Volatility Forecasting of Crude Oil, Gold, and Silver Futures: A Case of Pakistan Mercantile Exchange

SHAMSUL NAHAR ABDULLAH[1], IQRA KHAN[2], FARAH NAZ[2], KANWAL ZAHRA[3], TOOBA LUTFULLAH[2]

[1]Faculty of Business and Communication,
INTI International University,
Nilai,
MALAYSIA

[2]Department of Accounting and Finance,
Kinnaird College for Women,
Lahore,
PAKISTAN

[3]Faculty of Management Sciences,
University of Central Punjab,
Lahore,
PAKISTAN

Abstract: - The volatility of commodity prices has been a topic of interest for researchers and investors for decades. In recent years, the prices of key commodities have shown significant fluctuations, causing challenges for market participants to make informed investment decisions. Therefore, this paper provides an understanding of forecasting and modeling the volatility of commodity futures in the Pakistan Mercantile Exchange (PMEX) using GARCH and ARIMA models. The study aims to analyze and predict the volatility of three key commodities, namely Gold, Silver, and Crude Oil, and to compare the performance of the two models in forecasting their future prices. The study uses daily time-series data from 2010 to 2021 and finds that the prices of Gold and Crude Oil futures exhibit asymmetrical effects on their volatilities, while silver futures show stability over time. The results are useful for potential investors, economic agents, managers, financial researchers, and policymakers to analyze the volatility of commodity futures in the market. This will also help the investors to diversify their investments by analyzing the variation in such commodities in the international markets.

Key-Words: - ARIMA, GARCH, PMEX, GOLD futures, crude oil futures, volatility

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1 Introduction

Forecasting volatility is critical for financial analysts to examine the significant impact on the global economy and individual economies, [1]. It enables the analysis of various macroeconomic variables that depend on volatility factors. Financial markets trade in commodities that are valuable assets for investors, such as futures contracts. Therefore, derivative markets are highly volatile, which increases their risk. In these markets, forecasting is crucial, especially for highly volatile commodities like gold, silver, and crude oil. This price fluctuation can cause volatility spillover where the volatility of different commodities is interconnected, [2]. As a result, market participants must be well-informed about future uncertainties to make effective investment decisions. Financial researchers and investors have emphasized in-depth analysis of forecasting techniques in recent years to avoid the negative repercussions associated with future price uncertainty, [3]. Accurate forecasting can predict future uncertainties and prevent risky investments.

Forecasting the volatilities of various commodities plays a substantial role in the current era, where researchers must choose an efficient model to forecast data and identify any uncertainties related to it, [4]. Therefore, efficient analysis can be made using certain forecasting techniques like GARCH and ARIMA, which are widely used to predict macroeconomic variables, [5]. Several studies have used these models to predict volatilities and have compared them. However, the GARCH model is more accurate in the long run. In [6], the authors studied the comparison between the GARCH and ARIMA models for forecasting gold prices in Malaysia. The results found that the GARCH model was a more efficient model for forecasting prices than the ARIMA model. Crude oil is at the top of the traded items list in the world which has approximately 10% in the global trade volume due to the rise in its volatility prices over the past 30-40 years, [7]. Crude oil covers two-thirds of the world's energy demand, and its trade is conducted through various contracts, including spot prices and futures, [4].

Gold is a highly significant commodity in international trade. In 2008, following the recession, the price of gold experienced a 6% increase internationally, while the prices of other minerals decreased, and a 40% decline was recorded in the equity market. The study, [8], concluded that the behavior of gold prices does not correlate with the variations in other markets, and it behaves differently compared to other markets. Therefore, to analyze the volatility of such commodities, this study examined the Pakistan Mercantile Exchange (PMEX). In 2007, PMEX was established as the National Commodities Exchange of Pakistan and has since been extended to three digits. PMEX solely has all rights in dealing with the future markets of key commodities and is operated under the Securities and Exchange Commission of Pakistan (SECP), [9].

PMEX is a highly advanced and well-structured institution that has implemented efficient techniques to meet the demands of modern exchanges. Its robust infrastructure enables PMEX to offer a wide range of services, including commodity trading, trusteeship, and clearing. With a membership of 326, PMEX handles an average daily volume of 5 billion PKR. The international membership of PMEX includes prestigious organizations such as Dubai's Gold and Commodities Exchange (DGCE), with whom they have signed Memorandums of Understanding (MoUs), [10], along with partnerships with the Iran Mercantile Exchange, Association of Futures Market (AFM), Borsa Istanbul, Futures Industry Association (FIA), Izmir Commodities Exchange (ICE), Hungary, and USA.

Initially, PMEX primarily focused on gold futures trading. However, in 2009, the Exchange introduced crude oil as a futures contract, [11], which has now become a prominent commodity traded on the platform. Alongside crude oil, silver and gold are considered the major products traded on the Mercantile Exchange of Pakistan.

This study makes a significant contribution in two aspects. First, it compares the models that can help in forecasting the future of commodities, and among them, the GARCH and ARIMA models are considered on top in predicting the futures of crude oil, silver, and gold. While previous literature has applied various statistical methods to analyze gold prices, this research focuses on utilizing the GARCH and ARIMA models to estimate and analyze the most recent future price results using current data. Second, the study also targets to capture the recent impact of the COVID-19 pandemic and major political or economic shocks on the volatility of the selected commodities by identifying a significant time-varying jump in the data. Previous research has also attempted to capture the effect of such jump behaviors in studying and forecasting volatility, [12], [13]. The study will add great credibility to the commodity market for investors, policymakers, government, and financial researchers in Pakistan. While many studies focus on forecasting a single commodity, the contribution of this study is to include three dominant commodities and perform simultaneous forecasting of these three dominant commodities traded on the Mercantile Exchange of Pakistan: crude oil, gold, and silver. The forecasting for these three major commodities is performed through the two famous models of volatility prediction i.e., ARIMA and GARCH models.

The structure of this paper is as follows. Section 2 reviews the prominent research and literature support that emphasized the forecasting and modeling of futures price volatility of the commodity market. Section 3 includes the description of the selected method used for the analysis of this research. While Section 4 illustrates the gold, crude oil, and silver futures prices time series from the PMEX database and presents its results produced by forecasting models GARCH and ARIMA. The conclusion of this research is drawn by section 5.

2 Literature Review

There is a vast amount of existing literature that covers the issue of volatility in the futures of key commodities in the financial assets that are being traded at stock exchanges and mercantile exchanges. Accurate forecasting related to volatility can not only prevent negative returns but can also provide an opportunity for the participants of the financial markets to gain huge positive returns as well. This study made a significant contribution in the finance domain because accurate forecasting would be of tremendous help to investors to manage their risk, select profitable portfolios, price the derivatives precisely, devise viable economic policies and formulate hedging techniques. Economic stability is significantly affected because of the existence of volatility in the prices of crude oil, [14]. The macroeconomic aspects can impact the future volatility in the stock exchange markets of the country leading to infer long-term effects, [15]. The study, [16], identified various issues related to the uncertain behaviors of crude oil prices and concluded that these uncertainties caused the economic recession of 1980 and 1982. Similarly, [17], [18], [19], [20], studied that crude oil prices can cast a significant influence, internationally, on different macroeconomic factors, for example, inflation, GDP, stock market performance, and exchange rate. However, [21], analyzed the relationship among silver, gold, oil, and energy sectors which depicts the positive dependence of energy uncertainty and crude oil prices in the medium and long run. Looking at the significance of forecasting in the financial world, vast literature exists on the topic. The studies aim to determine the most appropriate technique for forecasting volatility in asset prices. Some of the significant literature regarding the techniques of forecasting used in the financial world have been discussed as follows: [22], applied the GARCH family models on the electricity prices to capture volatility in the deregulated market of California. They ultimately concluded that GARCH performs comparatively better than ARIMA. The study, [23], referred to the involvement of the hedging concept to analyze the volatility of gold through the GARCH model and its impact to provide a better view to the investors. The volatility in the gold market may expose the risks for the currency of a country, however, it can protect the investor against the currency risks, [24].

Various studies have forecasted the volatility persistence in the equity market and foreign exchange market. However, very few studies have investigated the persistence in the prices of oil in international markets. The term persistence is often specifically referred to the GARCH models, where conditional variances shock increased to excessively high rates which are comparatively lower than exponential rates caused by decays in the shocks, [25], [26], [27]. Whereas, [2] found that the volatility models CGARCH and FIGARCH are more efficient in capturing persistence in the oil markets of Texas, Brut, and Dubai as compared to GARCH and IGARCH. The study, [28], elaborated on the research study of, [2], but the results of both studies varied considerably. Unlike previous findings, this study found that not a single model outperformed the loss function, but in general, results showed that the nonlinear model of GARCH is a comparatively more accurate predictor than that of the linear model. The spillover effect of gold may affect the performance which affects the analysis of the institutional investors to gain more returns however, the performance can be identified using the methodologies such as the GARCH model that provides more accurate predictor results, [29], [30]. The study, [31], analyzed the Chinese investors' sentiments and their impact on the volatility forecast of the prices of crude oil, gold, and silver. The study, [32], approached the time domain spillover method to investigate the connection between the crude oil prices and stock indexes with the metal prices where the results provide evidence that the stock index is not a contributor towards price spillover.

The study, [33], used crude oil data from China and analyzed the role of jump and leverage in the prediction of realized volatility (RV), the results revealed the useful effect of leverage and the significant effect of a jump in predicting the long-term information thereby leverage has the best predictive power for the oil futures. The study, [34], investigated the efficiency of various forecasting models that includes GARCH, EGARCH, APARCH, FIGARCH, and APARCH to forecast the volatility of oil prices in eleven international markets over a span of twelve years. APARCH performed better in comparison to the other models in line with it. Furthermore, the study stated that conditional standard deviations predict prices more accurately than those conditional variances. The study, [35], conducted a study by making a comparison between the models of GARCH & ARIMA for forecasting the gold prices of Malaysia. The study concluded that the GARCH model is a comparatively more efficient model to forecast prices than the ARIMA model. Few of the studies figured out that the combination extensions of ARIMA models and GARCH models, particularly the FIGARCH and ARFIMA models, are identified as accurate estimators for accessing the influence of market instability on the volatility and returns of various commodities, [36], [37], [38], [39], [40]. Whereas, [10], modeled and forecasted the market risks and the conditional volatility of highly volatile commodities such as natural gas, silver, crude oil, and gold by the GARCH family models. The results revealed that the non-linear models of GARCH can more efficiently forecast the volatility of all these commodities compared to the other available methods. FIPARCH, an extension of the GARCH model, is identified as more accurate for predicting the volatilities in both the long and short run. Oil is one of the commodities that are most important in international trade therefore accessing its volatility is always a top priority task of investors, [1], [41].

Getting accurate estimates regarding the volatility of oil prices can not only help policymakers to hedge their foreign exchange risk, rather it can add tremendous economic stability, [42]. A wide array of studies has been conducted in this field. Like, [43], studied the influence of uncertainty in oil prices on the manufacturing industry of the South African region by using the model of GARCH. The study concluded that uncertainty in oil prices could negatively affect the manufacturing sector of South Africa substantially. The study, [44], investigated the significance of gold and crude oil on the stock market index of Pakistan which depicts the insignificant relation of oil towards the index as compared to gold which showed significant relation to the market. The study, [37], demonstrated the phenomenon that the reduction in the crude oil price of 1985 did not lead to a significant increase in the output of the United States due to the high level of uncertainty associated with the crude oil price.

The study, [45], investigated that the market participants and policymakers also have an important role to reach the correct forecast in the stock market of the United States where the volatility can be analyzed effectively by using the information of futures of key commodities. On the contrary, [46], investigated that forecasts are not improved by volatilities that are linked with the commodities and thus no volatility spillovers can be detected using the heterogeneous autoregressive (HAR) models. The study conducted on the derivatives of oil prices is done by, [4]. The study aimed to analyse and forecast the volatility in the spot prices and future prices of crude oil being generated by any political or economic changes in Pakistan. The research used the GARCH family models for the evaluation of volatility in the prices of crude oil. The study concluded that oil price derivatives tend to remain volatile in the long run. The prices would increase the shocks of a negative nature and would decrease the shocks of a positive nature. Hence, the study concluded that there is an impact of economic and political changes on crude oil volatility.

Gold plays a key function in the international financial sector and has long been utilized as a measure of wealth and a method of income. Investors usually use gold as a method to buffer inflation in their portfolios due to its diversified portfolio, [28]. Previous research has extensively studied the modeling of price volatility of key commodities, using various frameworks and perspectives, yielding valuable insights. However, the focus of these studies has been limited to markets other than Pakistan, leaving a gap in the literature regarding the analysis of Pakistan's market. Additionally, while some studies have examined the price volatility of individual commodities, such as gold or oil, there is a dearth of research that considers the price volatility of all three commodities, including silver, to predict their futures in Pakistan. This study aims to bridge the existing gap by examining the price volatility of gold, silver, and crude oil and predicting their futures using appropriate modeling techniques in the context of Pakistan's market and providing a comparative analysis of ARIMA and GARCH models for forecasting.

3 Data and Methodology

The primary objective of this research is to analyze and forecast the volatility in the prices of three significant commodities: gold, silver, and crude oil. To achieve this objective, price data for these commodities spanning from January 2010 to January 2021 has been collected from the derivatives market of Pakistan. The monthly data has been sourced from the official database of the Pakistan Mercantile Exchange (PMEX), which is known as a futures exchange in Pakistan. This data not only allows for the examination of volatility in commodity prices but also facilitates an evaluation of the impact of the COVID-19 pandemic on these prices.

Given the high volatility and fluctuations in gold prices over time, both the heteroscedasticity approach of ARIMA and the GARCH models are considered suitable for testing the entire dataset. The ARIMA model, a type of linear model, is capable of representing both stationary and non-stationary time series data, making it useful for prediction purposes. On the other hand, the GARCH model specifically tests for volatility in gold prices, capturing the clusters present in the data resulting from pronounced fluctuations over time. Therefore, both models will be compared to assess their performance during the period from January 2010 to January 2021.

Furthermore, this study aims to examine the impact of the coronavirus pandemic on the prices of crude oil, gold, and silver. To ensure the validity of the time series models employed, the stationarity of the selected variables is checked using the ADF unit root test. Stationarity is a crucial assumption for most time series models, including ARIMA and GARCH models. If the data is found to be non-stationary, these models cannot be effectively applied. Thus, it is imperative to assess the stationarity of the data before proceeding with the modeling phase.

3.1 The Box-Jenkins (ARIMA) Methodology

The ARIMA model is used for this research, which is being introduced by Box and Jenkins, [47]. Thus, the model is represented with the Box-Jenkins methodology because it provides the efficient and effective analysis and testing of forecasting techniques for a univariate time period or the type of data which involves the univariate series as it was in our study, [48]. The major aim of the research was to depict whether this model is suitable to effectively test the volatility issues in commodities such as gold, silver, and oil therefore the ARIMA model is tested which is the generalized form of the Box-Jenkins methodology, [49], [50]. The model tested with its referred form as ARIMA(p,d,q) where the ‘p’ shows the autoregressive order, ‘d’ required the integration, and ‘q’ means the moving average parts involved in the model. This referred to the form which helps in testing the results for the time-series data because if any term is missing or zero it will automatically remove from the output results. For example, ARIMA(0,0,1) results when the MA(1) model is in the data, and on the other hand ARIMA(0,1,0) comes when the I(1) model is in the data.

To predict the natural phenomena of different types to utilize the modeling of the random process an autoregressive (AR) model is better suitable where the AR(p) denoted the order ‘p’ of the autoregressive model. Hence, the AR (p) can be defined using the equation:

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} \dots \dots + \alpha_p Y_{t-p} + \varepsilon_t$$

The analysis of univariate time series models can be analyzed with the moving average (MA) of the ARIMA model with the order of ‘q’ and the notation for this is referred to as the moving average model MA (q):

$$Y_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} \dots \dots + \beta_q \varepsilon_{t-q}$$

The ARIMA model with the order (p, q) is defined as:

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} \dots \dots + \alpha_p Y_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} \dots \dots + \beta_q \varepsilon_{t-q}$$

The Box-Jenkin methodology helps in depicting the generalized form of the ARIMA model where the order of p, d, and q provides the non-negative integers of the ARIMA model. This order refers to the autoregressive, integrated, and moving average parts to approach and analyze the time series modeling with the methodology of Box-Jenkins respectively.

$$\Delta^d Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} \dots \dots + \alpha_p Y_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} \dots \dots + \beta_q \varepsilon_{t-q}$$

The methodology of Box-Jenkins is an important technique as compared to other methodologies and helps in analyzing the performance of variables selected for the forecasting model as compared to the performance of the same variables in the past to check the generalizability of the models under each class, [51]. However, the strategies can be applied which are mainly depicted with the four-step strategy of identifying, estimating, and diagnosis for checking and forecasting. Thus, the patterns of the time series can b represented by the categories mentioned:

- Autoregressive models (AR): It is a linear function to provide the basis for the past values of variables.
- Moving Average model (MA): It is the linear combination that provides the basis for past errors.

- Autoregressive-Moving Average models (ARMA): It is the combination of categories mentioned in the above two points of AR and MA, [52].

3.2 ARCH Model

The volatility clustering can be observed in the time series data of financials. However, the big shocks in these volatile clusters might follow the other big shocks and the same goes for the small shocks that are mainly known as the residuals. To effectively assess the variances in these types of models, it is necessary to consider the historical patterns and model them accordingly. For this purpose, the previous literature can help such as the autoregressive conditional heteroscedastic (ARCH) model proposed by, [53], to pattern such volatility in the market. This model is dependent on the squared errors with the error term of 't' to analyze the variances with the specification given below for ARCH (1) as:

$$\sigma^2 = \varpi + a\varepsilon_{t-1}^2 \quad \varpi \geq 0, a \geq 0$$

The big shocks in the time 't' can be depicted with the model of ARCH (1) to analyze the absolute large values when the variances of such values are also large.

To check the order 'p' in the ARCH (1) model, it can be represented as:

$$\sigma^2 = \varpi + \beta_1\varepsilon_{t-1}^2 + \beta_2\varepsilon_{t-2}^2 + \cdots \beta_p\varepsilon_{t-p}^2$$
$$\varpi \geq 0, \beta_j \geq 0 j = 1,2, \ldots., p$$

Thus, the shocks in period 'p' can be analyzed through the ARCH model because the shocks that are older than period 'p' may not affect the volatility of the current shocks in the market because the ARCH model adjusts them in the results, [53].

3.3 GARCH Model

The GARCH model introduced by [54], commonly named Generalized Autoregressive conditional Heteroscedastic provides the extension to the ARCH model discussed previously. This model helps in analyzing the lagged squared error terms with its lagged terms because error variances are regressed under the processing of lagged squared error terms. Thus, the equations mentioned provide the GARCH model with the order 'p' and 'q' where the first equation shows the conditional mean equation, and the second equation shows the conditional variance equation:

$$Y_t = \mu + \phi Y_{t-1} + \varepsilon_t$$

$$V\left[{}^{\varepsilon_t}/_{\varepsilon_{t-1}}\right] = h_t = \varpi_0 + \sum_{i=1}^{0} a_1\, \varepsilon_{t-1}^2 + \sum_{j=1}^{i} \beta_1\, h_{i-j}$$
$$\varpi_0 > 0, a_1 \geq 0, \beta_1 \geq 0 \qquad i = 1,2, \ldots., pj = 1,2, \ldots., p$$

With the above equations, the GARCH(p, q) can be analysed as:

$$Y_t = \mu + \phi Y_{t-1} + \varepsilon_t$$
$$V\left[{}^{\varepsilon_t}/_{\varepsilon_{t-1}}\right] = h_t = \varpi_0 + a\varepsilon_{t-1}^2 + \beta h_{t-1}$$
$$\varpi_0 > 0, a_1 \geq 0, \beta_1 \geq 0$$

The GARCH model is named Vanilla in the literature because it is commonly used for the financial time series.

Most of the prominent research on forecasting used the GARCH and ARIMA models, [4], [22], [28], [55], [56]. An econometric model GARCH was introduced by, [54], and is now widely utilized by many researchers to forecast the prices of different financial instruments. For estimating GARCH, the initial step is the estimation of the best fit and appropriate autoregressive model. A further step is the estimation of autocorrelation for error terms. The last step in GARCH modeling is analyzing the significance level. ARIMA is an econometric term introduced by Box Jenkins, that's why this model is often called the Box-Jenkin Method. This method is used for predicting the time series data where the data shows non-stationarity. So, In ARIMA, the initial step is to make different series. By breaking down the term ARIMA, the AR term is referred to as an Auto-regressive term, I is referred to as the differencing term and MA indicates the number of moving averages, [57].

4 Empirical Results

This study employs GARCH and ARIMA models to facilitate the forecasting process, specifically to predict future prices of gold, silver, and crude oil. A comparative analysis is conducted among the selected models to determine the most suitable approach for accurate price prediction.

Before commencing the modeling phase, the volatility patterns within the datasets are visually represented through graphical representations. Additionally, the stationarity of the datasets is examined to ensure reliable forecasting results.

To forecast the futures prices of gold, silver, and crude oil, a comprehensive comparison is performed between the GARCH and ARIMA models. The objective is to identify the best-fit model that exhibits optimal performance in forecasting the volatility associated with the futures prices of these commodities.

The initial step in the analysis involves assessing the stationarity of the data through unit root tests. The p-values for all three datasets are found to be less than 0.05, indicating that the datasets exhibit stationarity at the level. Table 1 presents the results of the unit root tests conducted for the silver, gold, and crude oil futures datasets, confirming their stationarity.

Table 1. Unit Root Test ADF

Variable	**Unit root Test**	**P value**	**Decision**
Crude oil	On level	0.0314<0.05	Data is Stationary
Gold	On level	0.0023<0.05	Data is Stationary
Silver	On level	0.00433<0.05	Data is Stationary

To proceed with GARCH modeling, the first step is to examine the presence of ARCH effects by conducting a heteroscedasticity test. This test is performed on all three commodities, and the results indicate that the p-values for all cases are below 0.05, suggesting that the data exhibits homogeneity. Table 2 displays these findings.

Next, the analysis checks for serial correlation by applying the Breusch-Godfrey LM test to the crude oil, gold, and silver futures datasets. The results reveal that the p-value is 0.00 for all three cases, indicating the presence of serial correlation.

Fig. 1(a): Gold price series

Fig. 1(b): Silver price series

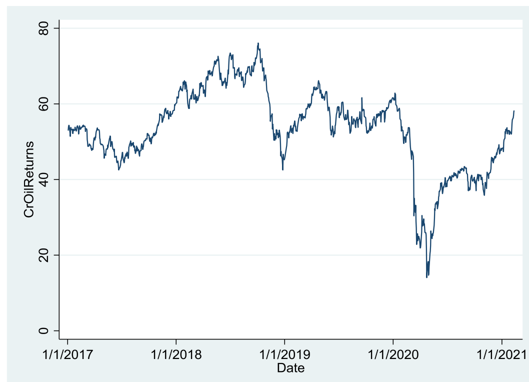

Fig. 1(c): Crude oil price series

The LM values (Obs*R-squared) are 20.124, 52.431, and 47.986, respectively, suggesting that past values positively influence future values. Table 3 provides a summary of these results.

Furthermore, one advantage of utilizing GARCH models over ARIMA models is that GARCH can capture non-linear data patterns, while ARIMA is more suitable for linear data.

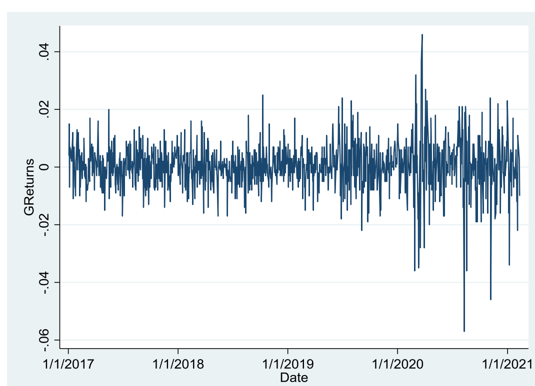

Fig. 1(d): Gold return series

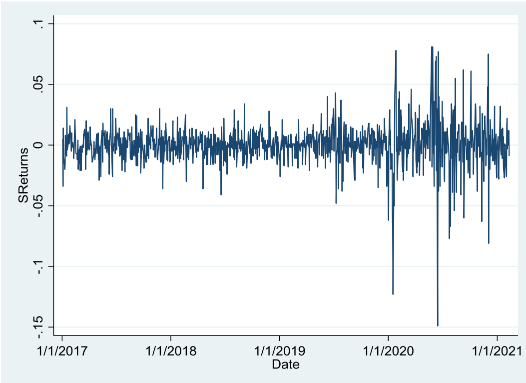

Fig. 1(e): Gold return series

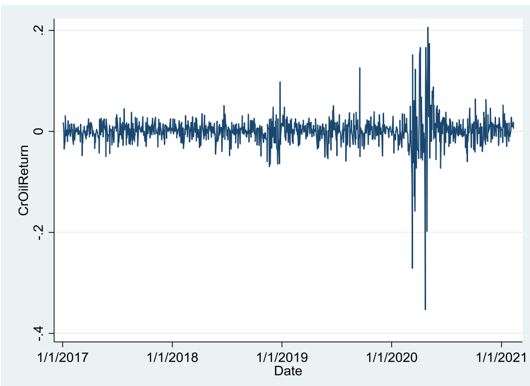

Fig. 1(f): Crude oil return series

Fig. 1: Graphical Representation of Authors' Calculations showing Gold, Silver, and Crude Oil Prices and Return Series

Figure 1(a) to Figure 1(f) illustrates the price and line chart series of commodities, highlighting notable fluctuations, particularly during the COVID-19 period in 2020. Specifically, the trend of crude oil prices exhibits a downward trajectory during this time, largely influenced by the impact of the pandemic. The demand side analysis of the commodity indicates that containment measures and economic disruptions stemming from the pandemic resulted in reduced production and mobility, consequently leading to a substantial decline in global oil demand, [58]. Furthermore, the figure provides a clear representation of the daily log return series of commodities futures, revealing significant clusters that attest to the occurrence of volatility throughout the year 2020.

Table 2. ARCH and LM Test Statistics

Commodity	ARCH		Breusch-Godfrey LM Test	
Crude Oil			Obs*R Squared	20.124
	Prob. F(1,1099)	0.0062	Prob. F(2,1098)	0.000
	Prob. Chi-Square(1)	0.0049	Prob. Chi-Square(2)	0.0000
Gold			Obs*R Squared	52.431
	Prob. F(1,1099)	0.0042	Prob. F(2,1098)	0.0000
	Prob. Chi-Square(1)	0.0003	Prob. Chi-Square(2)	0.0000
Silver			Obs*R Squared	47.986
	Prob. F(1,1060)	0.0000	Prob. F(2,1059)	0.0000
	Prob. Chi-Square(1)	0.0000	Prob. Chi-Square(2)	0.0000

The uncorrelated time series exhibit serial dependence as a result of the dynamic conditional variance process. The Autoregressive Conditional Heteroscedastic (ARCH) effects manifest in the time series, introducing heteroscedasticity or autocorrelation in the squared series. When assessing the significance of these ARCH effects, Engle's ARCH test is more suitable than the Lagrange Multiplier Test.

Upon analyzing the results, it is evident that the ARCH LM test provides compelling evidence for rejecting the null hypothesis concerning the series of commodity futures returns. The results demonstrate significance at the 1% level, leading to the rejection of the null hypothesis that suggests the absence of ARCH effects. Consequently, this rejection signifies the presence of ARCH effects within the mean equation of the residual series.

To further measure the forecasting ability, this paper estimates the forecasts that are within the sample. This sample forecasting shows the predictability power of the model. If there is a small difference in the magnitude of actual and forecasted value, then this results in good forecasting power. However, for this case, the GARCH (1,1) shows good results which can be evident from Table 3.

Based on the analysis, the GARCH model has been identified as the most suitable model for Crude oil futures. This determination is based on several factors. Firstly, the Akaike, Hannan, and Schwarz values of the GARCH model are lower than the corresponding values of the ARIMA model. Additionally, the Durbin Watson statistic for the GARCH model is closer to two, indicating a better fit compared to the ARIMA model.

Table 3. Results of ARIMA and GARCH

Model	ARIMA			GARCH	
CrudeOil		Coefficient	P value	Coef.	P value
	AR(1)	-0.449	0.00	1.66	0.00
	MA(3)	-0.334	0.00	0.13	0.13
Gold	AR (4)	0.158	0.02	0.68	0.01
	MA(1)	0.542	0.00	-0.04	0.17
Silver	AR(2)	0.552	0.00	0.87	0.00
	MA(2)	-1.000	0.99	0.38	0.00

Furthermore, the estimated results of the GARCH model provide additional evidence supporting its suitability. The p-value of 0.04, which is less than the significance level of 0.05, indicates the overall goodness of fit for the study. With a sample size of 84 observations, the GARCH(1,1) coefficient of 0.386 is highly significant, as evidenced by its p-value of 0.000.

Conversely, the estimated results of the ARIMA model suggest that it is the best-fit model for Crude oil futures. This conclusion is based on the lower values of Akaike, Hannan, and Schwarz criteria compared to the GARCH model. Additionally, the Durbin Watson statistic for the ARIMA model is closer to two and smaller than the corresponding value for the GARCH model. These findings are summarized in Table 4, which provides a comprehensive comparison of the GARCH and ARIMA models.

Table 4. Selected Model based on Minimum Criterion

Model	ARIMA	GARCH(1,1)
Crude Oil Futures	(1,0,3)	
Akaike info criterion	48.52934	48.97884
Schwarz criterion	49.06509	49.09460
Hannan-Quinn criter.	49.05757	49.22538
Durbin-Watson stat	2.107575	2.962236
Gold Futures	(4,0,1)	
Akaike info criterion	48.42438	48.44187
Schwarz criterion	48.54014	48.55762
Hannan-Quinn criter.	48.47092	48.48840
Durbin-Watson stat	1.941293	2.757628
Silver Futures	(2,0,2)	
Akaike info criterion	46.37752	44.99191
Schwarz criterion	46.49328	45.10766
Hannan-Quinn criter.	46.42405	45.03844
Durbin-Watson stat	2.060311	1.974451

After estimating the best-fit model, the next step is to compare the efficiency of both models GARCH and ARIMA fact that which model suits which commodity, and this is done based on some important statistics which are Hannan, Schwarz, Akaike, and Durbin Watson. For this, we needed to run the GARCH model on all three commodities. So, after this next step is the estimation of the GARCH model on these data sets. GARCH (1,1) model is applied to conduct both the in-sample and out-of-sample forecasting for Gold, Silver, and Crude oil futures. After the application of the ARIMA and GARCH models, a further step is to conduct the comparison between both model sets based on the above-mentioned facts. The next step is to predict the best-fit model of ARIMA for each commodity. Different models were generated to find out an accurate one. The model is selected based on some statics of the ARIMA model. The model with the maximum R-square, Adjusted R-square, Likelihood values, and Minimum Hannan, Schwarz, and Akaike values and the Durbin Watson should be around two, and the p-value less than 0.05 is selected for each commodity. Detailed modeling was done to predict the best-fit model for crude oil, gold, and silver futures. The model is selected based on some statics of the ARIMA model. The model with the maximum R-square, Adjusted R-square, Likelihood values, and Minimum Hannan, Schwarz, and Akaike values and the Durbin Watson should be around two and the p-value less than 0.05 is selected for each commodity. Detailed modeling was done to predict the best-fit model for crude oil, gold, and silver futures. The following table represents the best-fit models of ARIMA for all these commodities. The ARIMA model is applied to the data of crude oil futures and consists of 84 observations. The coefficients of AR(1) are -0.449 and MA(3) is -0.334. Both statics are identified as highly significant because their p values are 0.000 and 0.0006 respectively, which are lower than 0.05. Based on the results shown in Table 4, ARIMA is identified as the best-fit model for gold futures because Akaike, Hannan, and Schwarz values are less than the predicted values in the GARCH model. And the value of Durbin Watson is closer to 2 as well and less than the value derived by GARCH. By analyzing the estimated results of the ARIMA model for gold, results depict that the p-value is 0.0257 which is less than 0.05 which concludes that this model is an overall good fit for the study and for efficiently analyzing the variables consisting of 84 observations. The coefficients of AR (4) are 0.1588 and MA(1) are 0.542. Both statistics are identified as highly significant because their p values are 0.02 and 0.000 respectively, which are lower than 0.05. A comparison of the ARIMA and GARCH models for forecasting volatility in crude oil, gold, and silver futures shows that both models can provide accurate forecasts, but their performance may depend on the specific market under consideration. While the ARIMA model is suitable for modeling short-term volatility, the GARCH model is more effective in capturing the long-term persistence of volatility. Moreover, the GARCH model can capture the asymmetric response of volatility to shocks, which is prevalent in futures markets.

5 Conclusion

The study tried to recommend a suitable model for the prediction of the volatility of futures prices of Crude oil, Gold, and silver from PMEX to provide insights to the investors. For forecasting, GARCH and ARIMA Models are used to conduct the comparison among them and to recommend the best model for the prediction of the futures of three key commodities of PMEX. The results by taking the daily data from 2010-2021 revealed a mixed recommendation about the suitable model and identified ARIMA as the most beneficial model for the prediction of futures of crude oil and gold while for the prediction of silver futures, GARCH(1,1) is recommended as the most suitable model. Based on the trend line, the results of this research also revealed an asymmetrical effect on the volatility of futures of crude oil and gold. However, for silver, the trend line shows persistence over time. The study recommends that gold and crude oil futures prices are more volatile and unstable as compared to the silver futures price in Pakistan. The time jump like political or economic shift affects the volatility of the futures price of these commodities positively or negatively depending on the nature of the shocks. The results are useful for the economic agents, managers, and policymakers that have an interest in commodities trading, and its volatility affects their potential to invest. Thus, the international markets that trade in such commodities may analyze the dynamics of fluctuation in the commodity futures to build efficient risk hedging models by implementing the appropriate policies to suppress the financial stress in the markets. The results will also help in analyzing the intensity of the variation of commodities that will provide them to diversify their investments with the strategies and identify the information across the commodity future markets. Future research may include the macroeconomic effect of variables like interest rate, international reserves, trade flows, and openness on volatility in developing countries like Pakistan. Similarly, researchers can consider the spillover effect, leverage effect, and geopolitical risks on oil prices and metals futures volatility. Different methodologies can also be utilised to study the leverage effect and to predict gold, silver, and crude oil futures price volatility.

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