Travelling wave-like solutions for some nonlinear equations with cubic nonlinearities

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Abstract: we exercised the series-expansion method to extract solitary wave solutions for complex nonlinear evolution equations (NLEEs) such as the coupled Higgs equation (CHE), the (3+1)-dimensional dynamical system, the (3+1)-dimensional nonlinear Schrodinger(NLSE) equation which have many physical importance in different branches. Varieties of periodic and solitonic wave-like solutions are extracted. Computational work has been done and plots and counter graphs are plotted using Wolfram Mathematica 11.

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1 Introduction

Importance of nonlinear evolution equations (NLEEs) in different breaches such as physical sciences, engineering, and biological sciences cannot be ignored. Some well known phenomena in chemical kinetics, nonlinear optics [1], flow mechanics [2], nuclear physics [3], condense matter physics [5], etc population dynamics etc [6], are described by NLEEs. Once a problem is identified, then its mathematical models can be traced with a suitable equations, preferably in NLEEs [7]. Further we generally wish to find exact algebraic and numerical solutions of such nonlinear equations in order to predict, control and quantify the underlying features of the system under study. In last few decades, numerous exercise have been made to extract the exact algebraic and numerical solutions of NLEEs and a many methods have been fabricated to cater the explicit travelling wave like solutions of NLEEs [8]. Recently, Wang *et al.* [2] developed a new formulation called $(\frac{F'}{F})$-expansion method for a dependable prescription of nonlinear wave equations. Consequently many more utilisation of this technique have also been reported [9]. A summarized version of $(\frac{F'}{F})$-expansion formalism is also prescribed recently [10]. In next section one, few equation [11] and [12] are identified and solved by $(\frac{F'}{F})$ method. At last few remarks are noted in summary section .

1.1 The Series-expansion method

In brief, here we prescribe the important steps of the $(\frac{F'}{F})$-expansion method [13]. Hypothesize that a nonlinear evolution differential equation (NLEEs) is of the form polynomial R as

$$R(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0, \qquad (1)$$

Step 1: The standard explication of (1) can be written by a polynomial in $(\frac{F'}{F})$ i.e.

$$u(\xi) = \alpha_m \left(\frac{F'}{F}\right)^m + \alpha_{m-1} \left(\frac{F'}{F}\right)^{m-1} + ..., \qquad (2)$$

where $F = F(\xi)$ satisfies the second order linear ODE of the form

$$F'' + \tau F' + \kappa F = 0, \qquad (3)$$

where α_m, α_{m-1},....., α_0, τ and κ are constants to be of single-minded and can ce calulated later with $\alpha_m \neq 0$.

Step 2: Substituting (2) into (1) and using (3), collecting all terms with the same order of $(\frac{F'}{F})$ together, and then equating each coefficient of the resulting polynomial to zero yields a set of algebraic equations for α_m, α_{m-1},....., α_0, c, τ and κ.Further depending on the sign of the discriminant $\triangle = \tau^2 - 4\kappa$ possess the many general solutions as

$$\left(\frac{F\prime}{F}\right) = \begin{cases} \frac{\sqrt{\tau^2-4\kappa}}{2}\left[\frac{A\sinh(\frac{\sqrt{\tau^2-4\kappa}}{2})\chi+B\cosh(\frac{\sqrt{\tau^2-4\kappa}}{2})\chi}{A\cosh(\frac{\sqrt{\tau^2-4\kappa}}{2})\chi+B\sinh(\frac{\sqrt{\tau^2-4\kappa}}{2})\chi}\right] - \frac{\kappa}{2}, \\ \frac{\sqrt{4\kappa-\tau^2}}{2}\left[\frac{-A\sinh(\frac{\sqrt{4\kappa-\tau^2}}{2})\chi+B\cosh(\frac{\sqrt{4\kappa-\tau^2}}{2})\chi}{A\cosh(\frac{\sqrt{4\kappa-\tau^2}}{2})\chi+B\sinh(\frac{\sqrt{4\kappa-\tau^2}}{2})\chi}\right] - \frac{\kappa}{2}, \\ \frac{B}{A+B\chi} - \frac{\kappa}{2} \end{cases}$$

or more simplified version as

$$\left(\frac{F\prime}{F}\right) = \begin{cases} \frac{\sqrt{\tau^2-4\kappa}}{2}\tanh\left(\frac{\sqrt{\tau^2-4\kappa}}{2}\chi+\chi_0\right) - \frac{\kappa}{2}, \\ \frac{\sqrt{\tau^2-4\kappa}}{2}\coth\left(\frac{\sqrt{\tau^2-4\kappa}}{2}\chi+\chi_0\right) - \frac{\kappa}{2}, \\ \frac{\sqrt{4\kappa-\tau^2}}{2}\cot\left(\frac{\sqrt{4\kappa-\tau^2}}{2}\chi+\chi_0\right) - \frac{\kappa}{2}, \\ \frac{B}{A+B\chi} - \frac{\kappa}{2} \end{cases}$$

Well known general solutions of eq.(3) have been familiar for us, then simulating α_m, α_{m-1},.....,α_0 with eq.(3) into eq.(2) we exract more traveling wave-like solutions of NLEEs eq.(1).

1.2 The Coupled Higgs Equation

Acknowledge the following form of coupled Higgs equation (CHE) [14]

$$u_{tt} - u_{xx} + |u|^2 u - 2uv = 0,$$
$$v_{tt} + v_{xx} - (|u|^2)_{xx} = 0. \quad (4)$$

Tajiri obtained N-solition solutions to the system (4) in[14]. Zhao calculated more accustomed traveling wave solutions of eq. (4) in [14].
By using the wave variables

$$u = e^{\imath\theta} U(\xi), v = V(\xi), \theta = px + rt, \xi = x + ct, \quad (5)$$

eq. (4) are carried to ODEs

$$(c^2 - 1)U'' + (p^2 - r^2)U + U^3 - 2UV = 0,$$
$$(c^2 + 1)V'' - (U^2)'' = 0. \quad (6)$$

Integrating the second equation of (6) twice and setting constants of integration is zero, then we get

$$V = \frac{U^2}{c^2 + 1}. \quad (7)$$

Substituting (7) into first equation of (6), we obtained

$$(c^4 - 1)U'' + (c^2 + 1)(p^2 - r^2)U + (c^2 - 1)U^3 = 0. \quad (8)$$

Now, balancing the terms u'' with u^3 we find, m=1.
Here, we suppose that

$$U(\xi) = \alpha_1 \left(\frac{F'}{F}\right) + \alpha_0, \quad \alpha_1 \neq 0, \quad (9)$$

where $F = F(\xi)$ satisfies the second order linear ODE

$$F'' + \tau F' + \kappa F = 0, \quad (10)$$

where α_1, α_0, τ and κ are constants to be determined later. Using (9) into (8) and rationalizing in power of $\frac{F'}{F}$ simultaneously, the LHS of (8) are converted into the polynomials in $\frac{F'}{F}$. Segregating each coefficient of the polynomials to zero, leads a set of concurrent analytic equations as follows:

$$2(c^4 - 1)\alpha_1 + (c^2 - 1)\alpha_1^3 = 0, \;\; 3\tau(c^4 - 1)\alpha_1 + 3(c^2 - 1)\alpha_1^2\alpha_0 = 0,$$

$$(2\kappa + \tau^2)(c^4 - 1)\alpha_1 + (c^2 + 1)(p^2 - r^2)\alpha_1 + 3(c^2 - 1)\alpha_1\alpha_0^2 = 0,$$

$$\tau\kappa(c^4 - 1)\alpha_1 + (c^2 + 1)(p^2 - r^2)\alpha_0 + (c^2 - 1)\alpha_0^3 = 0. \quad (11)$$

Further solving, we have

$$\alpha_0 = \pm\imath\sqrt{\frac{c^2 + 1}{2}}\tau, \alpha_1 = \pm\imath\sqrt{2(c^2 + 1)},$$
$$c = \pm\sqrt{\frac{2(p^2 - r^2)}{\tau^2 - 4\kappa} + 1}, \quad (12)$$

here τ, κ, p and r are arbitrary constant.
Again by taking (12) into (9), leads to

$$U(\xi) = \pm\imath\sqrt{2(c^2 + 1)}\left(\frac{F'}{F}\right) \pm \imath\sqrt{\frac{c^2 + 1}{2}}\tau. \quad (13)$$

Exchanging the second order LODE (10) into the (13) we got wide different category of solutions of (8) and the coupled Higgs equation (CHE) as follows:
With $\tau^2 - 4\kappa > 0$,

$$U(\xi) = \pm\imath\sqrt{\frac{c^2 + 1}{2}}\sqrt{\tau^2 - 4\kappa} \left(\frac{A\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}{A\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}\right), \quad (14)$$

$$V(\xi) = \frac{(4\kappa - \tau^2)}{2} \left(\frac{A\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}{A\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}\right)^2, \quad (15)$$

$$u(\xi) = \pm\imath\sqrt{\frac{c^2 + 1}{2}}\sqrt{\tau^2 - 4\kappa} \left(\frac{A\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}{A\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}\right) e^{\imath\theta}, \quad (16)$$

$$v(\xi) = \frac{(4\kappa - \tau^2)}{2} \left(\frac{A\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}{A\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}\right)^2, \quad (17)$$

here $\theta = px + rt$, ξ=x$\pm\sqrt{\frac{2(p^2 - r^2)}{\tau^2 - 4\kappa} + 1}$ t.
In Precisely, if $B \neq 0$, A=0, $\tau > 0$, $\kappa = 0$, then u and v becomes

$$u(\xi) = \pm\imath\sqrt{\frac{c^2 + 1}{2}}\tau\tanh(\frac{1}{2}\tau\xi)e^{\imath\theta}, \quad (18)$$

$$v(\xi) = -\frac{\tau^2}{2}\tanh^2(\frac{1}{2}\tau\xi). \quad (19)$$

These are the possible periodic wave-like solution of coupled Higgs equation (CHE).
With $\tau^2 - 4\kappa < 0$,

$$U(\xi) = \pm\iota\sqrt{\frac{c^2+1}{2}}\sqrt{4\kappa-\tau^2} \left(\frac{-A\sin(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi + B\cos(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi}{A\cos(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi + B\sin(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi}\right), \quad (20)$$

$$V(\xi) = \frac{(\tau^2-4\kappa)}{2} \left(\frac{-A\sin(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi + B\cos(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi}{A\cos(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi + B\sin(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi}\right)^2, \quad (21)$$

$$u(\xi) = \pm\iota\sqrt{\frac{c^2+1}{2}}\sqrt{4\kappa-\tau^2} \left(\frac{-A\sin(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi + B\cos(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi}{A\cos(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi + B\sin(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi}\right)e^{\iota\theta}, \quad (22)$$

$$v(\xi) = \frac{(\tau^2-4\kappa)}{2} \left(\frac{-A\sin(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi + B\cos(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi}{A\cos(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi + B\sin(\frac{1}{2}\sqrt{4\kappa-\tau^2})\xi}\right)^2, \quad (23)$$

where $\theta = px + rt$, ξ=x$\pm\sqrt{\frac{2(p^2-r^2)}{\tau^2-4\kappa}+1}$ t.
With $\tau^2 - 4\kappa = 0$,

$$U(\xi) = \pm\iota\sqrt{2(c^2+1)}\left(\frac{B}{A+B\xi}\right), \quad (24)$$

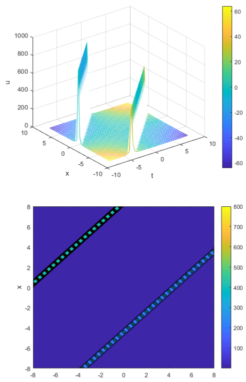
$$V(\xi) = -2\left(\frac{B}{A+B\xi}\right)^2, \quad (25)$$

$$u(\xi) = \pm\iota\sqrt{2(c^2+1)}\left(\frac{B}{A+B\xi}\right)e^{\iota\theta}, \quad (26)$$

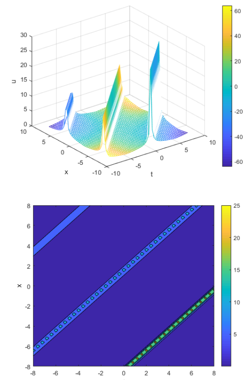
$$v(\xi) = -2\left(\frac{B}{A+B\xi}\right)^2, \quad (27)$$

where $p = r, c = 1, \theta = px + rt, \xi = x + ct$. Solutions for (CHE) are prepaired and curves are plotted for all possible wave-like solutions are crafted by their 3D plots and further their analogous contour plots.

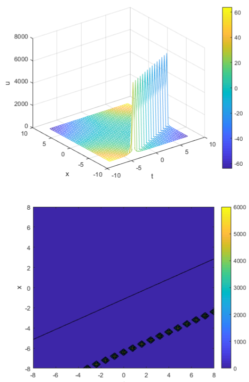
Figure 1: Traveling wave solution corresponding to the C. Higgs equation



(a) Plot of $u_1(\zeta)$, when $\triangle > 0, \tau = 0.1, \kappa = 0.02 A = 0.02, B = 0.03$



(b) Plot of $u_2(\zeta)$, when $\triangle < 0, \tau = 0.2, \kappa = 0.2, A = 0.02, B = 0.03$



(c) Plot of $u_3(\zeta)$, when $\triangle = 0, \tau = 0.2, \kappa = 0.01, A = 0.02, B = 0.03$

1.3 The (2+1)-dimensional NLS Equation

Consider the following (2+1)-dimensional NLS equation [15]

$$\iota u_t + u_{xx} + u_{yy} + a|u|^2 u = 0, \quad (28)$$

We take the transformation $u = e^{\iota\theta}U(\xi)$, $\theta = \alpha x + \beta y + \tau t$, ξ=x+y+ct. The substitution of transformation into (28) yields the ODE

$$2U'' - (\alpha^2 + \beta^2 + \tau)U + aU^3 = 0. \tag{29}$$

under the condition c=-2$(\alpha + \beta)$.
Now, balancing the terms u'' with u^3 we find, m=1. Here, we suppose that

$$U(\xi) = \alpha_1\left(\frac{F'}{F}\right) + \alpha_0, \quad \alpha_1 \neq 0, \tag{30}$$

where $F = F(\xi)$ satisfies the second order linear ODE

$$F'' + \tau F' + \kappa F = 0, \tag{31}$$

where α_1, α_0, τ and κ are well known to us.
Following the steps and equating each coefficient leads to group of equations as follows:

$$4\alpha + \alpha_1^3 = 0,, \quad 6\tau\alpha + 3a\alpha_1^2\alpha_0 = 0,$$

$$2(2\kappa + \tau^2)\alpha - (\alpha^2 + \beta^2 + \tau)\alpha + 3a\alpha\alpha_0^2 = 0,$$

$$2\tau\kappa\alpha - (\alpha^2 + \beta^2 + \tau)\alpha_0 + a\alpha_0^3 = 0. \tag{32}$$

further, one can have

$$\alpha_0 = \pm\frac{\iota\tau}{\sqrt{a}}, \alpha = \pm\frac{2\iota}{\sqrt{a}}, \tau^2 - 4\kappa = -(\alpha^2 + \beta^2 + \tau), \tag{33}$$

where τ, κ, α, β, τ are well known to us.
By using (33) in expression (30), we get

$$U(\xi) = \pm\frac{2\iota}{\sqrt{a}}\left(\frac{F'}{F}\right) \pm \frac{\iota\tau}{\sqrt{a}}. \tag{34}$$

following the earlier steps (31) into the (34) we have three types of travelling wave solutions of (2+1)-dimensional NLS equation (28) as follows:
With $\tau^2 - 4\kappa > 0$,

$$U(\xi) = \pm\sqrt{\frac{(4\kappa - \tau^2)}{a}} \left(\frac{A\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}{A\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}\right), \tag{35}$$

$$u(\xi) = \pm\sqrt{\frac{(4\kappa - \tau^2)}{a}} \left(\frac{A\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}{A\cosh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi + B\sinh(\frac{1}{2}\sqrt{\tau^2 - 4\kappa})\xi}\right)e^{\iota\theta}, \tag{36}$$

where $\theta = \alpha x + \beta y + \tau t$,$\xi$=x+y-2$(\alpha + \beta)$ t.
Precisely,, if $A \neq 0$, $B = 0$, $\tau > 0$, $\kappa = 0$, then u and v becomes

$$u(\xi) = \pm\frac{\iota\tau}{\sqrt{a}}\tanh(\frac{\tau\xi}{2})e^{\iota\theta}, \tag{37}$$

which are the periodic wave solution of (2+1)-dimensional NLS equation.
With $\tau^2 - 4\kappa < 0$,

$$U(\xi) = \pm\sqrt{\frac{(\tau^2 - 4\kappa)}{a}} \left(\frac{-A\sin(\frac{1}{2}\sqrt{4\kappa - \tau^2})\xi + B\cos(\frac{1}{2}\sqrt{4\kappa - \tau^2})\xi}{A\cos(\frac{1}{2}\sqrt{4\kappa - \tau^2})\xi + B\sin(\frac{1}{2}\sqrt{4\kappa - \tau^2})\xi}\right), \tag{38}$$

$$u(\xi) = \pm\sqrt{\frac{(\tau^2 - 4\kappa)}{a}} \left(\frac{-A\sin(\frac{1}{2}\sqrt{4\kappa - \tau^2})\xi + B\cos(\frac{1}{2}\sqrt{4\kappa - \tau^2})\xi}{A\cos(\frac{1}{2}\sqrt{4\kappa - \tau^2})\xi + B\sin(\frac{1}{2}\sqrt{4\kappa - \tau^2})\xi}\right)e^{\iota\theta}, \tag{39}$$

With $\tau^2 - 4\kappa = 0$,

$$U(\xi) = \pm\frac{2\iota}{\sqrt{a}}\left(\frac{B}{A + B\xi}\right), \tag{40}$$

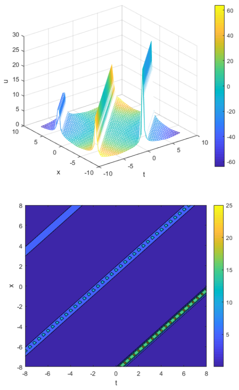
$$u(\xi) = \pm\frac{2\iota}{\sqrt{a}}\left(\frac{B}{A + B\xi}\right)e^{\iota\theta}, \tag{41}$$

where c=$\mp 2\sqrt{2\alpha\beta - \tau}$, $\theta = \alpha x + \beta y + \tau t$, $\xi = x + y \mp 2\sqrt{2\alpha\beta - \tau}$ t. Solutions of (FNE) are prepaired and curves are plotted for all possible wave-like solutions are crafted by their 3D plots and further their analogous contour plots.

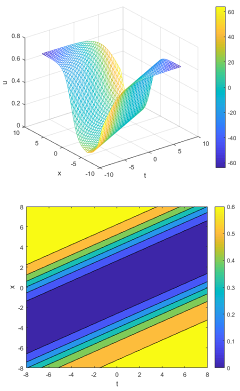
2 Conclusion

We can expand this work and rterritory applications of the $(\frac{F'}{F})$-expansion method. During this work we tried to obtain new solutions of some NLEES with complex NLEEs well known the coupled Higgs equation, the (3+1)-dimensional nonlinear Schrodinger equation(NLSE). The result we got are purely in traveling wave nature and can further leads to solitary wave or periodic solutions under wide parametric restrictions. Further interesting to claim here the from of the generic results, we can easily extract the solutions which are accomplished from different methods.

Figure 2: Traveling wave solution corresponding to the NLS Equation



(a) Plot of $u_1(\chi)$, when $\triangle > 0, \tau = 0.1, \rho = 0.02, A = 0.02, B = 0.03$



(b) Plot of $u_2(\chi)$, when $\triangle = 0, \tau = 0.2, \rho = 0.01, A = 0.01, B = 0.02$

References:

[1] J S Virdi, *On different approaches for integrals of physical dynamical systems*, Discontinuity, Nonlinearity, and Complexity **9(2)** (2000) , pp.299-307.,

[2] Wang M.: Solitary Wave Solutions for Variant Boussinesq Equations, *Phys. Lett. A.* **199**, (1995) 169.

[3] Swain S.N, Virdi J.S., (2019) *Lecture Notes in Mechanical Engineering*, doi.org 10.1007-978-981-15-0287-3-15, 187-200.

[4] J.S. Virdi, *Some New Solutions of Non Linear Evolution Equations With Mutable Coefficients.* Front. Appl. Math. Stat. (2021) https://doi.org/10.3389/fams.2021.631052.

[5] S Behera, J S Virdi, *Some More Solitary Traveling Wave Solutions of Nonlinear Evolution Equations,* Discontinuity, Nonlinearity, and Complexity **12(1)** (2023) 75-85.

[6] Virdi J.S, Some new solutions of nonlinear evolution equations with variable coefficients, *AIP Proceedings* **1728**, (2016), 020039.

[7] S.Behera, N.H.Aljahdaly, J.S.Virdi, *On the modified* $(\frac{G'}{G^2})$*-expansion method for finding some analytical solutions of the traveling waves*, J. Ocean Eng. Sci. (2022) https://doi.org/10.1016/j.joes.2021.08.013

[8] S. Behera and J.S. Virdi, (2022) *Generalized soliton solutions to Davey-Stewartson equation*, Nonlinear Optics, Quantum Optics, **57**, 325 (2023).

[9] Behera, S. and Virdi, J. P. S. (2021). Travelling wave solutions of some nonlinear systems by Sine-cosine Approach. *Proceedings of the Seventh International Conference on Mathematics and Computing, Advances in Intelligent Systems and Computing 1412*, https://doi.org/10.1007/978-981-16-6890-6_66.

[10] S Behera, J S Virdi, *Some More Solitary Traveling Wave Solutions of Nonlinear Evolution Equations,* Discontinuity, Nonlinearity, and Complexity **11(4)** (2022).

[11] W Yao, S Behera, Mustafa, H Rezazadeh, J S Virdi, W. Mahmoud, Omar Abu Arqub, M.S. Osman, Analytical solutions of conformable Drinfel'd–Sokolov–Wilson and Boiti Leon Pempinelli equations via sine–cosine method, Results in Physics **42**, 105990 (2022)

[12] S. Behera and J.S. Virdi, Generalized soliton solutions to Davey-Stewartson equation, Nonlinear Optics, Quantum Optics, vol. **56** 5, (2023).

[13] Behera, S. and Virdi, J. S. (2021). Travelling wave solutions of some nonlinear systems by Sine-cosine Approach. Proceedings of the Seventh International Conference on Mathematics and Computing, Advances in Intelligent Systems and Computing 1412,https://doi.org/10.1007/978-981-16-6890-6 66.

[14] S Behera, J S Virdi, Some More Solitary Traveling Wave Solutions of Nonlinear Evolution Equations, Discontinuity, Nonlinearity, and Complexity **12**(3) 75-85, (2023).

[15] J.S.Virdi, Some Study On Solitary Traveling Wave Solutions For Nonlinear Evolution Equations, Journal of Tianjin University Science and Technology, Vol:**55** 03, (2022)

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