A Stochastic Model for the Impact of Climate Change on Temperature and Precipitation

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Abstract: The variations from year to year of the monthly average temperatures are modelled as a discrete-time Markov chain. By computing the limiting probabilities of the Markov chain, we can see the impact of climate change on these temperatures. The same type of model is proposed for the variations of the monthly amounts of precipitation. An application to Jordan is presented.

Key-Words: Markov chain, forecasting, limiting probabilities, geometric distribution, goodness-of-fit tests, climate change.

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1 Introduction

Because of the great importance of climate change, there have been numerous papers written on prediction models for this phenomenon; see, [1] and the long list of references therein, [2] and, [3] for a recent example of such a model.

The author has used stochastic processes, in particular diffusion processes, as models in hydrology and meteorology. For example, [4] used filtered Poisson processes to forecast river flows. This problem has also been addressed by various authors; see, [5], [6], [7], [8], [9], [10], [11], [12] and, [13].

In, [14], the author considered the problem of modelling and forecasting the variations from year to year of the monthly average temperatures and amounts of precipitation in Italy. In both cases, he proposed a discrete-time Markov chain as a model. After having justified the validity of the model by making use of real-life data, he computed the limiting time probabilities of the Markov chains, from which one can forecast the long-term behaviour of the chains. Then, he divided the data set into two equal parts and computed the limiting probabilities for each part in order to check whether they had varied significantly in the time period considered. He concluded that there were some signs of the effects of climate change on the averages of interest, particularly on the monthly average amounts of precipitation. Related papers on this topic are, [15], [16], [17], [18] and, [19].

In this paper, the aim is to carry out the same type of analysis for Jordan, a country having a climate which is quite different from that of Italy. The climate in Jordan is not uniform, but it can be considered in general to be subtropical arid. Moreover, most of the time rainfall is scarce. According to the experts, temperatures are likely to increase due to climate change, while the total annual precipitation will probably decrease. We will see whether the model that we propose agrees with these statements. Furthermore, we will be able to quantify the expected changes.

Let $\{X_n, n = 0, 1, \ldots\}$ be a discrete-time stochastic process having state space S. We recall that $\{X_n, n = 0, 1, \ldots\}$ is a Markov chain if

$$\begin{aligned} p_{i,j}(n) &:= P[X_{n+1} = j \mid X_n = i, \\ &\qquad X_{n-1} = i_{n-1}, \ldots, X_0 = i_0] \\ &= P[X_{n+1} = j \mid X_n = i] \end{aligned} \tag{1}$$

for all states $i_0, \ldots, i_{n-1}, i, j$ in S, and for any $n \in \{0, 1, \ldots\}$. In most applications, it is also assumed that the Markov chain is time-homogeneous or stationary, so that the *one-step transition probabilities* $p_{i,j}(n)$ do not depend on n:

$$p_{i,j}(n) \equiv p_{i,j} := P[X_1 = j \mid X_0 = i] \tag{2}$$

for any states $i, j \in S$. In practice, this property may not hold for any n. For instance, in hydrology the Markov chain might be almost stationary during a given season, but not over the whole year. In such a case, a different model may be used for each season.

The matrix $\mathbf{P} := (p_{i,j})_{i,j \in S}$ is called the *one-step transition probability matrix*. We define the *n-step* *transition probability matrix*

$$\mathbf{P^{(n)}} = \left(p_{i,j}^{(n)}\right)_{i,j\in S}, \quad (3)$$

where

$$p_{i,j}^{(n)} := P[X_n = j \mid X_0 = i]. \quad (4)$$

It can be shown that $\mathbf{P^{(n)}} = \mathbf{P^n}$. An important question is to determine whether the limit $\lim_{n\to\infty} \mathbf{P^{(n)}}$ exists and, if so, to compute the *limiting probabilities*

$$\pi_j := \lim_{n\to\infty} P[X_n = j], \quad (5)$$

which must not depend on the initial state i.

When the Markov chain is irreducible and ergodic (see, [20], for example), it can be shown that the π_j's exist and are the unique positive solution of the linear system

$$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}, \quad (6)$$

subject to

$$\sum_{i=0}^{\infty} \pi_i = 1, \quad (7)$$

where

$$\boldsymbol{\pi} := (\pi_0, \pi_1, \ldots). \quad (8)$$

Remark. In theory, we can also obtain the limiting probabilities by computing the limit $\lim_{n\to\infty} \mathbf{P^{(n)}}$. If the size of the state space S is large, or even infinite, this task is quite difficult. However, making use of a mathematical software program, we can compute $\mathbf{P^{(n)}}$ for n large enough, in order to see how fast $p_{i,j}^{(n)}$ converges to π_j.

In the next section, we will propose a particular Markov chain as a model for the variations from year to year of the monthly average temperatures in Jordan. Next, we will confirm the validity of the model by making use of real-life data, and we will compute the limiting probabilities of the chain in order to forecast what is likely to happen when the process stabilizes. The model will enable us to look for signs of climate change in Jordan. Similarly, in Section 3 a Markov chain will serve as a model for the variations from year to year of the monthly average amounts of precipitation. We will conclude this paper with a few remarks in Section 4.

2 Variations of Monthly Average Temperatures

We will define the state X_n of the stochastic process $\{X_n, n = 0, 1, \ldots\}$ after n months in terms of the difference D_n between the average temperature (in degrees Celsius) for this month and the corresponding one of the previous year. Because the experts consider that a one-degree change in the average temperature is already significant, we consider the following states:

$$X_n = \begin{cases} 1 & \text{if } D_n \leq -1, \\ 2 & \text{if } D_n \in (-1, 1), \\ 3 & \text{if } D_n \geq 1. \end{cases} \quad (9)$$

In practice, we cannot check whether Eqs. (1) and (2) are indeed satisfied for any states and any n. Actually, in real-life applications, it is impossible to exactly satisfy these properties. Our aim is to propose a model that is at least *realistic*. To do so, we can consider the random variable K_i that denotes the number of months that the process $\{X_n, n = 0, 1, \ldots\}$ spends in state i before making a transition to another state. If $\{X_n, n = 0, 1, \ldots\}$ is indeed a Markov chain, then by independence we can write that K_i has a *geometric distribution* with parameter $p := 1 - p_{i,i}$, so that

$$P[K_i = k] = p_{i,i}^{k-1}(1 - p_{i,i}), \quad \text{for } k = 1, 2, \ldots \quad (10)$$

When we have a set of real-life data, we can perform Pearson's goodness-of-fit statistical test to check whether the geometric distribution is an *acceptable* model for K_i, $i = 1, 2, 3$. More simply, we can look at the histograms of the random variables to see whether their form is that of a decreasing exponential function, which should be the case if they have a geometric distribution.

The historical monthly average temperatures and rainfalls for all the countries in the world can be found on the website *Climate Change Knowledge Portal (CCKP)* (https://climateknowledgeportal.worldbank.org/) that was created by the World Bank. To estimate the transition probabilities $p_{i,j}$ of the Markov chains, the data for the years 1991-2016 will be used. Then, we will compute the limiting probabilities π_j, which will enable us to forecast the long-term behaviour of the variations of the monthly average temperatures and rainfalls. Finally, we will divide the dataset into two equal subsets: from 1991 to 2003, and from 2004 to 2016. We will compute the limiting probabilities for each subset and compare them to see if there are significant signs of climate change.

The differences in monthly average temperatures seem to be quite random and symmetrical, as can be seen in Figure 1 and Figure 2 produced by the statistical software program *Minitab*. Moreover, based on the Anderson-Darling normality test performed by *Minitab*, we can easily accept that the differences follow a Gaussian distribution, with a p-value greater than 0.58; see Figure 3. Hence, it is a *good* dataset.

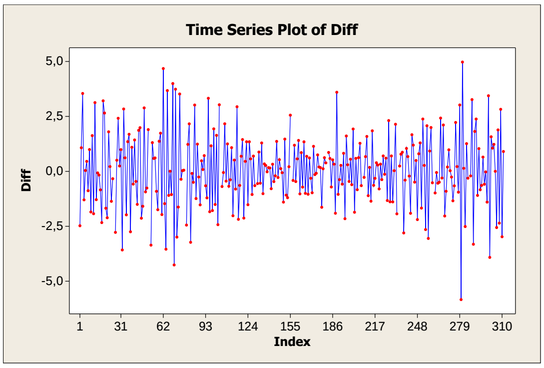

Fig. 1: Time series plot of the differences in monthly average temperatures

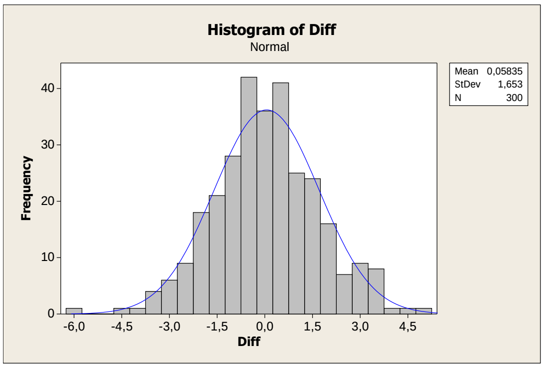

Fig. 2: Histogram of the differences in monthly average temperatures

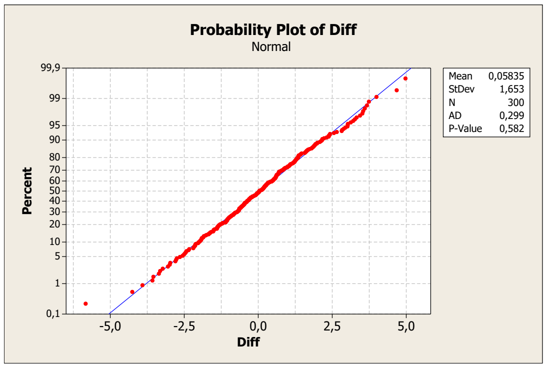

Fig. 3: Probability plot of the differences in monthly average temperatures

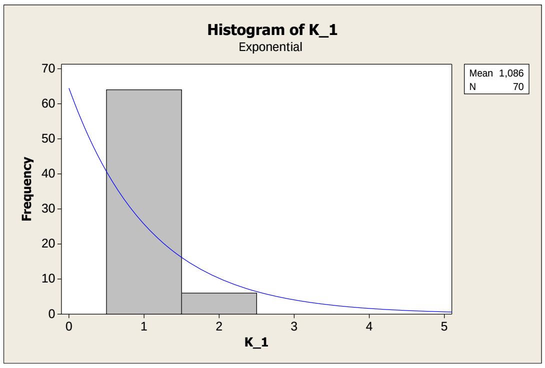

Fig. 4: Histogram of the random variable K_1 in the case of temperature

Using the whole dataset, *Minitab* produced the histograms of the random variables K_1, K_2 and K_3 that are shown in Figure 4, Figure 5, and Figure 6, together with an exponentially decreasing function. Although K_1 and K_3 took only respectively two and three different values, we can state that the form of each histogram (especially that of K_2, which took 7 different values) is in agreement with that of a geometric distribution. Based on the histograms, a Markov chain with the above states seems to be appropriate to serve as an *approximate* model for the *variations* of the monthly average temperatures in Jordan between 1991 and 2016. To make this statement more precise, we performed Pearson's chi-square statistical test to check the goodness of fit of a geometric distribution to the data. The statistics χ^2 and the p-values are given in Table 1. From these values, one can indeed accept that each random variable K_i has *approximately* a geometric distribution.

Table 1. Pearson's chi-square statistical test for a geometric distribution in the case of temperature

Variable	χ^2	p-value
K_1	0.60	0.74
K_2	1.72	0.89
K_3	0.02	0.99

Remarks. (i) Because of the small number of values taken by K_1 and K_3, the p-values given in Table 1 are only approximate. Moreover, in order to perform the tests, we assumed that the point estimate $\hat{p}$ of the parameter p of the geometric distribution was in fact the known value of p (otherwise, the number of *degrees of freedom* in the case of K_1 and K_3 would be equal to 0). Nevertheless, the p-values in Table 1 are so high that the geometric distribution is surely a valid model for the random variables K_i.

(ii) In the case when at least one of the variables K_i cannot be assumed to have a geometric distribution, we can try to define a new state space in order to satisfy this fundamental property.

Next, we can estimate the transition probabilities $p_{i,j}$ of the Markov chain. We find that the estimated one-step transition probability matrix is given by

$$\mathbf{P} = \begin{bmatrix} 6/74 & 27/74 & 41/74 \\ 27/137 & 81/137 & 29/137 \\ 39/77 & 28/77 & 10/77 \end{bmatrix}. \tag{11}$$

Because $p_{i,j} \neq 0$ for any pair of states i, j, the Markov chain $\{X_n, n = 0, 1, \ldots\}$ is both irreducible and aperiodic. Moreover, the number of states being finite, it is also positive recurrent, and thus ergodic. Hence, the limiting probabilities exist. Solving the system (6), (7), we get

$$\pi_1 = 0.2530, \quad \pi_2 = 0.4712, \quad \pi_3 = 0.2758. \tag{12}$$

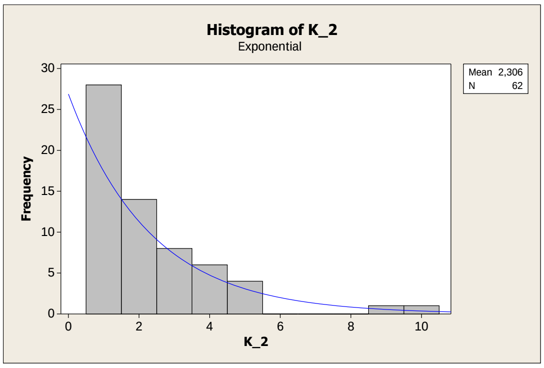

Fig. 5: Histogram of the random variable K_2 in the case of temperature

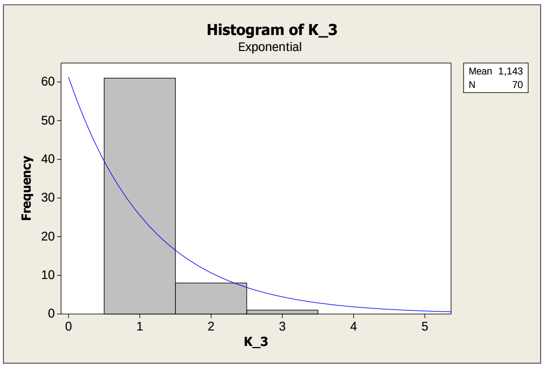

Fig. 6: Histogram of the random variable K_3 in the case of temperature

According to the limiting probabilities, it would be slightly more likely to observe in the future an increase of the average temperature of at least one degree during a given month, compared to the previous year, than a decrease of at least one degree. This is consistent with the fact that the average of the 300 monthly differences is equal to 0.0583. However, the standard deviation is equal to 1.653, which is relatively large and creates uncertainty.

Now, our aim is to detect signs of climate change. Therefore, we performed the same analysis as above for each of the two equal subsets of data. The transition probability matrices for the periods 1991-2003 and 2004-2016 are respectively

$$\mathbf{P} = \begin{bmatrix} 1/31 & 13/31 & 17/31 \\ 12/77 & 50/77 & 15/77 \\ 18/36 & 12/36 & 6/36 \end{bmatrix} \tag{13}$$

and

$$\mathbf{P} = \begin{bmatrix} 5/43 & 14/43 & 24/43 \\ 15/60 & 31/60 & 14/60 \\ 21/41 & 16/41 & 4/41 \end{bmatrix}. \tag{14}$$

The resulting limiting probabilities are presented in Table 2.

We see that the value of π_3 has increased by about 8.5% in the second time period considered, which is significant, but the value of π_1 has increased by a staggering 30.5%! The long-term probabilities that the Markov chain will be in state 1 or in state 3 are almost equal. These results are quite comparable to the corresponding ones in Italy over the period 1991-2015. However, the changes observed in Jordan are more pronounced.

Table 2. Limiting probabilities for the time periods 1991-2016, 1991-2003 and 2004-2016 in the case of monthly average temperatures

Period	π_1	π_2	π_3
1991-2016	0.2530	0.4712	0.2758
1991-2003	0.2199	0.5150	0.2651
2004-2016	0.2870	0.4255	0.2875

According to the website CCKP, the mean annual temperature has increased by 0.89 degrees Celsius in Jordan since 1900. We can conclude from the above data that the variations in monthly average temperatures have greatly accelerated in recent years. Thus, the impact of climate change is observable in Jordan.

Remarks. (i) We find that if we raise the matrix $\mathbf{P}$ given in Eq. (11) to the power 16, then its three lines are practically equal to the limiting probabilities π_1, π_2 and π_3. Therefore, these limiting probabilities could be observed after 16 months. Hence, the process is already almost in steady state. However, the limiting probabilities should be updated when new observations become available.

(ii) The time period considered, namely from 1991 to 2016, is relatively short. In order to make the above conclusions more credible, we calculated the differences in monthly average temperatures from 1901 onwards. The mean difference for the period 1901-1990 is equal to 0.0056, compared to 0.0583 between 1991 and 2016. It is therefore clear that the increase in the average temperature has been felt mainly since the 1990s. More precise results can be found in Table 3. Notice the steady increase in the average temperature. Finally, we also computed the probability that the average monthly temperature will increase by at least 1 degree Celsius, compared to the previous year, if we assume that the average monthly difference follows a Gaussian distribution. The various probabilities are given in Table 4. After a marked increase between the years 1961 and 1990, the probability of interest stabilized in the time period

Table 3. Differences in monthly average temperatures for the time period 1901-2016

Period	1901-1930	1931-1960	1961-1975
Mean difference	-0.0170	0.0133	0.0140
Period	1976-1990	1991-2003	2004-2016
Mean difference	0.0260	0.0570	0.0600

1991-2016. This probability for 1991-2016 is close to the value of π_3 in Table 2. Thus, we may conclude that these results are consistent with the above conclusions.

Table 4. Probability that the monthly average temperature will increase by at least 1 degree Celsius for various time periods

Period	1931-1960	1961-1990	1991-2016
Probability	0.1897	0.2756	0.2845

(iii) Our aim was to determine whether we can expect the average annual temperature to increase by at least 1 degree Celsius in the next few years. We can be more precise and look for changes in the average temperatures during the various seasons. Because we need enough data to reach reliable conclusions, we divided the data set into two equal parts: from November to April (*winter*), and from May to October (*summer*). Proceeding as above, we obtained the following results:

$$\mathbf{P}_{\text{winter}} = \begin{bmatrix} 5/47 & 11/47 & 31/47 \\ 13/48 & 24/48 & 11/48 \\ 26/49 & 14/49 & 9/49 \end{bmatrix} \tag{15}$$

and

$$\mathbf{P}_{\text{summer}} = \begin{bmatrix} 1/28 & 18/28 & 9/28 \\ 13/87 & 54/87 & 20/87 \\ 15/29 & 13/29 & 1/29 \end{bmatrix}. \tag{16}$$

The corresponding limiting probabilities are given in Table 5. We see that it is actually during the winter months that the average monthly temperature is more likely to increase than to decrease by at least 1 degree Celsius. The temperatures during the summer months are expected to remain fairly stable.

Table 5. Limiting probabilities for the monthly average temperatures during the winter and summer months for the time period 1991-2016

Season	π_1	π_2	π_3
Winter	0.3099	0.3433	0.3468
Summer	0.2028	0.5894	0.2078

In the next section, the same analysis will be performed to see whether the conclusions are similar in the case of monthly average rainfalls in Jordan.

3 Variations of Monthly Average Rainfalls

Contrary to Italy, there is very little precipitation in Jordan from about April to October. In Figure 7, we present the differences in monthly average rainfalls in the period 1991-2016. We cannot model these differences as a stationary Markov chain. Instead, we will consider the months of interest, namely from November to March. The new time series is shown in Figure 8, as well as the corresponding histogram and probability plot in Figure 9 and Figure 10. As in the previous section, we can surely accept the hypothesis that the differences follow a Gaussian distribution; this time the p-value is greater than 0.63. The mean difference is equal to -0.8434 millimetres of precipitation, and the standard deviation to 13.46. Therefore, precipitation has decreased during the time period considered. On the website CCKP, we can read that *Global Historical Climatology Network (GHCN) data for the country indicates a 2.92 mm/month per century reduction in average annual precipitation since 1900. The majority of local station records indicate that precipitation dropped from 94 mm to 80 mm during the last 10 years for the period 1937/38 to 2004/2005.* The minimum and

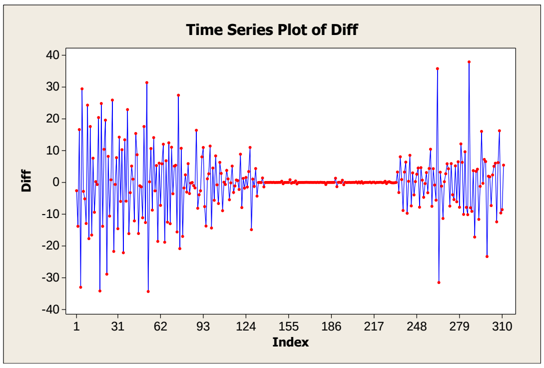

Fig. 7: Time series plot of the differences in monthly average rainfalls

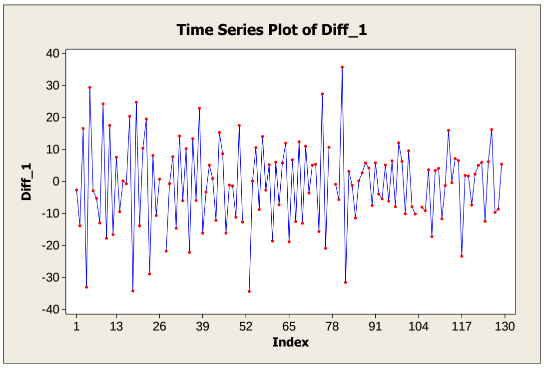

Fig. 8: Time series plot of the differences in monthly average rainfalls for the months from November to March

maximum differences are equal to -34.30 and 35.79 respectively. The median difference is -0.28.

It is less obvious than in the case of temperatures how we should define the states. We denote by D_n^* the difference between the average rainfall (in mm) for a

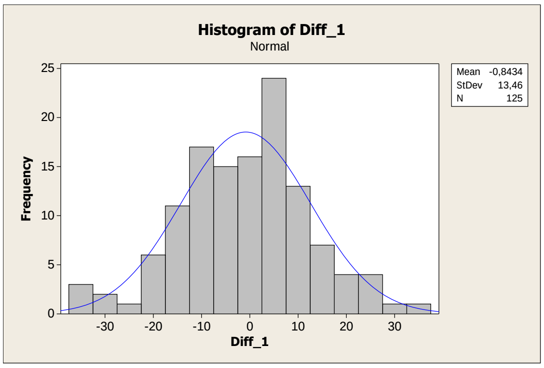

Fig. 9: Histogram of the differences in monthly average rainfalls for the months from November to March

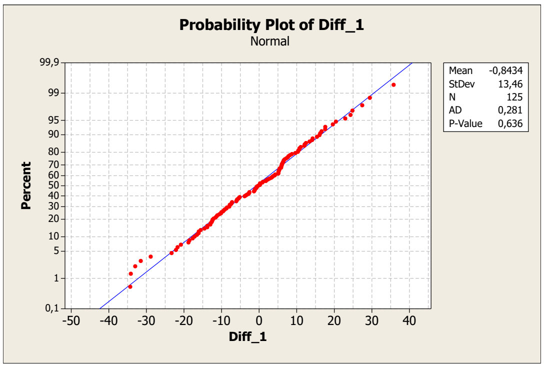

Fig. 10: Probability plot of the differences in monthly average rainfalls for the months from November to March

month of a given year and the corresponding month of the previous year. We define

$$X_n = \begin{cases} 1 & \text{if } D_n^* \leq -5, \\ 2 & \text{if } D_n^* \in (-5, 5), \\ 3 & \text{if } D_n^* \geq 5. \end{cases} \qquad (17)$$

The histograms of the random variables K_1, K_2 and K_3 are presented in Figure 11, Figure 12 and Figure 13. We see that each variable took only two values, namely 1 and 2. Therefore, the stochastic process $\{X_n, n = 0, 1, \ldots\}$ did not remain for more than two months in the same state, which concurs with the volatility observed in Figure 8 and the fact that the standard deviation is large (13.46). Still, the required exponential decrease can be observed in each case, so that the fact that $\{X_n, n = 0, 1, \ldots\}$ is approximately a Markov chain is at least plausible. As in Section 2, we performed Pearson's chi-square statistical test for a geometric distribution; see Table 6. The (approximate) p-values are all large enough to indeed consider $\{X_n, n = 0, 1, \ldots\}$ as a Markov chain.

Next, with the help of our real-life dataset, we calculated the estimated one-step transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} 4/47 & 14/47 & 29/47 \\ 12/29 & 6/29 & 11/29 \\ 31/45 & 7/45 & 7/45 \end{bmatrix}. \qquad (18)$$

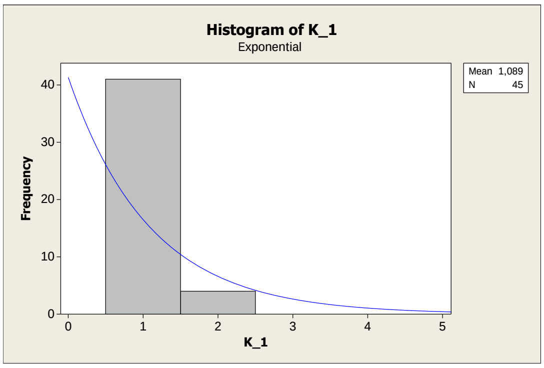

Fig. 11: Histogram of the random variable K_1 in the case of monthly average rainfalls

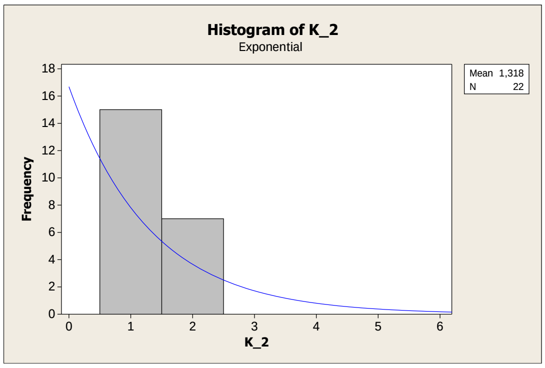

Fig. 12: Histogram of the random variable K_2 in the case of monthly average rainfalls

As in the previous section, we can state that the limiting probabilities exist. By solving the system (6), (7), we find that

$$\pi_1 = 0.3913, \quad \pi_2 = 0.2227, \quad \pi_3 = 0.3860. \qquad (19)$$

According to the limiting probabilities, it would be slightly more likely to observe a drop of the monthly average precipitation of at least 5 mm during a given month, compared to the previous year, than an increase of at least 5 mm. This is consistent with the mean monthly difference which is equal to -0.8434 mm, thus pointing towards a small decrease.

Finally, for the periods 1991-2003 and 2004-2016 the transition probability matrices are respectively

$$\mathbf{P} = \begin{bmatrix} 2/23 & 7/23 & 14/23 \\ 7/16 & 3/16 & 6/16 \\ 15/21 & 4/21 & 2/21 \end{bmatrix} \qquad (20)$$

and

$$\mathbf{P} = \begin{bmatrix} 2/24 & 7/24 & 15/24 \\ 5/13 & 3/13 & 5/13 \\ 16/24 & 3/24 & 5/24 \end{bmatrix}. \qquad (21)$$

The limiting probabilities are given in Table 7.

We see that the value of π_3 has increased by more than 10% in the second time period considered, which is really significant, while the probability of a difference of less than 5 mm in absolute value has decreased also by more than 10%. Moreover, rather unexpectedly, it is now slightly more likely to observe

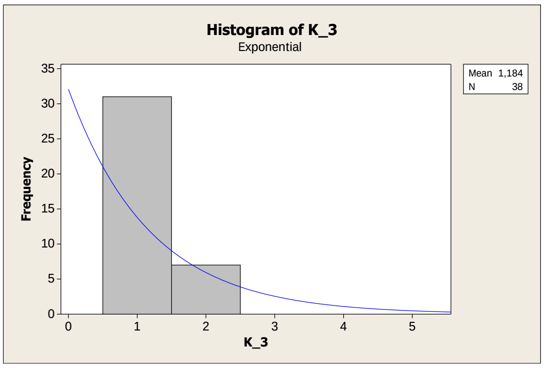

Fig. 13: Histogram of the random variable K_3 in the case of monthly average rainfalls

Table 6. Pearson's chi-square statistical test for a geometric distribution in the case of precipitation

Variable	χ^2	p-value
K_1	0.42	0.81
K_2	0.71	0.40
K_3	1.76	0.41

in the future a monthly increase rather than a decrease of at least 5 mm of precipitation. Remember however that these conclusions are only valid for the months from November to March. Even if there is a small increase in precipitation during the winter, the total precipitation for the whole year may still decrease.

Remarks. (i) As in the case of temperature, the three rows of the matrix $\mathbf{P}^{16}$, where $\mathbf{P}$ is defined in Eq. (18), are almost equal to limiting probabilities for the time period 1991-2016. Hence, we may state that our forecasts should be valid after at most 16 months. This is due to the fact that the Markov chain is almost in steady state since 1991.

(ii) We computed the mean differences in monthly average precipitation for various time periods from 1901 to 2016; see Table 8. After a steady increase between 1901 and 1960, we observe that the trend is reversed in the time period 1991-2016. There is therefore more volatility than in the case of temperature. Notice however than the mean differences are rather small.

In Table 9, we present the probability that the monthly average amount of precipitation will decrease by at least 5 mm for various time periods, assuming that the monthly average differences follow a Gaussian distribution. Because the standard deviation of the observations was larger in the time periods 1931-1960 and 1961-1990 than between 1991 and 2016, we find that the probability of interest is actually quite stable. This probability is close to the value of π_1 in Table 7, which again strengthens the validity of our forecasts.

Table 7. Limiting probabilities for the time periods 1991-2016, 1991-2003 and 2004-2016 in the case of monthly average precipitation

Period	π_1	π_2	π_3
1991-2016	0.3913	0.2227	0.3860
1991-2003	0.3989	0.2352	0.3659
2004-2016	0.3834	0.2112	0.4053

Table 8. Mean differences in monthly average precipitation between 1901 and 2016

Period	1901-1930	1931-1960
Mean difference	0.008	0.02
Period	1961-1990	1991-2016
Mean difference	0.21	-0.84

4 Conclusion

In this paper, we proposed a simple stochastic model to forecast the impacts of climate change in Jordan. We were interested in forecasting the expected changes in monthly average temperature and precipitation. Thanks to a website created by the World Bank containing real-life data for all the countries in the world, we were able to both justify the validity of the Markov chains that we put forward as models and to estimate their various transition probabilities. Then, it was a simple matter to compute the limiting probabilities of the Markov chains, which enable us to forecast the future variations of the variables of interest. These limiting probabilities should be recalculated when new observations become available.

Because of the scarcity of precipitation in Jordan, especially in the summer, defining a state space of the stochastic process $\{X_n, n = 0, 1, \ldots\}$ such that this process could indeed be considered as a Markov chain was more difficult than in the case of temperature. By restricting the period during which the model was used, namely from November to March, we were able to justify the validity of this assumption and then to quantify the expected effects of climate change on precipitation in Jordan.

Jordan is a small enough country for our study to make sense. Indeed, the statistical data analysis that we carried out would not be very useful for a really large country like Russia or Canada. For large countries, we would need data for their various parts, which can also be found on the website of the World Bank.

Table 9. Probability that the monthly average amount of precipitation will decrease by at least 5 mm for various time periods

Period	1931-1960	1961-1990	1991-2016
Probability	0.3791	0.3835	0.3786

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