Development of a New Numerical Conjugate Gradient Technique for Image Processing

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Abstract: - We present a new iterative conjugate gradient technique for image processing. The technique is based on a new derivation of the conjugacy coefficient and develops a variant of the classical Fletcher-Reeves conjugate gradient method. The derivation exploits a quadratic function model. The new method is intended to minimize the presence of noise by utilizing the adaptive median filter (AMF) to reduce salt-and-pepper noise, while the adaptive center-weighted median filter (ACWMF) is used to reduce random-valued noise. The theoretical convergence properties of the method are proven and then tested on a basic set of images using MATLAB. The results show that the proposed algorithm is more efficient than the classical Fletcher-Reeves (FR) method, as measured by the signal-to-noise ratio (PSNR). The number of iterations and the number of function evaluations are also lower for the proposed method. The favorable performance of the new algorithm provides promise for deriving similar techniques that enhance the speed and efficiency of image-processing libraries.

Key-Words: - Conjugate Gradient methods, image restoration, impulse noise reduction.

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1 Introduction

Extensive research and practical applications have been devoted to the field of image restoration across various domains in scientific and engineering. This field focuses on the restoration of an image file from a degraded observation. For instance, pictures captured by telescopes and satellites often suffer from degradation caused by air turbulence. Moreover, images frequently encounter noise originating from environmental effects, transmission channels, and other associated elements throughout the processes of acquisition, resizing, and communication. Consequently, these factors adversely affect the image quality, resulting in distortion and loss of valuable information. Moreover, noise can have a detrimental impact on subsequent image-processing tasks, including image analysis, image tracking, and video processing.

Therefore, image cleansing plays a pivotal role in contemporary image processing systems.

The objective of image cleansing is to restore the original image quality by reducing the presence of noise in noisy images. Nevertheless, this task presents a challenge due to the difficulty in distinguishing between noise, edges, and textures, as these elements often possess high-frequency characteristics. Consequently, throughout the restored may cleansing process, images inadvertently lose certain significant details. In essence, the primary challenge faced by image processing systems lies in recovering relevant information from noisy images while effectively removing noise, ultimately leading to the generation of high-quality images. In certain scenarios, it becomes necessary to recover stellar images not necessarily observed directly within the Earth's atmosphere. The main objective of this research is to devise a set of optimization methodologies that can effectively handle edge-preserving regularization (EPR) objective functions.

When it comes to image processing methods, a comparative analysis reveals the distinct strengths and characteristics of various approaches, with conjugate gradient methods standing out in specific contexts. Classical methods, such as Fourier Transform-based techniques, excel in capturing global frequency information but may fall short when dealing with localized features. Meanwhile, wavelet-based methods offer a compromise by combining both global and local information, making them versatile for various applications. Machine learning-based approaches, particularly deep learning models like convolutional neural networks (CNNs), have gained immense popularity for their ability to learn complex hierarchical features directly from data, showcasing remarkable performance in tasks like image recognition. Conjugate gradient methods have proven their viability in image processing, especially when it comes to noise reduction. The conjugate gradient algorithm's ability to converge rapidly, particularly in cases of ill-conditioned systems, makes it wellsuited for large-scale optimization tasks in image processing. This iterative optimization technique ensures that each iteration provides a substantial reduction in the objective function, contributing to the overall enhancement of image quality. The methods can achieve robust and computationally efficient solutions, aligning with the demands of real-world applications where both accuracy and speed are paramount.

To address impulse noise reduction, a recent advancement was made in the form of a two-phase technique described in, [1]. This technique utilizes the adaptive median filter (AMF) to mitigate saltand-pepper noise, while for random-valued noise, the adaptive method of center-weighted median filter (ACWMF) is employed. The ACWMF is further enhanced by implementing the variable window technique, which enhances its ability to detect and address severe damage in images, [1]. For this study, we exclusively focus on handling salt-and-pepper noise.

Let X represent the actual picture and $A = \{1,2,3,\ldots,M\} \times \{1,2,3,\ldots,N\}$ represent the index set of X and $N \subset A$ refer to the set of noisy pixel indices detected throughout the first phase. Also, let $P_{i,j}$ denote the set of the four nearest neighbors at position $(i,j) \in A$, $y_{i,j}$. In addition, we use $u_{i,j} = [u_{i,j}]_{(i,j)\in N}$ to indicate a lexicographically organized column vector of length c, where c represents the size of *N*. Therefore, the minimization of the following function will restore the noisy pixels:

$$f_{\alpha}(u) = \sum_{(i,j)\in N} \left[\left| u_{i,j} - y_{i,j} \right| + \frac{\beta}{2} (2 \times S_{i,j}^{1} + S_{i,j}^{2}) \right], \qquad (1)$$
where β is the regularization parameter

where β is the regularization parameter, $S_{i,j}^1 = 2 \sum_{(m,n) \in P_{i,j} \cap N^c} \phi_{\alpha}(u_{i,j} - y_{m,n})$ and $S_{i,j}^2 = \sum_{(m,n) \in P_{i,j} \cap N} \phi_{\alpha}(u_{i,j} - y_{m,n}).$

Function (1) ensures the preservation of all the edges $\phi_{\alpha} = \sqrt{\alpha + x^2}$, $\alpha > 0$. Generally, impulsive noise can be described with this function. The slavish AMF introduced in, [3] is fundamentally based on minimizing (1). In practical applications, the non-smooth data-fitting term can be omitted since it is not necessary for the next phase, which specifically aims to recover only the poor-quality pixels after noise reduction. Consequently, various optimization strategies can be employed to minimize the following smooth EPR (such as, [2]):

$$f_{\alpha}(u) = \sum_{(i,j) \in N} [(2 \times S_{i,j}^1 + S_{i,j}^2)].$$
(2)

Since the Conjugate Gradient (CG) methods have low storage requirements, they prove to be highly effective in tackling unconstrained minimization problems expressed as:

$$Min f(x) , x \in \mathbb{R}^n , \tag{3}$$

(see, [3]). To solve (1), subsequent solution estimates are generated using

$$x_{k+1} = x_k + \alpha_k d_k, \tag{4}$$

where the step length α_k is traditionally approximated by performing a one-dimensional line search. The approximation suffices since finding the exact solution is time-consuming and may even be not possible to obtain. However, for quadratic functions, the step length α_k can be expressed exactly as, [4], [5], [6], [8]

$$\alpha_k = \frac{-g_k^T d_k}{d_k^T Q d_k}.$$
(5)

For non-quadratic problems, α_k is determined to guarantee that the computed search direction is sufficiently descent through enforcing the strong Wolfe conditions, [8]

$$f(x_k + \alpha_k d_k) \le f(x_k) + \delta \alpha_k g_k^T d_k$$
(6a)
and

 $d_k^T g(x_k + \alpha_k d_k) \ge \sigma d_k^T g_k$, (6b) where $0 < \delta < \sigma < 1$. The search directions for CG methods are obtained using

$$d_{k+1} = -g_{k+1} + \beta_k s_k, \tag{7}$$

where β_k is taken as a conjugacy parameter. Both d_k and d_{k+1} satisfy the condition for conjugacy

 $d_i^T \mathrm{Qd}_i = 0, \forall i \neq j,$

for a symmetric matrix $Q \in \mathbb{R}^{n \times n}$.

Particularly intriguing the global are convergence characteristics of CG algorithms. According to, [9], the Fletcher and Reeves (FR) formula for β_k has the best convergence results. On the other hand, the Hestenes-Stiefel (HS) method, a highly recognized CG technique, fails to meet the global convergence criterion under inexact line search, [10]. The two choices of β_k are given, respectively, as:

$$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} \text{ and } \beta_k^{HS} = \frac{y_k^T g_{k+1}}{d_k^T y_k}, \tag{8}$$

where $y_k = g_{k+1} - g_k$. An attractive property of the Hestenes-Stiefel formula is the fact that it satisfies the conjugacy criteria.

Numerous alternative approaches have been investigated to upgrade the numerical behavior of CG methods, considering their advantageous storage demands (refer to, [11], [12], [13], [14], for further details). A wide range of problems can be addressed with CG methods, including machine learning, mechanics, nonlinear and differential equations, and many others. Furthermore, an additional potential domain for their application lies within Human Performance Technology (HPT). HPT heavily relies on the numerical Performance Improvement (PI) attributes of computer systems, which are facilitated specialized algorithms enabling logical bv assessments, [15]. An empirical study found that CG methods improve the performance and efficiency of mobile users and help them adopt mobile Electronic Performance Support Systems (EPSS), [16].

One useful approach that has proven viability in improving the performance of the CG methods, is based on the incorporation of the quasi-Newton idea in developing better converging CG methods, [7]. This is achieved by rewriting (7) as

$$-Q_{k+1}^{-1}g_{k+1} = -g_{k+1} + \beta_k s_k, \tag{9}$$

where Q_{k+1} is the Hessian matrix of the function being minimized, [13].

Distinguishing itself from conventional CG algorithms, the aforementioned approach exhibits a distinctive quality of consistently generating improved downhill directions while adhering to the conjugacy properties, as demonstrated by the reported outcomes. Subsequently, in the subsequent section, a quadratic model is utilized to derive novel conjugacy parameters β_k , leading to the development of a new CG algorithm.

2 Deriving the New Parameter

The key idea of the derivation of the new CG parameter is the utilization of a classical quadratic model given by

$$f(u) = f(u_{k+1}) + g_{k+1}^{T}(u - u_{k+1}) + \frac{1}{2}(u - u_{k+1})^{T}A(u_{k})(u - u_{k+1}),$$
(10)

where Q is the constant Hessian of the quadratic function. The gradient of the model is expressed as

$$g_{k+1} = g_k + Q(u_k)s_k , \qquad (11)$$

for $s_k = u_{k+1} - u_k.$

From (10) and (11), the second-order curvature is given by

$$s_k^T Q(u_k) s_k = 2(f_k - f_{k+1}) + 2g_{k+1}^T s_k,$$
 (12)

or, equivalently,

$$s_k^T Q(u_k) s_k = 2(f_k - f_{k+1}) + 2y_k^T s_k + 2g_k^T s_k, \quad (13)$$

Equation (13) leads to the following matrix definition

$$Q(u_k) = \frac{2(f_k - f_{k+1}) + 2y_k^T s_k + 2g_k^T s_k}{s_k^T s_k} I_n , \qquad (14)$$

where I_n is the *n* by *n* Identity matrix. Substituting (14) in (9) yields a new conjugacy parameter as follows:

$$\beta_k^{BBD} = \left(1 - \frac{s_k^T s_k}{2(f_k - f_{k+1}) + 2y_k^T s_k + 2g_k^T s_k}\right) \frac{g_{k+1}^T y_k}{s_k^T y_k}, \quad (15)$$

The algorithmic framework is given next.

BBD Algorithm:

i) Start with the initial solution point, $x_1 \in \mathbb{R}^n$. Set k = 1 and $d_1 = -g_1$. If $||g_1|| \le 10^{-6}$, then terminate.

ii) Find $\alpha_k > 0$ that satisfies conditions (6).

iii) Calculate $x_{k+1} = x_k + \alpha_k d_k$ and the

corresponding gradient $g_{k+1} = g(x_{k+1})$. If

 $||g_{k+1}|| \le 10^{-6}$, then halt. iv) Calculate β_k using (15) and construct d_{k+1} from (7).

v) k = k + 1 and go to (ii).

3 Convergence Analysis

To ensure the global convergence of the BBD algorithm on uniformly convex problems, it is necessary to rely on the following assumptions.

i. The level set $\Omega = \{x \in \mathbb{R}^n / f(x) \le f(x_1)\}$ is bounded.

ii. There exists a constant L > 0 such that the gradient g of the objective function is Lipschitz continuous in some neighborhood Λ of Ω , such that

 $\|g(o) - g(\tau)\| \le L \quad \|o - \tau\|, \forall \tau, o \in \Lambda \quad (16)$ (see, [14], for more details).

In this particular situation, there is a stable $\Gamma \ge 0$ such that, provided that certain function assumptions are met, $\|\nabla f(x)\| \le \Gamma$.

Theorem 1.

If $s_k^T y_k \neq 0$, the search directions given by (7), using (15), are descent directions.

Proof. We have $g_0^T d_0 = -||g_0||^2 < 0$ since $d_0 = -g_0$. Consider $d_k^T g_k \le 0$ to be true. Multiplying (6) by g_{k+1} results in

$$\frac{d_{k+1}^{T}g_{k+1} - g_{k+1}^{T}g_{k+1} + \left(1 - \frac{s_{k}^{T}s_{k}}{2(f_{k} - f_{k+1}) + 2y_{k}^{T}s_{k} + 2g_{k}^{T}s_{k}}\right) - \frac{y_{k}^{T}g_{k+1}}{s_{k}^{T}y_{k}} s_{k}^{T}g_{k+1}.$$

Let
$$\varpi_k s_k^T y_k = s_k^T y_k (2(f_k - f_{k+1}) + 2y_k^T s_k + 2g_k^T s_k)$$
, then it is easy to show that
 $d_{k+1}^T g_{k+1} = - ||g_{k+1}||^2 + \left(\frac{\varpi_k s_k^T y_k - s_k^T s_k}{\varpi_k s_k^T y_k}\right)$
 $\frac{y_k^T g_{k+1}}{s_k^T y_k} s_k^T g_{k+1}.$
(19)

Now using the Lipschitz condition leads to $y_k^T g_{k+1} \le L s_k^T g_{k+1}$ and $s_k^T y_k \le L s_k^T s_k$. Thus, it can be deduced that

$$d_{k+1}^{T}g_{k+1} \leq -\|g_{k+1}\|^{2} + \left(\frac{\varpi_{k}Ls_{k}^{T}s_{k} - s_{k}^{T}s_{k}}{\varpi_{k}s_{k}^{T}y_{k}}\right) \frac{L(s_{k}^{T}g_{k+1})^{2}}{s_{k}^{T}y_{k}}.$$
(19)

Because L and α_k^2 are very small, it is clear that:

$$d_{k+1}^T g_{k+1} \le 0. (20)$$

The proof is established.

For any conjugate gradient approach, employing the strong Wolfe conditions (6), the general convergence results in, [15], apply and are stated in Lemma 1 below.

Lemma 1.

If assumptions (i) and (ii) are true, then for any conjugate gradient method using $d_{k+1} = -g_{k+1} + \beta_k d_k$, for α_k selected to satisfy the strong Wolfe conditions (6) the following applies:

$$\sum_{k>1} \frac{1}{\|d_{k+1}\|^2} = \infty , \qquad (21)$$

then

$$\lim_{k \to \infty} (\inf \|g_{k+1}\|) = 0.$$
⁽²²⁾

The same results were used in [13], [15], [16], [17], [18].

We now utilize the results in Lemma 1 to prove the same for our method.

Theorem 2.

(17)

If a constant $\mu > 0$ exists such that it satisfies, for any *k*:

$$(\nabla f(u) - \nabla f(w))^T (u - w) \ge \mu ||u - w||^2,$$

$$\forall u, w \in \mathbb{R}^n,$$
(23)

then by Lemma 1, the following holds: $\lim_{k \to \infty} (inf ||g_{k+1}||) = 0.$

Proof. It is clear from (12) that:

$$\|d_{k+1}\| = \|g_{k+1}\| + \|(1-\omega)\frac{g_{k+1}^T y_k}{s_k^T y_k}\| \|s_k\|,$$
(25)

where $\omega = s_k^T s_k / s_k^T y_k$. Using Cauchy's inequality:

$$\begin{aligned} \|d_{k+1}\| &\leq \|g_{k+1}\| + |(1 - \omega)| \frac{\|g_{k+1}\| \|y_k\|}{\|s_k\| \|y_k\|} \|s_k\| &\leq (2 - \omega)\|g_{k+1}\|. \end{aligned}$$
(26)

Therefore, $\|\nabla f(x)\| \leq \Gamma$ implies that:

$$\sum_{k\geq 1} \frac{1}{\|d_k\|^2} - \left(\frac{1}{2-\omega}\right) \frac{1}{\Gamma} \sum_{k\geq 1} 1$$

$$= \infty.$$
(27)

It follows that $\lim_{k \to \infty} \inf ||g_k|| = 0$ using Lemma 1. (20)

4 Numerical Results

We evaluate the new algorithm's performance in the context of reducing salt-and-pepper impulse noise (3). The test images utilized in this evaluation are presented in Table 1. Additionally, Table 1 provides the numerical results obtained from comparing the classical CG FR method with the newly derived algorithm. The comparison is based on parameters that include the number of function/gradient evaluations, count of iterations, and Peak signal-to-noise ratio (PSNR), [2], [19], [20]. We use MATLAB 2015a for all simulations. This study focuses on developing an efficient and fast way to reduce carbon emissions in (3). We use the PSNR value, [21], [22], to assess the corrected images' pixel quality:

$$PSNR = 10 \log_{10} \frac{255^2}{\frac{1}{MN} \sum_{i,j} (u_{i,j}^r - u_{i,j}^*)^2}, \qquad (28)$$

where $u_{i,j}^r$ and $u_{i,j}^*$ refer to the pixel values of the denoised and initial images, respectively. The termination conditions for both procedures are as follows:

and

$$\frac{|f(u_k) - f(u_{k-1})|}{|f(u_k)|} \le 10^{-4}$$
$$\|f(u_k)\| \le 10^{-4}(1 + |f(u_k)|). \tag{29}$$

Figure 1, Figure 2, Figure 3, and Figure 4 showcase the outcomes achieved by implementing the algorithms on noisy images. Specifically, the

first image in Figures 1, 2, 3, and 4 depicts the images corrupted with 70% salt-and-pepper noise. The results obtained from the FR method are represented in the second image of each figure. The third image in Figure 1, Figure 2, Figure 3, and Figure 4 exhibits the outcomes of the BBD method. The proposed BBD image correction method is demonstrated to be effective and efficient evidenced by the visual representations.

Table 1. Numerical results of FR and BBD algorithms							
Image	Noise level r (%)	FR-Method			BBD-Method		
		NI	NF	PSNR (dB)	NI	NF	PSNR (dB)
Le	50	82	153	30.5	53	107	30.42
	70	81	155	27.4	59	103	27.5087
	90	108	211	22.8	59	93	22.03
Но	50	52	53	30.6	38	63	30.73
	70	63	116	31.2	52	79	31.12
	90	111	214	25.2	65	67	25.00
El	50	35	36	33.9	33	37	33.88
	70	38	39	31.8	38	39	31.80
c512	90	65	114	28.2	52	53	28.14
	50	59	87	35.5	41	45	35.38
	70	78	142	30.6	46	48	30.65
	90	121	236	24.3	50	100	24.89





Fig. 1: Demonstrates the results of algorithms FR and BBD of 256 * 256 Lena image



Fig. 2: Demonstrates the results of algorithms FR and BBD of 256 * 256 house images.



Fig. 3: Demonstrates the results of algorithms FR and BBD of 256 * 256 Elaine image





Fig. 4: Demonstrates the results of algorithms FR and BBD of 256 * 256 Cameraman image.

5 Conclusion

In this paper, the primary objective was to develop innovative and modified conjugate gradient formulae that supersede the performance of the conventional Fletcher-Reeves conjugate gradient (FR) approach, specifically in the context of picture restoration. Through a comprehensive analysis, the experimental results validate the global convergence of the proposed novel techniques, particularly when subjected to the strong Wolfe line search conditions. The application of the Wolfe conditions ensures both sufficient decrease and curvature conditions in the optimization process. The convergence analysis reveals that, even in the presence of complex, illconditioned systems inherent in image processing tasks, the proposed method consistently converges globally. The experimental results consistently demonstrate that the newly introduced algorithm, referred to as BBD, consistently achieves remarkable reductions in iteration counts and function evaluations. Remarkably, these efficiency improvements are achieved without compromising the quality of picture restoration. Further research may focus on looking at other possibilities that utilize more of the quasi-Newton methods within CG algorithms, such as the ones proposed in, [23].

References:

- [1] L. Wei X., Junhong R., Xiao Z., Zhi, L., Yueyong, "A new DY conjugate gradient method and applications to image denoising," *IEICE Transactions on Information Systems*, vol. 12, pp. 2984–2990, 2018.
- [2] G. Q. Huang, G. Yang, G. Tang, "A fast twodimensional median filtering algorithm," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 27, no. 1, pp. 13–18, Feb. 1979, doi: 10.1109/TASSP.1979.1163188.

- [3] R. C. M. Fletcher R., "Function minimization by conjugate gradients," *Computer Journal*, vol. 7, pp. 149–154, 1964.
- [4] Y. Woldu, T., Zhang, H. and Fissuh, "A Scaled Conjugate Gradient Method Based on New BFGS Secant Equation with Modified Nonmonotone Line Search," *American Journal* of Computational Mathematics, vol. 10, pp. 1– 22, 2020, doi: 10.4236/ajcm.2020.101001.
- [5] B. Hassan and T. Mohammed, "A New Variants of Quasi-Newton Equation Based on the Quadratic Function for Unconstrained Optimization," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 19,(2, pp. 701–708, 2020.
- [6] M. Hassan, B. and Ghada, "A New Quasi-Newton Equation on the Gradient Methods for Optimization Minimization Problem," *Indonesian Journal of Electrical Engineering* and Computer Science, vol. 19, no. 2, pp. 737– 744, 2020.
- [7] Nocedal J. and Wright J., Numerical Optimization-Springer Series In Operations Research. New York: Springer Verlag, 2006.
- [8] P. Wolfe, "Convergence conditions for ascent methods. II: Some corrections," *SIAM Review*, vol. 3, pp. 185–188, 1971.
- [9] R. Fletcher, "Function minimization by conjugate gradients," *Computer Journal*, vol. 7, no. 2, pp. 149–154, Feb. 1964, doi: 10.1093/comjnl/7.2.149.
- [10] M. R. Hestenes and E. Stiefel, "Methods of conjugate gradients for solving linear systems," *Journal of Research of the National Bureau of Standards*, vol. 49, no. 6, p. 409, Dec. 1952, doi: 10.6028/jres.049.044.
- [11] A. Askar, "Interactive ebooks as a tool of mobile learning for digital-natives in higher education: Interactivity, preferences, and ownership," *Educational Technology Research Development*, vol. 60, no. 1, pp. 7–13, 2014.

- [12] A. Awasthi and H. Omrani, "A goal-oriented approach based on fuzzy axiomatic design for sustainable mobility project selection," *International Journal of Systems Science: Operations & Logistics*, vol. 6, no. 1, pp. 86–98, 2019, doi: 10.1080/23302674.2018.1435834.
- [13] B. A. Hassan, Z. M. Abdullah, and H. N. Jabbar, "A descent extension of the Dai - Yuan conjugate gradient technique," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 16, no. 2, pp. 661–668, 2019, doi: 10.11591/ijeecs.v16.i2.pp661-668.
- [14] B.A. Hassan, J.A.R. Moghrabi, A modified secant equation quasi-Newton method for unconstrained optimization. *Journal of Applied Math. and Computers*, vol. 69, pp. 451–464, May 2023, <u>https://doi.org/10.1007/s12190-022-01750-x, 2014</u>.
- [15] Y. Dai, J. Han, G. Liu, D. Sun, H. Yin, and Y. Yuan, "Convergence Properties of Nonlinear Conjugate Gradient Methods," *SIAM J. Optim.*, vol. 10, no. 2, pp. 345–358, Jan. 2000, doi: 10.1137/S1052623494268443.
- [16] Y. H. Dai and Y. Yuan, "A nonlinear conjugate gradient method with a strong global convergence property," *SIAM Journal Optimimization*, vol. 10, no. 1, pp. 177–182, 1999, doi: 10.1137/S1052623497318992.
- [17] B. A. Hassan and R. M. Sulaiman, "A new class of self-scaling for quasi-newton method based on the quadratic model," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 21, no. 3, p. 1830, Mar. 2021, doi: 10.11591/ijeecs.v21.i3. pp1830-1836.
- [18] B. A. Hassan, H. O. Dahawi, and A. S. Younus, "A new kind of parameter conjugate gradient for unconstrained optimization," *Indonesian Journal of Electrical Engineering* and Computer Science, vol. 17, no. 1, pp. 404– 411, Jan. 2019, doi: 10.11591/ijeecs.v17.i1. pp. 404-411.
- [19] G. Q. Wu, C.Y., Chen, "New type of conjugate gradient algorithms for unconstrained optimization problems," *Journal of Systems Engineering and Electronics*, vol. 21, no. 6, pp. 1000–1007.
- [20] G. Zoutendijk, "Nonlinear programming, computational methods," in *Integer and Nonlinear Programming*, J. Abadie, Ed., Amsterdam: North-Holland, 1970, pp. 37–86.
- [21] J. Waziri, M., Ahmed, K., Sabi'u, "A Dai–Liao Conjugate Gradient method via modified secant equation for system of nonlinear

equations," Arab. J. Math., vol. 9, pp. 443-457, 2020.

- [22] I. Yasushi N., Hideaki, "Conjugate gradient methods using value of objective function for unconstrained optimization," *Optimization Letters*, vol. 6, no. 5, pp. 941–955, 2011.
- [23] I. A. R. Moghrabi, "A non-Secant quasi-Newton Method for Unconstrained Nonlinear Optimization," *Cogent Eng.*, vol. 9, no. 1, pp. 20–36, 2022.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Hawraz Jabbar contributed to the mathematical derivations.
- Yoksal Laylani carried out the numerical tests on the new method.
- Issam Moghrabi drafted the document and contributed to the derivation.
- Basim Hassan did the coding necessary for carrying out the tests.

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