Optimizing the Energy Efficiency of a Lighting Network using Graph Theory

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Abstract: - In this paper, we discuss how to make electric street lighting systems more energy efficient by creating an algorithm and mathematical model for optimizing parameters, minimizing active power losses, and finding the best topology for the lighting network when it is being designed or updated. Scientific and technological progress has led to an increase in the complexity of every human being's daily life. Companies, institutions, and countries constantly need to find modern tools to help them make the best decisions. Graph theory has numerous applications to many everyday problems. It can resolve and simplify them. An algorithm was developed to determine the shortest length in the form of a modified Dijkstra graph, with nodes supporting the street lights and ribs being the wires connecting them.

Key-Words: - graph theory, algorithm Dijkstra, optimization, energy efficiency, lighting, mathematical model, topology.

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1 Introduction

A priority direction for energy development is the creation of a new generation of urban energy systems based on modern technological means and distributed intelligent management systems. One of the main components of the city's energy system is street lighting. In the context of this work, a power supply set of street lighting systems, consisting of power and lighting networks, as well as the installations themselves, is considered. Their operational life and the safety and comfort of city dwellers depend on the reliability of their functioning. These systems are energy-intensive objects; electricity consumption for lighting needs can reach 30-40% of the total energy consumption in the city. Because saving energy is one of the priorities of developed economies, ensuring energy efficiency is one of the most important areas to modernize the city's electricity system. By energyefficient operation, we mean the lowest electricity consumption while providing the standard level of road and sidewalk lighting, [1]. The analysis of scientific publications and technical solutions in this field showed that several approaches are currently used to increase the energy efficiency of street lighting, namely: replacing light sources with other more efficient

ones; optimizing the lighting network; and algorithms for energy-efficient management. It should be noted that in the existing scientific studies, the application of methods for optimizing the parameters of network elements of street lighting was carried out without formalizing boundary conditions that reflect the requirements of regulatory documents in the field of electricity. Additionally, some solutions consider simulating how all the parts of street lighting work together. This lets you model different systems and pick the best elements and parameters for controlling and figuring out how energy-efficient each mode is based on the results of optimizing the lighting network's parameters. The calculation can be aimed at reducing active power losses by reducing the power of street lighting installations, increasing the cross-section of power supply lines, and compensating reactive power, [1], [2], [3]. The results of the study show that these measures can reduce active power losses by up to 45%. Noting the undeniable practical importance of the listed methods, it is worth mentioning that they analyze specific ways to reduce losses in existing networks but do not offer ways to optimize street lighting parameters at the design stage. In the analyzed developments, the only criterion for choosing the measures to reduce losses is maximization, but the costs of introducing energy-saving measures and their impact on illumination are not taken into account, [4]. Mathematical models of the system are also proposed to support decision-making for the design and modernization of street lighting systems that allow for maximizing energy efficiency. Even though in the scientific community, the issue of optimizing the parameters of distribution network elements has been developed quite thoroughly, [5], [6], the authors do not know of any works that offer mathematical models and methods for optimizing the parameters of electrical networks for street lighting.

There are automated systems for controlling lighting fixtures for open spaces, such as "Arman", "Helios", "Zora", and "Modul-C." All of them were found to use deterministic algorithms for operating mode selection with a single input variable, either readings of light sensors, geographic coordinates of a point, or a predefined schedule for operating light heating installations. The comparative analysis of control systems showed that the most promising should be the development of control systems with a mode of operation according to sensor readings and the use of intelligent algorithms. It is noted that the lighting network with the highest energy efficiency will have light sources that allow smooth regulation of energy and light flow consumption. Papers, [2], are devoted to the development of street lighting control systems using artificial intelligence methods. Currently, such management systems are being implemented as pilot projects and are not widely used, despite their considerable economy.

The purpose of this paper is to present a theoretical approach to graph theory and a solution to the routing problem for wiring in a street lighting network. For a fixed location (the lighting node), the algorithm will be designed to optimize a specific wiring route. A changed version of Dijkstra's algorithm is suggested for a brand-new network. This algorithm can figure out a topology with the fewest number of wires by using the known locations of the power sources on streetlight poles, [7], [8], [9].

At the same time, the relationship between the number M of light poles and the distance V between them will be studied, and the reduction of the distance between them will be sought as a function of the distance V, [10]. Finally, the total distance between the nodes will be calculated. The most difficult process—the mathematical formula for node selection—will also be presented, and an optimal topology will be found.

2 A Model for Optimizing the Parameters of the Lighting Network

We assume that the lighting network will be represented as a graph in the process of developing a methodology for determining the ideal cross-section of the wires and the number of lighting fixtures. The nodes (n) in the graph will be the street lighting poles, and the ribs will be the wires connecting them. In this instance, a matrix connection C [n x n] can be used to depict the topology of the lighting network. As an optimization criterion, we provide:

1. Electrotechnical standard for reducing active power losses (Figure 1):



Fig. 1: Network lighting area

$$\Delta P = \sum_{k=1}^{n} \sum_{j=1}^{n} I_{k,j}^2 R_{k,j} \to min \tag{1}$$

where n is the total number of nodes and k and j are the number of nodes; In the lighting network's regular operating state, $I_{k,j}$ is the current flowing along the branch linking nodes k and j, and $R_{k,j}$ is the resistance of the branch that connects them, Om, [11].

2. The financial standard for reducing operating and capital expenses related to energy loss while building the network is:

$$S_{capital} = S_c + S_p + S_l \to min \tag{2}$$

$$S_{exploitation} = \Delta P * \tau_{max} * S_l \to min \tag{3}$$

$$S_{conductors} = \sum_{k=1}^{n} \sum_{j=1}^{n} l_{k,j} S_{uc} \tag{4}$$

$$S_{light} = n(P(n) \neq 0 * S_i \tag{5}$$

$$S_{pole} = n * S_{sp} \tag{6}$$

where S_p is the price of street lighting poles, and S_c is the cost of cables. S_1 is the price of lamps, k,j is the distance in kilometers that a segment of the network connects nodes j and k, S_{uc} is a wire's unit cost per kilometer. S_{sp} is the price of a single pole, including installation. Ssl is the price of a single lamp, installed. P[n] is the load vector's number of non-zero entries, where n $\neq 0$.

3. Light standard to optimize roadway lighting:

$$D = l * A * n(P)n \neq 0) \rightarrow max$$
(7)

where A is the width of the illuminated road, sidewalk, etc., and I is the distance between the lamps, m. We get the goal function $S'(x) \rightarrow max$ for the task of finding the best cross-section of wires and the number of lamps in the lighting network by using multiplicative convolution as a scalarization method:

$$S'(x) = S_1^{P_1} * S_2^{P_2} \dots * S_n^{P_n}$$
(8)

where $x=(x_1,x_2)$ is a vector of desired values (wire cross-section, number of devices), S'(x) is a super criterion, S₁,S₂...,S_n are the values of optimization criteria, and P₁,P₂...,P_n are the weights of corresponding criteria. As a result, the optimization problem's boundary conditions can be defined as follows:

- Long-term wire current that is permitted:

$$\frac{S_{k,j}}{U_k} \le I_a$$
(9)

where I_a is the allowed current for the chosen conductor segment, U_k is the voltage at node k, and $S_{k,j}$ is the power of the branch linking nodes k and j, VA.

- voltage dropouts inside the illumination system:

$$\Delta U = \frac{U_1 - \min(U(n))}{U_1} \le 5\%$$
(10)

where U_1 is the supply voltage, V;

An extra prerequisite for the border is that the required quantities must be discrete.

We suggest determining the type of wire and its cross-section, for which the maximum operating current defines the resistance and its specific pricing, as the standard series sets the cross-section of the power supply wires of the lighting network. By progressively adding installations and ensuring that boundary conditions are met, the mode of the lighting network is determined. The values of the optimization criteria and the super criterion S'(x) are computed for this variation if the boundary requirements are met. The option with the highest super criterion value among all the options is chosen if no boundary criteria are satisfied. The electrical part's generated simulation model consists of:

1. A model of an electricity source Since the lighting network is not the only burden on the substation, I suggest that the supply voltage value serve as a representation of the power source while taking the external load into account.

2. A lighting network model. Given that insulated wire fills the supply lines and that a 0.4 kV voltage powers the lighting installations, the model advises ignoring the section currents, which are negligible due to the high insulation resistance, and the wire's capacitance, which is suitably low due to the low network voltage. Consequently, a series connection of active and inductive resistance can be used to mimic power line sections. It is suggested that a matrix C of dimension nxn, where n is the number of nodes in the network, be used to mathematically depict the topology of the lighting network. The lighting network branch, the power connection point, and the SL connection point are all regarded as nodes. C(j,k) = 1 if nodes k and j are directly connected; otherwise, C(j,k) = 0. In the same way, the resistances and lengths of each section of the network can be set using the matrix as electric load vectors S[n], P[n], and Q[n]. Each vector element represents the load in the node that corresponds to that number. The voltage vector at the network node U [n] and the matrix branch currents I [n x n] define the network's mode of operation.

3. The comparison of the load models showed that when the constant power load model is used, the voltages in the nodes and the power losses in their branches are calculated with an error of no more than 3%. Based on this, it was determined to carry out additional research using the mathematical constant power load model Sn=const.

By combining the mathematical models of the power supply, lighting network, and load, it is possible to compute the lighting's operational parameters and simulate the ES SL with various lighting network settings. The study's goal is to determine the mode parameters of the currents in the branches and the voltages in the network nodes since lighting networks have a lot of nodes and their current load varies between the first and last sections. I suggest utilizing an iterative computation approach, which has the advantages of being highly accurate and simple to formulate. The lighting network mode is iteratively calculated using a function block from the Simulink simulation

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environment and an m-function in the MATLAB environment.

To get both the natural and surface illumination values for different operating modes, the Simulink simulation environment uses a mathematical model that is built as a functional block (Figure 2).



Fig. 2: Components of the functioning simulation model

3 Dijkstra's Algorithm

Finding the best pathways is a crucial part of using graphs to solve problems. The Dijkstra and Bellman-Ford algorithms are the most commonly utilized methods for resolving this issue. Both directed and undirected coherent networks with positive weights can be solved using Dijkstra's technique. Bellman-Ford's technique should be applied when there are negative weights because Dijkstra's algorithm will yield inaccurate results, [12]. It is a greedy algorithm because, up until it reaches the terminal node, it chooses the locally optimal solution at each step and then synthesizes all of the earlier answers that have led to the best result. The following needs to be defined before the algorithm is executed, [12], [13], [14]:

• G(V, E) graph

• N(n): Nodes in the system.

 \bullet W (v, u) stands for the weights assigned to nodes v and u.

• Launch: Launch node.

• A vector of size N that holds the node's distance from the origin.

• A vector of size N that stores the shortest path to each node that comes right before the preceding one.

All of the vertices are split into two groups while the algorithm runs:

• T = 0, which keeps track of every tested node as well as the best route that led to their discovery.

• All of the network nodes that need to be checked are contained in Set F = V.

A node is eliminated from the current set after each test. The method stops when there are no more nodes to test or when the set F goes to zero. To keep the path as short as possible, a node v that is not part of the set T is selected at each step. The vector is then updated with a new value for each neighbor of the node that, if it turns out to be the shortest path, is not in the set T.

Before we can formulate a lighting network routing problem mathematically, we need to establish the following assumptions:

• One of a kind is the delivery node, or light pole.

•The cables that link the poles have the same capacity and are similar.

•A single wire can be used to connect two poles.

• The number of pupils positioned atop the pillars is unalterable.

• One network covers a single section of the city;

• Networks can be joined to one another.

The overall distance between the street light poles will serve as the primary optimization criterion.

4 Mathematical Model for Optimization

The following function reduces the total distance between street light poles, [15], [16], [17]:

$$\min(\sum_{x \in V} \sum_{y \in V} f_{xy} \sum_{m=1}^{n} z_{xym}, \qquad (11)$$

One significant drawback is that lighting fixtures require k poles, meaning that only one wire may be placed at the separation between x and y, [18], [19].

s.t.
$$\sum_{x \in V} z_{xym} = e_{iym}, \forall x \in V, m = 1, ..., n$$
 (12)

Route connectivity m, or the number of networks in each area:

$$\sum_{x,y \in Q} z_{xym} \le |Q| - 1, \forall Q \subseteq V, \forall m$$
(13)

Meeting the restriction that node visits can only be made once, except the city's collecting point for all networks, [20], [21], [22]:

$$\sum_{m=1}^{n} e_{xym} \le 1, \forall x \in V \tag{14}$$

Making certain that every pillar was linked to the one that was nearest to it, [23], [24], [25]:

$$\sum_{m=1}^{n} d_{xkm} \le r_{xk}, \forall k \in R, \forall x \in V$$
(15)

Applying the constraint that the capacity of a wire is not exceeded:

$$\sum_{x \in V} \sum_{k \in R} d_{xkm} \le C, m = 1, \dots, n$$
(16)

Using the restriction that if the equivalent pole is not present in this network, pole X should not be connected to pole x:

$$d_{xkm} \le e_{xm}, \,\forall x, k, m \tag{17}$$

imposing the limitation that a pair of pillars can only be joined once [26], [27].

$$\sum_{x \in V} \sum_{m=1}^{n} d_{xkm} = 1, \forall k \in V$$
(18)

The choice variables that follow have to be binary. In other words, they have to choose between 1 and 0.

$$e_{xm} \in \{0, 1\}, \forall x \in V, m = 1, ..., n$$

$$z_{xym} \in \{0, 1\}, \forall x, y \in V, x \neq y, m = 1, ..., n$$

$$d_{xkm} \in \{0, 1\}, \forall x, y \in V, x \neq y, k \in R$$
(19)

Below are the main criteria for selecting an element and placing it in the corresponding m sets. The main criterion for element selection is the objective function. It is defined as the minimum distance between the sums of the shortest distances contained in Q_i . In this order, we will analyze and present the process of finding a minimal number of subgraphs N containing Q_x . Through an algorithmic procedure, we will place suitable nodes in the corresponding subsets $Q_x \subseteq R$. The elements of the nodes will be denoted as follows:

$$\begin{aligned} q_r^x \mu \varepsilon &\subset Q_x \\ q_y^x \left| V_x = \min(\min(\sum_{r \in Q_x} a_{yr}^x), \forall x = 1, \dots, m, \forall y \in \infty, \forall r \in Q_x \end{aligned} \end{aligned}$$

Only in cases where two or more potential nodes have the same value does the second criterion come into play. The node that has the least eccentricity in this instance will be chosen.

$$q_y^x \left| \min(\mathbf{e}(q_y^x)), \forall x = 1, \dots, m, \forall y \in \infty \right.$$
(21)

In recent years, the stability results of divergent systems have had a strong effect on impulsive profits. All impulse gains should be stabilizing impulses. In the convergent behavior of some divergent systems, most impulse controllers allow a finite number of destabilizing impulses. It follows from the obtained results that an infinite number of destabilizing pulses can be allowed to reduce these conservatives using the applied method.

In this instance, the element that is chosen will be the node that has the most neighbors. The following is computed in each step:

• The objective function is used to determine the potential elements at nodes qi and r.

• The main component is:

• Next, determine whether nodes satisfy the maximum distance requirement;

- Aligning them with the proper Qi group;
- Locating the following lamppost;

• Determining the shortest path between two adjacent poles;

• The addition of a node, or multiple nodes, to set

F; that is, their classification from single to commit.

To develop the model, specific economic indicators related to the creation and implementation of such a smart grid were identified. The models chosen for the study are multifactorial and regression. The lack of incentives to build is due to technology versus transformation and energy efficiency policies.

The advantages are: the ability to proactively remove potential causes of accidents, prevent unplanned power outages, and replace depreciated assets before they actually fail. It is also possible to perform energy monitoring, which gives you the flexibility to determine the power supply according to your needs. The energy community frequently updates the Energy Efficiency Roadmap, which grows in scope with the adoption of new legislation in the field of electricity. The goal of the economic design of a smart grid is to find the lowest costs for the system. Overall, the problem is multifaceted, with all components in the system and system configuration having some impact on performance and thus cost. Therefore, the optimal design must find the optimal number of components and, at the same time, provide the possibility for the system to reduce the electrical load. Such a street lighting network is usually characterized by high installation costs and low operating costs. It enables limited network losses and the forecasting of energy consumption and production levels according to climatic conditions.

5 Application of the Algorithm to Solve a Street Lighting Network's Route

This is the task that this essay will go on to detail. The issue is specifically with an integrated system that delivers uniform objects (light fixtures) to permanent sites in an urban environment called light poles and electrical sources. We'll examine the path taken by the pillars and the feeder, which serves as the starting point. There will be a map available in the form of a graph that includes all light poles, or nodes, as well as the beginning position. All node weights and distances will also be provided.

6 A Lighting Network's Topology and Its Shortest Wire Lenght Determination

Description of the problem

The following factors are considered when determining the routes that will connect the nodes:

• The primary goal is to shorten the wires' travel distance.

• The maximum amount of space that can be allowed between street lighting poles and the power source

• Within a predetermined window of time, power transmission must be finished.

• electricity with a fixed capacity.

A modified Dijkstra algorithm has been developed as a methodology to solve the problem of determining the optimal topology of a newly designed street lighting network. This allows one to determine the topology of the lighting network with the least number of wires required, given the coordinates of the power source and the street lighting poles. The following steps are part of the methodology:



Fig. 3: Summarizing the nodes

1. Building the lighting network schedule, accounting for the lighting project's maximum allowable wire sag and the spacing between street lighting installations (Figure 3) and (Table 1).

Table 1. Adjacency list

Ne node	address		node	severity	addaya		node	sevenity	oddresa		node	severity	address
1		-	2	2			3	4	1.1.112		4	3.	NULL
2		\rightarrow	5	5		+	7	4	NULL				
3		+	4	7	NUL		1000	1	1				
4		+	7	1		\rightarrow	. 6	1.		+	7	5	NULL
5		\rightarrow		8.			- X.	- 4	NULL				
6		+	3	2		\rightarrow	7	1	NULL				
- 14	ALC: NO.	_											

2. Creation of the resultant graph's adjacency matrix (Figure 4) and (Table 2).

3.Using a modified Dijkstra algorithm, determine the shortest path between each light fixture and the power source.

In contrast to Dijkstra's traditional algorithm, the label value of each node in the proposed t method is the entire length of the lighting network (the graph's edge), including the lighting bodies, rather than the distance from the power source to the street lighting installation (a node of the graph). Additionally, when a node's label is changed, the adjacency matrix is altered. The updated label's column is set to zero, and while searching for the path from which the update came, the node's connection is highlighted.

In one part, the updated adjacency matrix is used to make a graph network that shows the lighting network's topology from pole to pole with the shortest wire length (Figure 4).



Fig. 4: Graph - topology of the lighting network

The designed techniques are put into practice in the MATLAB environment and enable the identification of the lighting network's ideal version during the design stage.

After that, the adjacency matrix is computed using the appropriate weights. The calculation for each element $N_{xy} \in N$ is as follows:

$$N_{xy} = \begin{cases} d_{xy}, & \text{if } a_{xy} = 1\\ 0, & \text{if } a_{xy} = 0 \end{cases}$$
(22)

Given the orientation of the graph, $N_{xy} \neq N_{yx}$

- There are nodes in the urban network N.
- The set S n expresses the nodes of urban areas. There are a total of 7 regions: |S|=|N|
 -1 = m ⇒ m².
- The origin-destination node in a city network N is taken to be: network node = 7.
- There are k = 1 networks in each region.
- Lighting pole capacity for the regional node: C = 15.
- The maximum separation between pillars that is allowed is W =2.

Table 2. Adjacency Matrix with Weights for the Network

(0	0	7	15	0	0	0
	2	0	0	0	0	0	0
	7	1	0	0	0	15	0
	1	0	0	0	7	0	0
	0	1	2	15	0	0	0
(0	0	0	0	7	0	0
(0	0	0	0	15	0	0

7 Conclusion

A review of the available technical fixes and a rationale for the indicator system were conducted. In the process of designing or updating the street lighting network, an ideal topology was looked after, and a technique for optimizing the characteristics of its parts with the least amount of active power loss was created. The following elements of the street lighting electrical system were included in the simulation model that was developed: power supply, lighting network, street lighting installation, and a system for controlling the system's operating mode. Unlike current methods, the optimization process for the network elements is based on a selection of parameters based on the least capital cost, regulatory compliance, and maximum lit area criteria.

Multifactorial problem-solving is required. To determine the best solution, the problem's needs must be completely specified from the beginning. Finding the best answer will take less time if the suggested approach is implemented using a programming language like Python. Finding the differences between the routing problems, analyzing them, and then making any necessary adjustments to the algorithm to enable its execution are the objectives. References:

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