All-to-All Broadcast in optical WDM ring with 2-length extension and 3-length extension

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Abstract: - All-to-all broadcast communication is to transmit a distinctive data from each node to every other node available in the network. This is a fundamental prerequisite in high performance computing and telecommunication networks including enterprises operating datacenters which may spread across thousands of nodes by means of WDM optical networks. Wavelengths are scarce resources in WDM optical networks. Reducing the wavelengths usage is essential to decrease the price and complication of the network. In this article, a ring network is extended by additionally connecting alternate nodes in order to provide alternate paths so as to reduce the effective number of hops between the communicating nodes and also to reduce the wavelength requirements and this network is referred as a ring network with 2-length extension. Similarly, a ring network is extended by directly linking all nodes which are separated by two intermediate nodes with additional fibers and this network is referred as a ring network with 3-length extension. For the ring network with 2-length extension the optimum wavelength number necessary to establish all-to-all broadcast under unidirectional routing is derived by grouping nonoverlapping connections on a common wavelength. The wavelength number necessary atmost to establish all-to-all broadcast in a unidirectional ring with 3-length extension is derived using longest link first routing algorithm.

Key-Words: - All-to-All Broadcast, WDM Optical Network, Linear Array, Wavelength Assignment, RWA, Modified Linear Array

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1 Introduction

A WDM optical network has the potential of interconnecting thousands of users covering local to wide area networks. The WDM optical network employs numerous optical nodes and nodes are interconnected using optical fibers in some fashion. WDM technology permits the passage for multiple wavelength optical signals through the same fiber and thus provides abundant bandwidth. Each optical node employs required optical sources (Ex: laser diodes) at the transmitter section to modulate the input electrical signals with light signal as carrier and required optical detectors (Ex: photo diodes) at the receiver section to demodulate the received signal and extract the input signal that was fed at the transmitter. Though the same fiber can be used for signal transmission in both forward and reverse directions, it is normally assumed that each optical link is a set of two fibers, with one fiber dedicated to forward transmission and another one for reverse transmission. An optical connection (lightpath) (m, n) corresponds to the establishment of an optical path for transfer information from source m to destination n on a distinctive wavelength. In the absence of wavelength converters at the intermediate optical nodes, each lightpath needs to be on the same wavelength from source to destination. All-to-all broadcast communication, distributing messages from each node to every other node, is a opaque communication pattern and finds abundant applications from network control plane to datacenters[1-3]. In general, all-to-all broadcast is employed for numerous applications in advanced distributed computing and communication systems which employ WDM optical networks comprising hundreds of optical nodes at the backbone and involving huge number of operating wavelengths [4-18]. Wavelength need to be assigned for various lightpaths in such a way that no two lightpaths are established using the same wavelength, if they share any common link along entire route. Wavelengths being a scarce and costly resource, hence their utilization must be limited to reduce the complexity and cost of the network. WDM optical all-to-all broadcast was extensively analysed by many researchers but still it contains so many research challenges. All-to-all broadcast was studied for numerous optical networks like ring, linear-array, torus, mesh and tree under all optical routing models. Preceding research works [19-22] propose interconnecting the alternate nodes of primary ring network with additional link, and called as modified / extended ring topology to establish enormous traffic requirement with high speed, enlarged call connection probability and improved survivability. The link and node failure analysis are studied for the modified/ extended ring networks topology [23-24]. Also, the wide-sense nonblocking multicast communication for modified/ extended ring is studied [25]. In this workr, we examine WDM all optical all-to-all broadcast in a ring with 3-length extension network, as it guarantees lower wavelength necessities and emerge attractive for optical control plane.

In this article, a study on all-to-all broadcast communication in a ring with 2-length extension and ring with 3-length extension network was performed under all optical routing model. The wavelength assignment for all-to-all broadcast is studied for the unidirectional ring with 2-length extension and unidirectional ring with 3-length extension network using longest link first routing algorithm. The wavelength number necessary for all-to-all broadcast is derived. Section 2 gives the preliminaries necessary to study the unidirectional ring with 2 length extension and 3 length extension. Section 3 provides the wavelength number necessary to establish all-to-all broadcast in ring with 2 length extension and 3-length extension under uni directional communication. Section 4 discusses the results obtained the in the work. Section 5 concludes the article highlighting future research avenues.

2 Preliminaries

2.1 Unidirectional Ring with 2-length Extension

Fig. 1 shows an eight node (0 to 7) ring with 2-length extension [19-22]. Every node of the ring network is additionally interconnected to an alternate node using additional link.

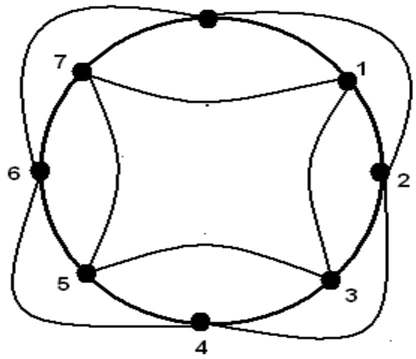

Fig. 1: An eight node Ring with 2 length extension

At each node, data can move from node x to node $x \oplus 1$ and node $x \oplus 2$ where $\oplus$ denotes modulo N addition. This offers alternate paths and it decrease the number of hops and also to decrease the wavelength necessities for all-to-all broadcast.

Example 1: Wavelength assignment for all connections of all-to-all broadcast in an eight-node unidirectional ring network using longest link first routing algorithm.

Consider the eight-node ring with 2-length extension shown in Fig. 1. All-to-all broadcast connections are listed as shown below,

(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(0,7)
(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),(1,0)
(2,3),(2,4),(2,5),(2,6),(2,7),(2,0),(2,1)
(3,4),(3,5),(3,6),(3,7),(3,0),(3,1),(3,2)
(4,5),(4,6),(4,7),(4,0),(4,1),(4,2),(4,3)
(5,6),(5,7),(5,0),(5,1),(5,2),(5,3),(5,4)
(6,7),(6,0),(6,1),(6,2),(6,3),(6,4),(6,5)
(7,0),(7,1),(7,2),(7,3),(7,4),(7,5),(7,6)

The non-overlapping connections are grouped in the above set of connections and prevalent wavelength is allocated as shown below:

$\{(0,1),(4,0),(1,2),(2,1)\} - \lambda_1$
$\{(2,3),(3,2),(3,4),(4,3)\} - \lambda_2$
$\{(4,5),(5,4),(5,6),(6,5)\} - \lambda_3$
$\{(6,7),(7,6),(7,0),(0,7)\} - \lambda_4$
$\{(0,2),(2,0),(1,3),(3,1)\} - \lambda_5$
$\{(2,4),(4,2),(3,5),(5,3)\} - \lambda_6$
$\{(4,6),(6,4),(5,7),(7,5)\} - \lambda_7$
$\{(6,0),(0,6),(7,1),(1,7)\} - \lambda_8$
$\{(0,3),(3,0),(1,4),(4,1)\} - \lambda_9$
$\{(2,5),(5,2),(3,6),(6,3)\} - \lambda_{10}$
$\{(4,7),(7,4),(5,0),(0,5)\} - \lambda_{11}$
$\{(6,1),(1,6),(7,2),(2,7)\} - \lambda_{12}$
$\{(0,4),(4,0),(1,5),(5,1)\} - \lambda_{13}$
$\{(2,6),(6,2),(3,7),(7,3)\} - \lambda_{14}$

Thus, 14 wavelength numbers are necessary atmost for an eight-node unidirectional ring with 2-length extension to establish all-to-all broadcast.

2.2 Unidirectional Ring with 3-length Extension

Fig. 2 shows a 12-node (indexed from 0 to 11) ring with 3-length extension. A ring network is extended by additionally connecting two nodes which are separated by two intermediate nodes with additional fibers. This network is referred as ring with 3-length extension. That is, each nodex is straightly connected to node $(x \oplus 1)$and node $(x \oplus 3)$ where $\oplus$ denotes addition modulo N. This provides alternate paths so as to aid reduce the effective number of hops for communicating nodes and also to reduce the wavelength number necessary for all-to-all broadcast.

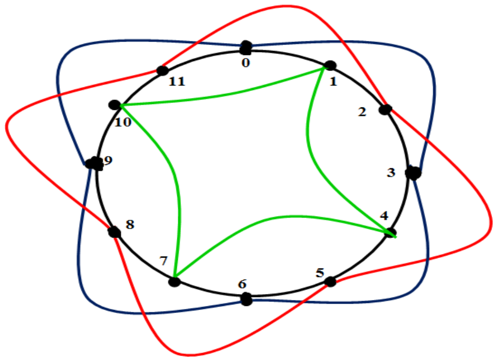

Fig. 2: A 12-node ring with 3-length extension

Example 2: Wavelength allotment for all-to-all broadcast in a 12-node unidirectional ring with 3-length extension using longest link first routing algorithm.
Consider the 12-node ring with 3-length extension shown in Fig. 2 All-to-all broadcast connections can be listed as shown below:
(0,1), (0,2), (0,3), (0,4), (0,5), (0,6), (0,7), (0,8), (0,9), (0,10), (0,11)
(1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (1,10), (1,11), (1,0)
(2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (2,9), (2,10), (2,11), (2,0), (2,1)
(3,4), (3,5), (3,6), (3,7), (3,8), (3,9), (3,10), (3,11), (3,0), (3,1), (3,2)
(4,5), (4,6), (4,7), (4,8), (4,9), (4,10), (4,11), (4,0), (4,1), (4,2), (4,3)
(5,6), (5,7), (5,8), (5,9), (5,10), (5,11), (5,0), (5,1), (5,2), (5,3), (5,4)
(6,7), (6,8), (6,9), (6,10), (6,11), (6,0), (6,1), (6,2), (6,3), (6,4), (6,5)
(7,8), (7,9), (7,10), (7,11), (7,0), (7,1), (7,2), (7,3), (7,4), (7,5), (7,6)
(8,9), (8,10), (8,11), (8,0), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7)
(9,10), (9,11), (9,0), (9,1), (9,2), (9,3), (9,4), (9,5), (9,6), (9,7)(9,8)
(10,11), (10,0)(10,1), (10,2), (10,3), (10,4), (10,5), (10,6), (10,7),
(10,8), (10,9)
(11,0)(11,1)(11,2), (11,3), (11,4), (11,5), (11,6), (11,7), (11,8),
(11,9), (11,10)
The non-overlapping connections are grouped in the above set of connections and prevalent wavelength is allocated as shown below:

{(0,1),(1,0),(4,5),(5,4),(8,9),(9,8)} -λ1
{(1,2),(2,1),(5,6),(6,5),(9,10),(10,9)} -λ2
{(2,3),(3,2),(6,7),(7,6),(10,11),(11,10)} -λ3
{(3,4),(4,5),(7,8),(8,7),(11,0),(0,11)} -λ4
{(0,2),(2,0),(4,6),(6,4),(8,10),(10,8)} -λ5
{(1,3),(3,1),(5,7),(7,5),(9,11),(11,9)} -λ6
{(2,4),(4,2),(6,8),(8,6),(10,0),(0,10)} -λ7
{(3,5),(5,3),(7,9),(9,7),(11,1),(1,11)} -λ8
{(0,3),(3,0),(4,7),(7,4),(8,11),(11,8)} -λ9
{(1,4),(4,1),(5,8),(8,5),(9,0),(0,9)} -λ10
{(2,5),(5,2),(6,9),(9,6),(10,1),(1,10)} -λ11
{(3,6),(6,3),(7,10),(10,7),(11,2),(2,11)} -λ12
{(0,4),(4,0),(4,8),(8,4),(8,0),(0,8)} -λ13
{(1,5),(5,1),(5,9),(9,5),(9,1),(1,9)} -λ14
{(2,6),(6,2),(6,10),(10,6),(10,2),(2,10)} -λ15
{(3,7),(7,3),(7,11),(11,7),(11,3),(3,11)} -λ16
{(0,5),(5,0),(4,9),(9,4),(8,1),(1,8)} -λ17
{(1,6),(6,1),(5,10),(10,5),(9,2),(2,9)} -λ18
{(2,7),(7,2),(6,11),(11,6),(10,3),(3,10)} -λ19
{(3,8),(8,3),(7,0),(0,7),(11,4),(4,11)} -λ20
{(0,6),(6,0),(1,7),(7,1),(2,8),(8,2)} -λ21
{(3,9),(9,3),(4,10),(10,4),(5,11),(11,5)} -λ22

Thus, 22 wavelength numbers are necessary atmost for a 12-node unidirectional ring with 3-length extension to establish all-to-all broadcast.

The following definitions are necessary to understand the analysis done in this chapter.
Definition 1: "A link that joins the nodes x and (x ⊕ 1) is said to be shorter link. A link that directly joins the nodes x and (x ⊕ 2) and nodes x and (x ⊕ 3) is said to be longer link" [22].
Definition 2: "A connection is the set of all links that joins source node and destination node following a prescribed routing method" [22].
Definition 3: "A connection that selects longer link over a shorter link at the source node and at various intermediate nodes to reach the destination node is said to follow 'longest link first routing'. For example, in Figure 1, under longest link first routing, a connection from source node 2 to destination node 5 selects first the longer link inter connecting the node 2 with 4 and then the shorter link joining node 4 with 5" [22].
Definition 4: "If the number of intermediate nodes between the source node and destination node in the primary ring is $l - 1$, then the connection is called a length l connection. For example, in Figure 4.1, if the source node is indexed 2 and the destination node is indexed 5, then the length of the connection is 3" [22].

Lemma 1: Under the longest link first routing algorithm, for $1 \le l \le \left\lfloor \frac{N}{2} \right\rfloor$, a l length connections initiating from a source node with even index do not interfere with another same length connection initiating from a source node with odd index.

Proof: It can be observe that all connections of length $l = 2a$, where a is positive integer use only longer links inter connecting two even numbered nodes, if the index of source node is an even number whereas all connections of same length l, use onlylonger links inter connecting two odd numbered nodes, if the index of source node is an odd number. Also, all connections of length $l = 2a + 1$, use first only longer links inter connecting two even numbered nodes and finally end with a shorter link inter connecting even numbered node with an odd numbered node, if the index of source node is an even number. Whereas, all connections of same length use first only longer links inter connecting two odd numbered nodes and finally end with a shorter link inter connecting odd numbered node with an even numbered node, if the index of source node is an odd number. Hence, all the l length connections initiating from source node with even index do not interfere with all the same length connections initiating from source node with odd index because they never share any common link.

Lemma 2: Based on longest link first routing algorithm, for $3 \leq l \leq \left\lfloor \frac{N}{2} \right\rfloor$, and $(l \bmod 3 = 0)$, three connections of length l, and initiating (source) from any 3 consecutive nodes never interfere with each other.

Proof: Let $a, a \oplus 1, a \oplus 2$ be the index of the three consecutive nodes where $a \geq 0$. A connection of length l, initiating from node index a, first use the longer links inter connecting the nodes a and $a \oplus 3$, then nodes $a \oplus 3$ with $a \oplus 6$ and so on. Similarly, l length connections initiating from node index $a \oplus 1$, first use the longer links inter connecting the nodes $a \oplus 1$ and $a \oplus 4$, then nodes $a \oplus 4$ with $a \oplus 7$, and so on. Also, length l connections initiating from node index $a \oplus 2$, first use the longer links inter connecting the nodes $a \oplus 2$ and $a \oplus 5$, then nodes $a \oplus 5$ with $a \oplus 8$, and so on. Hence, these 3 sets of connections never share any common link and hence they do not interfere with each other.

Lemma 3: Under longest link first routing algorithm, for $4 \leq l \leq \left\lfloor \frac{N}{2} \right\rfloor$, and $(l \bmod 3 = 1)$, three connections of length l, and initiating (source) from any 3 consecutive nodes never interfere with each other.

Proof: Let $a, a \oplus 1, a \oplus 2$ be the index of the three consecutive nodes where $a \geq 0$. A connection of length l, initiating from node index a, first use the longer links inter connecting the nodes a and $a \oplus 3$, then nodes $a \oplus 3$ with $a \oplus 6$, and so on and finally end with one shorter link. Similarly, length l connections initiating from node index $a \oplus 1$,first use the longer links inter connecting the nodes $a \oplus 1$ and $a \oplus 4$, then nodes $a \oplus 4$ with $a \oplus 7$, and so on and finally end with one shorter link. Also, length l connections initiating from node index $a \oplus 2$, first use the longer links inter connecting the nodes $a \oplus 2$ and $a \oplus 5$, then nodes $a \oplus 5$ with $a \oplus 8$, and so on and finally end with one shorter link. As the longer links involved in the 3 sets of connections are completely different, the shorter link immediately following the last longer link in the 3 sets of connections will also be different (as the source node of shorter links are not same). Hence, these 3 sets of connections never share any common link and hence they do not interfere with each other.

Lemma 4: Let$N$ be a positive integer. Then, two wavelengths are sufficient to establish all connections of length $l = 2$ in a N node ring with 3-length extension using longest link first routing algorithm.

Proof: Let $a, a \oplus 2$ be the index of the two nodes (where $a \geq 0$) which are separated by exactly one intermediate node indexed $a \oplus 1$. A connection of length $l = 2$, initiating from node index a, involve two consecutive shorter links, first the link inter connecting the nodes a and $a \oplus 1$, then the link inter connecting the nodes $a \oplus 1$ and $a \oplus 2$. Similarly, connections of length $l = 2$, initiating from node index $a \oplus 2$, involve two consecutive shorter links, first the link inter connecting the nodes $a \oplus 2$ and $a \oplus 3$, then the link inter connecting the nodes $a \oplus 3$ and $a \oplus 4$. Hence, these 2 sets of connections never share any common link and hence they do not interfere with each other. Hence, two wavelengths are sufficient to route all length 2connections.

Lemma 5: Under longest link first routing algorithm, for $5 \leq l \leq \left\lfloor \frac{N}{2} \right\rfloor$, and $(l \bmod 3 = 2)$, connections of same length l, and initiating (source) from any 3 nodes which are separated by exactly one intermediate node they never interfere with each other.

Proof: Let $a, a \oplus 2, a \oplus 4$ be the indices of the three nodes (where $a \geq 0$) which are separated by one intermediate nodes. A connection of length l, initiating from node index a,first use the longer links inter connecting the nodes a and $a \oplus 3$, then nodes $a \oplus 3$ with $a \oplus 6$, and so on and finally end with two consecutive shorter links. Similarly, length l connections initiating from node index $a \oplus 2$, first use the longer links inter connecting the nodes $a \oplus 2$ and $a \oplus 5$, then nodes $a \oplus 5$ with $a \oplus 8$, and so on and finally end with two consecutive shorter links. Similarly, length l, connections initiating from node index $a \oplus 4$, first use the longer links inter connecting the nodes $a \oplus 4$ and $a \oplus 7$, then nodes $a \oplus 7$ with $a \oplus 10$, and so on and finally end with two consecutive shorter links. As the longer links involved in the 3 sets of connections are different, the two consecutive shorter links immediately following the longer links in the 3 sets of connections would also be different (as the indices of the source node of the first shorter link in the 3 set of connections differ exactly by 2). Hence, these 3 sets of connections do not share any common link and hence they do not interfere with each other.

3 Main Results

In this section, first we derive the wavelength number necessary to establish all connections of all-to-all broadcast and then the link load of the ring network with 2-length extension and ring network with 2-length extension. The connections are routed based on the longest link first routing technique. The results of Theorems proved below are based on the principle on grouping non overlapping connections on a common wavelength.

3.1 Unidirectional Ring with 2-length Extension

Theorem 1: Let N be an odd integer and $N \geq 5$. Then, the wavelength number necessary for establishing all-to-all broadcast in a N node unidirectional ring with 2-length extension is at most $\frac{N^2-1}{4}$ using longest link first routing algorithm.

Proof: Define a group $G(i, j)$ as

$$G(i,j) = \{(2i, 2i \oplus j), (2i \oplus j, 2i), (2i \oplus 1, 2i \oplus 1 \oplus j), (2i \oplus 1 \oplus j, 2i \oplus 1)\}$$

for every integer i and j such that $0 \leq i \leq \frac{N-3}{2}$ and $1 \leq j \leq \frac{N-1}{2}$.

It can be observe that connections in $G(i, j)$ do not overlap. Hence, we assign a distinctive wavelength to all connections in a single $G(i, j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G(i, j)$ which is equal to

$$\sum_{i=0}^{\frac{N-3}{2}} \sum_{j=1}^{\frac{N-1}{2}} 1 = \sum_{i=0}^{\frac{N-3}{2}} \frac{N-1}{2} = \left(\frac{N-1}{2}\right)\left(\frac{N-3}{2}+1\right) = \left(\frac{N-1}{2}\right)^2 . (1)$$

The remaining connections can be grouped as $H(j) = \{(N-1, N-1 \oplus j), (N-1 \oplus j, N-1)\}$ for $1 \leq j \leq \frac{N-1}{2}$

It can be observe that connections in $H(j)$ do not overlap. Hence, we assign a distinctive wavelength to all connections in a single $H(j)$. Hence, the total wavelength number necessary to assign for all connections is equal to the number of groups $H(j)$, which is equal to $\left(\frac{N-1}{2}\right)$. (2)

Since the groups $G(i, j)$ and $H(j)$ defined above contains all the connections, by adding (1) & (2) we get $\frac{N^2-1}{4}$ wavelength number necessary to establish all-to-all broadcast communication.

Theorem 2: Let $N = 4m$ where mis a positive integer and $N \geq 8$. Then, the wavelength number necessary to establish all-to-all broadcast in a unidirectional N node ring with 2-length extension is at most $\frac{N(N-1)}{4}$ using longest link first routing algorithm.

Proof: Define a group $G(i, j)$ as

$$G(i,j) = \{(2i, 2i \oplus j), (2i \oplus j, 2i), (2i \oplus 1, 2i \oplus 1 \oplus j), (2i \oplus 1 \oplus j, 2i \oplus 1)\}$$

for every integer i and j such that $0 \leq i \leq \frac{N}{2} - 1$ and $1 \leq j \leq \frac{N-2}{2}$.

It can be observe that connections in $G(i, j)$ do not overlap. Hence, we assign a distinctive wavelength to all connections in a single $G(i, j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of group $G(i, j)$ which is equal to

$$\sum_{i=0}^{\frac{N}{2}-1} \sum_{j=1}^{\frac{N-2}{2}} 1 = \sum_{i=0}^{\frac{N}{2}-1} \frac{N-2}{2} = \left(\frac{N-2}{2}\right)\left(\frac{N}{2}\right) = \frac{(N-2)(N)}{4}. \quad (3)$$

The remaining connections can be grouped as

$$H(j) = \left\{\left(2j, 2j \oplus \frac{N}{2}\right), \left(2j \oplus \frac{N}{2}, 2j\right), \left(2j \oplus 1, 2j \oplus 1 \oplus \frac{N}{2}\right), \left(2j \oplus 1 \oplus \frac{N}{2}, 2j \oplus 1\right)\right\}$$

for $0 \leq j \leq \frac{N}{4} - 1$.

It can be observe that connections in $H(j)$ do not overlap. Hence, we assign a distinctive wavelength to all connections in a single $H(j)$. Hence, the total wavelength number necessary to assign for all connections is equal to the number of groups $H(j)$, which is equal to $\left(\frac{N}{4}\right)$. (4)

Since the groups $G(i,j)$ and $H(j)$ defined above contains all the connections, by adding (3) & (4) we get $\frac{N(N-1)}{4}$ wavelength number necessary to establish all-to-all broadcast communication.

Theorem 3: Let $N = 4m + 2$ where m is a positive integer and $N \geq 6$. Then, the wavelength number necessary to establish all-to-all broadcast in a unidirectional N node ring with 2-length extension is at most $\frac{N^2-N+2}{4}$ using longest link first routing algorithm.

Proof: Define a group $G(i,j)$ as

$$G(i,j) = \{(2i, 2i \oplus j), (2i \oplus j, 2i), (2i \oplus 1, 2i \oplus 1 \oplus j), (2i \oplus 1 \oplus j, 2i \oplus 1)\}$$

for every integer i and j such that $0 \leq i \leq \frac{N}{2} - 1$ and $1 \leq j \leq \frac{N}{2} - 1$.

It can be observe that connections in $G(i,j)$ do not overlap. Hence, we assign a distinctive wavelength to all connections in a single $G(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N}{2}-1}\sum_{j=1}^{\frac{N}{2}-1} 1 = \sum_{i=0}^{\frac{N}{2}-1} \frac{N}{2} - 1$$

$$= \left(\frac{N}{2}\right)\left(\frac{N}{2} - 1\right). \quad (5)$$

The remaining connections can be grouped as

$$H(j) = \left\{\left(2j, 2j \oplus \frac{N}{2}\right), \left(2j \oplus \frac{N}{2}, 2j\right), \left(2j \oplus 1, 2j \oplus 1 \oplus \frac{N}{2}\right), \left(2j \oplus 1 \oplus \frac{N}{2}, 2j \oplus 1\right)\right\}$$

for $0 \leq j \leq \frac{N-6}{4}$ and

$H\left(\frac{N-2}{4}\right) = \left\{\left(\frac{N-2}{2}, N-1\right)\left(N-1, \frac{N-2}{2}\right)\right\}$ for $j = \frac{N-2}{4}$.

It can be observe that connections in $H(j)$ do not overlap. Hence, we assign a distinctive wavelength to all connections in a single $H(j)$. Hence, the total wavelength number necessary to assign for all connections is equal to the number of groups $H(j)$, which is equal to $\frac{N+2}{4}$. (6)

Since the groups $G(i,j)$ and $H(j)$ defined above contains all the connections, by adding (5) & (6) we get $\frac{N^2-N+2}{4}$ wavelength number necessary to establish all-to-all broadcast communication.

Now, let us derive the link load of the network of a network with N even. Consider a random longer link inter connecting the nodes x and $(x \oplus 2)$. It can be observe that for $1 \leq l \leq \frac{N-1}{2}$, nodes at a distance of length 2l, before node $(x \oplus 2)$ share the above link to transmit the message to $N - 2l$ number of nodes immediately after node x. Then the total number of connections that share the above link is $\sum_{l=1}^{\frac{N-1}{2}} N - 2l = \sum_{l=1}^{\frac{N-1}{2}} N - \sum_{l=1}^{\frac{N-1}{2}} 2l$ which is equal to $\left(\frac{N-1}{2}\right)^2$. Consider anarbitrary shorter link inter connecting the nodes x and $x \oplus 1$. It can be observe that for $0 \leq l \leq \frac{N-3}{2}$, nodes at a distance of length 2l, before node $x \oplus 1$, share the above link to transmit the message to $(x \oplus 1)$. Hence, the number of connections that share the above link is $\frac{N-3}{2} + 1 = \frac{N-1}{2}$.

Next, we derive the link load of the network with N odd. Consider a random longer link inter connecting the nodes x and $(x \oplus 2)$. It can be observe that for $1 \leq l \leq \frac{N-2}{2}$, nodes at a distance of length 2l, before node $(x \oplus 2)$ share the above link to transmit the message to $N - 2l$ number of nodes immediately after nodex. Then the total number of connections that share the above link is $\sum_{l=1}^{\frac{N-2}{2}} N - 2l = \sum_{l=1}^{\frac{N-2}{2}} N - 2\sum_{l=1}^{\frac{N-2}{2}} l$ which is equal to $\frac{N(N-2)}{4}$. Consider anarbitrary shorter link inter connecting the nodes x and $x \oplus 1$. It can be observe that for $0 \leq l \leq \frac{N-2}{2}$, nodes at a distance of length 2l, before node $x \oplus 1$, share the above link to transmit the message to $x \oplus 1$. Hence, the number of connections that share the above link is $\frac{N}{2}$. It can be noted that for a every N value, the link load of all longer links are same. Similarly, the link load of all shorter links are also same. Hence, the wavelength number derived in Theorems 1 through 3 is the optimum wavelength number using longest link first routing algorithm.

3.2 Unidirectional Ring with 3- length Extension

Theorem 1: Let $N = 12m$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2-N}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N}{12} - 1$, and $1 \le j \le \frac{N-2}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $G_1(i,j), G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$. By this way, the wavelength number necessary atmost to assign for all connections is equal to the number of groups of $G_1(i,j), G_2(i,j), G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-12}{12}} \sum_{j=1}^{\frac{N-2}{2}} 4 = 4 \sum_{i=0}^{\frac{N-12}{12}} \frac{N-2}{2} = 4\left(\frac{N-2}{2}\right)\left(\frac{N-12}{12}+1\right) = \frac{N(N-2)}{6} \quad (7)$$

The remaining connections can be grouped as

$$H(j) = \left\{\left(3j, 3j \oplus \frac{N}{2}\right), \left(3j \oplus \frac{N}{2}, 3j\right), \left(3j \oplus 1, 3j \oplus 1 \oplus \frac{N}{2}\right), \left(3j \oplus 1 \oplus \frac{N}{2}, 3j \oplus 1\right), \left(3j \oplus 2, 3j \oplus 2 \oplus \frac{N}{2}\right), \left(3j \oplus 2 \oplus \frac{N}{2}, 3j \oplus 2\right)\right\}$$

for $0 \le j \le \frac{N}{6} - 1$.

It can be observe that connections in H(j) do not overlap. Hence, a distinctive wavelength is allotted for all connections in a single H(j). Hence, the total wavelength number necessary to assign for all connections is equal to the number of groups H(j), which is equal to

$$\left(\frac{N}{6} - 1 + 1\right) = \frac{N}{6} \quad (8)$$

Since the groups $G_1(i,j), G_2(i,j), G_3(i,j), G_4(i,j)$ and H(j) defined above contains all the connections. Adding (7) & (8), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2-N}{6}$.

Theorem 2: Let $N = 12m + 1$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+N-2}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N-1}{12} - 1$, and $1 \le j \le \frac{N-1}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted for all connections in a single $G_1(i,j), G_2(i,j)$, $G_3(i,j)$and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j), G_2(i,j), G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-13}{12}} \sum_{j=1}^{\frac{N-1}{2}} 4 = 4 \sum_{i=0}^{\frac{N-12}{12}} \frac{N-1}{2} = 4\left(\frac{N-1}{2}\right)\left(\frac{N-13}{12}+1\right)$$

$$= \frac{(N-1)^2}{6} \quad (9)$$

The remaining connections can be grouped as $H(j) = \{(N-1, N-1 \oplus j), (N-1 \oplus j, N-1)\}$ for $1 \le j \le \frac{N-1}{2}$

It can be observe that connections in H(j) do not overlap. Hence, a distinctive wavelength is allotted for all connections in a single H(j).Hence, the wavelength number necessary to assign for all connections is equal to the number of groups H(j), which is equal to

$$\left(\frac{N-1}{2}\right) \quad (10)$$

Since the groups $G_1(i,j)$,$G_2(i,j)$,$G_3(i,j)$,$G_4(i,j)$ and H(j) defined above contains all the connections. Adding (9) & (10), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+N-2}{6}$.

Theorem 3: Let $N = 12m + 2$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+3N-4}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N-14}{12}$, and $1 \le j \le \frac{N-2}{2}$

Also, let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1)\}$$

for every integer i and j such that $\frac{N-14}{12} < i \le \frac{N-2}{12}$, and $1 \le j \le \frac{N-2}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j), G_2(i,j), G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-2}{12}} \sum_{j=1}^{\frac{N-2}{2}} 2 + \sum_{i=0}^{\frac{N-14}{12}} \sum_{j=1}^{\frac{N-2}{2}} 2 = 2\sum_{i=0}^{\frac{N-2}{12}} \frac{N-2}{2} + 2\sum_{i=0}^{\frac{N-14}{12}} \frac{N-2}{2}$$

$$= 2\left(\frac{N-2}{2}\right)\left(\frac{N-2}{12}+1\right) + 2\left(\frac{N-2}{2}\right)\left(\frac{N-14}{12}+1\right)$$

$$= \frac{N^2+2N-8}{6} \quad (11)$$

The remaining connections can be grouped as $H(j) = \left\{\left(3j, 3j \oplus \frac{N}{2}\right), \left(3j \oplus \frac{N}{2}, 3j\right), \left(3j \oplus 1, 3j \oplus 1 \oplus \frac{N}{2}\right), \left(3j \oplus 1 \oplus \frac{N}{2}, 3j \oplus 1\right), \left(3j \oplus 2, 3j \oplus 2 \oplus \frac{N}{2}\right), \left(3j \oplus 2 \oplus \frac{N}{2}, 3j \oplus 2\right)\right\}$

for $0 \le j \le \frac{N-8}{6}$.

$$H(j) = \left\{\left(3j, 3j \oplus \frac{N}{2}\right), \left(3j \oplus \frac{N}{2}, 3j\right)\right\}$$

for $\frac{N-8}{6} < j \le \frac{N-2}{6}$.

It can be observe that connections in H(j) do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single H(j). Hence, the total wavelength number necessary to assign for all connections is equal to the number of groups H(j), which is equal to

$$\left(\frac{N-8}{6}+1+1\right) = \frac{N+4}{6} \quad (12)$$

Since the groups $G_1(i,j)$,$G_2(i,j)$,$G_3(i,j)$, $G_4(i,j)$ and H(j) defined above contains all the connections. Adding (11) & (12), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+3N-4}{6}$.

Theorem 4: Let $N = 12m + 3$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+2N-3}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N-15}{12}$, and $1 \le j \le \frac{N-1}{2}$

Also, let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1)\}$$

for every integer i and j such that $\frac{N-15}{12} < i \le \frac{N-3}{12}$, and $1 \le j \le \frac{N-1}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $G_1(i,j), G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j), G_2(i,j), G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-3}{12}} \sum_{j=1}^{\frac{N-1}{2}} 2 + \sum_{i=0}^{\frac{N-15}{12}} \sum_{j=1}^{\frac{N-1}{2}} 2 = 2\sum_{i=0}^{\frac{N-3}{12}} \frac{N-1}{2} + 2\sum_{i=0}^{\frac{N-15}{12}} \frac{N-1}{2}$$
$$= 2\left(\frac{N-1}{2}\right)\left(\frac{N-3}{12}+1\right) + 2\left(\frac{N-1}{2}\right)\left(\frac{N-15}{12}+1\right)$$
$$= \frac{N^2+2N-3}{6} \quad (13)$$

Since the groups $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ defined above contains all the connections. From (13), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+2N-3}{6}$.

Theorem 5: Let $N = 12m + 4$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+N+4}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N-16}{12}$, and $1 \le j \le \frac{N-2}{2}$

Also, let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3)\}$$

for every integer i and j such that $\frac{N-16}{12} < i \le \frac{N-4}{12}$, and $1 \le j \le \frac{N-2}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single$G_1(i,j), G_2(i,j)$, $G_3(i,j)$and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j), G_2(i,j), G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-4}{12}}\sum_{j=1}^{\frac{N-2}{2}}2+\sum_{i=0}^{\frac{N-16}{12}}\sum_{j=1}^{\frac{N-2}{2}}2 = 2\sum_{i=0}^{\frac{N-4}{12}}\frac{N-2}{2}+2\sum_{i=0}^{\frac{N-16}{12}}\frac{N-2}{2}$$
$$=2\left(\frac{N-2}{2}\right)\left(\frac{N-4}{12}+1\right)+2\left(\frac{N-2}{2}\right)\left(\frac{N-16}{12}+1\right)$$
$$=\frac{N^2-4}{6} \quad (14)$$

The remaining connections can be grouped as

$$H_1(j)=\left\{\left(6j,6j\oplus\frac{N}{2}\right),\left(6j\oplus\frac{N}{2},6j\right),\left(6j\oplus2,6j\oplus2\oplus\frac{N}{2}\right),\left(6j\oplus2\oplus\frac{N}{2},6j\oplus2\right),\left(6j\oplus4,6j\oplus4\oplus\frac{N}{2}\right),\left(6j\oplus4\oplus\frac{N}{2},6j\oplus4\right)\right\}$$
$$H_2(j)=\left\{\left(6j\oplus1,6j\oplus1\oplus\frac{N}{2}\right),\left(6j\oplus1\oplus\frac{N}{2},6j\oplus1\right),\left(6j\oplus3,6j\oplus3\oplus\frac{N}{2}\right),\left(6j\oplus3\oplus\frac{N}{2},6j\oplus3\right),\left(6j\oplus5,6j\oplus5\oplus\frac{N}{2}\right),\left(6j\oplus5\oplus\frac{N}{2},6j\oplus5\right)\right\}$$

for $0\le j\le\frac{N-16}{12}$.

$$H_1(j)=\left\{\left(6j,6j\oplus\frac{N}{2}\right),\left(6j\oplus\frac{N}{2},6j\right)\right\}$$
$$H_2(j)=\left\{\left(6j\oplus1,6j\oplus1\oplus\frac{N}{2}\right),\left(6j\oplus1\oplus\frac{N}{2},6j\oplus1\right)\right\}$$

for $\frac{N-16}{12}<j\le\frac{N-4}{12}$.

It can be observe that connections in $H_1(j)$ do not overlap and connections in $H_2(j)$ also do not overlap. Hence, a distinctive wavelength is allotted for all connections in a single $H_1(j)$ and $H_2(j)$.Hence, the total wavelength number necessary to assign for all connections is equal to the number of groups $H_1(j)$ and $H_2(j)$, which is equal to

$$2\left(\frac{N-4}{12}+1\right)=\frac{N+8}{6} \quad (15)$$

Since the groups $G_1(i,j)$,$G_2(i,j)$,$G_3(i,j)$, $G_4(i,j)$, $H_1(j)$ and $H_2(j)$ defined above contains all the connections. Adding (14) & (15), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+N+4}{6}$.

Theorem 6: Let $N=12m+5$ where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+3N-4}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j)=\{(12i,12i\oplus j),(12i\oplus j,12i),(12i\oplus4,12i\oplus4\oplus j),(12i\oplus4\oplus j,12i\oplus4),(12i\oplus8,12i\oplus8\oplus j),(12i\oplus8\oplus j,12i\oplus8)\}$$
$$G_2(i,j)=\{(12i\oplus1,12i\oplus1\oplus j),(12i\oplus1\oplus j,12i\oplus1),(12i\oplus5,12i\oplus5\oplus j),(12i\oplus5\oplus j,12i\oplus5),(12i\oplus9,12i\oplus9\oplus j),(12i\oplus9\oplus j,12i\oplus9)\}$$
$$G_3(i,j)=\{(12i\oplus2,12i\oplus2\oplus j),(12i\oplus2\oplus j,12i\oplus2),(12i\oplus6,12i\oplus6\oplus j),(12i\oplus6\oplus j,12i\oplus6),(12i\oplus10,12i\oplus10\oplus j),(12i\oplus10\oplus j,12i\oplus10)\}$$
$$G_4(i,j)=\{(12i\oplus3,12i\oplus3\oplus j),(12i\oplus3\oplus j,12i\oplus3),(12i\oplus7,12i\oplus7\oplus j),(12i\oplus7\oplus j,12i\oplus7),(12i\oplus11,12i\oplus11\oplus j),(12i\oplus11\oplus j,12i\oplus11)\}$$

for every integer i and j such that $0\le i\le\frac{N-17}{12}$, and $1\le j\le\frac{N-1}{2}$

Also, let

$$G_1(i,j)=\{(12i,12i\oplus j),(12i\oplus j,12i),(12i\oplus4,12i\oplus4\oplus j),(12i\oplus4\oplus j,12i\oplus4)\}$$
$$G_2(i,j)=\{(12i\oplus1,12i\oplus1\oplus j),(12i\oplus1\oplus j,12i\oplus1),(12i\oplus3,12i\oplus3\oplus j),(12i\oplus3\oplus j,12i\oplus3)\}$$
$$G_3(i,j)=\{(12i\oplus2,12i\oplus2\oplus j),(12i\oplus2\oplus j,12i\oplus2)\}$$

for every integer i and j such that $\frac{N-17}{12}<i\le\frac{N-5}{12}$, and $1\le j\le\frac{N-1}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-5}{12}}\sum_{j=1}^{\frac{N-1}{2}}3+\sum_{i=0}^{\frac{N-17}{12}}\sum_{j=1}^{\frac{N-1}{2}}1 = 3\sum_{i=0}^{\frac{N-5}{12}}\frac{N-1}{2}+\sum_{i=0}^{\frac{N-17}{12}}\frac{N-1}{2}$$
$$=3\left(\frac{N-1}{2}\right)\left(\frac{N-5}{12}+1\right)+\left(\frac{N-1}{2}\right)\left(\frac{N-17}{12}+1\right)$$
$$=\frac{N^2+3N-4}{6} \quad (16)$$

Since the groups $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ defined above contains all the connections. From (16), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+3N-4}{6}$.

Theorem 7: Let $N = 12m + 6$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+5N-12}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$
$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$
$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$
$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer iand j such that $0 \le i \le \frac{N-18}{12}$, and $1 \le j \le \frac{N-2}{2}$

Also, let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4)\}$$
$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5)\}$$
$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2)\}$$
$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3)\}$$

for every integer i andj such that $\frac{N-18}{12} < i \le \frac{N-6}{12}$, and $1 \le j \le \frac{N-2}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single$G_1(i,j), G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j), G_2(i,j), G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-6}{12}}\sum_{j=1}^{\frac{N-2}{2}}4 = 4\sum_{i=0}^{\frac{N-6}{12}}\frac{N-2}{2}$$
$$= 4\left(\frac{N-2}{2}\right)\left(\frac{N-6}{12}+1\right)$$

$$=\frac{(N+6)(N-2)}{6} \quad (17)$$

The remaining connections can be grouped as

$$H(j) = \left\{\left(3j, 3j \oplus \frac{N}{2}\right), \left(3j \oplus \frac{N}{2}, 3j\right), \left(3j \oplus 1, 3j \oplus 1 \oplus \frac{N}{2}\right), \left(3j \oplus 1 \oplus \frac{N}{2}, 3j \oplus 1\right), \left(3j \oplus 2, 3j \oplus 2 \oplus \frac{N}{2}\right), \left(3j \oplus 2 \oplus \frac{N}{2}, 3j \oplus 2\right)\right\}$$

for$0 \le j \le \frac{N}{6} - 1$.

It can be observe that connections in $H(j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $H(j)$. Hence, the total wavelength number necessary to assign for all connections is equal to the number of groups $H(j)$, which is equal to

$$\left(\frac{N}{6}-1+1\right)=\frac{N}{6} \quad (18)$$

Since the groups $G_1(i,j)$,$G_2(i,j)$,$G_3(i,j)$,$G_4(i,j)$ and $H(j)$ defined above contains all the connections. Adding (17) & (18), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+5N-12}{6}$.

Theorem 8: Let $N = 12m + 7$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+4N-5}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N-19}{12}$, and $1 \le j \le \frac{N-1}{2}$

Also, let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3)\}$$

for every integer i and j such that $\frac{N-19}{12} < i \le \frac{N-7}{12}$, and $1 \le j \le \frac{N-2}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single$G_1(i,j)$, $G_2(i,j)$,$G_3(i,j)$and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-7}{12}} \sum_{j=1}^{\frac{N-1}{2}} 4 = 4 \sum_{i=0}^{\frac{N-7}{12}} \frac{N-1}{2}$$

$$= 4\left(\frac{N-1}{2}\right)\left(\frac{N-7}{12} + 1\right)$$

$$= \frac{N^2 + 4N - 5}{6} \qquad (19)$$

Since the groups $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ defined above contains all the connections. From (19), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+4N-5}{6}$.

Theorem 9: Let $N = 12m + 8$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+3N-4}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N-8}{12}$, and $1 \le j \le \frac{N-2}{2}$

Also, let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7)\}$$

for every integer i and j such that $\frac{N-20}{12} < i \le \frac{N-8}{12}$, and $1 \le j \le \frac{N-2}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$,

$G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $G_1(i,j), G_2(i,j)$, $G_3(i,j)$and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j), G_2(i,j), G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-8}{12}}\sum_{j=1}^{\frac{N-2}{2}} 4 = 4\sum_{i=0}^{\frac{N-8}{12}}\frac{N-2}{2} = 4\left(\frac{N-2}{2}\right)\left(\frac{N-8}{12}+1\right)$$

$$= \frac{N^2+2N-8}{6} \qquad (20)$$

The remaining connections can be grouped as

$$H(j) = \left\{\left(3j, 3j \oplus \frac{N}{2}\right), \left(3j \oplus \frac{N}{2}, 3j\right), \left(3j \oplus 1, 3j \oplus 1 \oplus \frac{N}{2}\right), \left(3j \oplus 1 \oplus \frac{N}{2}, 3j \oplus 1\right), \left(3j \oplus 2, 3j \oplus 2 \oplus \frac{N}{2}\right), \left(3j \oplus 2 \oplus \frac{N}{2}, 3j \oplus 2\right)\right\}$$

for$0 \le j \le \frac{N-8}{6}$.

$$H(j) = \left\{\left(3j, 3j \oplus \frac{N}{2}\right), \left(3j \oplus \frac{N}{2}, 3j\right)\right\}$$

for $\frac{N-8}{6} < j \le \frac{N-2}{6}$.

It can be observe that connections in H(j) do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single H(j). Hence, the total wavelength number necessary to assign for all connections is equal to the number of groups H(j), which is equal to

$$\left(\frac{N-2}{6}+1\right) = \frac{N+4}{6} \qquad (21)$$

Since the groups $G_1(i,j), G_2(i,j)$, $G_3(i,j)$, $G_4(i,j)$ and H(j) defined above contains all the connections. Adding (20) & (21), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+3N-4}{6}$.

Theorem 10: Let $N = 12m + 9$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+2N-3}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N-21}{12}$, and $1 \le j \le \frac{N-1}{2}$

Also, let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7)\}$$

for every integer i and j such that $\frac{N-21}{12} < i \le \frac{N-9}{12}$, and $1 \le j \le \frac{N-2}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $G_1(i,j), G_2(i,j)$, $G_3(i,j)$and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j), G_2(i,j), G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-9}{12}}\sum_{j=1}^{\frac{N-1}{2}} 4 = 4\sum_{i=0}^{\frac{N-9}{12}}\frac{N-1}{2}$$

$$= 4\left(\frac{N-1}{2}\right)\left(\frac{N-9}{12}+1\right)$$

$$= \frac{N^2 + 2N - 3}{6} \qquad (22)$$

Since the groups $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ defined above contains all the connections. From (22), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+2N-3}{6}$.

Theorem 11: Let $N = 12m + 10$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2+N-2}{6}$ using longest link first routing algorithm.

Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N-22}{12}$, and $1 \le j \le \frac{N-2}{2}$

Also, let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7)\}$$

for every integer i and j such that $\frac{N-22}{12} < i \le \frac{N-10}{12}$, and $1 \le j \le \frac{N-2}{2}$

It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of groups $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-10}{12}} \sum_{j=1}^{\frac{N-2}{2}} 4 = 4 \sum_{i=0}^{\frac{N-10}{12}} \frac{N-2}{2}$$

$$= 4\left(\frac{N-2}{2}\right)\left(\frac{N-10}{12} + 1\right)$$

$$= \frac{N^2 - 4}{6} \qquad (23)$$

The remaining connections can be grouped as

$$H_1(j) = \left\{\left(6j, 6j \oplus \frac{N}{2}\right), \left(6j \oplus \frac{N}{2}, 6j\right), \left(6j \oplus 2, 6j \oplus 2 \oplus \frac{N}{2}\right), \left(6j \oplus 2 \oplus \frac{N}{2}, 6j \oplus 2\right), \left(6j \oplus 4, 6j \oplus 4 \oplus \frac{N}{2}\right), \left(6j \oplus 4 \oplus \frac{N}{2}, 6j \oplus 4\right)\right\}$$

for $0 \le j \le \frac{N-10}{12}$.

$$H_2(j) = \left\{\left(6j \oplus 1, 6j \oplus 1 \oplus \frac{N}{2}\right), \left(6j \oplus 1 \oplus \frac{N}{2}, 6j \oplus 1\right), \left(6j \oplus 3, 6j \oplus 3 \oplus \frac{N}{2}\right), \left(6j \oplus 3 \oplus \frac{N}{2}, 6j \oplus 3\right), \left(6j \oplus 5, 6j \oplus 5 \oplus \frac{N}{2}\right), \left(6j \oplus 5 \oplus \frac{N}{2}, 6j \oplus 5\right)\right\}$$

for $0 \le j \le \frac{N-22}{12}$.

$$H_2(j) = \left\{\left(6j \oplus 1, 6j \oplus 1 \oplus \frac{N}{2}\right), \left(6j \oplus 1 \oplus \frac{N}{2}, 6j \oplus 1\right), \left(6j \oplus 3, 6j \oplus 3 \oplus \frac{N}{2}\right), \left(6j \oplus 3 \oplus \frac{N}{2}, 6j \oplus 3\right)\right\}$$

for $\frac{N-22}{12} < j \le \frac{N-10}{12}$.

It can be observe that connections in $H_1(j)$ do not overlap and connections in $H_2(j)$ also do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $H_1(j)$ and $H_2(j)$. Hence, the wavelength number necessary to assign for all connections is equal to the number of groups $H_1(j)$ and $H_2(j)$, which is equal to

$$2\left(\frac{N-10}{12}+1\right)=\frac{N+2}{6} \quad (24)$$

Since the groups $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$, $G_4(i,j)$ and $H(j)$ defined above contains all the connections. Adding (23) & (24), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2+N-2}{6}$.

Theorem 12: Let $N = 12m + 11$, where m is a positive integer. Then, the wavelength number necessary atmost to establish all-to-all broadcast in a N node unidirectional ring with 3-length extension is $\frac{N^2-1}{6}$ using longest link first routing algorithm.
Proof: Let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7), (12i \oplus 11, 12i \oplus 11 \oplus j), (12i \oplus 11 \oplus j, 12i \oplus 11)\}$$

for every integer i and j such that $0 \le i \le \frac{N-23}{12}$, and $1 \le j \le \frac{N-1}{2}$
Also, let

$$G_1(i,j) = \{(12i, 12i \oplus j), (12i \oplus j, 12i), (12i \oplus 4, 12i \oplus 4 \oplus j), (12i \oplus 4 \oplus j, 12i \oplus 4), (12i \oplus 8, 12i \oplus 8 \oplus j), (12i \oplus 8 \oplus j, 12i \oplus 8)\}$$

$$G_2(i,j) = \{(12i \oplus 1, 12i \oplus 1 \oplus j), (12i \oplus 1 \oplus j, 12i \oplus 1), (12i \oplus 5, 12i \oplus 5 \oplus j), (12i \oplus 5 \oplus j, 12i \oplus 5), (12i \oplus 9, 12i \oplus 9 \oplus j), (12i \oplus 9 \oplus j, 12i \oplus 9)\}$$

$$G_3(i,j) = \{(12i \oplus 2, 12i \oplus 2 \oplus j), (12i \oplus 2 \oplus j, 12i \oplus 2), (12i \oplus 6, 12i \oplus 6 \oplus j), (12i \oplus 6 \oplus j, 12i \oplus 6), (12i \oplus 10, 12i \oplus 10 \oplus j), (12i \oplus 10 \oplus j, 12i \oplus 10)\}$$

$$G_4(i,j) = \{(12i \oplus 3, 12i \oplus 3 \oplus j), (12i \oplus 3 \oplus j, 12i \oplus 3), (12i \oplus 7, 12i \oplus 7 \oplus j), (12i \oplus 7 \oplus j, 12i \oplus 7)\}$$

for every integer i and j such that $\frac{N-23}{12} < i \le \frac{N-11}{12}$, and $1 \le j \le \frac{N-2}{2}$
It can be observe that connections in $G_1(i,j)$ do not overlap. Similarly, the connections in $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ do not overlap. Hence, a distinctive wavelength is allotted to all connections in a single $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$. By this way, the wavelength number necessary to assign for all connections is equal to the number of $G_1(i,j)$, $G_2(i,j)$, $G_3(i,j)$ and $G_4(i,j)$ which is equal to

$$\sum_{i=0}^{\frac{N-11}{12}} \sum_{j=1}^{\frac{N-1}{2}} 4 = 4 \sum_{i=0}^{\frac{N-11}{12}} \frac{N-1}{2} = 4\left(\frac{N-1}{2}\right)\left(\frac{N-11}{12}+1\right) = \frac{N^2-1}{6} \quad (25)$$

Since the groups $G_1(i,j)$, $G_2(i,j)$ $G_3(i,j)$ and $G_4(i,j)$ defined above contains all the connections. From (25), the wavelength number necessary atmost to establish all-to-all broadcast is obtained as $\frac{N^2-1}{6}$.

Link Load
Now, the link load of the ring with 3-length extension for all-to-all broadcast is derived. Let π denote link load of ring with -length extension, which is the maximum number of paths that share a common link.

Case i) $N = 3m$.
Consider a random longer link inter connecting the nodes x and $(x \oplus 3)$. It can be observe that for $1 \le l \le \frac{N-3}{3}$, nodes at a distance of length $3l$, before node $(x \oplus 3)$ share the above link to transmit the message to $N - 3l$ number of nodes immediately after node x. Then the total number of connections that share the above link is

$$\sum_{l=1}^{\frac{N-3}{3}} N - 3l = \sum_{l=1}^{\frac{N-3}{3}} N - \sum_{l=1}^{\frac{N-3}{3}} 3l = \frac{N^2-3N}{6} \quad (26)$$

Consider a random shorter link inter connecting the nodes x and $x \oplus 1$. It can be observe that for $0 \le$

$l \leq \frac{N-3}{3}$, nodes which are present at a distance of length $3l$ prior to node x, share the above link to transmit the message to $(x \oplus 1)$ and $(x \oplus 2)$node. Similarly, nodes which are present at a distance of length $(3l-1)$ prior to node x, share the above link to transmit the message to$(x \oplus 1)$ node. Hence, the number of connections that share the above link is $2*\left(\frac{N-3}{3}+1\right)+\left(\frac{N-3}{3}+1\right) = N$ (27)

The link load of longer link is higher than that of shorter link, so the link load of a longer link is the link load of the network. From the equitation (26) & (27), the link load of the network $\pi = \frac{N^2-3N}{6}$.

Case ii) $N = 3m+1$.

Consider a random longer link inter connecting the nodes x and $(x \oplus 3)$. It can be observe that for $1 \leq l \leq \frac{N-1}{3}$, nodes at a distance of length $3l$, before node $(x \oplus 3)$ share the above link to transmit the message to $N-3l$ number of nodes immediately after node x. Then the total number of connections that share the above link is

$$\sum_{l=1}^{\frac{N-1}{3}} N-3l = \sum_{l=1}^{\frac{N-1}{3}} N - \sum_{l=1}^{\frac{N-1}{3}} 3l = \frac{N^2-3N+2}{6} \quad (28)$$

Consider a random shorter link inter connecting the nodes x and $(x \oplus 1)$. It can be observe that for $0 \leq l \leq \frac{N-4}{3}$, nodes which are present at a distance of length $3l$ prior to node x, share the above link to transmit the message to $(x \oplus 1)$ and $(x \oplus 2)$ node. Similarly, nodes which are present at a distance of length $(3l-1)$ prior to node x, share the above link to transmit the message to$(x \oplus 1)$ node. Hence, the number of connections that share the above link is $2*\left(\frac{N-4}{3}+1\right)+\left(\frac{N-4}{3}+1\right) = N-1$ (29)

The link load of longer link is higher than that of shorter link, so the link load of a longer link is the link load of the network. From the equitation (28) & (29), the link load of the network $\pi = \frac{N^2-3N+2}{6}$.

Case iii) $N = 3m+2$.

Consider a random longer link inter connecting the nodes x and $(x \oplus 3)$. It can be observe that for $1 \leq l \leq \frac{N-2}{3}$, nodes at a distance of length $3l$, before node $(x \oplus 3)$ share the above link to transmit the message to $N-3l$ number of nodes immediately after node x. Then the total number of connections that share the above link is

$$\sum_{l=1}^{\frac{N-2}{3}} N-3l = \sum_{l=1}^{\frac{N-2}{3}} N - \sum_{l=1}^{\frac{N-2}{3}} 3l = \frac{N^2-3N+2}{6} \quad (30)$$

Consider a random shorter link inter connecting the nodes x and $(x \oplus 1)$. It can be observe that for $0 \leq l \leq \frac{N-5}{3}$, nodes which are present at a distance of length $3l$ prior to node x, share the above link to transmit the message to $(x \oplus 1)$ and $(x \oplus 2)$ node. Also, node $(N-2)$ prior to node x, share the above link to transmit the message to $(x \oplus 1)$. Similarly nodes which are present at a distance of length $(3l-1)$ prior to node x, share the above link to transmit the message to $(x \oplus 1)$ node. Hence, the number of connections that share the above link is

$$2*\left(\frac{N-5}{3}+1\right)+1+\left(\frac{N-5}{3}+1\right) = N-1. \quad (31)$$

The link load of longer link is higher than that of shorter link, so the link load of a longer link is the link load of the network. From the equitation (30) & (31), the link load of the network $\pi = \frac{N^2-3N+2}{6}$. It is to be observed that for a particular value of N, the link load of all longer links is same. Similarly, the link load of all shorter links is also same.

4 Results and Discussion

4.1 Unidirectional Ring with 2-length Extension

Table 1 compares the results obtained in the previous section with that of a unidirectional primary ring. It can be easily observed that the wavelength number necessary atmost to establish all-to-all broadcast in a ring with 2-length extension is reduced by a minimum of 46% and a maximum of 50% than that necessary for a primary ring. Table 2 shows the values of wavelength number necessary atmost to establish all-to-all broadcast along with link load for certain values of node number N. It can be observed that the difference between the wavelength number necessary and link load is very small, which indicates that the derived results are either equal or nearer to the minimum wavelength number.

Table 1. Wavelength number necessary atmost to establish all-to-all broadcast in a unidirectional ring and unidirectional ring with 2-length extension.

Network Topology with N node	Wavelength number required atmost to establish all-to-all broadcast
Unidirectional ring [5]	$\frac{N(N-1)}{2}$
Unidirectional ring with 2-length extension	$\frac{N^2-N+2}{4}$ if $N = 4m+2$, $\frac{N(N-1)}{4}$ if $N = 4m$ *and* $\frac{N^2-1}{4}$ if N is odd.

Table 2. Wavelength number required atmost to establish all-to-all broadcast along with link load for certain values of node number N in a unidirectional ring with 2-length extension.

Node number N	Wavelength number required	Link load	Difference between wavelength number and link load
25	156	144	12
30	218	210	8
40	390	380	10
55	756	729	27
70	1208	1190	18
85	1806	1764	42
90	2003	1980	23
100	2475	2450	25
120	3570	3540	30
150	5588	5550	38
175	7656	7569	87
225	12656	12544	112
350	30538	30450	88
500	62375	62250	125

Table 3. Comparison of wavelength number required atmost to establish all-to-all broadcast for certain values of node number **N** in a unidirectional ring, unidirectional ring with 2-length extension.

Node number N	Wavelength number required atmost to establish all-to-all broadcast	
	Unidirectional ring (Sabrigiriraj et al. 2009)	Unidirectional ring with 2-length extension
12	66	33
13	78	42
14	91	46
15	105	56
16	120	60
17	136	72
18	153	77
19	171	90
20	190	95
21	210	110
22	231	116
23	253	132
24	276	138
27	351	182
30	435	218
33	528	272
36	630	315
39	741	380
42	861	431
45	990	506
48	1128	564
51	1275	650
60	1770	885
70	2415	1208
80	3160	1580
90	4005	2003
100	4950	2475
125	7750	3906
150	11175	5588
175	15225	7656
200	19900	9950
300	44850	22425
500	124750	62375
700	244650	122325
1000	499500	249750

From Table 3, it can be observed that the wavelength number necessary atmost to establish all-to-all broadcast in a unidirectional ring with 2-length extension is reduced by a minimum of 46% and a maximum of 50% than that necessary for a unidirectional primary ring network.

4.2 Unidirectional Ring with 3-length Extension

Table 4 compares wavelength number necessary atmost to establish all-to-all broadcast in a unidirectional ring with 3-length extension, unidirectional ring with 2 length extension and unidirectional primary ring.

Table 4 Wavelength number required atmost to establish all-to-all broadcast in a unidirectional ring, unidirectional ring with 2-length extension and unidirectional ring with 3-length extension.

Network Topology with N node	Wavelength number required atmost to establish all-to-all broadcast
Unidirectional ring [5]	$\frac{N(N-1)}{2}$
Unidirectional ring with 2-length extension	$\frac{N^2-N+2}{4}$ if $N=4m+2$ $\frac{N(N-1)}{4}$ if $N=4m$ $\frac{N^2-1}{4}$ if N is odd.
Unidirectional ring with 3-length extension	$\frac{N^2-N}{6}$ if $N=12m$ $\frac{N^2+N-2}{6}$ if $N=12m+1$ $\frac{N^2+3N-4}{6}$ if $N=12m+2$
Unidirectional ring with 3-length extension	$\frac{N^2+2N-3}{6}$ if $N=12m+3$ $\frac{N^2+N+4}{6}$ if $N=12m+4$ $\frac{N^2+3N-4}{6}$ if $N=12m+5$ $\frac{N^2+5N-12}{6}$ if $N=12m+6$ $\frac{N^2+4N-5}{6}$ if $N=12m+7$ $\frac{N^2+3N-4}{6}$ if $N=12m+8$ $\frac{N^2+2N-3}{6}$ if $N=12m+9$ $\frac{N^2+N-2}{6}$ if $N=12m+10$ $\frac{N^2-1}{6}$ if $N=12m+11$

Table 5. Wavelength number required atmost to establish all-to-all broadcast along with link load for certain values of node number **N** in a unidirectional ring with 3-length extension.

Node number *N*	Wavelength number required	Link load	Difference between wavelength number required and link load
25	108	92	16
30	173	135	38
40	274	247	27
55	540	477	63
70	828	782	46
85	1218	1162	56
90	1423	1305	118
100	1684	1617	67
120	2380	2340	40
150	3873	3675	198
175	5220	5017	203
225	8512	8325	187
350	20591	20242	349
500	41916	41417	499

Table 5 shows the values of wavelength number necessary atmost to establish all-to-all broadcast along with link load for certain values of N. It can be observed that the difference between the wavelength number necessary and link load and is little, which indicates that the derived results are either equal or nearer to the minimum wavelength number.

Table 6. Comparison of wavelength number required atmost to establish all-to-all broadcast for certain value of node number N in a unidirectional ring, unidirectional ring with 2-length extension and unidirectional ring with 3-length extension.

Node number N	Comparison of wavelength number required atmost to establish all-to-all broadcast		
	Unidirectional ring [5]	Unidirectional ring with 2-length extension	Unidirectional ring with 3-length extension
12	66	33	22
13	78	42	30
14	91	46	39
15	105	56	42
16	120	60	46
17	136	72	56
18	153	77	67
19	171	90	72
20	190	95	76
21	210	110	80
22	231	116	84
23	253	132	88
24	276	138	92
27	351	182	130
30	435	218	173
33	528	272	192
36	630	315	210
39	741	380	266
42	861	431	327
45	990	506	352
48	1128	564	376
51	1275	650	450
60	1770	885	590
70	2415	1208	828
80	3160	1580	1106

Node number N	Comparison of wavelength number required atmost to establish all-to-all broadcast		
	Unidirectional ring [5]	Unidirectional ring with 2-length extension	Unidirectional ring with 3-length extension
90	4005	2003	1423
100	4950	2475	1684
125	7750	3906	2666
150	11175	5588	3873
175	15225	7656	5220
200	19900	9950	6766
300	44850	22425	14950
500	124750	62375	41916
700	244650	122325	81784
1000	499500	249750	166834

From Table 6, it can be easily observed that the wavelength number necessary atmost to establish all-to-all broadcast in a ring with 3-length extension is reduced by a minimum of 56% and a maximum of 66% when compared to primary ring. Similarly, the wavelength number necessary atmost to establish all-to-all broadcast in a unidirectional ring with 3-length extension is reduced by a minimum of 13% and a maximum of 33% when compared to unidirectional ring with 2-length extension.

5 Conclusion and Future Work

In this article, all-to-all broadcast is studied for unidirectional ring with 2 length extension and unidirectional ring with 3 length extension. The wavelength number necessary atmost to establish all-to-all broadcast in a unidirectional ring with 2-length extension and 3-length extension is derived. The proof of various Theorems clearly displays the method of wavelength allotment. The result obtained shows that the wavelength number necessary atmost to establish all-to-all broadcast in a unidirectional ring with 2-length extension is reduced by a minimum of 46% and a maximum of 50% than that necessary for a unidirectional primary ring network. Also, the result obtained shows that the wavelength number necessary atmost to establish all-to-all broadcast in a unidirectional ring with 3-length extension is reduced by a minimum of 56% and a maximum of 66% when compared to primary ring. Similarly, the wavelength number necessary atmost to establish all-to-all broadcast in a unidirectional ring with 3-length extension is reduced by a minimum of 13% and a maximum of 33% when compared to unidirectional ring with 2-length extension.

Wavelength number requirement needs to be investigated with still higher order extensions, to judge the rate of reduction in wavelength number requirements with increasing extension and is a challenging issue. Also, deriving a generalized expression for wavelength number requirement in a linear array/ring network with k-length extension (k is any positive integer and $k < N$ where N is the total number of nodes in the network) is another interesting and challenging future work. Examining the effects of physical layer impairments and network survivability on routing and wavelength assignment are other research issues with these extended networks.

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