Weld Defect Radiographic Image Segmentation with Finite Mixture Model (FMM)

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***Abstract*— Image segmentation is an important task in computer vision. This paper aims at studying two image segmentation methods based on the mixture of two probability density functions. We explore here the exploitation of a Finite Mixture Model (FMM), particularly Gaussian and Student's t - mixtures models (GMM, SMM) in histogram classification-based image segmentation. The expectation maximization (EM) algorithm is used to estimate the parameters of each model that maximize the log-likelihood function. Experiments have been conducted to segment real industrial radiography images. Comparison of results is achieved for GMM and SMM image segmentation. The obtained results show that the SMM is more robust that the GMM when segmenting real industrial radiography images.**

Keywords: Image segmentation, Finite Mixture Model (FMM), GMM, SMM, Gaussian distribution, Student's t-distribution.

I. Introduction

Segmentation is one of the most difficult problems in image analysis and pattern recognition systems, because its outcomes govern the final quality of the interpreta-tion [1][2]. Image segmentation consists of partitioning the image pixels into non-overlapping regions named segments. Each segment will represent some kind of in-formation to the user in the form of color, intensity or texture. Image segmentation is considered as a low level image processing operation that precedes some high level processing to analyze the image. Many image segmentation techniques have been developed by researchers and scientists[3][4][5][6], based on gray level thresholding, edge detection, region growing, region splitting and merging, relaxation, fuzzy set theory… etc. It is well known in image segmentation research that there is no single algorithm that can effectively segment all types of images. Furthermore, the applica-tion of different algorithms to the same image generally produces different segmenta-tion results. The selection of the adequate segmentation algorithm depends mainly on the type of images under investigation and the application areas.

levels can be modeled with known probability density function (pdfs) or a mixture of them, for example Gaussian, Rayleigh, t-Student, Beta, Gama, etc. [7]. The classes defining the resulting image regions are then, obtained considering the chosen model and Bays Rules [8].

The Finite Mixture Model (FMM) method is one of the most common histogram-based image segmentation, thanks to the ease of its implementation. Because of its flexibility, the Gaussian Mixture Model (GMM) is a widely used mixture model. The applications of GMM have shown to give good segmentation results when applied on images in which the gray levels distribution follows the Gaussian low. Nevertheless, GMM -based methods perform poorly when the inherent distribution is not a Gaussi-an one, which limit its use [9].

The Student's-t mixture model (SMM) has also been widely used for robust statistical modeling, which has received great attention on image segmentation; it has come to be regarded as an alternative to the GMM. The use of SMM components is justified by their advantages over GMM components; they have heavier tails, thus providing robustness to outliers. The SMM overcome, also, the binning problem of histogram-based methods and provide a continuous model of the image density [10].

In this paper, we propose to test two histogram-based segmentation methods that rely on Gaussian mixture model, and on Student's t-mixture one. The efficiency of the algorithms is shown on real industrial radiography images.

The structure of the remainder of the paper is as follows. In Section 2, we introduce the mathematical foundation of a mixture model, the Expectation Maximization (EM) and the two segmentation algorithms based on Gaussian mixture model and student's t-mixture model. In section 3 we present the experimental results. In section 4 we draw the main conclusion

II. Finite Mixture model (FMM)

Let $X=\{x_1, x_2, ..., x_N\}$ be a set of N realizations of a random a vector χ, with a probability density function (*pdf*) $f(x_i)$ where $i= 1,\dots,N$. Thus the parameterized *pdf* can be written as a combination of *pdfs* of the M components C_m ($m = 1,\dots,M$) characterizing the finite mixture model [7]:

ISSN: 2692-5079

$$f(x_i/\Theta) = \sum_{m=1}^{m=M} \pi_m f_m(x_i/\theta_m) \quad (1)$$

Where $\Theta = (\pi_1, \pi_2, \dots \pi_M, \theta_1, \theta_2, \dots \theta_M)$ is the vector of parameters to estimate, with π_m is the prior probability of the m^{th} component that satisfies the following conditions:

$$\pi_m \geq 0 \quad and \quad \sum_{m=1}^{m=M} \pi_m = 1$$

In presence of independent and identically observations (*iid*) the likelihood function can be expressed as:

$$L(\Theta) = \prod_{i=1}^{i=N} f(x_i/\Theta) = \prod_{i=1}^{i=N} \sum_{m=1}^{m=M} \pi_m f_m(x_i/\theta_m) \quad (2)$$

Consider the vector X as a partial observation of the considered phenomenon, then the maximization of $L(\Theta)$ is difficult to perform directly. A random variable Z_i introduced. It corresponds to the latent data such as:

$$Z_i = \{z_{i,1}, z_{i,2}, \dots, z_{i,M}\} \quad \text{where } z_{i,m} = \begin{cases} 1 & if x_i \in C_m \\ 0 & elsewhere \end{cases}$$

Let $Y = (X,Z)$ be the complete data. Thus, EM algorithm consists of maximizing the likelihood in presence of incomplete data by maximizing iteratively the expectation of the complete log-likelihood given by:

$$logL_c(\Theta|\mathrm{Y}) = \sum_{i=1}^{i=N} \sum_{m=1}^{m=M} z_{i,m} log[\pi_m f_m(x_i/\theta_m)] \quad (3)$$

Hereafter $L_c(\Theta)$ indicates $logL_c(\Theta|Y)$. The parameters Θ are estimated iteratively by using expectation and maximization steps.

E-step: compute the $L_c(\Theta)$ expectation:

$Q(\Theta/\Theta^{(t)}) = E\left[L_c(\Theta), \Theta^{(t)}\right]$ by using the posterior probability $Z_{im}{}^{(t)}$, which depends on the current parameters $\Theta^{(t)}$.

M-step: comes to maximize the Q function in relation to Θ to estimate the new model parameter values $\Theta^{(t+1)}$. These steps are repeated iteratively until a convergence criterion is reached.

The algorithm of EM can be summarized as follows:

Inputs: Number of M modes; Initial parameters $\Theta^{(0)}$; the algorithm convergence threshold ε

Outputs: Parameters $\widehat{\Theta}$ giving a maximum of the likelihood Function t ⟵ 0; Initialization of the model $\widehat{\Theta}^{(t)} = \Theta^{(0)}$;

Repeat

(E-Step) Compute posterior probabilities $\hat{z}_{im}^{(t)}$

$$\hat{z}_{im}^{(t)} = \Pr\big(m/x_i, \Theta(\mathrm{t})\big) = \frac{\pi_m f(x_i/\theta_m^{(t)})}{\sum_{m=1}^{m=M} \pi_m f_m(x_i/\theta_m^{(t)})} \quad (4)$$

(M-Step) Maximization of $L_c(\Theta)$

$$\widehat{\Theta}^{(t+1)} = argmax\, Q(\Theta/\widehat{\Theta}^{(t)})$$

If $\left\| Q(\Theta/\widehat{\Theta}^{(t+1)}) - Q(\Theta/\widehat{\Theta}^{(t)}) \right\| < \varepsilon$ end, else repeat $t \leftarrow t+1$;

Let hg ($g \in [1,L]$ where L is the number of gray levels) be the normalized histogram of an image I which can be seen as an estimate of the true *pdf* $f(g)$ of the image. In case of histogram fitting–based image segmentation with GMM and SMM, the gray level histogram is approximated by a Gaussian Mixture Model and a Student's t-Mixture Model, respectively.

A. Gaussian Mixture Model(GMM)

In case of GMM, the m^{th} Gaussian *pdf* is given by:

$$f_m(x_i|\theta_m) = \frac{1}{\sigma_m\sqrt{2\pi}} exp\left[-\frac{(x_i-\mu_m)^2}{2\sigma_m^2}\right] \quad (5)$$

Where μ_m and σ_m^2 are is the class mean and variance respectively. The estimated parameters in this case are:

$$\widehat{\Theta} = (\hat{\pi}_m, \hat{\theta}_m = (\hat{\sigma}_m, \hat{\mu}_m))$$

After the application of the EM algorithm, the parameters estimations are given by:

$$\hat{\pi}_m^{(t+1)} = \frac{\sum_{g=1}^{L} z_{gm}}{L} = \sum_{g=1}^{L} z_{gm}{}^{(t)} h_g \quad (6)$$

$$\hat{\mu}_m^{(t+1)} = \frac{\sum_{g=1}^{L} z_{gm} h_g x_g}{\sum_{g=1}^{L} z_{gm}} \quad (7)$$

$$\hat{\sigma}_m^{2\,(t+1)} = \frac{\sum_{g=1}^{L} z_{gm} h_g x_g{}^2}{\sum_{g=1}^{L} z_{gm}} \quad (8)$$

B. Student 's t- Mixture Model (SMM)

In case of the mixture of Student's t, the mth Student's t pdfs is given by:

$$f_m(x_i/\theta_m) = \frac{\Gamma\left(\frac{v_m}{2}+\frac{1}{2}\right)}{\sigma_m(\pi v_m)^{1/2}\,\Gamma\left(\frac{v_m}{2}\right)\left[1+\frac{(x_i-\mu_m)^2}{v_m\sigma_m^2}\right]^{\frac{(1+v_m)}{2}}} \quad (9)$$

Where, v_m is the degree of freedom, $\Gamma(x)$ is the gamma function. Here, the complete data is (X, Z, U), where u is a latent weight variable distributed as: u~Gamma $(v/2, v/2)$.

In this case, h_g is approximated by a mixture of M univariate Student-t functions [12]. In addition to$\left(z_{gm}^{(t)}\right)$, the expectation of the weight $(u_{gm}^{(t)})$ is computed in E-step as follows:

$$u_{gm}^{(t)} = \frac{v_m^{(t)}+1}{v_m^{(t)} + \frac{\left(x_g-\mu_m^{(t)}\right)^2}{\sigma_m^{2(t)}}} \quad (10)$$

In M-step, equation (6) is used to calculate $\hat{\pi}_m^{(t+1)}$ whereas $\hat{\mu}_m^{(t+1)}$, $\hat{\sigma}_m^{2(t+1)}$ and $\hat{v}_m^{(t+1)}$ are computed as follows:

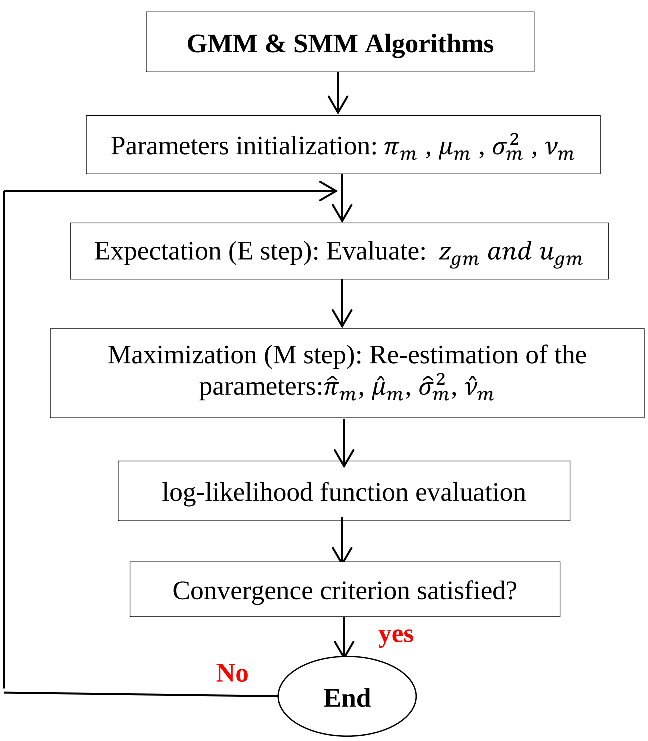
$$\hat{\mu}_m^{(t+1)} = \frac{\sum_{g=1}^{L} h_g \hat{z}_{gm}^{(t)} u_{gm}^{(t)} x_g}{\sum_{g=1}^{L} h_g \hat{z}_{gm}^{(t)} u_{gm}^{(t)}} \quad (11)$$

$$\hat{\sigma}_m^{2(t+1)} = \frac{\sum_{g=1}^{L} h_g Z_{gm}^{(t)} u_{gm}^{(t)} \left(x_g - \mu_m^{(t+1)}\right)^2}{\sum_{i=1}^{N} h_g Z_{gm}^{(t)}} \quad (12)$$

The degree of freedom of each component is estimate by solving the following nonlinear equation (13):

$$\log\left(\frac{\nu_m^{(t+1)}}{2}\right) - \psi\left(\frac{\nu_m^{(t+1)}}{2}\right) - \log\left(\frac{\nu_m^{(t)} + 1}{2}\right) + \psi\left(\frac{\nu_m^{(t)} + 1}{2}\right) + \frac{\sum_{g=1}^{L} h_g Z_{gm}^{(t)}\left(\log u_{gm}^{(t)} - u_{gm}^{(t)}\right)}{\sum_{g=1}^{L} h_g Z_{gm}^{(t)}} = 0 \quad (13)$$

where $\psi(x) = \partial \ln \Gamma(x)/\partial(x)$ denotes the digamma function.We obtain the approximate solution of each nonlinear equation using Newton-Raphson method [13].
The various steps of the two GMM and SMM algorithms can be summarized in the following flowchart:



III. Results and Discussions

In order to evaluate the efficiency of the two studied methods and to compare them to each other's, we first begin by applying the image segmentation on a sample of radiographic images. The initialization of the common parameters ($\hat{\pi}_m$, $\hat{\mu}_m$, $\hat{\sigma}_m^2$), is the same for the two methods, we have chosen to exploit three components; one for the defect and two for the variable background. Implementation of algorithms has been developed with Matlab R2013a. The segmentation results are shown in fig. 1. Moreover, to compare the two methods, we have applied Yasnoff evaluation segmentation method [14]. As the method needs reference images, the sample of radiographic images have been segmented manually by experienced persons in the field of radiographic testing you.

A. Evaluating Defect Segmentation

To assess segmentation results, Yasnoff [14] has exploited the distance between the miss-segmented pixel and the nearest pixel that actually belongs to the miss-segmented class [15]. Let P be the number of miss-segmented pixels for the whole image and d(i) be a distance metric from the ith miss -segmented pixel and the nearest pixel that actually is of the miss-classified class; a discrepancy measure (D) based on this distance is defined by the author of [14] as:

$$D = \sum_{i=1}^{P} d^2(i) \quad (14)$$

In equation (14), each distance is squared. This measure is further normalized (*ND*), to exempt the influence of image size and to give it a suitable value range by: [14],[15]

$$ND = \frac{100\sqrt{D}}{A} \quad (15)$$

Where *A* is the total pixels number of the image.
Here the miss-segmented pixels are the false negative (FN) (pixels belonging to the defect that have been affected to the background) and false positive (FP) (false detection or pixels belonging to the background that have been affected to the defects). As the images are too large comparing to the defects size, we have chosen to compute *A* as the sum of all defects sizes. This will permit to better assess the influence of the segmentation errors on the defects. Thus, a measure error that is greater than "1" , means that the amount of the erroneous segmentation in terms of pixels exceeds the defects in size. In the following table, we give the evaluation of the segmentation results of only the defects (one class), with this measure. Execution time, is also, shown on this table (*T*(*s*), second). NFP and NFN are the evaluation measures related to the false positive and the false negative, respectively.

From the results given by the Table 1 and visually from the Fig. 2, we can conclude that the two methods tend to produce more over segmentation than under segmentation. On the other hand, the two methods have proven to be efficient in presence of relatively uniform background, as shown by the segmentation results of Img2 in Fig.2. Nevertheless, when faced to a high intensity variability of the background (Img1 and Img3), the GMM method fails to extract the defects. This is shown by the high values the NFP measures of the GMM for these two images. However, the SMM is more time consuming than the GMM, as it is shown on Table I. This fact is the result of the nonlinear equations of that have to be solved by iterative procedures in SMM.

Table. 1. NFP, NFN and T values for GMM and SMM models

Model/ images		Img1	Img2	Img3
GMM	*NFN*	0.0086	0.0269	0.0000
	NFP	2.4947	0.2173	1.3002
SMM	*NFN*	0.0086	0.0252	0.0058
	NFP	0.7707	0.0652	0.0250
GMM	*T(s)*	0.1180	0.1222	0.1278
SMM		2.1579	2.1086	2.0831

IV. Conclusion

In this paper, we have tested two finite mixture models for segmenting real industrial radiography images. The first one is based on Gaussian distribution and the other one on Student's t-distribution. The EM (Expectation Maximization) algorithm has been used to estimate the parameters of these mixtures. Experiment results have shown the efficiency of the two mixture models when the background does not present a high variability. Nevertheless, when faced to a high intensity variability of the background, the GMM method fails to extract the defects and presents an over segmentation. We have demonstrated the robustness, the accuracy and the effectiveness of SMM model when faced to a moderate intensity variability of background

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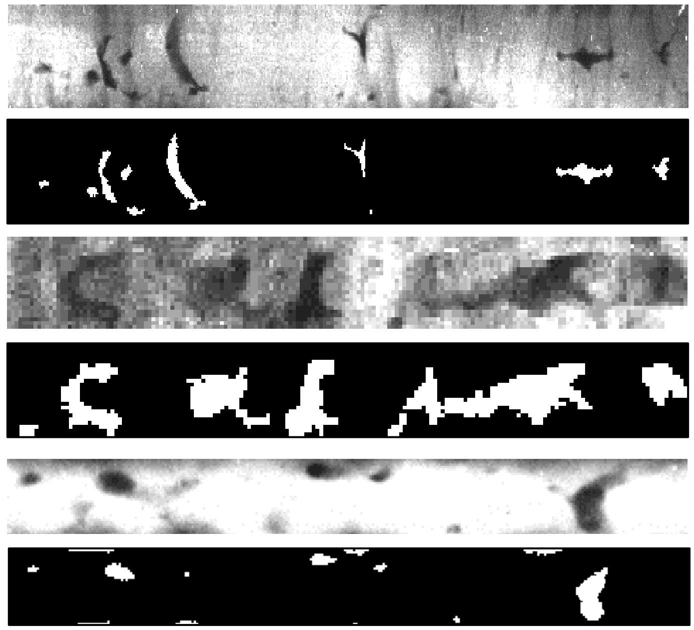

Fig. 1. Radiographic images and the corresponding hand segmented Images (Reference Images)

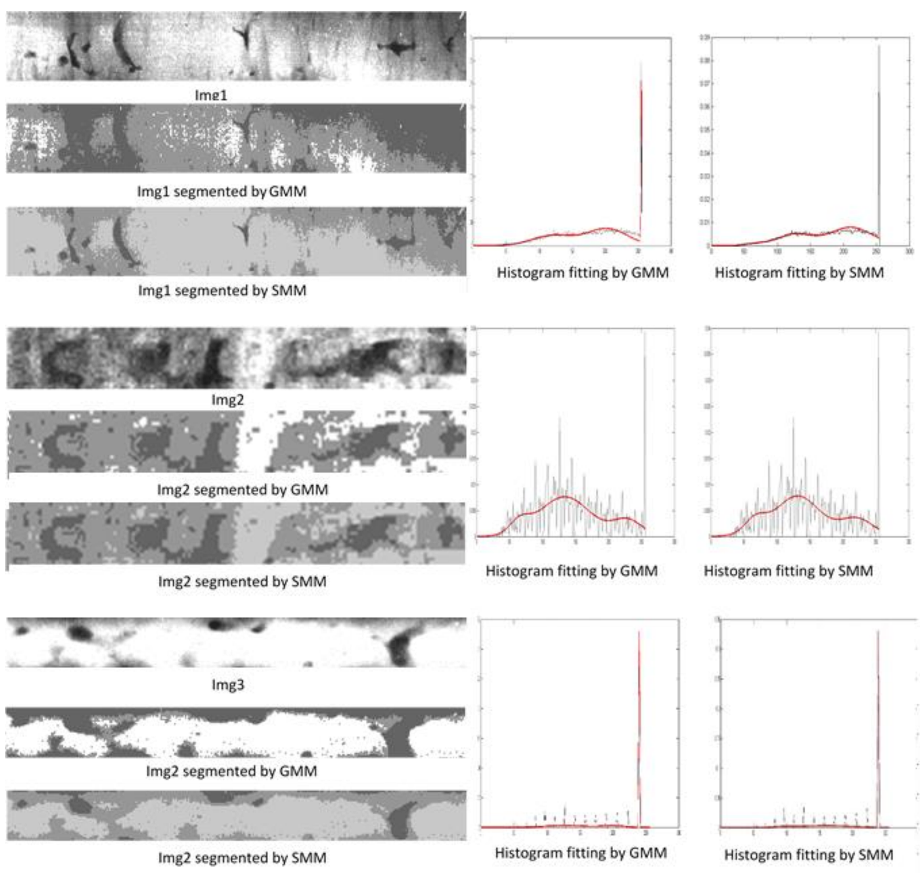

Fig. 2. Industrial radiography images segmented by GMM and SMM, Histogram fitting by GMM and SMM,

——— : Histogram of original image
——— : Pdfs estimate