Wideband Chebyshev Impedance Transformer Set with Very Low Reflection Coefficient

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Abstract: - In various electronic and electrical devices for communication systems, impedance transformers with wide bandwidths are important. In this article, Chebyshev impedance transformers for ohmic loads with order 3 are presented at different frequencies and load impedance values using Duroid and FR4 substrates (εr = 2.2 and εr = 4.4 respectively). Simulated results are shown using the software Advanced Design System (ADS). The thickness of the substrate for FR4 dielectric material is 1.544 mm and for Duroid material is 1.27mm. The matching transformers were performed at 1 GHz, 1.5 GHz, 2 GHz, and 1.5 GHz for two load impedance 100 Ohms and 75 Ohms from 50 Ohms showing a reflection coefficient Γm = 0.05. The S_{11} scattering parameter was obtained.

Key-Words: - Chebyshev, matching Networks, Duroid substrate, FR4 substrate, ADS, reflection coefficient.

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1 Introduction

The reduction of coupling losses demands the design of networks that allow, on both sides of the network, to present the complex conjugate of the impedances of the blocks to which it is intended to be coupled. The above is not easy, especially when the blocks operate in a large bandwidth, because that the various electrical and electronic elements of the blocks have different impedances within the bandwidth, achieving maximum power transfer between adjacent blocks is difficult.

In 1945 Bode began the study of the coupling between block impedances and later Fano in 1950 reinforced the analysis of the coupling between passive loads with a resistive generator that, together with the techniques used by Youla, helped the analysis and design of the coupling networks, [1], [2], [3], [4], [5], [6], [7].

There are narrowband and wideband impedance transformers. Among the wide bandwidth impedance transformers, we can mention the Binomial transformers and the Chebyshev transformers.

The Chebyshev transformer is based on the λ/4 transformer, that is, it is used to couple real impedances. However, this allows coupling within a wide bandwidth.

Chebyshev transformers are calculated by equating the reflection coefficient Γ(θ) with the Chebyshev polynomial, the reflection coefficient establishes the relationship between the incident wave and the reflected wave, so depending on the application the reflection coefficient will be required, which the Chebyshev type transformer must be coupled within the required bandwidth, [2], [8], [9], [10], [11], [12], [13].

The use of multi-section impedance transformers allows the bandwidth to be increased based on the requested reflection coefficient. From here the Chebyshev polynomials are used, [2].

The Chebyshev polynomial is shown in equation 1.

$$T_n(x) = 2xT_{n-1} - T_{n-2}(x) \tag{1}$$

It is necessary to find a solution of the type shown in equation 2.

$$x = \sec\theta_m \cos\theta \tag{2}$$

Where:

θ_m is the electrical length when you have a maximum reflection coefficient.
θ is the electrical length of the entire system.

In equations 3, 4, and 5, the first three polynomials required for the design of the transformers are shown:

$$T_1(x) = x \tag{3}$$

$$T_2(x) = 2x^2 - 1 \tag{4}$$

$$T_3(x) = 4x^3 - 3x \tag{5}$$

Therefore, for the polynomial of order three, equation 5 will be used, resulting in what is shown in equation 6:

$$T_3(\sec\theta_m \cos\theta) = sec^3\theta_m(\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta \tag{6}$$

From here:

$$\Gamma(\theta) = 2e^{-jN\theta}[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2n)\theta + \cdots + \Gamma_n \cos(N-2n)\theta + \cdots] = Ae^{-jN\theta}T_N(\sec\theta_m \cos\theta) \tag{7}$$

where:

$$A = |\Gamma_m| \tag{8}$$

and

$$\sec\theta_m = \cosh\left[\frac{1}{N} arc\ \cosh\left(\left|\frac{\ln {Z_L}/{Z_0}}{2\Gamma_m}\right|\right)\right] \tag{9}$$

The fractional bandwidth (FBW) is given by equation 10:

Finally, for each impedance per the attached section they are related by equation 11:

$$\Gamma_n = \frac{1}{2}\ln\frac{Z_{n+1}}{Z_n} \tag{11}$$

In this article, the Chebyshev coupling network is presented to get a maximum reflection coefficient in two frequencies and two load impedances, the number of sections proposed was three for different substrate materials.

The simulation of the scattering parameter S_{11} was carried out using ADS (Advanced Design System) with the *Linecalc* function to get the dimensions of the three stages according to their impedance values. The purpose of carrying out the calculation, simulation, and later construction and validation of said high bandwidth impedance transformers is for use in research, as well as for educational purposes.

2 Coupling Network Design

The Chebyshev transformer was designed with three stages or sections, therefore, the descriptive polynomial of the reflection coefficient with equation 7 is obtained:

$$\Gamma(\theta) = 2e^{-j3\theta}\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\big(3-2(1)\big)\theta + \Gamma_2 \cos\big(3-2(2)\big)\theta\right] = Ae^{-j3\theta}T_3(\sec\theta_m \cos\theta)$$

From here, the terms are eliminated which they turn out to be negative ($\Gamma_2 \cos\big(3-2(2)\big)\theta$), therefore:

$$\Gamma(\theta) = 2[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta] = AT_3(\sec\theta_m \cos\theta)$$

Now the polynomial of degree 3 is replaced

$$\Gamma(\theta) = 2[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta] = A\left[sec^3\theta_m(\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta\right]$$

The terms that correspond to Γ_0 and Γ_1 are identified (equations 12 and 13), resulting in:

$$2\Gamma_0 = Asec^3\theta_m \tag{12}$$

$$2\Gamma_1 = 3A(sec^3\theta_m - \sec\theta_m) \tag{13}$$

Using a source impedance of $Z_0 = 50\Omega$ and a load impedance of $Z_L = 100\Omega$ with the reflection coefficient $\Gamma_1 = 0.05$, the calculation of the electrical length is obtained using equation 9.

$$\sec\theta_m = \cosh\left[\frac{1}{3} arc\ \cosh\left(\left|\frac{\ln {}^{100}/_{50}}{0.1}\right|\right)\right]$$

Therefore:

$$\theta_m = 44.72^o$$

Now, the fractional bandwidth is calculated:

$$FBW = 1.00606$$

From here, the coefficients of each section of the impedance transformer are obtained (equations 12 and 13):

$$\Gamma_0 = 0.0697$$

$$\Gamma_1 = 0.1035$$

Among the characteristics of Chebyshev-type impedance transformers is their symmetrical nature given their structure and therefore it is not necessary to perform the calculation for Γ_2 and Γ_3 since they will have the same values of Γ_1 and Γ_0 respectively.

$$\Gamma_2 = \Gamma_1 = 0.1035$$

$$\Gamma_3 = \Gamma_0 = 0.0697$$

From equation 11, the different values of the impedances for each section of the Chebyshev transformer are obtained:

For

$n = 0$ $\quad Z_1 = 57.4806\Omega$

$n = 1$ $\quad Z_2 = 70.7106\Omega$

$n = 2$ $\quad Z_3 = 86.9857\Omega$

With these values, the *Linecal* function of the ADS software was used to obtain the dimensions of the transmission lines of each section of the Chebyshev transformer. Here you have to specify the central operating frequency, as well as the substrate material (Duroid and FR4).

3 Results

The set of Chebyshev transformers using Duroid and FR4 substrates were developed for frequencies of 1 GHz, 1.5 GHz, 2 GHz, and 2.5 GHz to match impedances of 50 Ω with impedances of 75 Ω and 100 Ω and are shown in Table 1, Table 2 and Table 3.

Table 1. Z_1 Parameters

	Parameters		**Freq. (GHz)**	**Z_1 (Ω)**	**W_1 (mm)**	**L_1 (mm)**
1			1	57.480	3.152	54.976
2		ε_r=2.2	1.5	57.480	3.151	36.633
3		h=1.27	2	57.480	3.152	27.459
4	**Z_0**=50Ω		2.5	57.480	3.152	21.955
5	**Z_L**=100Ω		1	57.480	2.313	41.430
6		ε_r=4.4	1.5	57.480	2.313	27.579
7		h=1.54	2	57.480	2.314	20.650
8			2.5	57.480	2.316	16.491
9			1	55.122	3.366	54.868
10		ε_r=2.2	1.5	55.122	3.366	36.561
11		h=1.27	2	55.122	3.366	27.405
12	**Z_0**=50Ω		2.5	55.122	3.367	21.911
13	**Z_L**=75Ω		1	55.122	2.493	41.300
14		ε_r=4.4	1.5	55.122	2.492	27.492
15		h=1.54	2	55.122	2.494	20.584
16			2.5	55.122	2.496	16.437

Table 2. Z_2 Parameters

	Parameters		**Freq. (GHz)**	**Z_2 (Ω)**	**W_2 (mm)**	**L_2 (mm)**
1			1	70.710	2.231	55.520
2		ε_r=2.2	1.5	70.710	2.231	36.998
3		h=1.27	2	70.710	2.231	27.736
4	**Z_0**=50Ω		2.5	70.710	2.232	22.178
5	**Z_L**=100Ω		1	70.710	1.550	42.074
6		ε_r=4.4	1.5	70.710	1.550	28.015
7		h=1.54	2	70.710	1.551	20.982
8			2.5	70.710	1.552	16.761
9			1	61.237	2.846	55.140
10		ε_r=2.2	1.5	61.237	2.846	36.743
11		h=1.27	2	61.237	2.847	27.543
12	**Z_0**=50Ω		2.5	61.237	2.847	22.022
13	**Z_L**=75Ω		1	61.237	2.059	41.627
14		ε_r=4.4	1.5	61.237	2.059	27.713
15		h=1.54	2	61.237	2.060	20.752
16			2.5	61.237	2.062	16.573

Table 3. Z_3 Parameters

	Parameters		Freq. (GHz)	Z_3 (Ω)	W_3 (mm)	L_3 (mm)
1			1	86.985	1.513	56.080
2		ε_r=2.2	1.5	86.985	1.513	37.374
3		h=1.27	2	86.985	1.513	28.020
4	**Z_0=50Ω**		2.5	86.985	1.513	22.407
5	**Z_L=100Ω**		1	86.985	0.971	42.685
6		ε_r=4.4	1.5	86.985	0.971	28.429
7		h=1.54	2	86.985	0.971	21.298
8			2.5	86.985	0.972	17.018
9			1	68.029	2.386	55.417
10		ε_r=2.2	1.5	68.029	2.386	36.929
11		h=1.27	2	68.029	2.386	27.684
12	**Z_0=50Ω**		2.5	68.029	2.387	22.135
13	**Z_L=75Ω**		1	68.029	1.678	41.954
14		ε_r=4.4	1.5	68.029	1.678	27.934
15		h=1.54	2	68.029	1.678	20.921
16			2.5	68.029	1.680	16.711

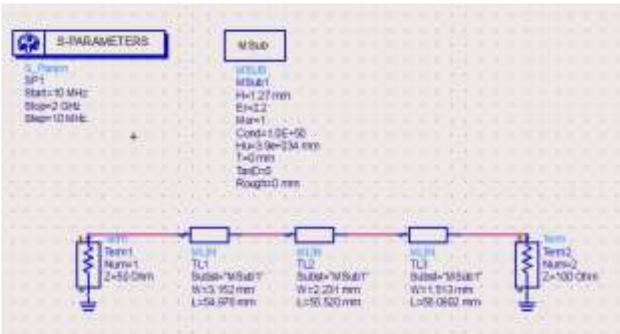

Fig. 1: Schematic of the Chebyshev impedance transformer.

Figure 1 shows the schematic of the circuit in ADS, where the values of the dimensions of the transmission lines of the stages of the Chebyshev-type transformer are introduced, as well as the impedances to be coupled, analysis frequencies, and information on the substrate.

Below are the simulated results in the ADS for the different values of impedances to be coupled, frequencies, and type of substrate, here the scattering parameter S_{11} is obtained, [3].

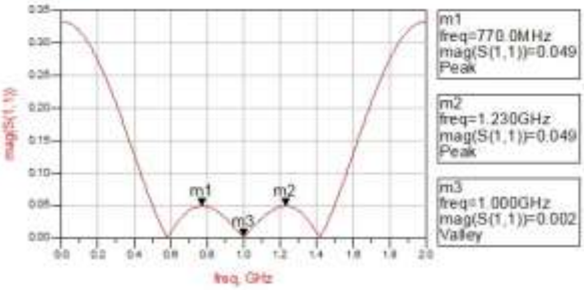

Fig. 2. Scattering parameter S_{11} with Z_o=50 Ω, Z_L=100 Ω at 1 GHz with Duroid substrate.

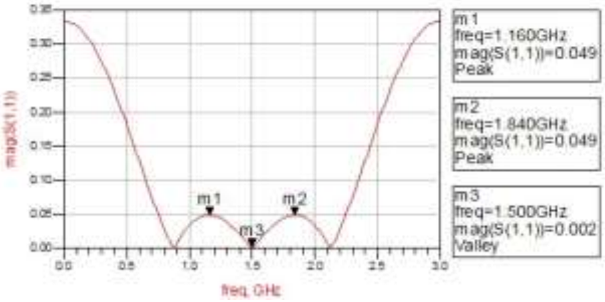

Fig. 3: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=100 Ω at 1.5 GHz with Duroid substrate.

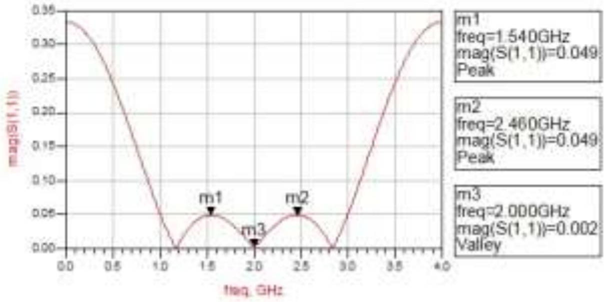

Fig. 4: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=100 Ω at 2 GHz with Duroid substrate.

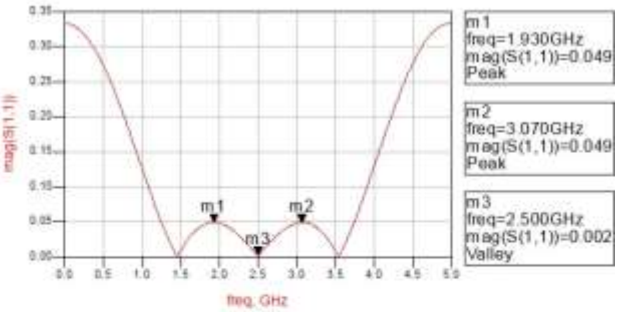

Fig. 5: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=100 Ω at 2.5 GHz with Duroid substrate.

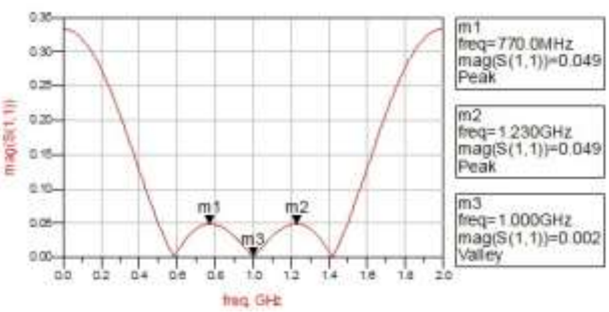

Fig. 6: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=100 Ω at 1 GHz with FR4 substrate.

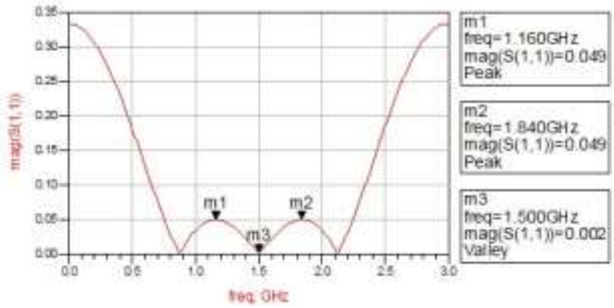

Fig. 7: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=100 Ω at 1.5 GHz with FR4 substrate.

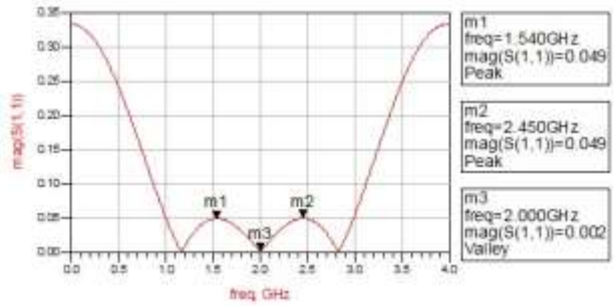

Fig. 8: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=100 Ω at 2 GHz with FR4 substrate.

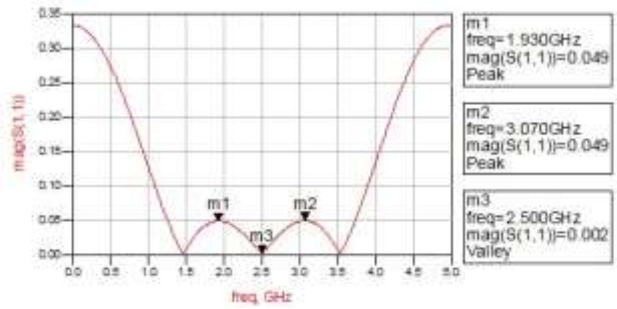

Fig. 9: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=100 Ω at 2.5 GHz with FR4 substrate.

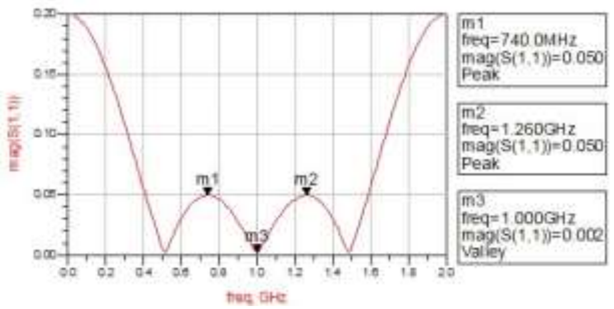

Fig. 10: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=75 Ω at 1 GHz with Duroid substrate.

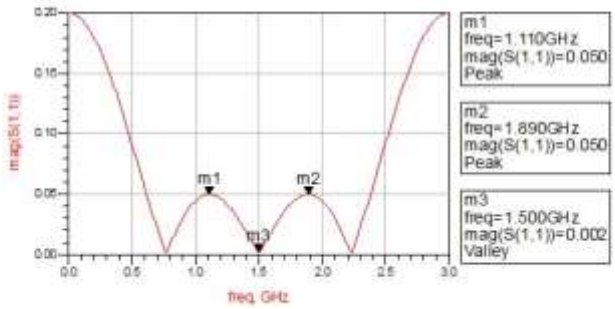

Fig. 11: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=75 Ω at 1.5 GHz with Duroid substrate.

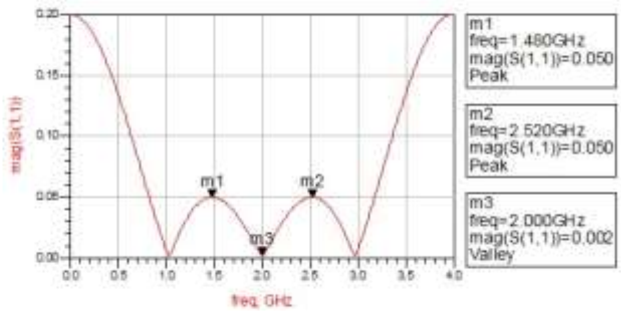

Fig. 12: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=75 Ω at 2 GHz with Duroid substrate.

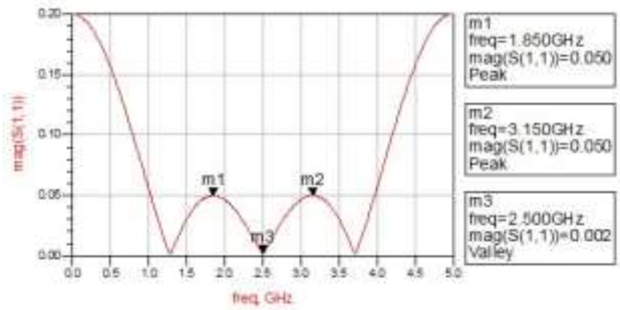

Fig. 13: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=75 Ω at 2.5 GHz with Duroid substrate.

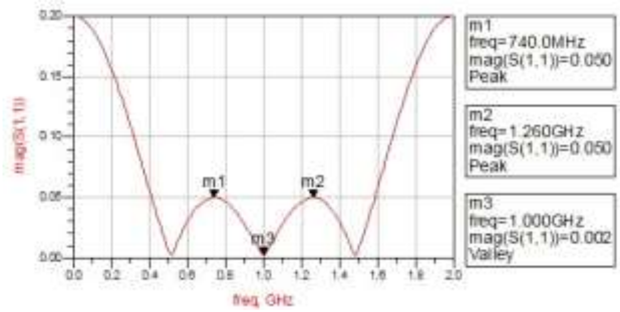

Fig. 14: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=75 Ω at 1 GHz with FR4 substrate.

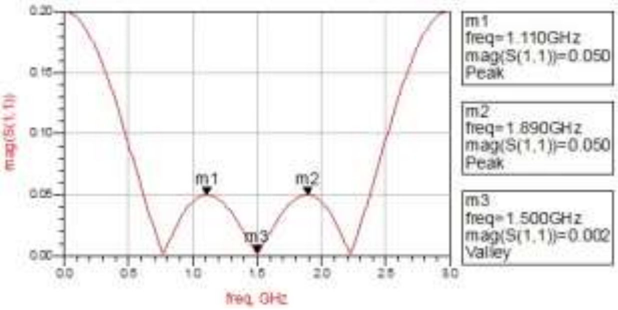

Fig. 15: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=75 Ω at 1.5 GHz with FR4 substrate.

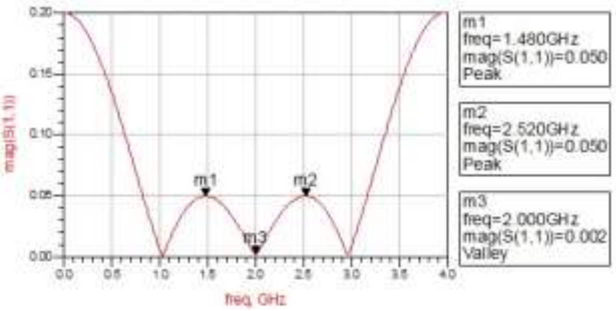

Fig. 16: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=75 Ω at 2 GHz with FR4 substrate.

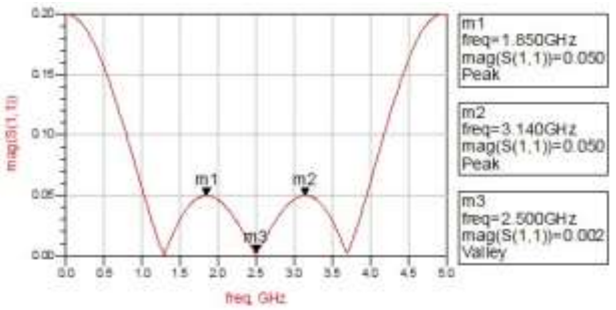

Fig. 17: Scattering parameter S_{11} with Z_o=50 Ω, Z_L=75 Ω at 2.5 GHz with FR4 substrate.

From the results from Figure 2, Figure 3, Figure 4, Figure 5, Figure 6, Figure 7, Figure 8, Figure 9, Figure 10, Figure 11, Figure 12, Figure 13, Figure 14, Figure 15, Figure 16, Figure 17, it is verified that in all cases maximum values of the reflection coefficient of 0.05 are obtained at the desired frequency and within bandwidths of up to and maximum values of Γ=0.049 when coupled the 50 Ω impedance with the 100 Ω impedance and a maximum value of Γ=0.05 to couple the 50 Ω impedance with the 75 Ω impedance.

In all cases, reflection coefficients of $\Gamma = 0.002$ were obtained at the central frequency and with bandwidths of at least 1 GHz and a maximum of 3 GHz.

The technique used for the development of high bandwidth impedance transformers with real impedances is useful and widely used, however, techniques for coupling real and complex impedances through the use of stubs are important for ultra-wideband (UWB) and super wideband (SWB) communications systems, [14].

4 Conclusion

The use of impedance transformers is important to reduce the amount of signal losses. In various systems that work with large bandwidths, it is not easy to achieve this purpose since for each frequency the value of the elements of the impedance transformers varies significantly, however, the use of impedance transformers such as Binomial and in the case of our study the Chebyshev transformer, allow not only to maintain at most a required reflection coefficient, but also present low values in wide bandwidths as we work with higher frequencies.

It was observed that like the Duroid substrate (which is not cheap), the FR4 substrate presents an almost identical behavior, which makes it ideal for frequency response given a low reflection coefficient.

For future work, impedance transformers will be made for complex impedances in UWB and SWB systems.

References:

[1] Chen W. K., "Broadband Matching: Theory and Implementations," *World Scientific Publishing*, USA, 2016.

[2] Yuqiang X., Xiuping L., Qingwen L. and Zihang Q., "A direct analytical and exact synthesis for multi-section impedance transformers," *International Journal of RF and Microwave Computer-Aided Engineering*, Vol. 31, No. 12, pp. December 2021.

[3] Pérez J. M., Torres A. E., Gonzalo R. and Ederra I, "A Chebyshev Transformer-Based Microstri-to-Groove-Gap-Waveguide Inline Transition for MMIC Packaging," *IEEE Transactions on Components, Packaging and Manufacturing Technology*, Vol. 9, No. 8, pp. 1595-1602, August 2019.

[4] Orfanidis S. J., "A Two-Section Dual-Band Chebyshev Impedance Transformer," *IEEE Microwave and Wireless Components Letters*, Vol. 13, No. 9, pp. 382-384, September 2003.

[5] Pérez J. M., Rebollo A., Gonzalo R. and Ederra I, "An Inline Microstrip-to-Waveguide Transition Operating in the Full W-Band Based on a Chebyshev Multisection

Transformer," 2016 *10th European Conference on Antennas and Propagation (EuCAP)*, April 2006.

[6] Satitchantrakula T., Akkaraekthalinb P., Silapunta R. and Torrungrueng D., "Compact wideband multi-section quarter-wave-like transformers," *Journal of Electromagnetic Waves and Applications*, Vol. 32, No. 15, pp. 1911–1924, June 2018.

[7] Byeong J. J. and Soo J. K., "Metasurface Modeled as Chebyshev Impedance Transformer for Super-Broadband Anti-Reflection of Visible and Near Infrared Light," *IEEE Photonics Journal*, Vol. 13, No. 3, June 2021.

[8] Castaldi G., Fiumara V. and Pinto I. M., "A Dual-Band Chebyshev Impedance Transformer," *Microwave And Optical Technology Letters*, Vol. 39, No. 2, pp. 141-145, October 2003.

[9] Biggs A. W., "Slow Wave Transmission Line Transformers," *IEEE Transactions On Electromagnetic Compatibility*, Vol. 34, No. 3, pp. 242-251, August 1992.

[10] Collin R. E., "Theory and Design of Wide-Band Multisection Quarter-Wave Transformers," *Proceedings of the I. R. E.*, Vol. 43, No. 2, pp. 179-185, February 1955.

[11] Monzon C., "Analytical derivation of a two-section impedance transformer for a frequency and its first harmonic," *IEEE Microwave And Wireless Components Letters*, Vol. 12, No. 10, pp. 381-382, October 2002.

[12] Ebnabbasi K., Busuioc D., Birken R. and Wang M., "Taper Design of Vivaldi and Co-Planar Tapered Slot Antenna (TSA) by Chebyshev Transformer," *IEEE Transactions on Antennas and Propagation*, Vol. 60, No. 5, pp. 2252-2259, May 2012.

[13] Darraji R., Honari M. M., Mirzavand R., Ghannouchi F. M. and Mousavi P., "Wideband Two-Section Impedance Transformer With Flat Real-to-Real Impedance Matching," *IEEE Microwave and Wireless Components Letters*, Vol. 26, No. 5, pp. 313-315, May 2016.

[14] Sun Z., Yang X., Wang Y., Ma J., Wang X., Chen Ch., Milinevsky G. and Lu G., "A Dual-Band Filtering Impedance Transformer With Real-to-Complex Impedance Loads," *IEEE 6th International Conference on Electronic Information and Communication Technology (ICEICT)*, pp. 793-795, 2023.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Karol Yamel Bautista Ramírez, Edgar Alejandro Andrade González, and Mario Reyes Ayala carried out the Chebyshev transformer design. They established the limits of the reflection coefficient for applications in UWB systems and the operation conditions.
- Hilario Terres and Sandra Chávez simulated ADS to get de scattering parameter S_{11}.
- Gerardo Salgado Guzmán and José Ignacio Vega Luna described the introduction of the paper and got the dimension of the Chebyshev impedance transformer using *Linecalc* with ADS.
- Héctor Bolivar Olmos Ramírez checked spelling and grammar. Also, he wrote, review and edited. Finally, he helped with supervision.

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Conflict of Interest

The authors have no conflict of interest to declare.