H-Infinity Tracking Controller for Linear Discrete-Time Stochastic Systems with Uncertainties

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Abstract: - For linear discrete-time stochastic systems with uncertainties, this paper proposes a tracking control method based on the H-infinity tracking controller and the robust recursive least-squares (RLS) Wiener filter. In linear discrete-time deterministic systems without input and observation noises, the equations for the quantity $u(k)$ with the components of the control and exogenous inputs are as previously described. In Section 2, we show that $u(k)$ satisfies the same equations for linear discrete-time stochastic systems with white input and observation noises as for deterministic systems, based on the separation principle of control and estimation. The results show that the H-infinity tracking control algorithm for linear discrete-time stochastic systems is the same as that for linear discrete-time deterministic systems. The filtered estimate $\hat{x}(k)$ of the system state $x(k)$ is used to compute the estimate $\hat{u}(k)$ of $u(k)$. The robust RLS Wiener filter of Theorem 2 computes the filtered estimate $\hat{x}(k)$ of the system state $x(k)$ for degraded stochastic systems with uncertainties in the system and observation matrices. $\hat{x}(k)$ is updated from $\hat{x}(k-1)$ with the degraded observed value $\breve{y}(k)$, the filtered estimate $\hat{\breve{x}}(k-1)$ of the degraded state $\breve{x}(k-1)$, and the estimate $\hat{u}(k-1)$ of $u(k-1)$.

Key-Words: - H-infinity tracking controller, control input, exogenous input, robust recursive least-squares Wiener filter, discrete-time stochastic systems, uncertain parameters, separation principle.

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1 Introduction

Linear quadratic Gaussian (LQG) control issues have been thoroughly studied, for example, in [1–6]. The discrete-time LQG control problem is described in [2] for stochastic systems with input and observation noises. The optimal control law for the stochastic systems is the same as that for the discrete-time deterministic systems without input and observation disturbances. For state feedback, the control law uses the estimate of the state computed by the Kalman filter. In [6], the discrete-time LQG control that minimizes Massey's directed information from the plant observation output to the control input is studied to achieve the required control performance. The tracking control algorithm based on LQG is described, for instance, in [7-11]. A real-time trans-scale LQG tracking control algorithm for discrete-time stochastic systems is described in [11] and is based on wavelet packet decomposition (WPD). The stochastic systems in this case do not take into account the uncertain parameters. The covariance matrices of the input and observation noises are given. In [12], a controller with output feedback is studied for discrete-time stochastic systems with uncertainty and missing observations. Parameter uncertainty is bounded by the norm. The probability of the occurrence of missing data assumes that it is known. The problem is solved by linear matrix inequalities (LMIs). Based on the disturbance observer, studies in [13] propose a robust controller for linear continuous-time uncertain systems with a time delay. The observer parameters are determined by the solution of LMIs. State feedback control is treated in [13]. In [14], the H-infinity controller is designed for the state-space model with uncertain parameters in linear continuous-time stochastic systems. For linear discrete-time uncertain systems with nonlinear and unbounded uncertainties, the robust controller is developed in [15]. The reduced-order disturbance observer is shown, and the state-feedback controller is designed based on the LMI method. Subsection 5.3 of [16] describes the LMI approach for state feedback quadratic stabilization in linear continuous-time uncertain systems. The feedback gain for the control input is computed by solving LMIs. [17] proposes a mixed H_2/Passivity controller for linear discrete-time uncertain stochastic systems. Some sufficient conditions derived by Lyapunov theory are converted into LMIs. In [18], the iterative tracking controller is designed under the influence of an unknown disturbance with constrained frequency and parameter variations. State tracking control in uncertain stochastic time-varying delay systems is developed. [19] proposes the uncertainty and disturbance estimator for robust tracking control of the reference model in linear continuous-time uncertain stochastic systems.

In linear discrete-time degraded stochastic systems with uncertain parameters, this paper proposes an H-infinity tracking control method based on the H-infinity tracking controller [20] and the robust recursive least-squares (RLS) Wiener filter [21], [22]. The robust RLS Wiener fixed-point smoother is also presented in [21] and [22]. The robust RLS Wiener estimators in [21] are designed for signal estimation. The robust RLS Wiener estimators [22] estimate the state of the system using the degraded observations generated by the state and observation equations with uncertainties. Usually, the robust filter estimates the state of the system with uncertainties using the degraded observations [23]. The robust Kalman filter [24] for uncertain systems assumes multiplicative noise and norm-bounded time-varying uncertainty. For linear discrete-time deterministic systems without input and observation noises, $u(k)$ with control and exogenous input components satisfies (12), (10), and (11) in [20]. Based on the separation principle of control and estimation, it is demonstrated in Section 2 that $u(k)$ satisfies the same equations for linear discrete-time stochastic systems with white input and observation noises as for deterministic systems. As a result, the tracking control algorithm for linear discrete-time stochastic systems with white input and observation noises is the same as for linear discrete-time deterministic systems. The filtered estimate $\hat{x}(k)$ of the system state $x(k)$ is used to get the estimate $\hat{u}(k)$ of $u(k)$. From the state and observation equations (12) with uncertain parameters, the robust RLS Wiener filter of Theorem 2 computes the filtered estimate $\hat{x}(k)$, which is used as the filtered estimate of the system state $x(k)$ for the state equation (1). The robust RLS Wiener filter updates $\hat{x}(k)$ from $\hat{x}(k-1)$ with the degraded observed value $\breve{y}(k)$, the filtering estimate $\hat{\breve{x}}(k-1)$ of the degraded state $\breve{x}(k-1)$, and the estimate $\hat{u}(k-1)$ of $u(k-1)$. Then, the computation of the estimate $\hat{u}(k)$ of $u(k)$ in Theorem 1 uses the filtered estimate $\hat{x}(k)$ of the state $x(k)$ by the robust RLS Wiener filter.

In Section 4, a numerical simulation example compares the tracking control accuracy between the H-infinity tracking controller of Theorem 1 plus the robust RLS Wiener filter of Theorem 2 and the H-infinity tracking controller of Theorem 1 plus the RLS Wiener filter [25]. For the white Gaussian observation noises $N(0,0.1^2)$, $N(0,0.3^2)$, $N(0,0.5^2)$, and $N(0,1)$, the H-infinity tracking controller of Theorem 1 plus the robust RLS Wiener filter of Theorem 2 provides better tracking control accuracy than Theorem 1's H-infinity tracking controller plus the RLS Wiener filter [25].

2 H-Infinity Tracking Control Problem in Linear Discrete-Time Stochastic Systems

Let the nominal state-space model in linear discrete-time stochastic systems be given by (1).

$$\begin{aligned}
&y(k)=z(k)+v(k), z(k)=Cx(k),\\
&x(k+1)=Ax(k)+Gu(k)+\Gamma w(k),\\
&G=[G_1 \quad G_2], u(k)=\begin{bmatrix}u_1(k)\\ u_2(k)\end{bmatrix},\\
&x(0)=c, E[v(k)v^T(s)]=V\delta_K(k-s),\\
&E[w(k)w^T(s)]=W\delta_K(k-s),\\
&E[v(k)w^T(s)]=0, E[x(0)w^T(k)]=0,
\end{aligned} \tag{1}$$

Here, $x(k)\in R^n$ is the state vector; $u(k)\in R^m$ is the input vector; and $z(k)\in R^l$ is the signal vector. The control and exogenous input vectors are, respectively, $u_1(k)\in R^{m_1}$, and $u_2(k)\in R^{m_2}$, $m_1+m_2=m$. Both the input noise $w(k)\in R^p$ and the observation noise $v(k)\in R^l$ are zero-mean white noises that are mutually uncorrelated. $\delta_K(k-s)$ denotes the Kronecker delta function. C represents the $l\times n$ observation matrix. G stands for the $n\times m$ input matrix for $u(k)$, while Γ stands for the $n\times p$ input matrix for $w(k)$. The auto-covariance functions of the input and observation noises are given in (1). Let the expectation of $\|\tilde{z}(k)\|_2^2$ be given by (2), where $\tilde{z}(k)$ represents the performance output [26].

$$E[\|\tilde{z}(k)\|_2^2] = E[(\eta(k) - z(k))^T Q(k) \times \big(\eta(k) - z(k)\big) + u_1^T(k)\tilde{R}(k)u_1(k)] \quad (2)$$

Here, $\eta(k)$ is the desired value and the symmetric matrices $Q(k)$ and $\tilde{R}(k)$ are positive definite. The H-infinity optimal tracking control problem is to find the control input $u_1(k)$ and exogenous input $u_2(k)$ in the disturbance attenuation condition (3) for the minimum value of γ [20]. $\gamma > 0$ is referred to as the constant disturbance attenuation level.

$$\sum_{k=0}^{L} E\,[(\eta(k) - z(k))^T Q(k)\big(\eta(k) - z(k)\big)] + \sum_{k=0}^{L} E\,[u_1^T(k)\tilde{R}(k)u_1(k)] \le \gamma^2 \sum_{k=0}^{L} E\,[u_2^T(k)u_2(k)] \quad (3)$$

An equivalent transformation of the H-infinity tracking control problem for a finite horizon is a two-player, zero-sum linear quadratic dynamic game [27], [28]. That is, given γ^2, we investigate the minimax problem of minimizing the value function $J(x, \boldsymbol{u_1}, \boldsymbol{u_2})$ about $u_1(k)$ and maximizing $J(x, \boldsymbol{u_1}, \boldsymbol{u_2})$ about $u_2(k)$.

$$J(x, \boldsymbol{u_1}, \boldsymbol{u_2}) = \sum_{k=0}^{L} E\,[(\eta(k) - z(k))^T Q(k) \times (\eta(k) - z(k)) + u_1^T(k)\tilde{R}(k)u_1(k) - \gamma^2 u_2^T(k)u_2(k)] \quad (4)$$

Here, the worst-case disturbance $u_2(k)$ is the exogenous input, and $u_1(k)$ is the control input. (4) is expressed as (5) by introducing $R(k) = \begin{bmatrix} \tilde{R}(k) & 0 \\ 0 & -\gamma^2 I_{m_2 \times m_2} \end{bmatrix}$.

$$J(x, \boldsymbol{u_1}, \boldsymbol{u_2}) = \sum_{k=0}^{L} E\,[(\eta(k) - z(k))^T Q(k) \times (\eta(k) - z(k)) + u^T(k)R(k)u(k)] \quad (5)$$

In the value function (5), the discount factor is 1. $x(k)$ is represented as (6).

$$\begin{aligned} x(k) &= \Phi(k,0)c + \sum_{i=0}^{k-1} \Phi\,(k, i+1) \times (Gu(i) + \Gamma w(i)) \\ &= \Phi(k,0)c + \sum_{i=0}^{L} 1\,(k-i-1)\Phi(k, i+1) \times (Gu(i) + \Gamma w(i)) \\ 1(\alpha) &= \begin{cases} 1, 0 \le \alpha, \\ 0, \alpha < 0, \end{cases} \\ \Phi(k,s) &= \begin{cases} A^{k-s}, 0 \le s < k, \\ I, k = s. \end{cases} \end{aligned} \quad (6)$$

Here, $\Phi(k, s)$ represents the state-transition matrix, and $1(\alpha)$ denotes the discrete-time unit step sequence. Substituting (6) into (5), we get (7).

$$\begin{aligned} J(x, \boldsymbol{u_1}, \boldsymbol{u_2}) = \sum_{k=0}^{L} E\,[(&\eta(k) - C\Phi(k,0)c - C\sum_{i=0}^{L} 1\,(k-i-1)\Phi(k,i+1)(Gu(i) + \Gamma w(i)))^T Q(k)(\eta(k) \\ &- C\Phi(k,0)c - C\sum_{i=0}^{L} 1\,(k-i-1)\Phi(k,i+1) \times (Gu(i) + \Gamma w(i))) + u(k)^T R(k)u(k)] \end{aligned} \quad (7)$$

Let $\breve{u}(k)$ be the vector with the components of optimal control and exogenous inputs. By the calculus of variations [20], the necessary condition for $\breve{u}(k)$ to minimize the value function (7) about $u_1(k)$ and maximize (7) about $u_2(k)$ is satisfied by (8).

$$\begin{aligned} &R(k)\breve{u}(k) + \sum_{i=0}^{L}\sum_{j=0}^{L} 1(i-k-1)1(i-j-1)G^T \times \Phi^T(i,k+1)C^T Q(i)C\Phi(i,j+1)G\breve{u}(j) \\ &= \sum_{i=0}^{L} 1\,(i-k-1)G^T\Phi^T(i,k+1)C^T \times Q(i)(\eta(i) - C\Phi(i,0)c) \end{aligned} \quad (8)$$

By introducing

$$K(k,j) = \begin{cases} \sum_{i=k+1}^{L} G^T \Phi^T(i,k+1) C^T Q(i) C\Phi(i,j+1), \\ \qquad 0 \le j \le k \le L \\ \sum_{i=j+1}^{L} G^T \Phi^T(i,k+1) C^T Q(i) C\Phi(i,j+1), \\ \qquad 0 \le k \le j \le L, \end{cases} \quad (9)$$

and

$$m(k+1) = -\sum_{i=k+1}^{L} G^T \Phi^T(i,k+1) C^T Q(i) \times (C\Phi(i,0)c - \eta(i)), \quad (10)$$

the optimal $\breve{u}(k)$ satisfies

$$R(k)\breve{u}(k) + \sum_{j=0}^{L} K(k,j) G\breve{u}(j) = m(k+1). \quad (11)$$

Similar to [20], the sufficient condition for the value function $J(x, \boldsymbol{u_1 u_2})$ to be minimal for $\boldsymbol{u_1}(k)$ and maximal for $\boldsymbol{u_2}(k)$ is $R(k)\delta_K(k-s) + K(k,s)G > 0$. In [20], the integral equation is obtained instead of (11) for linear continuous-time systems. Recently the analysis of integral equations has been studied in [29], [30].

We should note that the results obtained in (9)–(11) for the H-infinity tracking control problem regarding the state-space model (1) are the same as the equations in (20) for the H-infinity tracking control problem in linear deterministic systems. Therefore, the H-infinity tracking control algorithm in [20] is equivalent to the H-infinity tracking control algorithm for the state-space model (1). Theorem 1 presents the H-infinity tracking control algorithm obtained from (9)–(11). The Kalman filter generates the filtered estimate for the discrete-time LQG tracking control algorithm in Section 2 of [10]. The LQG tracking control problem is solvable based on the separation principle of control and estimation. In [10], (11) and (12) compute the filtered estimate while taking the term of the control input into account. The filter gain in the Kalman filter is calculated by (13)–(15) in [10].

Suppose the degraded system of the nominal system (1) is given by (12).

$$\begin{aligned} &\breve{y}(k) = \breve{z}(k) + v(k), \breve{z}(k) = \overleftrightarrow{C}(k)\breve{x}(k), \\ &\overleftrightarrow{C}(k) = C + \Delta C(k), \\ &E[v(k)v^T(s)] = V\delta_K(k-s), \\ &\breve{x}(k+1) = \overleftrightarrow{A}(k)\breve{x}(k) + Gu(k) + \Gamma w(k), \\ &\overleftrightarrow{A}(k) = A + \Delta A(k), \\ &E[w(k)w^T(s)] = W\delta_K(k-s), \\ &E[v(k)w^T(s)], = 0, \\ &E[\Delta A(k)w^T(s)] = 0, E[\Delta A(k)v^T(s)] = 0, \\ &E[\Delta C(k)w^T(s)] = 0, E[\Delta C(k)v^T(s)] = 0, \\ &E[\breve{x}(0)w^T(s)] = 0, E[\breve{x}(0)v^T(s)] = 0 \end{aligned} \quad (12)$$

In (12), the system matrix A and the observation matrix C in (1) are replaced with the degraded system matrix $\overleftrightarrow{A}(k)$ and the degraded observation matrix $\overleftrightarrow{C}(k)$, respectively. It is assumed that $\Delta A(k)$ and $\Delta C(k)$ are uncorrelated with the input noise $w(k)$ and the observation noise $v(k)$. The initial system state $\breve{x}(0)$ is a random vector uncorrelated to both system and measurement noise processes. Under these assumptions, the separation principle of control and estimation can be applied to solve the H-infinity tracking control problem. In other words, the H-infinity tracking control algorithm in Theorem 1 for the nominal state-space model (1) utilizes the robust RLS Wiener filtered estimate in Theorem 2 for the degraded systems with uncertain parameters.

3 H-Infinity Tracking Controller and Robust RLS Wiener Filter

Fig.1 illustrates the structure of the H-infinity tracking controller of Theorem 1 and the robust RLS Wiener filter of Theorem 2.

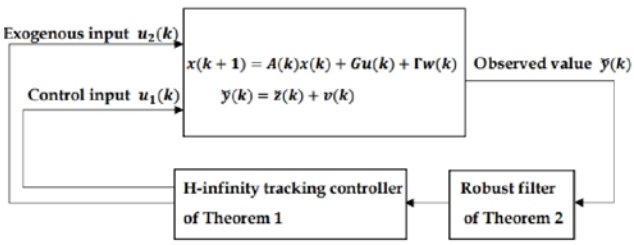

Fig. 1: Structure of the H-infinity tracking controller of Theorem 1 and robust RLS Wiener filter of Theorem 2.

Theorem 1 presents the H-infinity tracking control algorithm for the estimates of the control input, $u_1(k)$, and the exogenous input, $u_2(k)$, with the filtered estimate $\hat{x}(k)$ for $x(k)$. The estimates of

$u_1(k)$ and $u_2(k)$ are represented by $\hat{u}_1(k)$ and $\hat{u}_2(k)$, respectively. The robust RLS Wiener filter of Theorem 2 calculates the filtered estimate $\hat{x}(k)$ of the state $x(k)$ with the degraded observed value $\breve{y}(k)$.

Theorem 1 Assume that $R(k)$ is expressed as $R(k) = \begin{bmatrix} \tilde{R}(k) & 0 \\ 0 & -\gamma^2 I_{m_2 \times m_2} \end{bmatrix}$ and let $\eta(k)$ be the desired value. Assume that $u(k)$ has the components of the control input $u_1(k)$ and the exogenous input $u_2(k)$ as (1).

$$u(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \tag{13}$$

The estimate $\hat{u}(k)$ of $u(k)$ is then calculated by (14)–(17) regarding the nominal state-space model (1). In (14), $\hat{u}_1(k)$ is the estimate of the control input $u_1(k)$, and $\hat{u}_2(k)$ is the estimate of the exogenous input $u_2(k)$.

$$\hat{u}(k) = \begin{bmatrix} \hat{u}_1(k) \\ \hat{u}_2(k) \end{bmatrix} \tag{14}$$

$$\begin{aligned} &\hat{u}(k) = R^{-1}(k)G^T(A^T)^{-1}[A^T P(k+1) \\ &\times (I - GR^{-1}(k)G^T P(k+1))^{-1}A + C^T Q(k)C] \\ &-C^T Q(k)C\hat{x}(k) + R^{-1}(k)G^T(A^T)^{-1} \\ &\times A^T P(k+1)(I - GR^{-1}(k)G^T P(k+1))^{-1} \\ &\times GR^{-1}(k)G^T \xi(k+1) + A^T \xi(k+1) \\ &-C^T Q(k)\eta(k) + R^{-1}(k)G^T(A^T)^{-1} \\ &\times C^T Q(k)\eta(k) \end{aligned} \tag{15}$$

$$\begin{aligned} &P(k) = A^T P(k+1) \\ &\times (I - GR^{-1}(k)G^T P(k+1))^{-1}A \\ &-C^T Q(k)C,\ P(L+1) = 0 \end{aligned} \tag{16}$$

$$\begin{aligned} &\xi(k) = A^T P(k+1) \\ &\times (I - GR^{-1}(k)G^T P(k+1))^{-1} \\ &\times GR^{-1}(k)G^T \xi(k+1) \\ &+A^T \xi(k+1) + C^T Q(k)\eta(k),\ \xi(L+1) = 0 \end{aligned} \tag{17}$$

For the state $x(k)$, we utilize the filtered estimate $\hat{x}(k)$ in (15). The robust RLS Wiener filtering algorithm of Theorem 2 calculates $\hat{x}(k)$ using the degraded observed value $\breve{y}(k)$ in (12), the filtering estimate $\hat{\breve{x}}(k-1)$ of the degraded state $\breve{x}(k-1)$, and the estimate $\hat{u}(k-1)$ of $u(k-1)$. $\overline{P}$ and $\overline{\xi}$ are calculated in the time-reversed direction from time $k = L+1$ until the steady-state values, $\overline{P}$ and $\overline{\xi}$, respectively, are reached. The estimate $\hat{u}(k)$ of $u(k)$ is calculated by (15) using $\overline{P}$ and $\overline{\xi}$. $P(k+1)$ and $\xi(k+1)$ in (15) are replaced with their stationary values, $\overline{P}$ and $\overline{\xi}$, respectively. From the above considerations, the H-infinity tracking control algorithm of Theorem 1 and the robust RLS Wiener filtering algorithm of Theorem 2 adhere to the separation principle of control and estimation.

Let's now quickly review the robust RLS Wiener filter [21], [22]. Assume that an AR model of order N is used to fit the degraded signal sequence of $\breve{z}(k)$.

$$\begin{aligned} &\breve{z}(k) = -a_1\breve{z}(k-1) - a_2\breve{z}(k-2)\cdots \\ &-a_N\breve{z}(k-N) + \breve{e}(k), \\ &E[\breve{e}(k)\breve{e}^T(s)] = \breve{Q}\delta_K(k-s) \end{aligned} \tag{18}$$

The state vector $\breve{x}(k)$ can be used to represent $\breve{z}(k)$ as follows:

$$\begin{aligned} &\breve{z}(k) = \breve{C}\breve{x}(k), \\ &\breve{x}(k) = \begin{bmatrix} \breve{x}_1(k) \\ \breve{x}_2(k) \\ \vdots \\ \breve{x}_{N-1}(k) \\ \breve{x}_N(k) \end{bmatrix} = \begin{bmatrix} \breve{z}(k) \\ \breve{z}(k+1) \\ \vdots \\ \breve{z}(k+N-2) \\ \breve{z}(k+N-1) \end{bmatrix}, \\ &\breve{C} = [I_{l\times l} \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0] \end{aligned} \tag{19}$$

In light of this, the state equation for the state vector $\breve{x}(k)$ is given by

$$\begin{aligned} &\breve{x}(k+1) = \breve{A}\breve{x}(k) + \breve{\Gamma}\zeta(k), \\ &E[\zeta(k)\zeta^T(s)] = \breve{Q}\delta_K(k-s), \\ &\breve{A} = \begin{bmatrix} 0 & I_{l\times l} & 0 & \cdots & 0 \\ 0 & 0 & I_{l\times l} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_{l\times l} \\ -\breve{a}_N & -\breve{a}_{N-1} & -\breve{a}_{N-2} & \cdots & -\breve{a}_1 \end{bmatrix}, \\ &\breve{\Gamma} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ I_{l\times l} \end{bmatrix}, \zeta(k) = \breve{e}(k+N). \end{aligned} \tag{20}$$

The auto-covariance function $\breve{K}(k,s)$ of the state vector $\breve{x}(k)$ has the semi-degenerate functional form of

$$\breve{K}(k,s)=\begin{cases}\Psi(k)\Xi^T(s), 0\le s\le k,\\ \Xi(k)\Psi^T(s), 0\le k\le s,\end{cases}\quad (21)$$
$$\Psi(k)=\breve{\Phi}^k, \Xi^T(s)=\breve{\Phi}^{-s}\breve{K}(s,s).$$

Based on the wide sense stationarity of the auto-covariance function $\breve{K}(k,s)=E[\breve{z}(k)\breve{z}^T(s)]$ for the degraded signal $\breve{z}(k)$, (22) provides the auto-variance function $\breve{K}(k,k)$ of the state vector $\breve{x}(k)$.

$$\breve{K}(k,k)=E\left[\begin{bmatrix}\breve{z}(k)\\ \breve{z}(k+1)\\ \vdots\\ \breve{z}(k+N-1)\end{bmatrix}\times[\breve{z}^T(k)\quad \breve{z}^T(k+1)\quad\cdots\quad \breve{z}^T(k+N-1)]\right]$$
$$=\begin{bmatrix}K_{\breve{z}}(0) & K_{\breve{z}}(-1) & \cdots & K_{\breve{z}}(-N+1)\\ K_{\breve{z}}(1) & K_{\breve{z}}(0) & \cdots & K_{\breve{z}}(-N+2)\\ \vdots & \vdots & \ddots & \vdots\\ K_{\breve{z}}(N-2) & K_{\breve{z}}(N-3) & \cdots & K_{\breve{z}}(-1)\\ K_{\breve{z}}(N-1) & K_{\breve{z}}(N-2) & \cdots & K_{\breve{z}}(0)\end{bmatrix}\quad (22)$$

Using $K_{\breve{z}}(i)$, $0\le i\le N$, the Yule-Walker equation for the AR parameters $\breve{a}_i$, $1\le i\le N$, satisfies

$$\widehat{K}(k,k)\begin{bmatrix}a_1^T\\ a_2^T\\ \vdots\\ a_{N-1}^T\\ a_N^T\end{bmatrix}=-\begin{bmatrix}K_{\breve{z}}^T(1)\\ K_{\breve{z}}^T(2)\\ \vdots\\ K_{\breve{z}}^T(N-1)\\ K_{\breve{z}}^T(N)\end{bmatrix},$$
$$\widehat{K}(k,k)=\begin{bmatrix}K_{\breve{z}}(0) & K_{\breve{z}}(1) & \cdots & K_{\breve{z}}(N-1)\\ K_{\breve{z}}^T(1) & K_{\breve{z}}(0) & \cdots & K_{\breve{z}}(N-2)\\ \vdots & \vdots & \ddots & \vdots\\ K_{\breve{z}}^T(N-2) & K_{\breve{z}}^T(N-3) & \cdots & K_{\breve{z}}(1)\\ K_{\breve{z}}^T(N-1) & K_{\breve{z}}^T(N-2) & \cdots & K_{\breve{z}}(0)\end{bmatrix}.\quad (23)$$

The cross-covariance function of the state vector $x(k)$ with $\breve{x}(s)$ is represented by $K_{x\breve{x}}(k,s)=E[x(k)\breve{x}^T(s)]$. $K_{x\breve{x}}(k,s)$ has the form of

$$K_{x\breve{x}}(k,s)=\alpha(k)\beta^T(s), 0\le s\le k,$$
$$\alpha(k)=\mathrm{A}^k, \beta^T(s)=\mathrm{A}^{-s}K_{x\breve{x}}(s,s)\quad (24)$$

with the system matrix A for the state vector $x(k)$ in (1).

Theorem 2 presents the robust RLS Wiener filtering algorithm based on [22]. The filtered estimate $\hat{x}(k)$ is updated from $\hat{x}(k-1)$ with the degraded observed value $\breve{y}(k)$ in (12), the filtering estimate $\hat{\breve{x}}(k-1)$ of the degraded state $\breve{x}(k-1)$, and the estimate $\hat{u}(k-1)$ of $u(k-1)$. In comparison with the robust RLS Wiener filter in [22], the term $G\hat{u}(k-1)$ is inserted on the right-hand side of (26) for the filtered estimate $\hat{x}(k)$ of $x(k)$.

Theorem 2 Suppose (1) provides the state-space model for the state $x(k)$ in linear discrete-time stochastic systems. Assume that the sequence of the degraded signal $\breve{z}(k)$ is fitted to the AR model of order N. Assume that (22) represents the variance $\breve{K}(k,k)$ of the state $\breve{x}(k)$ concerning the degraded signal $\breve{z}(k)$. Let (24) represent the cross-variance $K_{x\breve{x}}(k,k)$ of the state vector $x(k)$, for the signal $z(k)$ in (1), with the state $\breve{x}(k)$ in (19). Let V denote the variance of the white observation noise $v(k)$. Thus, for the filtered estimate $\hat{x}(k)$ of the state $x(k)$, (25)–(31) constitute the robust RLS Wiener filtering algorithm.

Filtered estimate of the signal $z(k)$: $\hat{z}(k)$

$$\hat{z}(k)=C\hat{x}(k)\quad (25)$$

Filtered estimate of the state $x(k)$: $\hat{x}(k)$

$$\hat{x}(k)=A\hat{x}(k-1)+G\hat{u}(k-1)$$
$$+\Theta(k)(\breve{y}(k)-\breve{C}\breve{A}\hat{\breve{x}}(k-1)),\quad (26)$$
$$\hat{x}(0)=0$$

Filter gain for $\hat{x}(k)$ in (26): $\Theta(k)$

$$\Theta(k)=[K_{x\breve{z}}(k,k)-AS(k-1)\breve{A}^T\breve{C}^T]$$
$$\times\{V+\breve{C}[\breve{K}(k,k)-\breve{A}S_0(k-1)\breve{A}^T]\breve{C}^T\}^{-1}\quad (27)$$
$$K_{x\breve{z}}(k,k)=K_{x\breve{x}}(k,k)\breve{C}^T$$

Filtered estimate of $\breve{x}(k)$: $\hat{\breve{x}}(k)$

$$\hat{\breve{x}}(k)=\breve{A}\hat{\breve{x}}(k-1)$$
$$+g(k)(\breve{y}(k)-\breve{C}\hat{\breve{x}}(k-1)),\quad (28)$$
$$\hat{x}(0)=0$$

Filter gain for $\hat{\breve{x}}(k)$ in (28): $g(k)$

$$g(k)=[\breve{K}(k,k)\breve{C}^T-\breve{A}S_0(k-1)\breve{A}^T\breve{C}^T]$$
$$\times\{V+\breve{C}[\breve{K}(k,k)-\breve{A}S_0(k-1)\breve{A}^T]\breve{C}^T\}^{-1}\quad (29)$$
$$K_{x\breve{z}}(k,k)=K_{x\breve{x}}(k,k)\breve{C}^T$$

Auto-variance function of $\hat{\breve{x}}(k)$: $S_0(k)=E[\hat{\breve{x}}(k)\hat{\breve{x}}^T(k)]$

$$S_0(k) = \breve{A}S_0(k-1)\breve{A}^T$$
$$+g(k)\breve{C}[\breve{K}(k,k) - \breve{A}S_0(k-1)\breve{A}^T, \quad (30)$$
$$S_0(0) = 0$$

Cross-variance function of $\hat{x}(k)$ with $\hat{\breve{x}}(k)$: $S(k) = E[\hat{x}(k)\hat{\breve{x}}^T(k)]$

$$S(k) = AS(k-1)\breve{A}^T$$
$$+\Theta(k)\breve{C}[\breve{K}(k,k) - \breve{A}S_0(k-1)\breve{A}^T, \quad (31)$$
$$S(0) = 0$$

The tracking control algorithm of Theorem 1 calculates $\hat{u}(k)$ by (15) using the filtered estimate $\hat{x}(k)$ calculated by (26).

The flowchart in Fig. 2 is obtained by combining the robust H-infinity tracking controller of Theorem 1 with the RLS Wiener filter of Theorem 2.

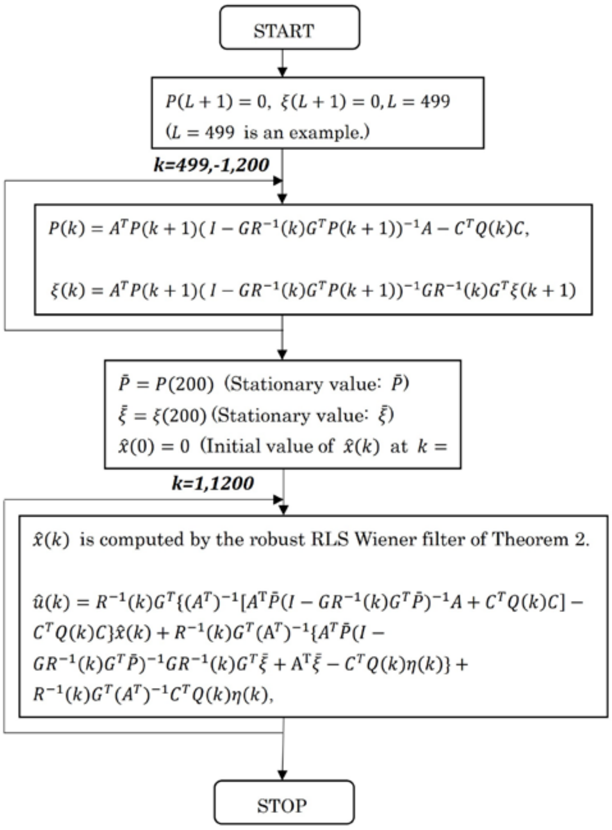

Fig. 2: Flowchart created using the H-infinity tracking controller from Theorem 1 and the robust RLS Wiener filter from Theorem 2.

Section 4 presents a numerical simulation example of the tracking control characteristics of the H-infinity tracking controller using the estimate $\hat{x}(k)$ of $x(k)$ by the robust RLS Wiener filter of Theorem 2 or the RLS Wiener filter [25].

4 A Numerical Simulation Example

Consider the observation and state equations given by

$$\begin{aligned}
&y(k) = z(k) + v(k),\ z(k) = Cx(k),\\
&C = [0.95 \quad -0.4],\\
&x(k+1) = Ax(k) + Gu(k) + \Gamma w(k),\\
&x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix},\\
&u(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}, A = \begin{bmatrix} 0.05 & 0.95 \\ -0.98 & 0.2 \end{bmatrix},\\
&G = \begin{bmatrix} 0.952 & 0 \\ 0.2 & 1 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.952 \\ 0.2 \end{bmatrix},\\
&E[v(k)v(s)] = V\delta_K(k-s),\\
&E[w(k)w(s)] = 0.5^2\delta_K(k-s).
\end{aligned} \quad (32)$$

In (32), $u_1(k)$ is the control input, and $u_2(k)$ is the exogenous input. Consider the following observation and state equations, assuming they produce degraded observations and a degraded signal.

$$\begin{aligned}
&\breve{y}(k) = \breve{z}(k) + v(k),\ \breve{z}(k) = \overleftrightarrow{C}(k)\breve{x}(k),\\
&\overleftrightarrow{C}(k) = C + \Delta C(k),\\
&\Delta C(k) = [0.3 * r(t) \quad 0],\\
&\breve{x}(k+1) = \overleftrightarrow{A}\,\breve{x}(k) + Gu(k) + \Gamma w(k),\\
&\breve{x}(k) = \begin{bmatrix} \breve{x}_1(k) \\ \breve{x}_2(k) \end{bmatrix}, \breve{x}(0) = \begin{bmatrix} 2.3 \\ 2.5 \end{bmatrix},\\
&\overleftrightarrow{A}(k) = A + \Delta A(k),,\\
&\Delta A(k) = \begin{bmatrix} 0.1 * r(t) & 0 \\ 0 & 0.2 * r(t) \end{bmatrix}\\
&E[\Delta A(k)w^T(s)] = 0, E[\Delta A(k)v^T(s)] = 0,\\
&E[\Delta C(k)w^T(s)] = 0, E[\Delta C(k)v^T(s)] = 0.
\end{aligned} \quad (33)$$

Here, "$r(t)$" refers to a MATLAB or GNU Octave function that generates uniformly distributed random numbers in the range (0,1) . In (33), conditions such as norm-bounded uncertainty [18] are not imposed on the uncertain matrices $\Delta A(k)$ and $\Delta C(k)$. The robust RLS Wiener filtering algorithm of Theorem 2 does not use the information on the uncertain matrices $\Delta A(k)$ and $\Delta C(k)$ at all. The robust RLS Wiener filter of Theorem 2 computes the filtered estimate $\hat{x}(k)$ in

(26) to get the estimate $\hat{u}(k)$ of $u(k)$ in (15). Given the desired value $\eta(k) = 10$, $\gamma = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$, Fig. 3 illustrates the signal $z(k) = Cx(k)$ and its filtered estimate $\hat{z}(k)$ by the H-infinity tracking controller of Theorem 1 and the robust RLS Wiener filter of Theorem 2 vs. k for the

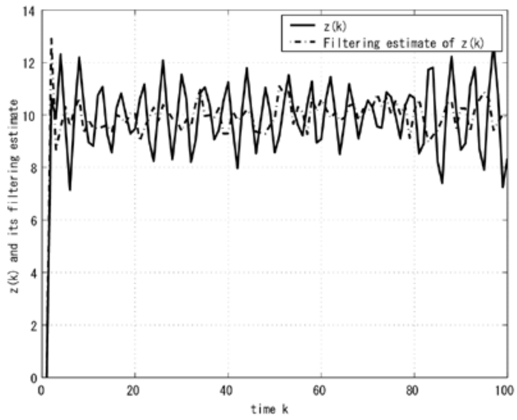

Fig. 3: $z(k) = Cx(k)$ and its filtered estimate $\hat{z}(k)$ vs. k for white Gaussian observation noise $N(0, 0.3^2)$, given the desired value $\eta(k) = 10$, $\gamma = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$.

white Gaussian observation noise $N(0, 0.3^2)$. From Fig. 3, the sequence of the filtered estimates $\hat{z}(k)$ is closer to the desired value of 10 than the signal $z(k)$. Fig. 4 illustrates the estimate $\hat{u}_1(k)$ of the control input $u_1(k)$ vs. k for the white Gaussian observation noise $N(0, 0.3^2)$, given $\eta(k) = 10$, $\gamma = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$. Fig. 5 illustrates the estimate $\hat{u}_2(k)$ of the exogenous input $u_2(k)$ vs. k for the white Gaussian observation noise $N(0, 0.3^2)$, given $\eta(k) = 10$, $\gamma = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$. In Figs 4 and 5, the H-infinity tracking controller of Theorem 1 and the robust RLS Wiener filter are used. Figs. 4 and 5 show that the $\hat{u}_2(k)$ sequence's amplitude is considerably smaller than that of the $\hat{u}_1(k)$ sequence.

Table 1 shows the mean-square values (MSVs) of the tracking errors $\eta(k) - z(k)$, $z(k) = Cx(k)$, and $\eta(k) - \hat{z}(k)$, $\hat{z}(k) = C\hat{x}(k)$, $1 \le k \le 1200$, by the H-infinity tracking controller of Theorem 1 and the robust RLS Wiener filter of Theorem 2 for $\gamma = 10$ and $\gamma = 0.01$, given $\eta(k) = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$. Here, the observation noises are $N(0, 0.1^2)$, $N(0, 0.3^2)$, $N(0, 0.5^2)$, $N(0,1)$ and $N(0, 5^2)$. The MSV of the tracking errors $\eta(k) - \hat{z}(k)$ is fairly smaller than the MSV of the tracking errors $\eta(k) - z(k)$ for each observation noise. This indicates that the filtered estimate $\hat{z}(k)$ tracks the desired value with high accuracy. For $\gamma = 10$ and $\gamma = 0.01$, the MSVs of the tracking errors $\eta(k) - z(k)$ are almost the same for each observation noise.

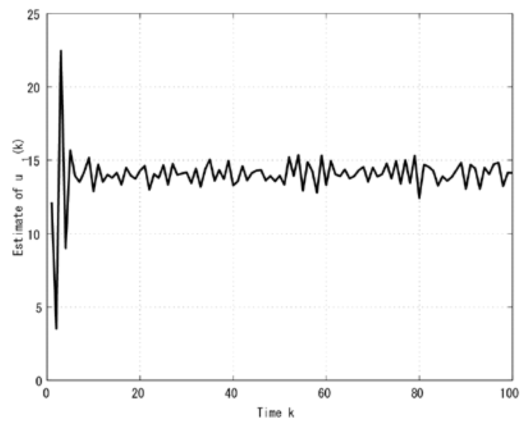

Fig. 4: Estimate $\hat{u}_1(k)$ of control input $u_1(k)$ vs. k for white Gaussian observation noise $N(0, 0.3^2)$, given the desired value $\eta(k) = 10$, $\gamma = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$.

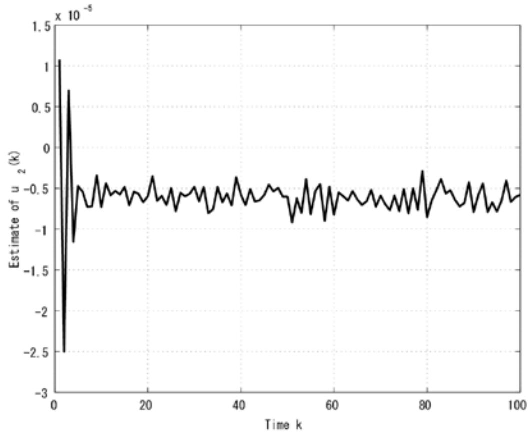

Fig. 5: Estimate $\hat{u}_2(k)$ of exogenous input $u_2(k)$ vs. k for white Gaussian observation noise $N(0, 0.3^2)$, given the desired value $\eta(k) = 10$, $\gamma = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$.

Similarly, for $\gamma = 10$ and $\gamma = 0.01$, the MSVs of the tracking errors $\eta(k) - \hat{z}(k)$ are almost the same for each observation noise. The MSVs of $\eta(k) - z(k)$ and $\eta(k) - \hat{z}(k)$ are minimums for the white Gaussian observation noise $N(0, 5^2)$, respectively. Table 2 shows the MSVs of the tracking errors $\eta(k) - z(k)$, $z(k) = Cx(k)$ and $\eta(k) - \hat{z}(k)$,

$\hat{z}(k) = C\hat{x}(k)$, $1 \le k \le 1200$, by the H-infinity tracking controller of Theorem 1 and the RLS Wiener filter [25] for $\gamma = 10$ and $\gamma = 0.01$, given $\eta(k) = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$. Table 2 shows that the tracking errors $\eta(k) - z(k)$ and $\eta(k) - \hat{z}(k)$ diverge for both $\gamma = 10$ and $\gamma = 0.01$ in the white Gaussian observation noises $N(0, 0.1^2)$ and $N(0, 0.3^2)$. The MSVs of the tracking errors $\eta(k) - z(k)$ and $\eta(k) - \hat{z}(k)$ are extremely large in both

Table 1. Mean-square values of tracking errors $\eta(k) - z(k)$, $z(k) = Cx(k)$and $\eta(k) - \hat{z}(k)$, $\hat{z}(k) = C\hat{x}(k)$, $1 \le k \le 1200$, by H-infinity tracking control algorithm plus robust RLS Wiener filter for $\gamma = 10$ and $\gamma = 0.01$, given $\eta(k) = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$.

White Gaussian observation noise	$\gamma = 10$		$\gamma = 0.01$	
	MSV of tracking errors $\eta(k) - z(k)$	MSV of tracking errors $\eta(k) - \hat{z}(k)$	MSV of tracking errors $\eta(k) - z(k)$	MSV of tracking errors $\eta(k) - \hat{z}(k)$
$N(0, 0.1^2)$	0.6507	0.0690	0.6485	0.0677
$N(0, 0.3^2)$	0.6583	0.0779	0.6605	0.0773
$N(0, 0.5^2)$	0.6114	0.0810	0.6215	0.0816
$N(0,1)$	0.6510	0.0879	0.5806	0.0814
$N(0, 5^2)$	0.2914	0.0460	0.2894	0.0465

Table 2. Mean-square values of tracking errors $\eta(k) - z(k)$, $z(k) = Cx(k)$and $\eta(k) - \hat{z}(k)$, $\hat{z}(k) = C\hat{x}(k)$, $1 \le k \le 1200$, by H-infinity tracking control algorithm plus RLS Wiener filter [25] for $\gamma = 10$ and $\gamma = 0.01$, given $\eta(k) = 10$, $\tilde{R} = 0.0001$, and $Q(k) = 1$.

White Gaussian observation noise	$\gamma = 10$		$\gamma = 0.01$	
	MSV of tracking errors $\eta(k) - z(k)$	MSV of tracking errors $\eta(k) - \hat{z}(k)$	MSV of tracking errors $\eta(k) - z(k)$	MSV of tracking errors $\eta(k) - \hat{z}(k)$
$N(0, 0.1^2)$	Diver-gence	Diver-gence	Diver-gence	Diver-gence
$N(0, 0.3^2)$	Diver-gence	Diver-gence	Diver-gence	Diver-gence
$N(0, 0.5^2)$	2.2315e+198	4.5986e+198	9.5268e+198	1.9712e+199
$N(0,1)$	7.9888	0.3956	7.7168	0.3835
$N(0, 5^2)$	0.4185	0.0411	0.4200	0.0411

$\gamma = 10$ and $\gamma = 0.01$ for the white Gaussian observation noise $N(0, 0.5^2)$. The MSV of the tracking errors $\eta(k) - z(k)$ by the tracking controller of Theorem 1 and the robust RLS Wiener filter of Theorem 2 is smaller than the MSVs by the tracking controller of Theorem 1 and the RLS Wiener filter [25] for the white Gaussian observation noises $N(0,1)$ and $N(0, 5^2)$ when $\gamma = 10$ and $\gamma = 0.01$. In the observation noise $N(0,1)$, for $\gamma = 10$ and $\gamma = 0.01$, the MSV of the tracking errors $\eta(k) - \hat{z}(k)$ in Table 1 is smaller than the MSV of the tracking errors $\eta(k) - \hat{z}(k)$ in Table 2. In the case of white Gaussian observation noise $N(0, 5^2)$, for $\gamma = 10$ and $\gamma = 0.01$, the MSV of the tracking errors $\eta(k) - \hat{z}(k)$ in Table 1 is almost the same as the MSV of the tracking errors $\eta(k) - \hat{z}(k)$ in Table 2. For the observation noises $N(0, 0.1^2)$, $N(0, 0.3^2)$, $N(0, 0.5^2)$, and $N(0,1)$, the H-infinity tracking controller of Theorem 1 and the robust RLS Wiener filter of Theorem 2 are superior in tracking control accuracy to the H-infinity tracking controller of Theorem 1 and the RLS Wiener filter [25].

5 Conclusion

This study proposed a tracking control technique based on the H-infinity tracking controller of Theorem 1 and the robust RLS Wiener filter of Theorem 2. In previous research, it has been shown that $\breve{u}(k)$ with control and exogenous input components satisfies (9), (10), and (11) in linear discrete-time deterministic systems. For the stochastic systems (1), based on the separation principle of control and estimation, $\breve{u}(k)$ also satisfies (9), (10), and (11). As a result, the tracking control algorithm of Theorem 1 is applied to the linear state-space model (1). For the degraded stochastic system (12), (26) in the robust RLS Wiener filter of Theorem 2 updates the filtered estimate $\hat{x}(k)$ of $x(k)$ from $\hat{x}(k-1)$ with the degraded observed value $\breve{y}(k)$, the filtered estimate $\hat{\breve{x}}(k-1)$ of the degraded state $\breve{x}(k-1)$, and the estimate $\hat{u}(k-1)$ of $u(k-1)$. Estimating $\hat{u}(k)$ of $u(k)$ in (15) uses the filtered estimate $\hat{x}(k)$ of the state $x(k)$ by the robust RLS Wiener filter.

The numerical simulation example compares the tracking control accuracy of the proposed method with that of the technique based on the H-infinity tracking controller of Theorem 1 and the RLS Wiener filter. As a result, the tracking controller of Theorem 1 and the robust RLS Wiener filter of Theorem 2 provide higher tracking control accuracy for the white Gaussian observation noises $N(0, 0.1^2)$, $N(0, 0.3^2)$, $N(0, 0.5^2)$, and $N(0,1)$. For $\gamma = 10$ and $\gamma = 0.01$, the MSV of the tracking errors $\eta(k) - \hat{z}(k)$ by the tracking controller and the robust RLS Wiener filter is almost the same as the MSV by the H-infinity tracking controller of Theorem 1 and the RLS Wiener filter in the observation noise $N(0, 5^2)$, respectively.

A future task is to design an H-infinity tracking controller with a robust RLS Wiener filter that estimates the degraded state in linear discrete-time uncertain systems.

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Conflict of Interest

The author has no conflict of interest to declare that is relevant to the content of this article.

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