Production Planning Through Multi-Objective De Novo Programming with Variable Prices

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Abstract: - This paper presents a novel multi-objective De Novo programming approach developed to address production planning problems. The proposed model incorporates increasing costs or quantity discounts for certain raw materials which were not considered in previous approaches presented in scientific literature. The efficiency of the proposed methodology was tested using a bakery production planning example. The multi-objective De Novo programming model was solved using various multi-objective programming approaches: the original De Novo programming approach, several goal programming approaches, and the global criterion method. The results indicate the successful application of the proposed methodology in solving production planning problems, with no significant difference in the efficiency of the applied multi-objective programming methods.

Key-Words: - De Novo programming, multi-objective, variable prices, bakery

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1 Introduction

About thirty-seven years ago, [1], introduced a new concept into mathematical programming, a special approach to optimization, and called it De Novo programming. In standard mathematical programming problems resources are set in advance and the work to be done is to "optimize a given system". However, the De Novo programming approach suggests a way of "designing an optimal system". In De Novo, resource quantities are not given, since they are available if we have enough money. The maximum quantities of resources are limited by the available budget, which is an important new element of De Novo.

De Novo is generally more effective in solving problems than the standard programming model. For example, multi-objective problems, [2], and price changing, i.e. increasing costs of raw materials, or quantity discounts, [3], [4], are the production situations that can be processed very successfully with the De Novo methodology providing satisfactory solutions.

Since this new approach was initiated, [1], De Novo programming has been developing rather slowly. Many articles and promising ideas related to De Novo come from authors from the Far East: [5], [6], [7], [8], [9], [10], [11]. However, the author of this approach has not abandoned the De Novo idea and has continued to engage with it in many of his later works, [12], [13], [14]. He also included it (as a single or multi-objective approach) among the eight concepts of optimization, [15], where classic optimization is only a special case. [16], introduced a meta-goal programming approach for solving the multi-objective De Novo programming problem. In [17], [18], authors tried to introduce some new constraints in De Novo multi-objective problems and presented a new way of solving the problem using an extension of the STEM method. In two recent papers, the author considers multi-objective De Novo linear programming problems, [19], [20], while in a third paper, [21], a project portfolio using the hybrid approach of data envelopment analysis and De Novo optimization was considered. Multi-objective De Novo programming problems were considered in recent times by [22], [23], while, [24], in 2021 presented the exhaustive literature review of De Novo programming.

This paper aims to present a multi-objective De Novo programming model that takes into account variable prices, which were not considered in previous approaches presented in scientific literature. While the transformation to the continuous knapsack problem is not possible in such cases, this paper proposes a solution with appropriate simplifications. Previous research, such as that by [3], [4], has focused on the single-objective De Novo programming model with variable prices, but the current study expands on this by considering a multi-objective approach.

After the introduction part of the paper, Section 2 gives a brief overview of the multi-objective De Novo programming problem, and in Section 3 situations with variable prices are presented. In Section 4 a multi-objective model in a bakery will be formulated while in Section 5 this model is solved using the founder Milan Zeleny's, original approach. Then, some different approaches involving goal programming and the global criterion method for solving multi-objective problems are presented. Section 6 presents the discussion of the results, and finally, in Section 7, there is the conclusion.

2 Multiple objective De Novo programming

The multiple-objective De Novo programming model, [14], has the following form:

$$\begin{aligned} &\text{Max } Z = CX \\ \text{s.t.} \quad &AX - b = 0 \\ &p^T b \le B \\ &X, b \ge 0 \end{aligned} \tag{1}$$

C is a (q, n) matrix comprising the coefficients of q objective functions, and A is a (m, n) matrix of technological coefficients, defining the usages of resource i upon producing the product type j. Vector b is the m-dimensional vector of unknown resource variables, X is the n-dimensional vector of decision variables, p is the m-dimensional vector of the unit prices of m resources, and B is the given total available budget. The solution to the problem (1) is to find the optimal allocation of budget B and the distribution of raw materials (resources) with which we can maximize the values $Z = CX$ of the product mix.

The main difference between the usual linear programming model and the De Novo formulation lies in the treatment of the resources. In the De Novo programming model resources are not given in advance but they become decision variables b_i.

2.1 Zeleny's Approach

From (1) follows

$$p^T AX = p^T b \le B$$

and, defining the n row vector of unit costs $V = p^T A,$ the problem (1) can be transformed into:

$$\begin{aligned} &\text{Max } Z = CX \\ \text{s.t.} \quad &VX \le B, \quad X \ge 0 \end{aligned} \tag{2}$$

where

$$Z = \left[z_1, \dots, z_q \right]^T \in R^q \qquad \text{and}$$

$$V = \left[V_1, \dots, V_n \right] = p^T A \in R^n .$$

Most authors solve multi-objective De Novo models according to suggestions from [1]. Namely, they construct an auxiliary model (the meta-optimum problem) involving the minimum budget quantity to achieve the ideal values of all of the objective functions. After that, the optimum-path ratio, which is the ratio of the given budget B and the minimum budget obtained by this auxiliary model (3), is calculated. This ratio is then used to obtain the final solution for the model with the previously given budget B.

Let $z_k{*} = \max z_k,\ k = 1, \dots, q$, be the optimal value for the k-th objective of the problem (1) subject to $VX \le B$, $X \ge 0$, and let $Z{*} = \left[z_1{*}, \dots, z_q{*} \right]^T$ be the q-objective value for the ideal system with respect to B. Then, the meta-optimum problem can be constructed as follows:

$$\begin{aligned} &\text{Min } VX , \\ \text{s.t.} \quad &CX \ge Z{*}, \ X \ge 0 \end{aligned} \tag{3}$$

After solving this problem optimal solution $X{*}$ is obtained and consequently $B{*} = VX{*}$, $b{*} = AX{*}$. The value $B{*}$ represents the minimum budget to achieve $Z{*}$ through $x{*}$ and $b{*}$. Since $B{*} \ge B,$ the optimum-path ratio for achieving ideal performance $Z{*}$ for a given budget level B can be defined as

$$r = \frac{B}{B{*}}$$

(4)

According to this ratio, the actual final solution (or optimal design) can be obtained by the following calculation:

$$\overline{X} = r \cdot X{*}, \ \overline{b} = r \cdot b{*} \text{ and } \overline{Z} = r \cdot Z{*} . \tag{5}$$

3 The Varying Cost of Raw Materials

In a situation of varying the prices of the same resource, model (1) can be transformed in such a way that it includes resources with variable prices.

Matrix A of technological coefficients is the (m, n) matrix. Let the matrix A_C of type (m_1, n) be the block matrix from matrix A which contains the technological coefficients of resources that have constant prices, and let block matrix A_V, type (m_2, n), be the matrix with coefficients of resources with variable prices. Of course, $m_1 + m_2 = m$. Without loss of generality, it can be supposed that matrix A_C is located in the first m_1 rows from matrix A, or the matrix form:

$$A = \begin{bmatrix} A_C \\ A_V \end{bmatrix}.$$

Besides vector b of resource variables, a new vector d can be included. It presents the quantity of resources if they cross the border values when the prices of resources become higher or lesser than the original prices. Both of these vectors are divided into two parts, one for resources with constant prices and the other for variable prices, i.e. in matrix form:

$$b = \begin{bmatrix} b_C \\ b_V \end{bmatrix}, \qquad d = \begin{bmatrix} 0 \\ d_V \end{bmatrix}.$$

Resource quantity for the resources with variable prices is also divided into two parts: first is the quantity that is purchased with the original price (b_V) and second is the additional quantity (d_V) that is purchased with higher or lesser price. Therefore, the total quantity of resources with variable prices will be $(b_V + d_V)$.

The price vector will be divided similarly. Let p_C be the first part of the price vector for resources that have constant prices, and p_V is the second part for resources that have variable prices. Besides that, the additional vector p_V' whose coefficients are the prices for an additional quantity of resources exists, or in matrix form:

$$p = \begin{bmatrix} p_C \\ p_V \end{bmatrix}, \qquad p' = \begin{bmatrix} 0 \\ p'_V \end{bmatrix}.$$

In these vectors, b_C and p_C are type $(m_1, 1)$ and b_V, d_V, p_V, p'_V are type $(m_2, 1)$ where, of course, $m_1 + m_2 = m$.

The model (1) with an additional quantity of resources for the resources with variable prices can now be transformed in:

$$\begin{aligned} &\text{Max } Z = CX \\ \text{s.t.} \quad &AX - b - d = 0 \qquad (6) \\ &p^T b + p'^T d \le B \\ &X, b, d \ge 0 \end{aligned}$$

or

$$\begin{aligned} &\text{Max } Z = CX \\ &\begin{bmatrix} A_C \\ A_V \end{bmatrix} \cdot X - \begin{bmatrix} b_C \\ b_V \end{bmatrix} - \begin{bmatrix} 0 \\ d_V \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ &\begin{bmatrix} p_C \\ p_V \end{bmatrix}^T \cdot \begin{bmatrix} b_C \\ b_V \end{bmatrix} + \begin{bmatrix} 0 \\ p'_V \end{bmatrix}^T \cdot \begin{bmatrix} 0 \\ d_V \end{bmatrix} \le B \\ &X, b_C, b_V, d_V \ge 0 \end{aligned}$$

or

$$\begin{aligned} &\text{Max } Z = CX \\ &A_C X - b_C = 0 \qquad (7) \\ &A_V X - b_V - d_V = 0 \\ &p_C{}^T b_C + p_V{}^T b_V + p'_V{}^T d_V \le B \\ &X, b_C, b_V, d_V \ge 0. \end{aligned}$$

For the resources with constant prices from (7), there follows: $p_C{}^T b_C = p_C{}^T A_C X$.

In that way, the budget equation in (7) can be written as:

$$p_C{}^T A_C X + p_V{}^T b_V + p'_V{}^T d_V \le B, \text{ or}$$

$$V_C X + p_V{}^T b_V + p'_V{}^T d_V \le B \qquad (8)$$

where $V_C = p_C{}^T A_C$ is the row vector of the unit costs of resources with constant prices.

In the end, the multi-objective model with variable prices in the simplified form can be presented as:

$$\begin{aligned} &\text{Max } Z = CX \\ &A_V X - b_V - d_V = 0 \qquad (9) \\ &V_C X + p_V{}^T b_V + p'_V{}^T d_V \le B \\ &X, b_V, d_V \ge 0 \end{aligned}$$

In this paper two cases of variable prices will be considered: the increasing costs of raw materials and the quantity discounts offered for volume purchases.

- *The increasing costs of raw materials*

Let us assume that k raw material can be purchased at the price p_k but only for a quantity lower (or equal) than Q. The price of k raw material above that quantity, Q is $p_k' > p_k$. The relation for the k raw material can now be transformed in:

$$a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n = b_k + d_k \qquad (10)$$

with the additional constraint $b_k \le Q_k$, where d_k is the additional quantity of the k raw material with the unit price p_k'.

There is no need to specify that b_k should reach the maximum value of Q_k first, before allowing d_k greater than zero. The optimization model ensures b_k reaches the maximum value of Q_k because of the lower penalty, i.e. lower price p_k.

- *Quantity discounts offered for volume purchases*

Quantity discounts offered for volume purchases may be formulated in a somewhat different way. Let us assume that for the *k* resource (b_k) the valid price is p_k as long as the purchased quantity is below Q_k, and the discounted price p_k' is valid for the entire quantity if the purchased quantity is higher than (or equal to) Q_k.

Let

b_k, p_k – the amount and price of *k* raw material if it is purchased at less than the quantity discount volume;

d_k, p_k' – the amount and price of *k* raw material if it is purchased at the quantity discount.

The new model for *k* raw material, instead of one equation (10), has relations:

$$b_k - Q_k^* y_{k1} \le 0 \tag{11}$$

$$d_k - Q_k y_{k2} \ge 0 \tag{12}$$

$$d_k - M y_{k2} \le 0 \tag{13}$$

where M is a very large positive number, or the upper limit for the procurement of the resource k, and Q_k* is a number that is slightly lower than Q_k. The variables y_{k1} and y_{k2} are binary variables (0 or 1), for which the following applies:

$$y_{k1} + y_{k2} = 1 \tag{14}$$

The problem of the mutual exclusivity of variables b_k and d_k can be introduced as follows: If $d_k = 0$ (there is no quantity discount) then the relation $b_k < Q_k$ has to be true (i.e. the necessary amount of resources is below the one needed for the discount). Similarly, if $b_k = 0$ then the relation $d_k \ge Q_k$ has to be true (we arrived at the quantity of resources needed for a discount).

Then, if $y_{k1} = 1$ from relation (14), it follows that $y_{k2} = 0$. The equations (11), (12), and (13) then become

$$b_k \le Q_k^*, \quad d_k \ge 0, \quad d_k \le 0.$$

The last two relations then assure that $d_k = 0$. Moreover, since Q_k* is slightly lower than Q_k, b_k is strictly lower than the limit Q_k.

In line with the same equation, if $y_{k2} = 1$ then $y_{k1} = 0$. Equations (11), (12), and (13) become

$$b_k \le 0, \quad d_k \ge Q_k, \quad d_k \le M.$$

Constraint $b_k \le 0$ and the non-negativity constraint on variable b_k ensure that $b_k = 0$, and d_k is greater than (or equal to) the discount limit Q_k and smaller than the big positive number M (or another upper limit defined in advance).

This way of introducing quantity discounts is useful especially if quantity discounts appear in several stages, i.e. if there are several classes in which a supplier approves different quantity discounts, [25].

Taking into consideration costs for additional quantities of raw material a new budget equation can be defined:

$$p^T b + p'^T d \le B$$

$$\sum_{i=1}^{m} p_i b_i + \sum_{k \in K} p_k' d_k \le B \tag{15}$$

Set K contains the indices of raw materials with increasing or discounted prices, i.e.

$$K = \{m_1 + 1, \cdots, m\}. \tag{16}$$

In accordance with the relation from the model (7) it follows:

$$p_C{}^T b_C + p_V{}^T b_V + p'_V{}^T d_V \le B \quad \text{or}$$

$$\sum_{i=1}^{m_1} p_i b_i + \sum_{k \in K} p_k b_k + \sum_{k \in K} p_k' d_k \le B \tag{17}$$

For resources with constant prices follows: $p_C{}^T b_C = p_C{}^T A_C X = V_C X$, and so the budget constraint becomes as in relation (8):

$$V_C X + p_V{}^T b_V + p'_V{}^T d_V \le B \qquad \text{or}$$

$$\sum_{j=1}^{n} v_{Cj} x_j + \sum_{k \in K} p_k b_k + \sum_{k \in K} p_k' d_k \le B \tag{18}$$

where v_{Cj} are the unit costs of the resources with constant prices used for producing product j.

Since the same raw material has a different price variable, the income from a product unit is no longer constant. Therefore, if among the objective functions, there exists a net income equation, maximizing the sum of $c_j\, x_j$, (where c_j is the unit profit for articles) would not be an accurate measure of net income. For that reason, the net income equation should be recalculated as the difference between sales and the total cost of materials, where the objective function will include materials at both prices. If S is the vector of the selling prices and s_j is the selling price of j product, the net income objective function is defined as follows:

$$Max \; z = S^T X - (p^T b + p'^T d), \text{ or}$$

$$Max\, z = \sum_{j=1}^{n} s_j x_j - \left(\sum_{i=1}^{m} p_i b_i + \sum_{k \in K} p_k' d_k \right) \tag{19}$$

In that equation, d_k ($k \in K$) remains for those materials which in additional quantities can be bought only at a higher price, or the quantities of raw materials if we bought them with quantity discounts.

Similarly, as in the budget equation, the second part of these equations follows:

$$p^T b + p'^T d = p_C{}^T b_C + p_V{}^T b_V + p'_V{}^T d_V ,$$

and, because $p_C{}^T b_C = p_C{}^T A_C X = V_C X$, from relation (8) follows:

$$p^T b + p'^T d = V_C X + p_V{}^T b_V + p'_V{}^T d_V .$$

In that way, the net income objective function becomes:

$$Max\ z = S^T X - V_C X - p_V{}^T b_V - p'_V{}^T d_V$$

or

$$Max\ z = \sum_{j=1}^{n} \left(s_j - v_{Cj}\right) x_j - \sum_{k \in K} p_k b_k - \sum_{k \in K} p_k{}' d_k \quad (20)$$

4 Case Study

A new multi-objective De Novo programming problem with the variable prices of raw materials will be explained using the example of production planning in a bakery, which produces twenty different products, [3]. Table 1 and Table 2 present the list of articles (Table 1) and the list of raw materials (Table 2) with some new data for formulating a new objective function. The tables are provided in the Appendix of the paper.

Table 1 presents the selling prices for each product, the weights of the articles, the amount of flour in each item, and the lower and upper bounds for monthly production. In Table 2, 27 different raw materials are used in the production of these articles. Table 2 also presents the purchasing prices for every one of them and for the last four raw materials for which we have variable prices. The technological coefficients, i.e. the amounts of raw materials in one unit of articles (a_{ij}) are presented in Table 3.

The raw materials R26 and R27 (Wheat flour T-850 and Wheat flour T-550) can be purchased at a discounted price if the purchased quantity is greater than 14200 kg (Q_{26} = 14200 kg or 14.2 t) for the first and greater than 60000 kg (Q_{27} = 60000 kg or 60.0 t) for the second raw material. This reduced price is valid for the entire quantity supplied, i.e. $p_{26}' = 2.3004$ and $p_{27}' = 2.244$.

In addition to this, let us consider the increasing costs for yeast (R24) and corn concentrate Aurelia (R25) as follows: The limit of yeast purchased at a lower price is Q_{24} = 2000 kg (2.0 t), while this limit for corn concentrate is Q_{25} = 1600 kg (1.6 t). The purchase price of the additional quantity of yeast is p_{24}' = 7.7616, and of corn concentrate p_{25}' = 14.6364 (12% higher) monetary units.

Suppose that the budget which is available for purchasing these raw materials is 300 000 m.u.

In accordance with Table 1, Table 2, and Table 3 a multiple objective linear programming problem with two objective functions (net income and total production measured by flour consumption) can be formulated. This problem has twenty-seven raw material constraints and one budget constraint. In addition to this, with the intention to introduce variable prices into the model, it appears some additional constraints for the raw materials that have quantity discounts - relations (11), (12), and (13). This model has 20 integer decision variables for the quantities of types of bread products x_j and 27 continuous resources variables b_i (among them b_{13} is also an integer – the number of eggs). Besides that, introducing variable prices requires some additional variables for resources that have variable prices, i.e. d_i is the additional quantity of the i-th raw material. In the end, there are some binary variables y_{ki}, which provide the mutual exclusivity of the amount of resources that would be purchased at lower or upper prices. In addition to that, there exist 40 lower and upper bounds constraints for the 20 types of bread products and two constraints for resources with increasing costs (b_{24} and b_{25}).

If x_j is the production quantity of j bakery product, then the first objective function, which takes increasing costs and quantity discounts into consideration, has the following form, as in equation (19):

$$Max\, z_1 = \sum_{j=1}^{20} s_j x_j - \left(\sum_{i=1}^{27} p_i b_i + \sum_{k \in K} p'_k\, d_k \right),$$

$$K = \{24, 25, 26, 27\}$$

The second objective function is the total bakery production measured by flour consumption. The coefficients for this objective function are given in Table 1 (c_{2j}), and this function is:

$$Max\, z_2 = \sum_{j=1}^{20} c_{2j} x_j$$

The budget and raw material constraints are as follows:

$$\sum_{i=1}^{27} p_i b_i + \sum_{k \in K} p'_k\, d_k \le B,\ K = \{24, \cdots, 27\}$$

$$\sum_{j=1}^{20} a_{ij}x_j - b_i - d_i = 0,\ \ i = 1,\cdots,27$$

The constraints for the discounted prices for the last two raw materials (k = 26, 27) in accordance with the relations (11) – (14) (b_k and d_k in kg) are:

$$b_{26} - 14199\, y_{26,1} \le 0\,,$$
$$d_{26} - 14200\, y_{26,2} \ge 0\,,$$
$$d_{26} - M\ y_{26,2} \le 0\,, \text{ and}$$
$$b_{27} - 59999\, y_{27,1} \le 0\,,$$
$$d_{27} - 60000\, y_{27,2} \ge 0\,,$$
$$d_{27} - M\ y_{27,2} \le 0\,.$$

For binary variables y_{k1}, y_{k2} holds
$y_{26,1} + y_{26,2} = 1$ and $y_{27,1} + y_{27,2} = 1$.

In addition, there are two constraints (in kg) for resources that have increasing costs:

$b_{24} \le 2000$ and, $b_{25} \le 1600$.

Of course, due to the nature of the data from Table 1, all articles have lower and upper bounds (40 additional constraints). If the lower and upper bounds are L_j and U_j, the complete De Novo programming multi-objective model for bakery production has the following form:

Model M1

$$Max\, z_1 = \sum_{j=1}^{20} s_j x_j - \sum_{i=1}^{27} p_i b_i - \sum_{k\in K} p'_k\, d_k\ ,$$

$$K = \{24, 25, 26, 27\} \tag{21}$$

$$Max\, z_2 = \sum_{j=1}^{20} c_{2j} x_j \tag{22}$$

subject to

$$\sum_{i=1}^{27} p_i b_i + \sum_{k\in K} p'_k\, d_k \le B \tag{23}$$

$$\sum_{j=1}^{20} a_{ij}x_j - b_i - d_i = 0,\ \ i = 1,\cdots,27 \tag{24}$$

$$b_k - Q_k^* y_{k1} \le 0,\ k = 26, 27 \tag{25}$$

$$d_k - Q_k\, y_{k2} \ge 0,\ k = 26, 27 \tag{26}$$

$$d_k - M y_{k2} \le 0,\ k = 26, 27 \tag{27}$$

$$y_{k1} + y_{k2} = 1,\ k = 26, 27 \tag{28}$$

$$b_k \le Q_k\,,\ \ k = 24, 25 \tag{29}$$

$$L_j \le x_j \le U_j\,,\ j = 1,\ldots,20 \tag{30}$$

where variables x_j, j = 1,…, 20, and b_{13}, are integers, while variables $y_{k1}, y_{k2};\ k = 26, 27$ are binary variables, and all decision variables are nonnegative.

Now some simplifications can be introduced. Namely, 27 constraints for raw materials can be reduced by the inclusion of the majority of these constraints in the budget constraint. Model (7) is as follows:

$$A_C X - b_C = 0\text{, or } b_C = A_C X$$

for resources with constant prices. This relation can be included in the budget constraint and thus the budget constraint from model (7) becomes

$$V_C X + p_V{}^T b_V + p'_V{}^T\, d_V \le B\,,$$

as in model (9), where $V_C = p_C{}^T A_C$.

Vector V_C can be calculated from initial data. Namely vector $p_C = [p_1,\cdots,p_{23}]^T$, i.e. the vector for the resources with constant prices, is presented in the first 23 rows the of price column in Table 2.

Matrix A_C is formed from the first 23 rows from Table 3 and components of vector V_C are calculated and presented in Table 4. All input data are presented in Appendix 1. Finally, budget constraint is:

$$\sum_{j=1}^{n} v_{Cj} x_j + \sum_{k\in K} p_k b_k + \sum_{k\in K} p_k{}' d_k \le B$$

as in relation (18).

Only four raw material constraints that have increasing costs (b_{24} and b_{25}) or discounted prices (b_{26} and b_{27}) will remain. In such a way the model will have 23 constraints and 23 decision variables less than the initial model, but the solution obtained will remain the same. Of course, the values of the b_i variable that are substituted have to be obtained by the inclusion of x_j variables (j = 1,…,20) in the initial relations (24) from the previous complete model M1.

Because the first objective function, in addition to twenty x_j variables, also contains raw material variables, the same relation from model (7) in the first objective function can be included. Then the new form of the first objective function as presented in relation (20) will be obtained:

$$Max\ z = \sum_{j=1}^{n} \left(s_j - v_{Cj}\right) x_j - \sum_{k\in K} p_k b_k - \sum_{k\in K} p_k{}' d_k$$

In the first objective function of the reduced model there now remains only eight raw material variables b_k and d_k, where $k \in K = \{24, 25, 26, 27\}$, as well as twenty x_j variables.

Following the substitution of these 23 raw material constraints in the budget constraint and in the first objective function a reduced and much simpler model is obtained.

The reduced model is as follows (the binary variables y_{ki} are now marked as y_1, y_2, y_3, and y_4):

Model M2

Max z_1 = Max $(9.452482x_1 + 10.629x_2 + 9.524034x_3 + 7.412533x_4 + 7.807031x_5 + 9.989509x_6 + 9.476482x_7 + 10.33086x_8 + 8.817266x_9 + 8.819602x_{10} + 4.528776x_{11} + 5.541641x_{12} + 5.501423x_{13} + 4.451883x_{14} + 4.004302x_{15} + 4.800361x_{16} + 5.484859x_{17} + 4.62133x_{18} + 4.824314x_{19} + 5.540952x_{20} - 6.93b_{24} - 7.7616d_{24} - 13.068b_{25} - 14.6364d_{25} - 2.706b_{26} - 2.3004d_{26} - 2.64b_{27} - 2.244\ d_{27})$

Max z_2 = Max $(0.41248x_1 + 0.29988x_2 + 0.31001x_3 + 0.4857x_4 + 0.486x_5 + 0.48592x_6 + 0.41248x_7 + 0.35115x_8 + 0.2145x_9 + 0.2413 + x_{10} + 0.086x_{11} + 0.0625x_{12} + 0.09354x_{13} + 0.06561x_{14} + 0.04335x_{15} + 0.06573x_{16} + 0.06327x_{17} + 0.03667x_{18} + 0.04592x_{19} + 0.053\ x_{20})$

s.t.

Budget constraint:

(B) $0.723518x_1 + 0.231x_2 + 1.791966x_3 + 0.471467x_4 + 0.076969x_5 + 0.066491x_6 + 0.723518x_7 + 0.52914x_8 + 2.498734x_9 + 1.356398x_{10} + 0.103224x_{11} + 0.110359x_{12} + 0.978577x_{13} + 0.060117x_{14} + 0.051698x_{15} + 0.059639x_{16} + 0.419141x_{17} + 0.0.41867x_{18} + 0.371686x_{19} + 0.939048x_{20} + 6.93b_{24} + 7.7616d_{24} + 13.068b_{25} + 14.6364d_{25} + 2.706b_{26} + 2.3004d_{26} + 2.64b_{27} + 2.244d_{27} \le 300000$

Constraints for resources with variable prices:

(R24) $0.008x_1 + 0.008x_2 + 0.0076x_3 + 0.0119x_4 + 0.01047x_5 + 0.01191x_6 + 0.008x_7 + 0.00896x_8 + 0.009x_9 + 0.0086x_{10} + 0.00252x_{11} + 0.00187x_{12} + 0.002\ x_{14} + 0.00133x_{15} + 0.00199x_{16} + 0.00186x_{17} + 0.0022x_{18} + 0.0027x_{19} + 0.003x_{20} - b_{24} - d_{24} = 0$

(R25) $0.12852x_2 + 0.013x_{20} - b_{25} - d_{25} = 0$

(R26) $0.184x_1 + 0.1551x_3 + 0.3399x_4 + 0.316x_5 + 0.184x_7 + 0.30165x_8 + 0.125x_9 - b_{26} - d_{26} = 0$

(R27) $0.102x_1 + 0.29988x_2 + 0.155x_3 + 0.170x_5 + 0.48592x_6 + 0.102x_7 + 0.241.3x_{10} + 0.086x_{11} + 0.0625x_{12} + 0.093.54x_{13} + 0.06561x_{14} + 0.04335x_{15} + 0.06573x_{16} + 0.06327x_{17} + 0.036.67x_{18} + 0.04592\ x_{19} + 0.053x_{20} - b_{27} - d_{27} = 0$

Constraints for the resources with quantity discounts:

$b_{26} - 14199\ y_1 \le 0$
$d_{26} - 14200\ y_2 \ge 0$
$d_{26} - M\ y_2 \le 0$
$y_1 + y_2 = 1$
$b_{27} - 59999\ y_3 \le 0$
$d_{27} - 60000\ y_4 \ge 0$
$d_{27} - M\ y_4 \le 0$
$y_3 + y_4 = 1$

Lower and upper bounds:

$$L_j \le x_j \le U_j,\ j = 1,\ldots,20$$

$$b_{24} \le 2000,\ \ b_{25} \le 1600$$

where variables x_j, $j = 1,\cdots,20,$ and b_{13}, are integers, while variables y_1, y_2, y_3, y_4 are binary, and M is a great positive number, and all decision variables are nonnegative.

5 Solving the Model

5.1 Zeleny's Approach

The first step in solving this model is to obtain the optimal solutions for each objective function separately. These optimal solutions are obtained with MATLAB and are presented in Table 5 and Table 6 (Appendix 2). Of course, the available budget is 300000 m.u. for both solutions.

The optimal values of the objective functions are:

z_1* = 2143888.1 m.u.; z_2* = 98457.5 (kg),

and in both solutions the available budget is completely spent.

In these tables, the required quantities of resources (in kg) are presented. In both solutions, we have quantity discounts for raw materials 26 and 27 since we purchase these two types of flour over limited quantities (Q_{26} = 14200 kg and Q_{27} = 60000 kg). The twenty-fourth raw material has to be purchased over the limited quantity and so the quantity over the limit (d_{24}) is purchased at a higher price. For R25 in both solutions, the limit is not exceeded (Q_{25} = 1600 kg) and the whole quantity (b_{25}) will be purchased at a lower price.

Of course the final values of the resource variables that are not presented in Model 2 (b_1 to b_{23}) have to be obtained by the inclusion of x_j variables (j = 1,…,20) in the initial relations (24) from the previous complete model M1.

After that, the meta-optimal problem (3) is formulated. It is shown in the relations below:

$$\text{Min } B = VX$$

$$\text{s.t.} \quad CX \ge Z^*$$

$$X \ge 0$$

where budget B is now the function of twenty x_j variables and eight raw material variables, as can be seen from relation (18) or the budget constraint in the simplified model M2.

Metaoptimal model – M3

Min *B* = $0.723518x_1 + 0.231x_2 + 1.791966x_3 + 0.471467x_4 + 0.076969x_5 + 0.066491x_6 + 0.723518x_7 + 0.52914x_8 + 2.498734x_9 + 1.356398x_{10} + 0.103224x_{11} + 0.110359x_{12} + 0.978577x_{13} + 0.060117x_{14} + 0.051698x_{15} + 0.059639x_{16} + 0.419141x_{17} + 0.0.41867x_{18} + 0.371686x_{19} + 0.939048x_{20} + 6.93b_{24} + 7.7616d_{24} + 13.068b_{25} + 14.6364d_{25} + 2.706b_{26} + 2.3004d_{26} + 2.64b_{27} + 2.244d_{27}$

s. t. $z_1 \geq z_1^* = 2143888.1$

$z_2 \geq z_2^* = 98457.5,$

and in addition to all the remaining constraints that are presented in Model M2 (constraints for raw materials with increasing costs and quantity discounts, and all lower and upper bounds constraints).

The optimal solution to this meta-optimal problem is presented in Table 7. The final values of the resource variables that are not presented in this model (resource variables with constant prices b_1 – b_{23}) have to be obtained by the inclusion of x_j variables (j = 1,…,20) in the relations (24) from model M1.

The optimum-path ratio is

$$r = B/B^* = 300000/308077 = 0.97378253$$

and the optimal design has to be obtained with the calculation $\overline{X} = r \cdot X^*$. Since the solutions for product quantity (x_j) must be integers, the obtained values for these variables are first rounded to integers. The quantities of raw materials (b_i) are obtained so that integers x_j are included in the initial raw material relations (24) from model M1.

According to that, the optimum-path ratio transformation $(\overline{b} = r \cdot b^*)$ was not used for obtaining the raw materials quantities. The problem is that the solution obtained in that way is not a feasible solution for the original problem. Namely, some product quantities have values lower than the lower bounds from Table 1 (x_1, x_2, x_3, x_9, x_{10}, x_{13}, x_{18}, and x_{20}). This final design is presented in Table 8.

Therefore, that way of solving this problem is not appropriate for problems that have integer variables and lower or upper bounds for the decision variables. For that reason, this multi-objective problem will be solved with some other approaches below.

Multi-objective De Novo programming with the meta-optimal solution is not the only available approach. Several multi-objective decision-making techniques can be used for obtaining the best compromise solution. Below some goal programming approaches will be presented and, in the end, an approach that uses the global criterion method has been shown.

5.2 Goal Programming Approaches

Goal programming is an extension of linear programming models and includes the achievement of target values (goals) for each objective, instead of the maximization or minimization of the objective functions.

The structure of the i-th goal is as follows, [26]:

$$f_i(X) + n_i - p_i = t_i \qquad (31)$$

where:

$f_i(X)$ - mathematical expression for the i-th attribute (X is the vector of decision variables).

t_i - target value for the i-th attribute, i.e. the achievement value that the DM considers as satisfying for the i-th attribute.

n_i - negative deviation variable, i.e. quantification of the under-achievement of the i-th goal.

p_i - positive deviation variable, i.e. quantification of the over-achievement of the i-th goal.

The first goal programming variant is known as weighted goal programming (WGP). In the WGP model deviations from the target values are assigned weights (u_i and v_i) according to their relative importance to the decision maker and minimized as an Archimedean sum. The formulation of this variant is as follows:

$$\min z = \sum_{i=1}^{q} \frac{1}{k_i}(u_i n_i + v_i p_i) \qquad (32)$$

s.t. $f_i(X) + n_i - p_i = t_i$, $i = 1,\ldots,q$, $X \in S$

where S is the set of model constraints.

The function z of weighted deviational variables is known as the achievement function.

Each deviational variable in the achievement function is divided by a normalization constant k_i. This allows us to overcome the problem of incommensurability. Namely, deviations from objectives measured in different units must not be summarized together. In this paper, a normalization technique based on the difference between the positive and negative ideal solution is presented [27]. For this task a payoff table is presented and from which it is easy to identify ideal and anti-ideal solutions or optimistic and pessimistic objective values. In the bakery problem, the payoff table is presented in Table 9. From that table, the ideal and anti-ideal solutions are as follows:

$Z^* = (2143888.10, 98457.49)$ - Ideal

$Z^- = (1895180.6.10, 92119.51)$ - Anti-ideal

The normalized constant is now calculated as follows: $k_i = z_i^* - z_i^-$, and in the bakery problem it is:

$$k_1 = 2143888.1 - 1895180.6 = 248707.5$$

$$k_2 = 98457.49 - 92119.51 = 6337.98$$

Finally, the weighted goal-programming model for solving multiple objective De Novo programming problems can be formulated. In our problem, there are two maximization objectives and for their target levels the ideal levels z_i* has been taken. If positive ideal values are used for maximization-type objectives, then the positive deviation (p_i) should be zero. This is because the objective values cannot be greater than the ideal values. In addition to that, let the weights u_i be equal for both objectives and *UI* = 10000. Consequently, the achievement function in the WGP model has the following form:

$$\min z = \frac{10000}{248707.5} n_1 + \frac{10000}{6337.98} n_2 \quad \Rightarrow$$

$$\min z = 0.040208 n_1 + 1.57779 n_2$$

s. t.

$$z_1(X) + n_1 = z_1^* = 2143888.1$$

$$z_2(X) + n_2 = z_2^* = 98457.49$$

and with all other constraints from model M2 (budget constraints, constraints for raw materials with increasing costs and quantity discounts, and all lower and upper bounds constraints).

The optimal solution of that model is presented in Table 10 and the values of the objective function are presented in the last row of the table. They are obtained with the following relations:

$$z_1(WGP) = z_1^* - n_1^* = 2066814.9 \text{ m.u.}$$

$$z_2(WGP) = z_2^* - n_2^* = 97426.54 \text{kg.}$$

Another goal programming variant, which was applied in the De Novo multi-objective model, is the so-called Min-max goal programming, [28], [29]. In Min-max goal programming, the maximum deviation from amongst the weighted set of deviations is minimised rather than the sum of the deviations themselves. The mathematical expression of that variant is as follows:

$$\min z = D$$

$$\text{s.t.} \quad \frac{1}{k_i}(u_i n_i + v_i p_i) \le D, \; i = 1, \ldots, q \tag{33}$$

$$f_i(X) + n_i - p_i = t_i, \; i = 1, \ldots, q, \; X \in S.$$

For the normalization constant k_i the same relation as in WGP has been taken. For the same reason, as in WGP (if we take $t_i = z_i^*$) $p_i = 0$. In addition to that, let the weights u_i be equal for both objectives and

u_i = 10000 like before in the WGP model. Consequently, our Min-max GP model is:

$$\min z = D$$

$$\text{s.t.} \; \frac{10000}{248707.5} n_1 \le D \;\Rightarrow\; 0.040208 \, n_1 \le D$$

$$\frac{10000}{5337.98} n_2 \le D \;\Rightarrow\; 1.57779 \, n_2 \le D$$

$$z_1(X) + n_1 = z_1^* = 2143888.1$$

$$z_2(X) + n_2 = z_2^* = 98457.49$$

with all other constraints from model M2.

The optimal result of that model is presented in Table 11. The objective functions values are presented in the last row of the table obtained with the following relations:

$$z_1(Min\max) = z_1^* - n_1^* = 2081877.21 \text{ m.u.}$$

$$z_2(Min\max) = z_2^* - n_2^* = 96877.21 \text{ kg.}$$

5.3 The Global Criterion Method

Lastly, a multi-objective De Novo model can be solved using the Global Criterion method, [30], [31].

This method develops a global objective function made up of the sum of the deviations of the values of the individual objective functions from their respective ideal values as a ratio to that of the ideal value, [32]. It can be said that the Global criterion model minimizes the distance to the ideal solution by using Minkowski's L_p metric. The mathematical formulation is as follows (the assumption is that all the objective functions have to be maximized).

$$\text{Min } F = \sum_{i=1}^{q} \left[\frac{z_i(X^*) - z_i(X)}{z_i(X^*)} \right]^p \tag{34}$$

where $z_i(X^*)$ is the value of the objective function i at its individual optimum X^*, $z_i(X)$ is the function itself, and p $(1 \le p \le \infty)$ is the integer value exponent that serves to reflect the importance of objectives. Of course, all constraints must be included in the model. Below the case p = 1 is presented, which implies that equal importance is given to all deviations.

The objective function in the bakery model is:

$$\text{Min } F = \left[2 - \sum_{i=1}^{2} \frac{z_i(X)}{z_i(X^*)} \right] =$$

$$= 2 - \text{Max} \sum_{i=1}^{2} \frac{z_i(X)}{z_i(X^*)}$$

where

$z_1(X^*) = z_1^* = 2143888.1$ and

$z_2(X^*) = z_2^* = 98457.49$.

Because the objective function coefficients in that function are too small, they are multiplied by 10^6 and solved only the maximum problem. The optimal solution remains the same but, in the end, the optimal value of the objective function has to be divided by 10^6 and subtracted from 2. In the end objective function for the global criterion method (maximisation) is as follows:

Max $(8.598459x_1 + 8.003596x_2 + 7.59108x_3 + 8.390612x_4 + 8.577669x_5 + 9.594857x_6 + 8.609654x_7 + 8.385363x_8 + 6.291351x_9 + 6.564639x_{10} + 2.985886x_{11} + 3.219647x_{12} + 3.516151x_{13} + 2.742925x_{14} + 2.308067x_{15} + 2.906689x_{16} + 3.200982x_{17} + 2.528028x_{18} + 2.716658x_{19} + 3.122838x_{20} - 3.23244b_{24} - 3.62034d_{24} - 6.09547b_{25} - 6.82704d_{25} - +1.26219b_{26} - 1.073d_{26} - 1.23141b_{27} - 1.0467d_{27})$

Of course, all other constraints from model M2 must be taken into consideration. The optimal solution of that model is presented in Table 12. Table 13 presents the optimal values of objective functions obtained by the three presented approaches.

6 Discussion of Results

From the results obtained, we can see that the multi-objective De Novo programming model has shown high application efficiency in solving production plan optimization problems. The efficiency and flexibility provided by the proposed model cannot be achieved by modeling the problem using standard mathematical programming models. In standard mathematical programming problems, resources are predetermined and the work to be done is to "optimize a given system." In contrast, the De Novo approach suggests a way of "designing an optimal system." In De Novo, resource quantities are not predetermined, as they are available if we have enough money. The maximum quantity of resources is limited by the budget, which is an important new element of De Novo. The presented model and obtained results show that variable resource prices can be successfully incorporated into the multi-criteria De Novo model.

From the results presented in Table 13, it can be seen that for all three approaches, the values of both objective functions are very close to the ideal value, especially for the second objective function. The Weighted Goal Programming approach provides a solution in which the first objective function achieves 96.4% of the ideal value, and the second objective function achieves 98.95% of the ideal value. The Min-max approach provides a solution in which the first objective function achieves 97.11% of the ideal value, and the second objective function achieves 98.39% of the ideal value. Finally, the third approach, Global Criterion Method, provides a solution in which the first objective function achieves 97.06% of the ideal value, and the second objective function achieves 98.45% of the ideal value.

This indicates that, according to the criterion of deviation of the obtained solution from the ideal values of the objective functions, it cannot be said that any one of these methods is more or less efficient than the others.

7 Conclusion

Compared to the standard programming model, De Novo is generally more effective in solving problems. For example, multi-objective problems, and price changing, i.e. the increasing costs of raw materials, or quantity discounts, are production situations that can be processed very successfully with De Novo methodology and provide satisfactory solutions.

The model presented in this paper (M1) indicates a high application efficiency when using De Novo multi-objective programming in solving production plan optimization problems in various production companies. Here is presented how this model can be applied to one such company, i.e. a bakery that produces twenty different articles and uses twenty-seven different raw materials. Since the prices of raw materials in the production program vary, the multi-objective model could not be easily transformed into a simple knapsack problem, as is usual with multi-objective De Novo problems. Namely, some of the raw materials have different price variables and their equations cannot be substituted in the budget equation. For that reason, a new simplification which reduces our set of constraints is introduced, so solving the model becomes much easier (in the case of the bakery the set of constraints and the set of decision variables by 23 constraints and variables altogether has been reduced).

The inclusion of variable prices in the multi-objective De Novo programming model is a further innovation. None of the papers that deal with multi-objective De Novo programming present a De Novo model which involves variable resource prices.

Future work on this topic will investigate other production situations in which the De Novo multi-objective programming model with increasing resource prices and quantity discounts can be applied.

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Appendix 1. Tables with input data

Table 1. List of articles

Mark	Article name	Selling prices s_j	Flour per unit (in kg) c_{2j}	Weight (in kg)	Monthly Amount	
					lower	upper
A1	Rye mixed round	10.176	0.41248	0.60	1580	4890
A2	Corn mixed	10.86	0.29988	0.60	10520	17100
A3	Bread with sunflower seeds	11.316	0.31001	0.50	1580	4210
A4	Wheat mixed semi-white	7.884	0.4857	0.65	13150	40390
A5	Wheat half-white bread - folk	7.884	0.486	0.65	9210	23670
A6	Wheat white sandwich	10.056	0.48592	0.65	65750	123080
A7	Rye mixed long	10.2	0.41248	0.60	2630	9460
A8	Wheat mixed Sun	10.86	0.35115	0.60	1710	6000
A9	Swedish bread	11.316	0.2145	0.50	1320	2760
A10	Wheat mixed bread - Zagora	10.176	0.2413	0.50	1710	4260
A11	White rolls of mini salty	4.632	0.086	0.10	9210	23670
A12	White rolls - milk roll	5.652	0.0625	0.08	1970	5670
A13	Stuffed pastry layered cheese	6.48	0.09354	0.10	2370	5260
A14	White rolls with salt	4.512	0.06561	0.07	2240	6310
A15	White rolls round kaiser	4.056	0.04335	0.06	7890	22090
A16	White mini rolls	4.86	0.06573	0.09	7890	24450
A17	White pastry croissant	5.904	0.06327	0.07	1970	4730
A18	Donut	5.04	0.03667	0.07	15780	42080
A19	White pastry - trace	5.196	0.04592	0.05	1580	4340
A20	Stuffed rolls	6.48	0.053	0.08	1970	7890

Table 2. List of raw materials

Mark	Raw material	Prices (per kg)	Mark	Raw material	Prices (per kg)	Variable prices
R1	Rye flour	3.96	R15	Rum aroma	49.8828	
R2	Wheat flour T-110	2.706	R16	Goldperle TBM	39.6	
R3	Kitchen salt	1.848	R17	Vanilla sugar	16.8828	
R4	Additive panifarin	36.3	R18	Butter aroma	151.3776	
R5	Wheat germs	19.6152	R19	Rye sourdough	15.6684	
R6	Grandma mix	26.3868	R20	Cheese for bakery	19.14	
R7	Suvita	20.7108	R21	Enhancer	24.948	
R8	Sugar	6.996	R22	Wiener note	30.228	
R9	Pure corn grits	9.24	R23	Grainpan Max	14.9952	
R10	Edible oil	9.24	R24	Yeast	6.93	7.7616
R11	Margarine BV	11.88	R25	Corn concentrate	13.068	14.6364
R12	Margarine Tropic	12.54	R26	Wheat flour T-850	2.706	2.3004
R13	Eggs (pieces)	0.924	R27	Wheat flour T-550	2.64	2.244
R14	Marmalade	10.824				

Table 3. Norms - The amount of raw material in grams in one unit of articles

Mark	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
R1	126.48						126.48	49.5	89.5	
R2				145.8						
R3	8	8	6.84	9.5	9.52	9.52	8	7.85		6.8
R4	4						4	7.14		
R5										34.4
R6			19							
R7			57							
R8										
R9										68.9
R10								3.57		
R11										
R12										
R13										
R14										
R15										
R16		4.2								
R17										
R18										
R19	4		3.8				4			
R20										
R21		2	1.52	2.38	2.38	1.96		1.06		1.3
R22										
R23									143	
R24	8	8	7.6	11.9	10.47	11.91	8	8.96	9	8.6
R25		128.52								
R26	184		155.1	339.9	316		184	301.65	125	
R27	102	299.88	155		170	485.92	102			241.3

Table 3. Norms - continuation

Mark	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20
R1										
R2										
R3	2	0.94	1.73	1.29	0.89	1.33	1.24	0.55	1	1
R4										
R5										
R6										
R7										
R8	2	0.94	1.73	1.29	1.33	1.33	3.1	1.47	5	
R9										
R10								3.67		
R11	3			2	1.33	1.93	5		5	3
R12			46.29				12.5			19
R13							0.15	0.2	0.1	
R14								10		
R15							0.31	0.73		
R16								0.8	0.8	0.8
R17								0.73		
R18									1	
R19										
R20			20							33
R21	2	0.31		1	1	1	1			
R22		3.12								
R23										
R24	2.52	1.87		2	1.33	1.99	1.86	2.2	2.7	3
R25										13
R26										
R27	86	62.5	93.54	65.61	43.35	65.73	63.27	36.67	45.92	53

Table 4. Selling prices and unit costs for the articles

Mark	Article name	Selling prices s_j	Unit costs for the first 23 raw materials – V_C $V_C = p_C^T \cdot A_C$	$s_j - v_{Cj}$
A1	Rye mixed round	10.176	0.723518	9.452482
A2	Corn mixed	10.86	0.231	10.629
A3	Bread with sunflower seeds	11.316	1.791966	9.524034
A4	Wheat mixed semi-white	7.884	0.471467	7.412533
A5	Wheat half-white bread - folk	7.884	0.076969	7.807031
A6	Wheat white sandwich	10.056	0.066491	9.989509
A7	Rye mixed long	10.2	0.723518	9.476482
A8	Wheat mixed Sun	10.86	0.52914	10.33086
A9	Swedish bread	11.316	2.498734	8.817266
A10	Wheat mixed bread - Zagora	10.176	1.356398	8.819602
A11	White rolls mini salty	4.632	0.103224	4.528776
A12	White rolls - milk roll	5.652	0.110359	5.541641
A13	Stuffed pastry layered cheese	6.48	0.978577	5.501423
A14	White rolls with salt	4.512	0.060117	4.451883
A15	White rolls round kaiser	4.056	0.051698	4.004302
A16	White mini rolls	4.86	0.059639	4.800361
A17	White pastry croissant	5.904	0.419141	5.484859
A18	Donut	5.04	0.41867	4.62133
A19	White pastry - trace	5.196	0.371686	4.824314
A20	Stuffed rolls	6.48	0.939048	5.540952

Appendix 2. Tables with optimal solutions

Table 5. Optimal solution for z_1 (max profit)

Variables	Optimal values	Variables	Optimal values	Variables	Optimal values	Variables	Optimal values
x_1	1580	x_{15}	22090	b_9	117.82	b_{23}	188.76
x_2	10520	x_{16}	24450	b_{10}	175.85	$\boldsymbol{b_{24}}$	**2000**
x_3	1580	x_{17}	4730	b_{11}	211.46	$\boldsymbol{d_{24}}$	**328.31**
x_4	13150	x_{18}	42080	b_{12}	205.26	$\boldsymbol{b_{25}}$	**1377.64**
x_5	21316	x_{19}	4340	b_{13}	9559	$\boldsymbol{d_{25}}$	**0**
x_6	122351	x_{20}	1970	b_{14}	420.80	$\boldsymbol{b_{26}}$	**0**
x_7	2630	b_1	947.62	b_{15}	32.18	$\boldsymbol{d_{26}}$	**14200.14**
x_8	6000	b_2	1917.27	b_{16}	82.90	$\boldsymbol{b_{27}}$	**0**
x_9	1320	b_3	1832.42	b_{17}	30.72	$\boldsymbol{d_{27}}$	**75054.63**
x_{10}	1710	b_4	59.68	b_{18}	4.34	y_1	0
x_{11}	23670	b_5	58.82	b_{19}	22.84	y_2	1
x_{12}	5670	b_6	30.02	b_{20}	112.41	y_3	0
x_{13}	2370	b_7	90.06	b_{21}	450.54	y_4	1
x_{14}	6310	b_8	225.03	b_{22}	17.69	**z_1* = 2143888.1**	

Table 6. Optimal solution for z2 (max flour consumption)

Variables	Optimal values	Variables	Optimal values	Variables	Optimal values	Variables	Optimal values
x_1	1580	x_{15}	7891	b_9	117.82	b_{23}	188.76
x_2	10520	x_{16}	7892	b_{10}	64.02	$\boldsymbol{b_{24}}$	**2000**
x_3	1580	x_{17}	1970	b_{11}	81.50	$\boldsymbol{d_{24}}$	**420.35**
x_4	35916	x_{18}	15780	b_{12}	171.76	$\boldsymbol{b_{25}}$	**1377.64**
x_5	23670	x_{19}	1580	b_{13}	3609	$\boldsymbol{d_{25}}$	**0**
x_6	123080	x_{20}	1970	b_{14}	157.80	$\boldsymbol{b_{26}}$	**0**
x_7	2630	b_1	735.27	b_{15}	12.13	$\boldsymbol{d_{26}}$	**21388.09**
x_8	1710	b_2	5236.55	b_{16}	59.65	$\boldsymbol{b_{27}}$	**0**
x_9	1320	b_3	1951.42	b_{17}	11.52	$\boldsymbol{d_{27}}$	**71097.73**
x_{10}	1710	b_4	29.05	b_{18}	1.58	y_1	0
x_{11}	9210	b_5	58.82	b_{19}	22.84	y_2	1
x_{12}	1970	b_6	30.02	b_{20}	112.41	y_3	0
x_{13}	2370	b_7	90.06	b_{21}	449.56	y_4	1
x_{14}	2243	b_8	85.46	b_{22}	6.15	**z_2* = 98457.5**	

Table 7. Metaoptimal solution (Min B, Z ≥ Z*)

Variables	Optimal values	Variables	Optimal values	Variables	Optimal values	Variables	Optimal values
x_1	1592	x_{15}	22090	b_9	117.82	b_{23}	188.76
x_2	10520	x_{16}	24450	b_{10}	79.33	$\boldsymbol{b_{24}}$	**2000**
x_3	1580	x_{17}	4730	b_{11}	211.46	$\boldsymbol{d_{24}}$	**433.66**
x_4	20126	x_{18}	15780	b_{12}	206.26	$\boldsymbol{b_{25}}$	**1377.64**
x_5	23670	x_{19}	4340	b_{13}	4300	$\boldsymbol{d_{25}}$	**0**
x_6	123080	x_{20}	1970	b_{14}	157.80	$\boldsymbol{b_{26}}$	**0**
x_7	8475	b_1	1688.41	b_{15}	12.99	$\boldsymbol{d_{26}}$	**18392.83**
x_8	6000	b_2	2934.37	b_{16}	61.86	$\boldsymbol{b_{27}}$	**0**
x_9	1320	b_3	1960.43	b_{17}	11.52	$\boldsymbol{d_{27}}$	**75442.03**
x_{10}	1710	b_4	83.11	b_{18}	4.34	y_1	0
x_{11}	23670	b_5	58.82	b_{19}	46.27	y_2	1
x_{12}	5670	b_6	30.02	b_{20}	112.41	y_3	0
x_{13}	2370	b_7	90.06	b_{21}	484.17	y_4	1
x_{14}	6310	b_8	186.37	b_{22}	17.69	***B**** **= 308077**	

Table 8. Optimal solution with optimal path ratio r = B/B* = 0.97378253
(Xj rounded to integer, b_i obtained from the original constraints)

Variables	Optimal values	Variables	Optimal values	Variables	Optimal values	Variables	Optimal values
$\boldsymbol{x_1}$	***1550***	x_{15}	21511	b_9	114.72	b_{23}	183.76
$\boldsymbol{x_2}$	***10244***	x_{16}	23809	b_{10}	77.25	$\boldsymbol{b_{24}}$	**2000**
$\boldsymbol{x_3}$	***1539***	x_{17}	4606	b_{11}	205.91	$\boldsymbol{d_{24}}$	**369.84**
x_4	19598	$\boldsymbol{x_{18}}$	***15366***	b_{12}	200.85	$\boldsymbol{b_{25}}$	**1341.49**
x_5	23049	x_{19}	4226	b_{13}	4187	$\boldsymbol{d_{25}}$	**0**
x_6	119853	$\boldsymbol{x_{20}}$	***1918***	b_{14}	153.66	$\boldsymbol{b_{26}}$	**0**
x_7	8253	b_1	1644.12	b_{15}	12.65	$\boldsymbol{d_{26}}$	**17910.46**
x_8	5843	b_2	2857.39	b_{16}	60.23	$\boldsymbol{b_{27}}$	**0**
$\boldsymbol{x_9}$	***1285***	b_3	1909.02	b_{17}	11.22	$\boldsymbol{d_{27}}$	**73463.90**
$\boldsymbol{x_{10}}$	***1665***	b_4	80.93	b_{18}	4.23		
x_{11}	23049	b_5	57.28	b_{19}	45.06		
x_{12}	5521	b_6	29.24	b_{20}	109.45		
$\boldsymbol{x_{13}}$	***2308***	b_7	87.72	b_{21}	471.48		
x_{14}	6145	b_8	181.48	b_{22}	17.23	***B**** **= 300000**	

Table 9. Pay-of-table for bakery problem

$z_1(x_1^*)$	$z_1(x_2^*)$	**2143888.10**	92119.51
$z_2(x_1^*)$	$z_2(x_2^*)$	1895180.6	**98457.49**

Table 10. Optimal solution with weighted goal programming

Variables	Optimal values	Variables	Optimal values	Variables	Optimal values	Variables	Optimal values
x_1	4080	x_{16}	20380	b_{11}	183.85	$\boldsymbol{b_{25}}$	**1377.64**
x_2	10520	x_{17}	4730	b_{12}	171.76	$\boldsymbol{d_{25}}$	**0**
x_3	1580	x_{18}	27566	b_{13}	3610	$\boldsymbol{b_{26}}$	**0**
x_4	33660	x_{19}	4337	b_{14}	157.80	$\boldsymbol{d_{26}}$	**18255.57**
x_5	19730	x_{20}	1970	b_{15}	12.13	$\boldsymbol{b_{27}}$	**0**
x_6	102570	b_1	735.27	b_{16}	59.65	$\boldsymbol{d_{27}}$	**74543.00**
x_7	7890	b_2	3892.86	b_{17}	11.52	y_1	0
x_8	5000	b_3	1936.16	b_{18}	1.58	y_2	1
x_9	1320	b_4	29.05	b_{19}	22.84	y_3	0
x_{10}	1710	b_5	58.82	b_{20}	112.41	y_4	1
x_{11}	19730	b_6	30.02	b_{21}	492.51	$\boldsymbol{n_1}$	**77073.20**
x_{12}	4730	b_7	90.06	b_{22}	17.69	$\boldsymbol{n_2}$	**1030.95**
x_{13}	2370	b_8	164.01	b_{23}	188.76		
x_{14}	5260	b_9	117.82	$\boldsymbol{b_{24}}$	**2000**		
x_{15}	18410	b_{10}	64.02	$\boldsymbol{d_{24}}$	**414.01**		
	z_1 (WGP) = 2066814.9			**z_2 (WGP) = 97426.54**			

Table 11. Optimal solution with goal programming – the Min-max approach (Min D)

Variables	Optimal values	Variables	Optimal values	Variables	Optimal values	Variables	Optimal values
x_1	1580	x_{16}	24450	b_{11}	185.74	$\boldsymbol{b_{25}}$	**1377.64**
x_2	10520	x_{17}	2344	b_{12}	176.44	$\boldsymbol{d_{25}}$	**0**
x_3	1580	x_{18}	15780	b_{13}	3666	$\boldsymbol{b_{26}}$	**0**
x_4	22418	x_{19}	1582	b_{14}	167.80	$\boldsymbol{d_{26}}$	**18094.20**
x_5	23670	x_{20}	1970	b_{15}	12.25	$\boldsymbol{b_{27}}$	**0**
x_6	123080	b_1	947.62	b_{16}	59.65	$\boldsymbol{d_{27}}$	**74567.01**
x_7	2630	b_2	3268.54	b_{17}	11.52	y_1	0
x_8	6000	b_3	1929.63	b_{18}	1.58	y_2	1
x_9	1320	b_4	59.68	b_{19}	22.84	y_3	0
x_{10}	1710	b_5	58.82	b_{20}	112.41	y_4	1
x_{11}	23670	b_6	30.02	b_{21}	457.24	n_1	62010.89
x_{12}	5670	b_7	80.06	b_{22}	17.69	n_2	1580.27
x_{13}	2370	b_8	165.18	b_{23}	188.76		
x_{14}	6310	b_9	117.82	$\boldsymbol{b_{24}}$	**2000**		
x_{15}	22090	b_{10}	79.33	$\boldsymbol{d_{24}}$	**402.19**	$\boldsymbol{D^*}$	**368.69**
	z_1 (WGP) = 2081877.21			**z_2 (WGP) = 96877.21**			

Table 12. Optimal solution with the Global criterion method

Variables	Optimal values	Variables	Optimal values	Variables	Optimal values	Variables	Optimal values
x_1	1580	x_{15}	22090	b_9	117.82	b_{23}	188.76
x_2	10520	x_{16}	24450	b_{10}	79.33	$\boldsymbol{b_{24}}$	**2000**
x_3	1580	x_{17}	1871	b_{11}	183.86	$\boldsymbol{d_{24}}$	**403.39**
x_4	22575	x_{18}	15780	b_{12}	171.77	$\boldsymbol{b_{25}}$	**1377.64**
x_5	23670	x_{19}	1580	b_{13}	3610	$\boldsymbol{d_{25}}$	**0**
x_6	123080	x_{20}	1970	b_{14}	157.80	$\boldsymbol{b_{26}}$	**0**
x_7	2633	b_1	948.00	b_{15}	12.13	$\boldsymbol{d_{26}}$	**18148.11**
x_8	6000	b_2	3291.44	b_{16}	59.65	$\boldsymbol{b_{27}}$	**0**
x_9	1320	b_3	1930.68	b_{17}	11.52	$\boldsymbol{d_{27}}$	**74543.63**
x_{10}	1710	b_4	59.69	b_{18}	1.58	y_1	0
x_{11}	23670	b_5	58.82	b_{19}	22.86	y_2	1
x_{12}	5670	b_6	30.02	b_{20}	112.41	y_3	0
x_{13}	2370	b_7	90.06	b_{21}	487.24	y_4	1
x_{14}	6310	b_8	164.01	b_{22}	17.69	$\boldsymbol{z^* = 0.045929}$	
$\boldsymbol{z_1}$ **(Global) = 2080933.08**				$\boldsymbol{z_2}$ **(Global) = 96931.03**			

Table 13. Comparisons of the results

	$\boldsymbol{z_1}$	$\boldsymbol{z_2}$
Ideal values	**2143888.1**	**98457.49**
WGP	2066814.9	97426.54
% from ideal values	0.964049803	0.989529
Minimax (Chebishev)	2081877.21	96877.21
% from ideal values	0.971075501	0.9839496
Global criterion method	2080933.08	96931.03
% from ideal values	0.97063512	0.984496

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