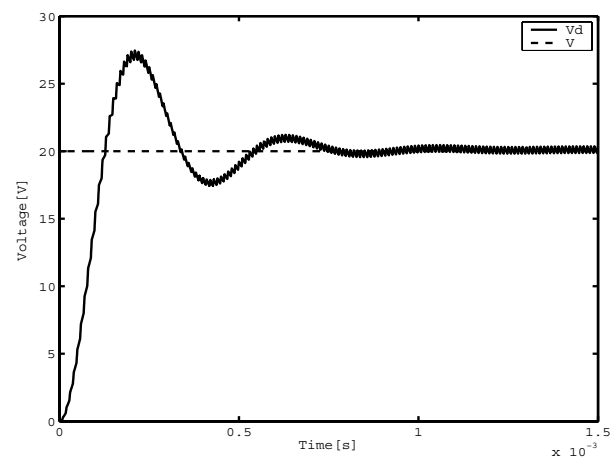


Fig. 5. Output voltage for change in reference output voltage.

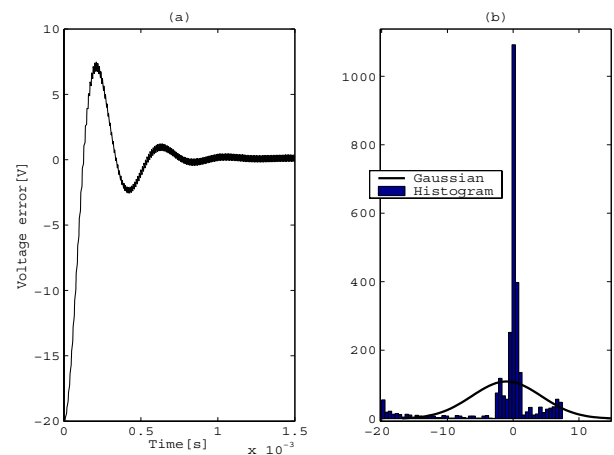


Fig. 6. Output voltage error with histogram and Gaussian distribution for change in reference output voltage.

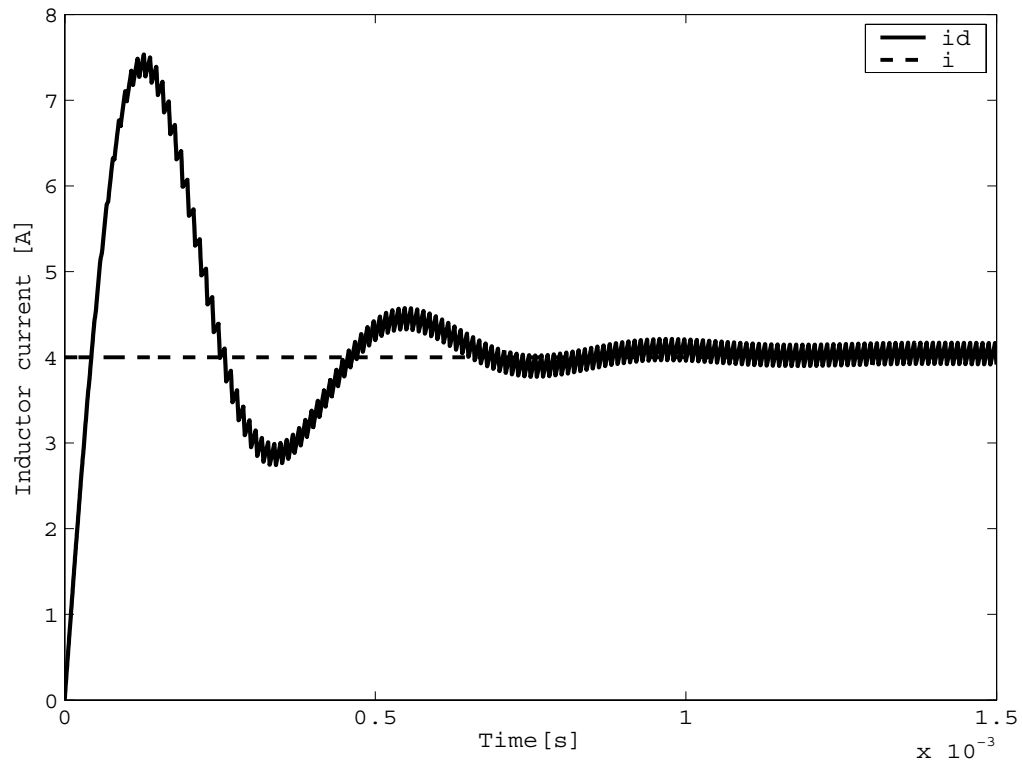


Fig. 7. Inductor current for change in reference output voltage.

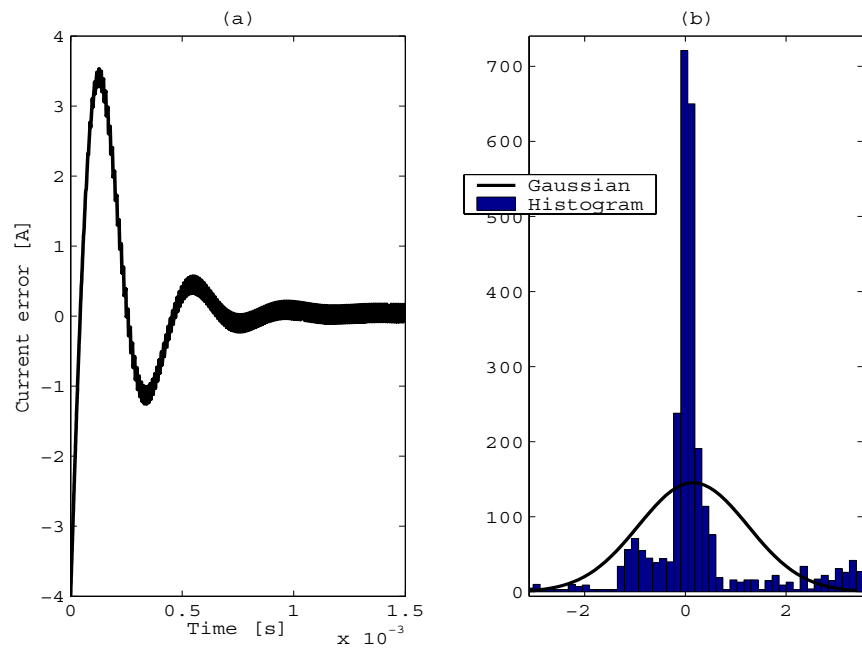


Fig. 8. Inductor current error with histogram and Gaussian distribution for change in reference output voltage.

## 5. Conclusion

In this paper, MRAC with MIT rule is chosen to control the DC-DC boost converter, this method is satisfied for its controller structure and good performance in various output voltages. The proposed system is stable and able to perfectly reach the model reference with a shorter recovery time. Adaptive gains determine the success of adaptive control. The adaptation gains of the proposed controller are obtained by empirical gains.

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