Research and Design of Coupled Magnetic Resonant Power Transfer System

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Abstract: - Coupled Magnetic Resonant Power Transfer (MRCPT) Technology is a kind of Wireless Power Transfer (WPT) technology which is flexible in space and has the advantage of transmission distance. It is suitable for the industrial and civil use in the future. In this paper a model of coupled magnetic resonant power transfer system is established and the features of the system is analyzed and a device based on E-Class amplifier is designed to verify the theoretical analysis. The results of this paper could provide a useful reference to design wireless power transfer system.

Key-Words: - Wireless power transfer, Coupled magnetic resonant, Class-E amplifier, Modeling, Electromagnetics, Mutual inductance, High frequency converter

1 Introduction

In November 2006 [1], Prof.Marin Soljačić and his research team in MIT put forward mid-range wireless power transfer technology based on coupled magnetic resonant and experimentally demonstrated a 60W bulb being lit up over 2m distance in June 2007 [2].

There are two ways of wireless power transfer technology used widely now. Electromagnetic induction technology features a larger transfer power, but due to the loose coupling between the coils, the transmission distance is limited to centimeters level. Electromagnetic induction technology have been used in daily life and can provide a huge power. Seokhwan, Lee provided the optional design for 100kw power with 5cm transmission distance [3] and Seungyong Shin designed a system of 480kw power [4].

On the other hand, the transmission distance of Coupled magnetic resonant technology is much longer which can reach meters level. In 2014, A4WP approved their specification version 1.0 [5].

Recent years RF technology and electromagnetics is used in coupled magnetic resonant power transfer technology. Shih-Hsiung Chang used franklin array antenna to improve transmission distance [6]. Bingnan Wang and his team created meta-materials based on electromagnetics theory to improve efficiency [7]. Yoon Do Chung and his team designed a wireless power system with high temperature superconducting resonance antenna [8].

In addition, there are more and more applications based on coupled magnetic resonant power transfer technology. F. Pellitteri offered an innovative battery charging solution for electric bicycles [9]. Anand Satyamoorthy designed a wireless power receiver that can operate in both low-frequency inductive and high-frequency resonant mode [10].

In this paper, a model of coupled magnetic resonant power transfer system is established and the relationship among output power, efficiency, resonance state, frequency, transmission distance, load resistance of the system is analyzed and a device based on E-Class amplifier is designed to verify the theoretical analysis. The results of this paper could provide a useful reference to design wireless power transfer system.

2 Principal and Model of Coupled Magnetic Resonant Power Transfer System

2.1 Principle of coupled magnetic resonances

Magnetic coupling is a physical phenomenon between the carrying current coils through each other's magnetic field linked to each other. In near-field [2], electromagnetic field energy periodically flows back and forth between the radiation sources internal and around space, and not radiates outward. When two matched objects resonant in the same frequency, there would be a strong coupling and the transfer would be more efficient.

Coupled magnetic resonant power transfer technology is to use magnetic coupling and resonance technology to realize the wireless transmission of power. The theory is based on coupled mode theory [11]. The diagram of coupled magnetic resonant power transfer system is shown in Fig.1.

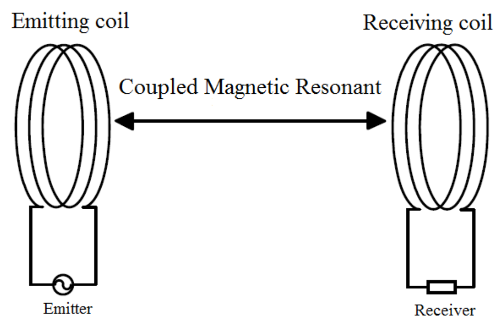

Fig1. Coupled magnetic resonant power transfer system diagram

Circuit theory is used to establish a model of the system. Its equivalent circuit is shown in Fig.2.

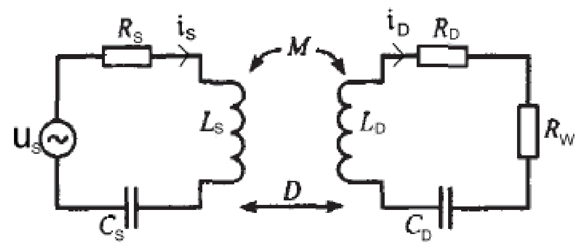

Fig2. Equivalent circuit diagram

As shown in Fig 2, system is divided into two parts: emitter and receiver. The power source of emitter is equivalent to ideal high-frequency source without internal resistance including R_S, L_S, R_D, L_D respectively as the parasitic parameter of emitting and receiving coil loop at high frequency(R_S, R_D is internal resistance of the coils and L_S, L_D is self-inductance of the coils), C_S, C_D respectively as resonant capacitance, R_W as load resistance, D as the distance between the two coils, M as mutual inductance between the emitting and receiving coil loop.

The state equation of equivalent circuit in resonance condition is:

$$\begin{bmatrix}\dot{U} \\ 0\end{bmatrix} = \begin{bmatrix} Z_S & -j\omega M \\ -j\omega M & Z_D\end{bmatrix}\begin{bmatrix} \dot{} \\ \dot{}\end{bmatrix} \tag{1}$$

The reactance of the emitter is:

$$X_S = \omega L_S - \frac{1}{\omega C_S} \tag{2}$$

The emitter is in resonance state, when $X_S = 0$, And it can be conclude that $\omega^2 L_S C_S = 1$.

The reactance of the receiver is

$$X_D = \omega L_D - \frac{1}{\omega C_D} \tag{3}$$

The receiver is in resonance state, when $X_D = 0$. And it can be conclude that $\omega^2 L_D C_D = 1$.

The current in receiver can be deduced by Eq. (4):

Z_D (4)

And the voltage in emitter can be deduced by Eq. (5):

Z_D (5)

Then it could be concluded that the equivalent impedance can be deduced by Eq. (6):

$$Z_{eq} = R_S + jX_S + \frac{(\omega M)^2}{R_D + jX_D + R_W} \tag{6}$$

According to the circuit theory, resistance consumes energy while inductance and capacitance transfer reactive energy. If the input power of circuit maintains a constant, the output power is less and the efficiency is lower with greater reactive power. Reactance will be zero and reactive power will be minimum when circuit is in resonance condition. So the efficiency of coupled magnetic resonant power transfer system will be maximum when emitting and receiving coil loop are in resonance condition.

2.2 The relationship among resonance state and output power and efficiency of coupled magnetic resonant power transfer system.

According to the circuit theory, the expression of input power of the system is:

$$P_{in} = U_S I_S \cos\theta \tag{7}$$

In the Eq. (7), $\cos\theta$ is the power factor of input. Substituting Eq.(5) into Eq.(7):

$$P_{in} = \frac{U_S^2\{R_S[(R_D+R_W)^2 + X_D^2] + (R_D+R_W)(\omega M)^2\}}{[R_S(R_D+R_W) + (\omega M)^2 - X_S X_D]^2 + [R_S X_D + (R_D+R_W)X_S]^2} \tag{8}$$

The output power can be deduced by Eq. (4) and Eq.(5):

$$P_{out} = \frac{U_S^2 R_W (\omega M)^2}{[R_S(R_D+R_W) + (\omega M)^2 - X_S X_D]^2 + [R_S X_D + (R_D+R_W)X_S]^2} \tag{9}$$

From Eq.(9) we can get that output power is related to many factors. And analyzing resonance state of system means analyzing reactance X_S and X_D. Therefore, other factors assumed to be a known value, and Eq.(9) is regarded as a function of two variables about X_S and X_D. As a result, numerator of Eq.(9) $U_S^2 R_W (\omega M)^2$ is a constant now and only denominator need be analyzed.

Let

$$f(X_S, X_D) = [R_S(R_D+R_W) - X_S X_D + (\omega M)^2]^2 + [R_S X_D + (R_D+R_W)X_S]^2 \tag{10}$$

Obviously, $f(X_S, X_D)$ has the first and second order continuous partial derivatives, so $f(X_S, X_D)$ has extremum.

Let partial derivatives of $f(X_S, X_D)$ equal zero:

$$\begin{cases} \dfrac{\partial f(X_S, X_D)}{\partial X_S} = 0 \\ \dfrac{\partial f(X_S, X_D)}{\partial X_D} = 0 \end{cases} \tag{11}$$

Then:

$$\begin{cases} (\omega M)^2 X_D = [(R_D+R_W)^2 + X_D^2] X_S \\ (\omega M)^2 X_S = (R_S^2 + X_S^2) X_D \end{cases} \tag{12}$$

Obviously, $X_S = 0, X_D = 0$ is a solution.

When $X_S \neq 0$ and $X_D \neq 0$, solving Eq.(12):

$$X_S^2 = \frac{R_S}{R_D + R_W}[(\omega M)^2 - R_S(R_D+R_W)] \tag{13}$$

Eq.(13) will be discussed on two cases:

When $(\omega M)^2 \leq R_S(R_D+R_W)$, $f(X_S, X_D)$ has only one minimum value, namely one maximum value of P_{out}, and $X_S = 0, X_D = 0$ which is shown in Fig.3.

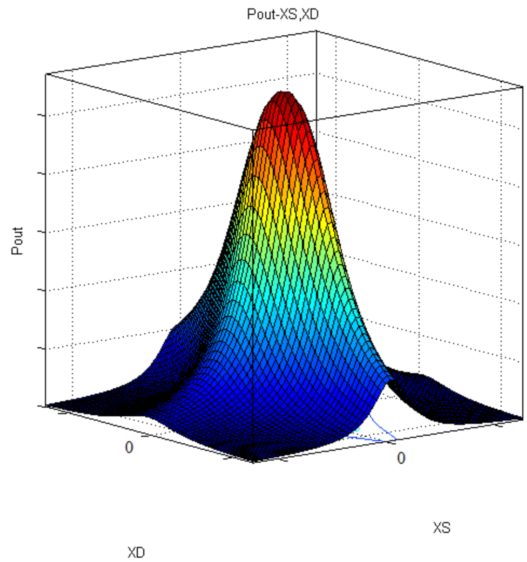

Fig.3. Only one maximum value

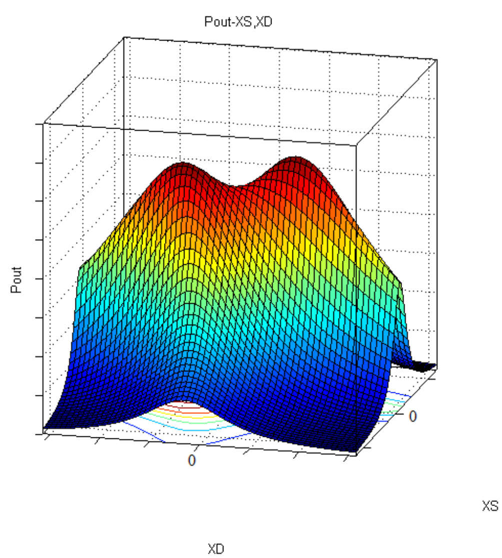

Fig.4. Two maximum value

When $(\omega M)^2 > R_S(R_D+R_W)$, $f(X_S, X_D)$ has two minimum values, namely two maximum values of P_{out}, and the two maximum values are:

$$\begin{cases} X_S = \sqrt{\dfrac{R_S}{R_D+R_W}[(\omega M)^2 - R_S(R_D+R_W)]} \\ X_D = \sqrt{\dfrac{R_D+R_W}{R_S}[(\omega M)^2 - R_S(R_D+R_W)]} \end{cases}$$

and

$$\begin{cases} X_S = -\sqrt{\dfrac{R_S}{R_D+R_W}[(\omega M)^2 - R_S(R_D+R_W)]} \\ X_D = -\sqrt{\dfrac{R_D+R_W}{R_S}[(\omega M)^2 - R_S(R_D+R_W)]} \end{cases}$$

The point of $X_S = 0, X_D = 0$ is the minimum value of P_{out} which is shown in Fig.4.

Through the above analysis it can be seen that when the mutual inductance M is less than a certain value, which means the distance is greater than a certain value, the power has one and only one maximum value. But if the mutual inductance M greater than a certain value, which means the distance is less than a certain value, the power has two maximum values which are not in resonance point. However, this certain value is too small and generally the transmission distance between resonance objects is far more lager than this value.

The efficiency can be deduced by Eq.(8) and Eq.(9):

$$\eta = \frac{P_{out}}{P_{in}} = \frac{R_W(\omega M)^2}{R_S[(R_D+R_W)^2 + X_D^2] + (R_D+R_W)(\omega M)^2} \tag{14}$$

When $X_D = 0$ which means receiver in resonance condition, the efficiency reach maximum value:

$$\eta_{\max} = \frac{(\omega M)^2 R_W}{(R_D+R_W)[R_S(R_D+R_W) + (\omega M)^2]} \tag{15}$$

2.3 The relationship among resonance frequency and output power and efficiency

It is assumed that the system has reached the resonant condition, namely $X_S = 0, X_D = 0$. When fixed the other parameters except ω in Eq.(9), the function image of P_{out} and ω can be drawn as shown in Fig.5.

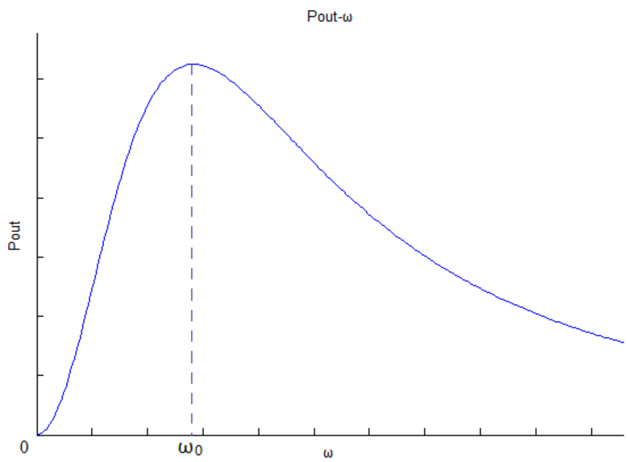

Fig.5. Relationship between output power and frequency

As shown in Fig.5, as the frequency increases, the output power will increase first and then decrease. The point of the maximum value can be deduced by differentiating P_{out} on ω as $\omega_0 = \frac{M}{6} + \frac{M^2}{36K} + K$, in which

$$K = \left(\sqrt{\left(\frac{R_S(R_D+R_W)}{4M} + \frac{M^3}{216}\right)^2 - \frac{M^6}{46656}} + \frac{R_S(R_D+R_W)}{4M} + \frac{M^3}{216}\right)^{\frac{1}{3}}$$

When fixed the other parameters except ω in Eq.(14) in resonance condition, the function image of η and ω can be drawn as shown in Fig.6.

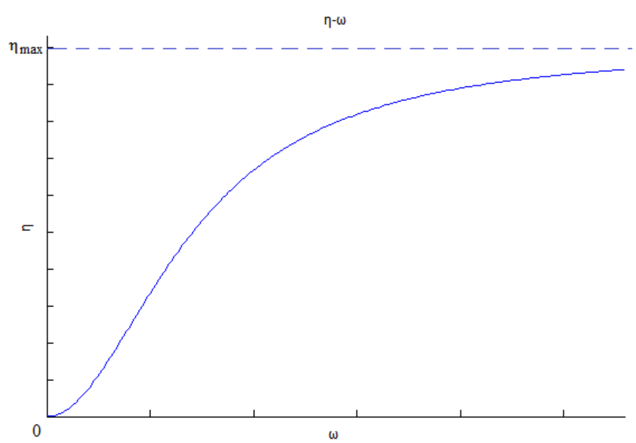

Fig.6. Relationship between efficiency and frequency

As shown in Fig.6, as the frequency increases, the efficiency will increase. According to Eq.(15), the efficiency is no more than $\eta_{\max}$, which $\eta_{\max} = \frac{(\omega M)^2 R_W}{(R_D+R_W)[R_S(R_D+R_W)+(\omega M)^2]}$.

It can be concluded that the resonance frequency has a certain relationship on the output power and efficiency of the system. So, choosing appropriate resonant frequency can improve the power output and the efficiency of the system. For Coupled Magnetic Resonant Power Transfer system, the typical frequency is in the range of 0.5 ~ 25 MHZ.

2.4 The relationship among transmission distance and output power and efficiency

With the increase of the distance, the interaction between emitting coil and receiving coil will decrease gradually. As a result, the mutual inductance between the emitter and the receiver will decrease. In the simplest of coaxial parallel coil, for example, the formula for the mutual inductance between coaxial parallel coils is:

$$M = \frac{\mu_0 \pi N_1 N_2 r_1^2 r_2^2}{2(r_1^1 + d^2)^{3/2}} (r_1 > r_2) \tag{16}$$

In Eq.(16), N means the number of turns in the coil, and r is radius of the coil, and d is transmission distance.

Substituting Eq.(16) into Eq.(9) in resonance condition. When fixed the other parameters except d in Eq.(14), the function image of P_{out} and d can be drawn as shown in Fig.7.

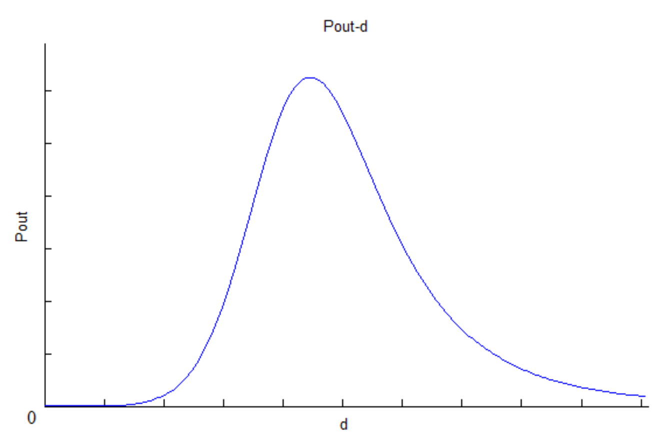

Fig.7. Relationship between output power and transmission distance

As shown in Fig.7, as the increase of transmission distance, the output power will increase first and then decrease. And the rate of increase and reduction is relatively close.

Substituting Eq.(16) into Eq.(14) in resonance condition. When fixed the other parameters except d in Eq.(14), the function image of η and d can be drawn as shown in Fig.8.

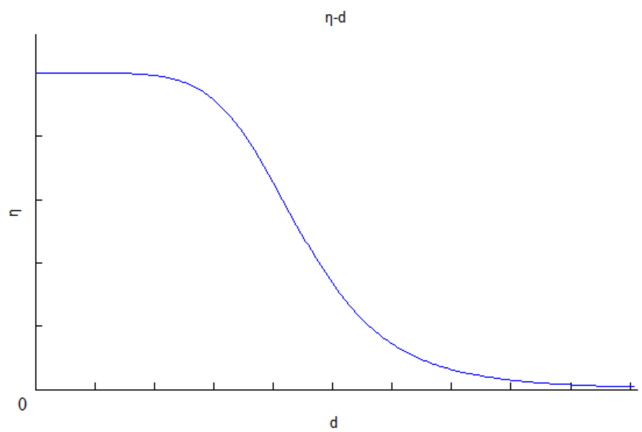

Fig.8. Relationship between efficiency and transmission distance

As shown in Fig.8, as the transmission distance increases, the efficiency will be decrease.

2.5 The relationship among load resistance and output power and efficiency

When fixed the other parameters except R_W in Eq.(9) in resonance condition, the function image of P_{out} and R_W can be drawn as shown in Fig.9 and the point of maximum value can be deduced as $R_{w0} = R_D + \frac{(\omega M)^2}{R_S}$.

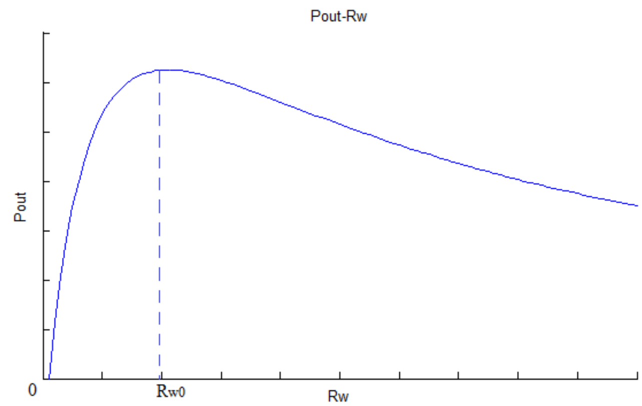

Fig.9. Relationship between output power and load resistance

As shown in Fig.9, as the load resistance increases, the output power will increase first and then decrease. When $R_w = R_{w0}$, the output power achieves the maximum value. In other words, if other parameters are fixed, there is optimum load resistance with which the output power can reach maximum.

When fixed the other parameters except R_W in Eq.(14) in resonance condition, the function image of η and R_W can be drawn as shown in Fig.10 and the point of maximum value can be deduced as $R_{W0} = \sqrt{\frac{R_D}{R_S}(\omega M)^2 + R_D^2}$.

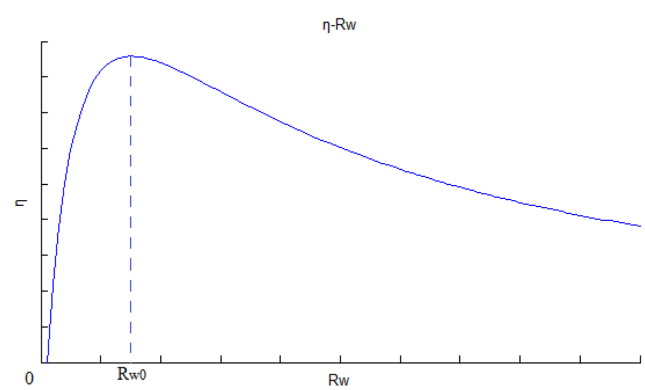

Fig.10. Relationship between efficiency and load resistance

As shown in Fig.10, as the load resistance increases, the efficiency will increase first and then decrease. When $R_W = R_{W0}$, the efficiency achieves the maximum value. In other words, if other parameters are fixed, there is optimum load resistance with which the efficiency can reach maximum.

3 Experiment and Analysis Of Coupled Magnetic Resonant Power Transfer System

Class-E Amplifier is used widely to design coupled magnetic resonant power transfer system [12].

The device is based on Class-E Amplifier with a frequency of 6.78MHz, which can transfer 52.9watt power and the transmission efficiency between coils can be 88.7%. The transmission distance could be 20cm. The device of the experiment is shown in Fig.11 and Fig.12 and the schematic of Class-E amplifier is shown in Fig.13.

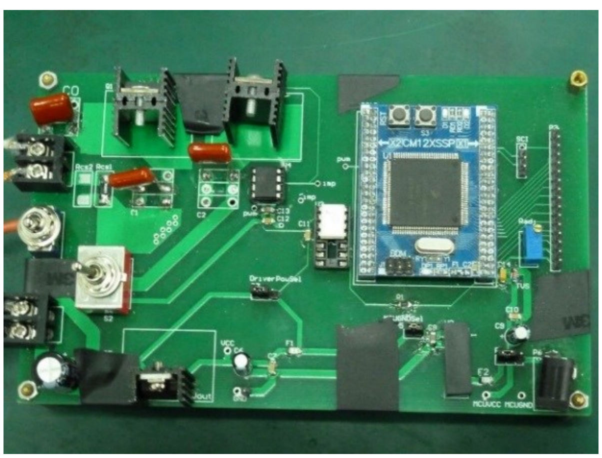

Fig.11. The device of the experiment based on Class-E amplifier

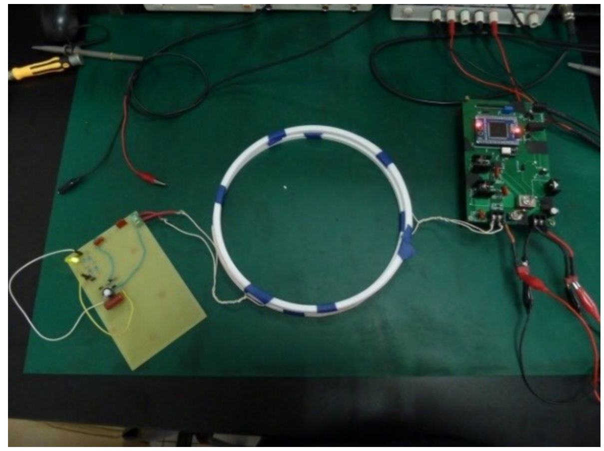

Fig.12. The device of the experiment with coil

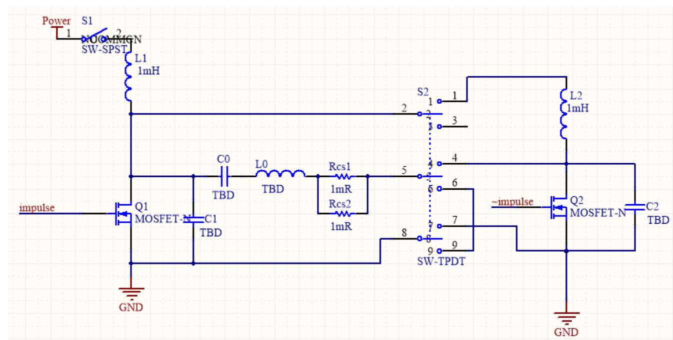

Fig.13. The schematic of Class-E amplifier

Input Power Supply: 0-30V, 0-3A Controllable DC power supply. The range of voltage in the experiment is from 9V to 30V. The parameters of coils are shown in Table 1.

coil	Emitter	Receiver
Radius(cm)	12	12
Turns	7	7
Theoretical value(uH)	34.81	34.81
practical value(uH)	34	35

Table 1. Resonant inductance value

3.1 The influence of the transmission distance to the system state

Fixed input voltage VIN=12V, load resistance RL=40Ω, the relationship among the transmission distance and output power and efficiency is shown in Fig.14:

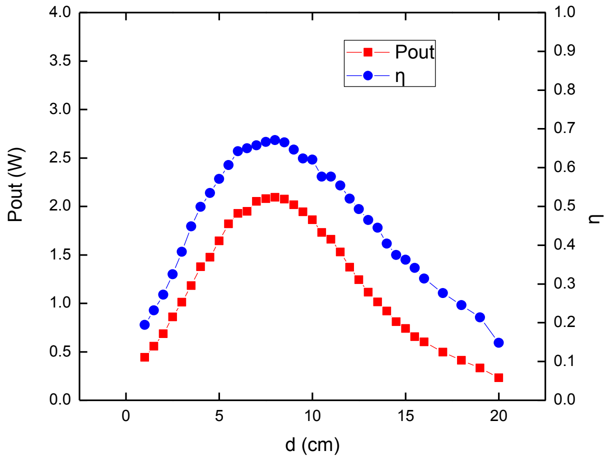

Fig.14. Relationship among output power and efficiency and transmission distance

It can be seen that as the transmission distance increases, output power and efficiency of the system will both increase first and then decrease. Besides, the maximum value of output power and the maximum value of efficiency do not appear at the same time. According to the previous theoretical analysis, the efficiency should be falling all the time with the increase of transmission distance. However, after many experiments, the change trend of efficiency always increases first and then decreases. The reason is that in the process of work, induced current generated in the receiving coil. Since the receiver includes a series resonance circuit, the induced current in the receiving coil generates a magnetic field. The original resonant state of the system is broken by the magnetic field, or this magnetic field influences the magnetic field of the emitter. As a result, when the transmission distance is too close, efficiency is low. In order to validate this idea, load resistance is increased to reduce induced current in the receiving coil and weaken the magnetic field and observe the relationship of efficiency and distance in such state. The result of the experiment is: When input voltage is 12V and load resistance is 1KΩ, the relationship among transmission distance and output power and efficiency is shown as Fig.15:

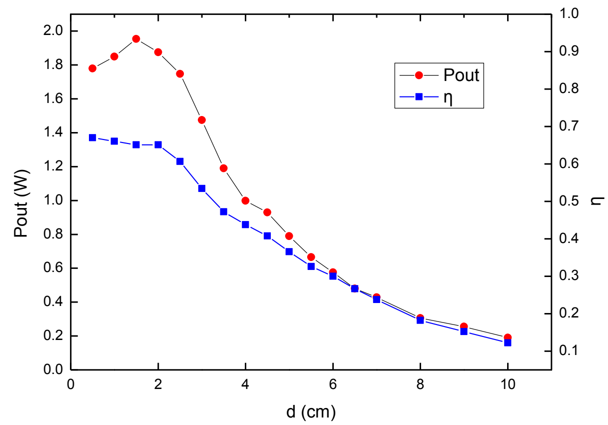

Fig.15. Relationship among output power and efficiency and transmission distance

As shown in Fig.10, when the load resistance increases, as the increase of transmission distance, output power will increase first and then decrease while the efficiency decrease all the time, which is in accord with Fig.7 and Fig.8.

3.2 The influence of the load resistance to the system state

Input voltage is 12V, and the transmission distance is 5cm, the relationship between efficiency and load resistance is shown in Fig.16.

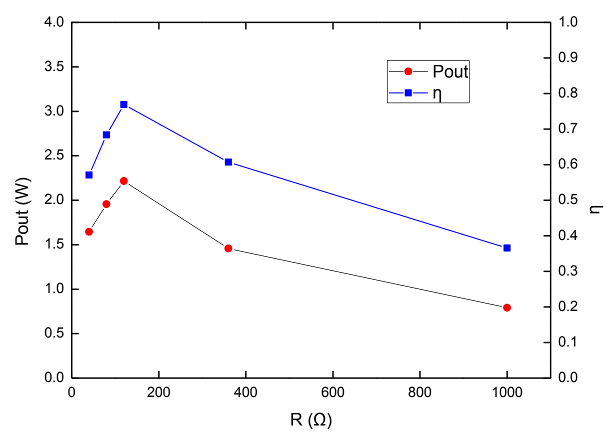

Fig.16. Relationship among output power and efficiency and load resistance

The results of the experiment verified the theoretical analysis results of Fig.9 Fig.10.

4. Conclusion

In this paper a model of Coupled Magnetic Resonant Power Transfer system is established and the relationship among output power, efficiency, resonance state, frequency, transmission distance, load resistance of the system is analyzed and a device based on E-Class amplifier is designed to verify the theoretical analysis. As the frequency increases, the output power will increase first and then decrease while the efficiency will increase all the time. As the transmission increases, the output power will increase first and then decrease while the efficiency will decrease all the time. As the load resistance increases, both output power and efficiency will increase first and then decrease, but do not reach maximum value at the same time.

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