Stochastic Models for the Fading Channel Success Rate of Sub-6G UWB Systems

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Abstract: - This paper presents different stochastic models for entangled UWB fading channels. Closed-form canonical expressions are derived for the statistical distribution of the fading power difference of interfering independent and dependent mixture of flat-fading Rayleigh and Rician channels. The link success probability is derived in terms of the cumulative distribution function of the channel fading power differences. An alternative metric for link success rate (LSR) is developed in terms of the statistical distribution of the ratio of the diffuse powers. Under this alternative formulation, and for the case of 2 independent Rayleigh paths, the LSR statistical distribution was consistent with the heavy-tail Lomax distribution. The scope of future research includes the validation of theoretical and simulated models with real measurements on actual multi-user massive multiple-input-multiple-output (MU-mMIMO) beam-forming sub-6G physical systems.

Keywords: - Circuits and Systems for Communications, Link success probability, MU-mMIMO, Rayleigh fading, Rician fading, Sub-6G UWB

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1 Introduction

The use of UWB systems at sub-6G spectral mid-bands, with operational frequency bands ranging from 3.4 GHz to 6GHz, has been advocated in light of the current explosion in 6G networks research. Wireless Body Area (WBA) networks and the Internet of Things (IoT) both use these systems extensively.

Four kinds of UWB channels, CM1, CM2, CM3, and CM4, are listed in IEEE 802.15.3.a standard. Table 1 describes the features of these channels together with the scattering multi-path fading statistics and matching transmission ranges. We note that the presence of a direct LOS (line of sight) drives the fading statistics to be governed by a Rician law.

The 6G communication system's performance thresholds are determined by channel statistical features. It is thus imperative to explore different sub-6G UWB channel models and develop stochastic models for random scattering elements distributed in space and time, [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14]. Radio propagation channel models, whether large-scale or small-scale fading, including MIMO fading channels, are established with coordinated multi-point transmissions at the allocated frequencies. In this work, we consider large-scale path models based on UWB multi-path scattering channels because they offer relative simplicity in performance calculations and predictions with high accuracy.

Table 1. IEEE 802.15.3a UWB channels

	CM1	CM2	CM3	CM4
	Line of sight	Non-line of Sight	Non-line of Sight	Non-line of Sight
Common Fading Statistics	Rician	Rayleigh	Nakagami-*m*	Lognormal
Transmission Range	0 to 4m	0 to 4m	4 to 10m	4 to 10m

2 Statistical Metrics for System Performance

The level crossing rate (LCR) and the average outage duration (AOD), that is, the average time for which the combined average signal-to-noise ratio (SNR) random process stays below a predetermined threshold level, were studied in [15] for pre-detection equal gain combining (EGC) of 2 correlated signal paths. Analysis and comparisons of the LCR and AOD for two-branch MRC, EGC, and selection combining (SC) are found in [16]. The LCR and AOD of the maximal ratio combining (MRC) over RAKE receivers for independent identically distributed (i.i.d.) Rayleigh fading path were examined by K. Young-Chai KO et. al in [17].

The wireless multi-diversity systems studied earlier suffer from drawbacks. There is a limitation to increasing the number of antennas and the increased multi-user interference noise which tends to limit the fading gain. To study the effect of co-channel interference (CCI) in multiple-input-multiple-output (MIMO) systems, we need to examine the statistical characteristics of internal multi-user and external interfering noise, and we thus propose a quality of service (QoS) metric under various fading models which allows us to dimension the multi-diversity systems, thereby effectively determining the number of antennas which will enhance the fading gain while guaranteeing a pre-set CCI-related quality of service metric.

Without loss of generality, the method in this paper can be extended to link success rates and interference margins in massive MIMO orthogonal frequency division multiplexing systems (mMIMO-OFDM), [18], [19], [20], [21]. When users are multiplexed using OFDM to increase system performance, MU-mMIMO systems are employed to reduce hardware costs, [22].

3 Stochastic Modeling of the Link Success Rate

3.1 Classical QoS Criteria

The outage probability, P_{out}, is a widely used standard performance criterion characteristic of UWB systems operating over fading channels. It is defined as the likelihood that the instantaneous error probability exceeds a specified value, or alternatively, the likelihood that the received SNR per bit, Γ, drops below a specified critical threshold Γ_{th}. From a mathematical standpoint, we have

$$P_{out} = \int_0^{\Gamma_{th}} p_\Gamma(\upsilon) d\upsilon, \tag{1}$$

where $p_\Gamma(\upsilon)$ is the pdf of the received SNR Γ, which represents the fading power distribution on a scaled version. For system performance analysis in sub-6G UWB systems, more elaborate QoS metrics need to be developed, as illustrated in the next section.

3.2 An Improved Metric for Link Success Rate based on Fading Statistics

In this work, we study the statistical distribution of the level crossing rate (LCR) of sub-6G UWB systems. The wireless multi-diversity systems mentioned in section 2 suffer from drawbacks. There is a limitation to increasing the number of antennas since the increased multi-user interference noise tends to limit the fading gain. To study the effect of CCI in MIMO systems, we examine the statistical characteristics of internal multi-user and external interfering noise, and we propose a QoS metric under various fading models, which in effect can be considered as an improved LSR measure. This improved LSR metric allows us to dimension the multi-diversity systems, thereby effectively determining the number of antennas that will enhance the fading gain while guaranteeing a pre-set CCI-related QoS.

The LSR metric is formulated by first defining

$$\begin{aligned} P_\delta &= P\left(\left|\Gamma_k^2 - \Gamma_{k\pm1}^2\right| > \delta\right) \\ &= P\left\{\left(\Gamma_k^2 - \Gamma_{k\pm1}^2 > \delta\right) \text{ or } \left(\Gamma_{k\pm1}^2 - \Gamma_k^2 > \delta\right)\right\}, \delta > 0 \end{aligned} \tag{2}$$

where P_δ denotes the probability that the fading power Γ_k^2 at the k-th receiver antenna exceeds the fading power $\Gamma_{k\pm1}^2$ at the next adjacent receiver antenna by a co-channel interference (CCI) robustness margin (or jamming or interference margin (IM)) δ. Let $\Upsilon = \Gamma_k^2 - \Gamma_{k\pm1}^2$, $|\Upsilon|$ is the interference margin (IM). IM in UWB and cellular systems is defined to be the amount by which the undesired interfering signal exceeds desired information-bearing signal and therefore causing the information signal to be jammed and the link to fail. We note that for $\delta = 0$, the P_δ represents the probability that the interference-to-source ratio (ISR) in UWB systems (or carrier-to-interference ratio (CIR) in more general wireless systems) exceeds 1 (or 0 dB). In fact, $P(\Gamma_k^2 - \Gamma_{k\pm1}^2 > 0) = P\left(\Gamma_k^2 / \Gamma_{k\pm1}^2 > 1\right)$ $= P(\text{ISR} > 1)$.

Let the link success probability be denoted by P_Λ, we have

$$\begin{aligned} P_\Lambda &= 1 - P_\delta \\ &= 1 - \left[P(\Upsilon > \delta) + P(-\Upsilon > \delta)\right] \\ &= 1 - \left[1 - P(\Upsilon \le \delta) + P(\Upsilon < -\delta)\right] \\ &= P(\Upsilon \le \delta) - P(\Upsilon < -\delta) \\ &= P_\Upsilon^+(\delta) - P_\Upsilon^-(-\delta),\ \ \delta > 0 \end{aligned} \tag{3}$$

In the sub-sections below, canonical closed-form expressions for $P_\Upsilon^+(\delta) = P(\Upsilon \le \delta)$ and $P_\Upsilon^-(-\delta) = P(\Upsilon < -\delta)$ are developed below for various stochastic fading models that are chosen for their conformity with the UWB IEEE 802.15.3a channel model. To the best of our knowledge, these developed closed-form expressions are novel and form the main contribution of this work. The performance metrics of wireless UWB systems are highly dependent on accurate and precise stochastic channel models.

3.2.1 Two Independent Rayleigh Channels

In this model, entangled UWB CM4 channels are stochastically modeled as 2 independent Rayleigh fading paths. A single-input-single-output path (SISO) fading power has a stochastic model $\Gamma_i \sim \mathcal{Rayleigh}\left(P_{dif}^{(i)}\right); i = k, k \pm 1$ with

$$p_\Gamma(\gamma) = \frac{2\gamma}{P_{dif}} e^{-\frac{\gamma^2}{P_{dif}}} I_{[0,\infty)}(\gamma); P_{dif} > 0 \tag{4}$$

where P_{dif} is the mean diffuse power and $I(.)$ is the indicator function defined as

$$I_\Omega(w) = \begin{cases} 1, w \in \Omega \\ 0, w \notin \Omega \end{cases} \tag{5}$$

The cumulative distribution function (CDF) of the fading power difference $\Upsilon = \Gamma_k^2 - \Gamma_{k\pm1}^2$ at 2 adjacent transmitting antennas is derived as

$$P_\Upsilon(\upsilon) = \begin{cases} P_\Upsilon^-(\upsilon) = \dfrac{P_{dif}^{(k\pm1)}}{P_{dif}^{(k)} + P_{dif}^{(k\pm1)}} \exp\left(\dfrac{\upsilon}{P_{dif}^{(k\pm1)}}\right), \upsilon < 0 \\ P_\Upsilon^+(\upsilon) = 1 - \dfrac{P_{dif}^{(k)}}{P_{dif}^{(k)} + P_{dif}^{(k\pm1)}} \exp\left(-\dfrac{\upsilon}{P_{dif}^{(k)}}\right), \upsilon \ge 0 \end{cases} \tag{6}$$

The link success probability as formulated in Eq. (3) is obtained by direct substitutions in the expressions of Eq. (6).

3.2.2 Two Dependent Rayleigh Channels

In this model, entangled UWB CM4 channels are stochastically modeled as 2 dependent Rayleigh fading paths with a low correlation factor, entangled CM3 channels are modeled as 2 dependent Rayleigh fading paths with a medium correlation factor, and entangled CM2 channels are modeled as 2 dependent Rayleigh fading paths with high correlation factor.

The channel independence assumption used in the previous model simplifies the analysis, but in real life, the diversity paths are correlated. The performance of diversity systems is affected by this fading correlation which can be characterized as co-channel interference (CCI). The maximum theoretical diversity gain cannot be attained when CCI is present in the channel characteristic features, but significant gain is nevertheless attained with an efficient receiver diversity design. The fading correlation does not invalidate our independent stochastic model since CCI generally follows a uniform decaying power profile with each additional interfering (correlated) diversity path introducing an extra dBm level (e.g. 10 dBm) to the underlying additive corruptive noise. In the context of our fading stochastic model, the focus is on multiplicative noise, so the fading correlation can be simply treated as additional additive noise and thus does not affect our derivations for the multiplicative fading model.

As mentioned, diversity techniques do not in practice produce independent signal paths as is commonly assumed. In reality, some dependence does exist between the diversity signals, especially adjacent ones. Hence, this model is more practical than the previous one, albeit more mathematically complex.

Denoting the correlation coefficient by ρ, with $|\rho_{k,k\pm1}| < 1$, the covariance matrix is derived as

$$\underline{\underline{C}} = \frac{1}{2}\begin{bmatrix} P_{dif}^{(k)} & \rho_{k,k\pm1}\sqrt{P_{dif}^{(k)}P_{dif}^{(k\pm1)}} \\ \rho_{k,k\pm1}\sqrt{P_{dif}^{(k)}P_{dif}^{(k\pm1)}} & P_{dif}^{(k\pm1)} \end{bmatrix} \quad (7)$$

Denoting the inverse of the covariance matrix by $\underline{\underline{C}}^{-1} = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix}$, the CDF of the fading power difference is

$$P_{\Upsilon}(\upsilon) = \begin{cases} P_{\Upsilon}^{-}(\upsilon) = \dfrac{2\left|\underline{\underline{C}}^{-1}\right|}{\mu\alpha}\exp\left(\dfrac{\alpha}{4}\upsilon\right), & \upsilon < 0 \\ P_{\Upsilon}^{+}(\upsilon) = 1 - \dfrac{2\left|\underline{\underline{C}}^{-1}\right|}{\mu\beta}\exp\left(-\dfrac{\beta}{4}\upsilon\right), & \upsilon \geq 0 \end{cases} \quad (8)$$

$$\mu = \sqrt{\left(c_{11}+c_{22}\right)^2 - 4c_{12}^2},$$
$$\alpha = \mu - (c_{11} - c_{22}),$$
$$\beta = \mu + (c_{11} - c_{22})$$

The link success probability as formulated in Eq. (3) is obtained by direct substitutions in the expressions of Eq. (8).

3.2.3 Two Independent Rayleigh and Rician Channels

This is a more practical model because of the dominance of a direct LOS in the Rician fading signal path. The model describes the stochastic distribution of fading power on 2 entangled CM1 and CM2 channels. The SISO fading power on the CM1 channel has the stochastic model $\Gamma_k \sim \mathcal{R}ician\left(P_{dif}^{(k)}, V_o\right)$ with

$$p_{\Gamma_k}(\gamma) = \frac{2\gamma}{P_{dif}^{(k)}} e^{-\frac{\left(\gamma^2+V_0^2\right)}{P_{dif}^{(k)}}} I_0\left(\frac{2V_0\gamma}{P_{dif}^{(k)}}\right) I_{[0,¥)}(\gamma),\ V_0 > 0,\ P_{dif}^{(k)} > 0, \quad (9)$$

where V_o is the coherent specular component, $I_o(.)$ is the modified Bessel function of the 1st kind of order 0. The other interfering SISO fading power $\Gamma_{k\pm1}$ on the CM2 path is Rayleigh distributed as in Eq. (4).

The CDF of the fading power difference at 2 adjacent transmitting antennas is derived as

$$P_{\Upsilon}(\upsilon) = \begin{cases} P_{\Upsilon}^{-}(\upsilon) = \mu\exp\left(\dfrac{\upsilon}{P_{dif}^{(k\pm1)}}\right), \ \upsilon < 0 \\ P_{\Upsilon}^{+}(\upsilon) = \mu\left\{1 - P_{dif}^{(k\pm1)} + \left(P_{dif}^{(k)} + P_{dif}^{(k\pm1)}\right)\times\right. \\ \times\exp\left(\dfrac{V_0^2}{P_{dif}^{(k)} + P_{dif}^{(k\pm1)}}\right)\times \\ \times\left(1 - Q\left(\dfrac{\sqrt{2}V_0}{\sqrt{P_{dif}^{(k)}}}, \dfrac{\sqrt{2\upsilon}}{\sqrt{P_{dif}^{(k)}}}\right)\right) \\ +P_{dif}^{(k\pm1)}Q\left(\sqrt{\dfrac{2V_0^2P_{dif}^{(k\pm1)}}{P_{dif}^{(k)}\left(P_{dif}^{(k)}+P_{dif}^{(k\pm1)}\right)}}, \sqrt{\dfrac{2\left(P_{dif}^{(k)}+P_{dif}^{(k\pm1)}\right)}{P_{dif}^{(k)}P_{dif}^{(k\pm1)}}\upsilon}\right)\times \\ \left.\times\exp\left(\dfrac{\upsilon}{P_{dif}^{(k\pm1)}}\right)\right\}, \ \upsilon \geq 0 \end{cases} \quad (10)$$

where

$$\mu = \frac{P_{dif}^{(k\pm1)}}{P_{dif}^{(k)} + P_{dif}^{(k\pm1)}}\exp\left(-\frac{V_0^2\left(0.5P_{dif}^{(k\pm1)} + P_{dif}^{(k)}\right)}{P_{dif}^{(k)}\left(P_{dif}^{(k)} + P_{dif}^{(k\pm1)}\right)}\right), \quad (11)$$

and

$$Q\left(x,\alpha\right) \square \int_{\alpha}^{\infty} e^{-\left(\frac{x^2+u^2}{2}\right)} I_0(xu)u\,du \quad (12)$$

is the Marcum Q-function. The link success probability as formulated in Eq. (3) is obtained by direct substitutions of the expressions in Eq. (10).

3.2.4 Two Independent Rician Channels

This is yet another very important model because of the possible presence of more than one line of sight in the fading signal paths. One line of sight can evidently be stronger than the other, and this is captured by a higher coherent specular component with mean power $|V_0|^2$. This model describes the stochastic distribution of fading power on 2 entangled CM1 channels. The SISO fading power on each of the interfering CM1 channels has the stochastic model $\Gamma_i \sim \mathcal{Rician}\left(P_{dif,i}, V_{0,i}\right); i = k, k \pm 1$, with probability density function given in Eq. (9).

The CDF of the fading power difference at 2 adjacent transmitting antennas is derived as

$$
P_{\Upsilon}(\upsilon) = \begin{cases}
P_{\Upsilon}^{-}(\upsilon) = \dfrac{\exp\left(-\left(\dfrac{V_{0,k}^2}{P_{dif,k}} + \dfrac{V_{0,k\pm1}^2}{P_{dif,k\pm1}}\right)\right)}{P_{dif,k\pm1} + P_{dif,k}} \times \\
\times \exp\left(-\dfrac{1}{2}\left(\dfrac{P_{dif,k\pm1} - P_{dif,k}}{P_{dif,k\pm1}P_{dif,k}}\right)\upsilon\right) \times \\
\times \displaystyle\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \dfrac{1}{m!(n!)^2}\left(-\dfrac{P_{dif,k\pm1}V_{0,k}^4}{P_{dif,k}^3\left(P_{dif,k} + P_{dif,k\pm1}\right)}\upsilon\right)^{m/2} \\
\times \left(-\dfrac{P_{dif,k}V_{0,k\pm1}^4}{P_{dif,k\pm1}^3\left(P_{dif,k} + P_{dif,k\pm1}\right)}\upsilon\right)^{n/2} \times \\
\times W_{(n-m)/2,(m+n+1)/2}\left(-\dfrac{\left(P_{dif,k} + P_{dif,k\pm1}\right)}{P_{dif,k\pm1}P_{dif,k}}\upsilon\right), \ \upsilon < 0 \\
P_{\Upsilon}^{+}(\upsilon) = \dfrac{\exp\left(-\left(\dfrac{V_{0,k}^2}{P_{dif,k}} + \dfrac{V_{0,k\pm1}^2}{P_{dif,k\pm1}}\right)\right)}{P_{dif,k\pm1} + P_{dif,k}} \times \\
\times \exp\left(\dfrac{1}{2}\left(\dfrac{P_{dif,k\pm1} - P_{dif,k}}{P_{dif,k\pm1}P_{dif,k}}\right)\upsilon\right) \times \\
\times \displaystyle\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \dfrac{1}{(m!)^2 n!}\left(\dfrac{P_{dif,k\pm1}V_{0,k}^4}{P_{dif,k}^3\left(P_{dif,k} + P_{dif,k\pm1}\right)}\upsilon\right)^{n/2} \\
\times \left(\dfrac{P_{dif,k}V_{0,k\pm1}^4}{P_{dif,k\pm1}^3\left(P_{dif,k} + P_{dif,k\pm1}\right)}\upsilon\right)^{m/2} \times \\
\times W_{(m-n)/2,(m+n+1)/2}\left(\dfrac{P_{dif,k\pm1} + P_{dif,k}}{P_{dif,k\pm1}P_{dif,k}}\upsilon\right), \upsilon \geq 0
\end{cases} \tag{13}
$$

where $W(.)$ is the transcendental Whittaker functions defined as

$$
\begin{aligned}
& W_{\alpha,\beta}(x) \triangleq \frac{x^{\beta+0.5}e^{-x/2}}{\Gamma\left(\beta - \alpha + 0.5\right)} \times \\
& \times \int_0^{\infty} t^{\beta-\alpha-0.5}(1+t)^{\beta+\alpha-0.5}e^{-xt}dt, \\
& \beta - \alpha + 0.5 > 0,
\end{aligned} \tag{14}
$$

and $\Gamma(.)$ is the Gamma function defined as

$$
\Gamma(z) \triangleq \int_0^{\infty} t^{z-1}e^{-t}dt. \tag{15}
$$

We note that the ratio $|V_0|^2 / P_{dif}$ is the Rician-K factor. The link success probability as formulated in Eq. (3) is obtained by direct substitutions of the expressions in Eq. (13).

The characteristic function of the fading power difference $\Upsilon = \Gamma_k^2 - \Gamma_{k\pm1}^2$ is given by

$$
\begin{aligned}
\phi_{\Upsilon}(jw) &= \frac{1}{(1 - jwP_{dif,k})(1 + jwP_{dif,k\pm1})} \times \\
&\times \exp\left(\frac{jwV_{o,k}^2}{1 - jwP_{dif,k}} - \frac{jwV_{o,k+1}^2}{1 + jwP_{dif,k+1}}\right),
\end{aligned} \tag{16}
$$

which is obtained by taking the Fourier transform of the cumulative distribution of Eq. (13) and multiplying the result by jw.

In Appendix A, we derive an alternative performance QoS metric based on the ratio of fading power of interfering channel paths.

4 Simulations

For comparative reasons, we show in Fig. 1 the CCI effect on the degree of statistical fluctuations of the received flat fading envelope at the input of a RAKE receiver using Monte Carlo simulations. The CCI is depicted as (i) Gaussian noise modulating a Rayleigh fading statistic, (ii) Gaussian noise superimposed on a Rayleigh fading statistic, and (iii) Gaussian noise modulating a Rayleigh fading statistic and superimposed on it. Clearly, the stochastic model exhibits the highest level of statistical fluctuations, all statistical parameters being equal.

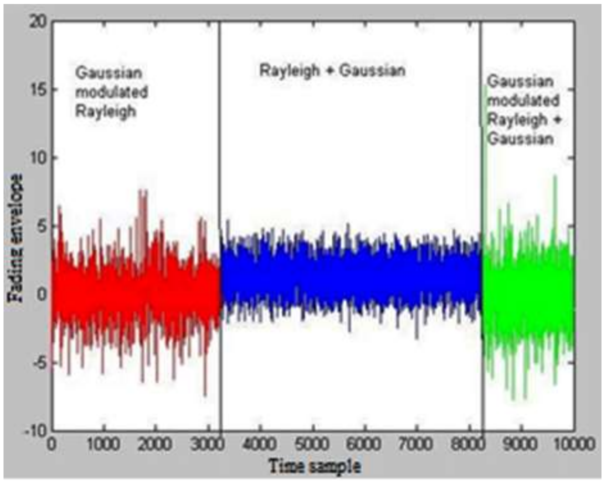

Fig. 1: Simulation of the received flat fading envelope samples under different CCI models

Simulation of the cumulative distribution functions developed in section 3 only serves to validate themselves) is a measurable function associated with the random fading envelope Υ with valid properties (monotonically increasing function from 0 to 1 over the support [0, ∞)).

The CDF models developed in this work can be incorporated into dimensioning models of MIMO systems where the number of UWB users can be optimized to guarantee a set of pre-set QoS metrics. The development of dimensioning models and algorithms for MIMO systems based on the stochastic models derived in this work is beyond the scope of this paper and is left for future research.

5 Conclusion and Future Scope

The examination of UWB channels in sub-6G systems was the primary focus of our work. Investigations were done into how co-channel interference affected UWB performance. The performance metrics of these systems are highly dependent on accurate and precise stochastic channel models. Further to the stochastic modeling study, canonical expressions closed-form were developed for the LSR metric using the cumulative distribution function of the differenced fading power distribution of two interfering channels and the probability density function of the ratio of 2 independent Rayleigh paths. The latter LSR distribution was consistent with the heavy-tail Lomax distribution.

Our research allows for the dimensioning of MU-mMassive wireless systems by quantifying the maximum number of interference sources that the system can accommodate without out degrading QoS metrics (in terms of link success probability). The dimensioning study will be considered in future research. Future research scope will also include conducting real-life experiments to validate that the theoretical and simulated models agree with real measurements on MU-mMIMO beamforming sub-6G physical systems.

6 Appendix A: An Alternative QoS Metric

In this appendix, we derive an alternative performance QoS metric. Assuming two independent interfering diversity Rayleigh paths on CM4 channels, we define an LSR-related metric in terms of the ratio of fading powers:

$$\text{QoS} = 1 - P\left(\frac{\psi_k}{\psi_{k+1}} > \delta\right), \delta > 0\,, \tag{17}$$

where the diffuse power on the k-th channel path $\psi_k = \Gamma_k^2 \sim \exp(1/P_{dif})$ is exponentially distributed, which conforms with a fading envelope $\Gamma_k \sim \text{Rayleigh}(P_{dif})$ that is Rayleigh distributed. The fading power distribution is given by

$$p_\psi(x) = \frac{1}{P_{dif}} \exp\left(-\frac{x}{P_{dif}}\right) I_{[0,\infty)}(x). \tag{18}$$

The QoS statistic of Eq. (17) is used instead of the difference of fading powers as analyzed in the body of this paper in section 3.

We now proceed into deriving a canonical expression for the QoS performance metric as follows:

$$\text{QoS} = 1 - P\left(\psi_k > \delta\psi_{k+1}\right). \tag{19}$$

By conditioning the QoS metric on the diffuse power of the k-th channel, we obtain the following expression:

$$
\begin{aligned}
\mathrm{QoS} \,|\, \psi_{k+1} &= 1 - \int_{\delta\psi_{k+1}}^{\infty} p_{\psi_k}(x)dx \\
&= 1 - \frac{1}{P_{dif,k}} \int_{\delta\psi_{k+1}}^{\infty} \exp\left(-\frac{x}{P_{dif,k}}\right)dx \\
&= 1 - \left[\exp\left(-\frac{x}{P_{dif,k}}\right)\right]_{x\to\infty}^{x=\delta\psi_{k+1}} \\
&= 1 - \exp\left(-\frac{\delta\psi_{k+1}}{P_{dif,k}}\right) + \lim_{x\to\infty}\exp\left(-\frac{x}{P_{dif,k}}\right) \\
&= 1 - \exp\left(-\frac{\delta\psi_{k+1}}{P_{dif,k}}\right).
\end{aligned}
\tag{20}
$$

The unconditional QoS is obtained by averaging the conditional QoS over random variates of the diffuse power of the *k*-th channel. This further develops the statistical analysis as shown below:

$$
\begin{aligned}
&\mathrm{QoS} = E_{\psi_{k+1}}\left(QoS \,|\, \psi_{k+1}\right) \\
&= \frac{1}{P_{dif,k+1}} \int_0^{\infty} \left(QoS \,|\, x = \psi_{k+1}\right) \exp\left(-\frac{x}{P_{dif,k+1}}\right)dx \\
&= 1 - \frac{1}{P_{dif,k+1}} \int_0^{\infty} \exp\left(-\frac{\delta x}{P_{dif,k}}\right)\exp\left(-\frac{x}{P_{dif,k+1}}\right)dx \\
&= 1 - \frac{1}{P_{dif,k+1}} \int_0^{\infty} \exp\left[-\left(\frac{\delta}{P_{dif,k}} + \frac{1}{P_{dif,k+1}}\right)x\right]dx \\
&= 1 - \frac{1}{P_{dif,k+1}}\left(\frac{1}{\frac{\delta}{P_{dif,k}} + \frac{1}{P_{dif,k+1}}}\right)\times \\
&\times \exp\left[-\left(\frac{\delta}{P_{dif,k}} + \frac{1}{P_{dif,k+1}}\right)x\right]\Bigg|_{x\to\infty}^{x=0} \\
&= 1 - \frac{1}{1 + \delta\frac{P_{dif,k+1}}{P_{dif,k}}} \\
&= \frac{\delta P_{dif,k+1}}{P_{dif,k} + \delta P_{dif,k+1}}.
\end{aligned}
\tag{21}
$$

The final expression for the QoS metric is given by

$$
\mathrm{QoS} = \frac{\delta P_{dif,k+1}}{P_{dif,k} + \delta P_{dif,k+1}}. \tag{22}
$$

We note that the statistical distribution of the ratio of diffuse powers $\upsilon = \psi_k / \psi_{k+1}$, under the assumptions stated above, is given by the heavy-tail Lomax distribution, also called the Pareto type II distribution

$$
p_\upsilon(v) = \frac{\left(P_{dif,k}P_{dif,k+1}\right)^{-1}}{\left(P_{dif,k+1}^{-1} + P_{dif,k}^{-1}v\right)^2} I_{[0,\infty)}(v). \tag{23}
$$

References

[1] J. S. Daba, M. R. Bell, “Segmentation of speckled images using a likelihood random field model,” *Optical Engineering*, vol. 47, no. 1, 2008.

[2] J. Dubois, “Scattering statistics of doppler faded acoustic signals using speckle noise models,” *IEEE International Conference on Direct and Inverse Problems of Electromagnetic and Acoustic Wave Theory* (*DIPED*), Lviv, Ukraine, pp. 185-189, 2003.

[3] J. S. Daba, M. R. Bell, “Estimation of the surface reflectivity of SAR images based on a marked Poisson point process model,” *IEEE International Symposium on Signals, Systems, and Electronics* (*ISSSE*'95), San Francisco, CA, pp. 183-186, 1995.

[4] J. S. Daba, M. R. Bell, A. Abdi, S. Nader-Esfahani, “Comments on statistics of the scattering cross-section of a small number of random scatterers," *IEEE Transactions on Antennas and Propagation*, vol. 48, no. 5, pp. 844-845, 2000.

[5] J. P. Dubois, O. M. Abdul-Latif, “SVM-based detection of SAR images in partially developed speckle noise,” IEC, Prague, pp. 321-325, 2005.

[6] J.P. Dubois, O. M. Abdul-Latif, “Detection of ultrasonic images in the presence of a random number of scatterers: A statistical learning approach,” IEC, Prague, pp. 326-329, 2005.

[7] J. Dubois, “Poisson modulated stochastic model for partially developed multi-look speckle,” *Proceedings of the American Conference on Applied Mathematics*, Harvard University, Cambridge, MA, USA, pp. 209-213, 2008.

[8] J. Dubois, "Segmentation of speckled ultrasound images based on a statistical model," *EURASIP Proceedings of the 16th International Biosignal Conference*, Czech Republic, vol. 16, 2002.

[9] J. S. Daba, M. R. Bell, "Statistics of the scattering cross-section of a collection of constant amplitude scatterers with random phase," *ECE Technical Reports*, Purdue University, p. 194, 1994.

[10] J. S. Daba, M. R. Bell, "Object discrimination and orientation determination in speckled images," *Optical Engineering*, vol. 33, no. 4, pp, 1287-1303, 1994.

[11] J. Dubois, "Traffic estimation in wireless networks using filtered doubly stochastic point processes," *Proceedings of IEEE International Conference on Electrical, Electronic, and Computer Engineering*, Cairo, Egypt, pp. 116-119, 2004.

[12] J. P. Dubois, "Statistical Modeling of Mobile Fading Channels Based on Triply Stochastic Filtered Marked Poisson Point Processes," *International Journal of Electronics and Communication Engineering*, vol. 11, no. 1, pp. 83-87, 2017.

[13] J. P. Dubois, "Improved Segmentation of Speckled Images Using an Arithmetic-to-Geometric Mean Ratio Kernel", *International Journal of Electrical and Computer Engineering*, vol. 1, no. 10, pp.1474-1477, 2007.

[14] J. P. Dubois, "Burstiness Reduction of a Doubly Stochastic AR-Modeled Uniform Activity VBR Video," *International Journal of Electrical and Computer Engineering*, vol. 1, no. 10, pp.1478-1482, 2007.

[15] G. J. Foshchini, M. J. Gans, "On the limits of wireless communication in a fading environment when using multi-element antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311-335, March 1998.

[16] M. D. Yacoub, "Fading Distributions and Co-Channel Interference in Wireless Systems," *Radio Science*, vol. 42, no. 1, pp. 150-158, Feb. 2000.

[17] Y. C. Ko, A. Abdi, M. Alouini, and M. Kaveh, "Average Outage Duration of Diversity Systems on Generalized Fading Channels," *IEEE Wireless Communications and Networking Conference*, Chicago, IL, Sept. 2000.

[18] J. Dubois, O. M. Abdul-Latif, "Novel diversity combining in OFDM-based MIMO systems," *Proceedings of the American Conference on Applied Mathematics*, Harvard University, Cambridge, MA, USA, pp. 189-194, 2008.

[19] J. Dubois, O. Abdul-Latif, "Improved receiver diversity processing over SIMO fading channels," *Proceedings of the IEEE International Conference on Signal Processing and Communications*, 2007.

[20] J. P. Dubois and H. M. Chiu, "High Speed Video Transmission for Telemedicine Using ATM Technology," *International Journal of Electrical and Computer Engineering*, vol. 1, no. 12, pp. 515-519, 2007.

[21] O. M. Abdul-Latif, J. Dubois, "Performance of UWB system in a partially developed fading channel with CCI," *Proceedings of the 5th IEEE GCC Communication and Signal Processing Conference*, Kuwait, pp.1-5, 2009.

[22] R. Dilli, "Design and feasibility verification of 6G wireless communication systems with state of the art technologies," *International Journal of Wireless Information Networks*, vol. 29, pp. 93–117, 2022.

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