ISSN: 2692-5079

Temperature distributions in a parallel flat plate microchannel with electroosmotic and magnetohydrodynamic micropumps

Juan Pablo Escandón[a], Fernando Santiago[b] and Oscar Bautista[c]
SEPI-ESIME Azcapotzalco del Instituto Politécnico Nacional, Av. de las Granjas No. 682,
Col. Santa Catarina, Del. Azcapotzalco, México, D.F. 02250, Mexico
Email: [a]jescandon@ipn.mx, [b]sant_dot@hotmail.com, [c]obautista@ipn.mx

***Abstract*—In this work, the steady heat transfer process to find the temperature distribution through of a viscoelastic fluid flow in a parallel flat plate microchannel was solved. The fluid flow is based in a combination of electroosmotic and magnetohydrodynamic driven forces. A fully-developed flow is considered and the fluid obeys a constitutive relation based in a simplified Phan-Thien-Tanner model. The non-dimensional fluid flow model is expanded in a regular expansion series in powers of small Hartmann numbers. The effect of certain non-dimensional parameters on the fluid flow is predicted and we report advantages in the simultaneous application of electroosmotic and magnetohydrodynamic forces due to significant increase in the flow rate and by the diminishing of the unnecessary Joule heating in microfluidic devices. Therefore, by employed low magnetic fields with low electrical conductivities in the buffer solution, is possible that the electric and magnetic effects can be used to move a charged solution in the flow control and sample handling in biomedical and chemical analysis.**

I. INTRODUCTION

The great potential of microscale and nanoscale technology for chemical and biological analysis is reflected by recent explosive growth in research on laboratory-on-a-chip and miniature diagnostic devices. The terms laboratory-on-a-chip and micro total analysis system stand synonymous for devices that use fluids as a working medium and integrate a number of different functionalities on a small scale. Advances in microfabrication and microelectromechanical systems (MEMS) over the past few decades has allowed miniaturized devices of growing complexity and sophistication to be developed for various applications [1]. Microchip devices have already been developed for drugs screening, electrochemical immunoassays, drug delivery, point-of-care-medical diagnostics, bacteria detection, environmental monitoring, and the detection of explosives and biological warfare agents [2], [3]. Also, Microfluidics devices based in the Polymerase Chain Reaction (PCR) chips are used in amplification of DNA and separate species through capillary electrophoresis [4]–[6].

Many microfluidic applications would benefit from a laboratory-on-a-chip active pump with size comparable to the small volume of fluid to be pumped, i.e. an integrated micropump. A number of micropumps have been designed and built using emerging microfabrication technologies. Laser and Santiago [3] classifying many of these into two groups: membrane-displacement pumps and field-induced flow pumps. Membrane pumps may be further classified based on how membranes are actuated, including piezoelectric, thermopneumatic, electromagnetic and photothermally actuated pumps. Field-induced flow pumps include electroosmotic, electrohydrodynamic and magnetohydrodynamic pumps. A clear advantage of field-induced pumps over membrane pumps is that they do not require moving parts. However, the moving parts in displacement micropumps make the fabrication and operation delicate [7]. In this context, the requirement of an integrated micropump with no moving parts can be fulfilled by using electrohydrodynamic and magnetohydrodynamic micropumps [5].

The term electrokinectics concerns the use of applied electric fields to impart a net electrostatic force in polarized surface regions such that fluid motion or the motion of particles suspended within the bulk of a liquid is induced [8]. Afonso et al. [9] present an analytical solution of mixed electroosmotic and pressure driven flows of viscoelastic fluids in microchannels, which rheological behavior can be described by the constitutive equation of the simplified Phan-Thien-Tanner model, as blood, saliva and synovial fluids. The solution is non-linear with a significant contribution arising from the coupling between the electric and pressure potentials. Others studies about fluid flow characteristics of purely electroosmotic flows in microchannels [10]–[12] with complex fluids has been developed already.

Magnetohydrodynamic is the study of flow of electrically conducting liquids in electric and magnetic fields and the basic principle of the magnetohydrodynamic micropump is the generated Lorentz force [13]. Compared with other types of nonmechanical micropumps, the magnetohydrodynamic micropump has several advantages, such as simple fabrication process, bidirectional pumping ability, and the usability of medium conducting liquid. It is believed that the magnetohydrodynamic micropump can be used in biomedical devices or microfluidic propulsion application [14]–[17].

In many of these applications, an upper limit of the device performance has often been restrained by the limitations of the strength of the axial electric field that can be employed to actuate the flow, in order to minimize the Joule heating effects and the associated adverse consequences. Has been appreciated that combined electromagnetohydrodynamic effects can potentially be utilized to enhance the liquid flow rates in microchannels. In all of them, electric current flows through the working liquid and a variety of pumping techniques is encountered depending on the conductivity of the liquids [18]. This authors have examined the influence of the combined electromagneto-hydrodynamic forces in controlling the Newtonian fluid flow through parallel plate rectangular microchannels. In this study shows that, with the aid of a relatively low-magnitude magnetic field, a substantial augmentation in the volumetric flow rates can be achieved, and certain dimensionless parameters are also identified, which can play significant roles in the overall flow augmentation mechanism.

The main goal of the present work is to describe the physical principles that describes the performance of the electroosmotic and magnetohydrodynamic micropumps, and what physical parameters affect the performance of these devices with the consideration of Joule heating effects. The dimensionless temperature profiles in the fluid and the microchannel wall are obtained as function of the dimensionless parameters involved in the analysis, and the interactions between the coupled momentum and thermal energy equations are examined.

II. Mathematical formulation

A. Physical model

The physical model shown in Figure 1 consider a flat parallet plate microchannel of height $2b$, width w and length L, with $w \gg 2a$ and $L \gg 2b$. The origin of the rectangular coordinate system is placed in the following way, the x-axis is in the transverse direction, normal to the surface of the microchannel, the y-axis is in the symmetry line of the microchannel, and the z-axis is in the inlet of the microchannel. The system described above is filled with fluid medium based of a mixture of an ionized solution and solutes that exhibits non-Newtonian behavior. The fluid flow is driven by the combination of an electric fields and a magnetic field. The component E_y along the axis of the microchannel provides an electroosmotic force when acting over the high concentration of free electric charges within the electrical double layer. The interaction between the magnetic field B_x and electric field E_z produces the magnetohydrodynamic force in the flow direction. Because of the symmetry of the physical model, we consider only the upper half of this configuration. The high concentration of electric charge is placed into the Debye length, κ^{-1}, in the electric double layer. The temperature of the fluid at the inlet and outlet of microchannel is assumed to be at the ambient temperature, T_0. The external surfaces of the microchannel walls are subjected to convection boundary conditions with temperature T_0 and heat transfer coefficient h.

B. Governing equations

Assumptions are adopted in order to simplify the analysis:

- Incompressible fluid.
- Steady, fully-developed flow ($u = 0, v(x) = 0, w = 0$).
- The EDL's does not overlap on the centerline of the microchannel.
- The wall potentials are low enough ($\leq 25mV$) that the Debye-Hückel linearization approximation is valid.
- Constant thermophysical properties in the system for smaller temperature changes (less than 10 K).

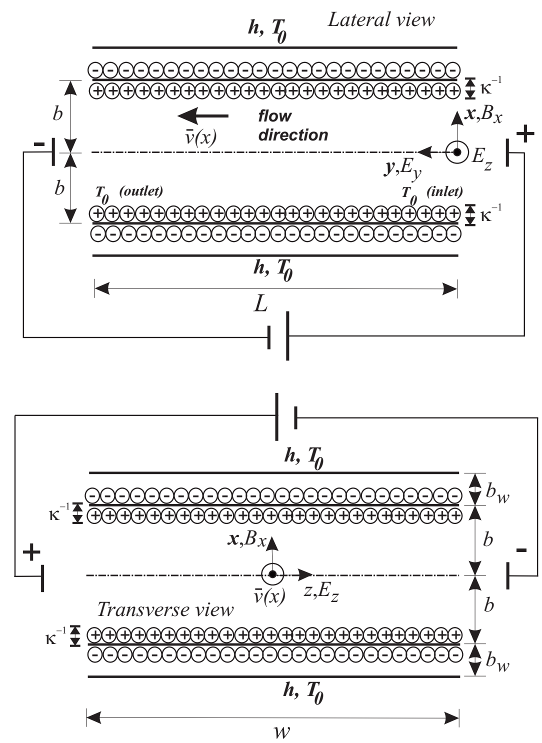

Fig. 1. Electroosmotic and magnetohydrodynamic flows in a parallel flat plates microchannel.

The flow field is governed by the momentum equation, which takes de following form

$$\frac{d\tau_{yx}}{dx} + \sigma B_x^2 v + \sigma E_z B_x + \rho_e E_y = 0, \quad (1)$$

where τ_{yx} is the shear stress, σ is the electrical conductivity of the medium and $\rho_e = -\epsilon\kappa^2\zeta\frac{\cosh(\kappa x)}{\cosh(\kappa b)}$ is the net free electric charge within the electrical double layer in the called Debye length, which is obtained from the Poisson-Boltzmann equation for electroosmotic flows and low zeta potentials [8]. Where ζ is the zeta potential in the microchannel walls and ϵ is the dielectric permittivity. From the previous equation, the velocity gradient of the flow is obtained as

$$\frac{dv}{dx} = \frac{1}{\sigma B_x^2}\left[\frac{d^2\tau_{yx}}{dx^2} - \epsilon\kappa^3\zeta E_y \frac{\sinh(\kappa x)}{\cosh(\kappa b)}\right]. \quad (2)$$

Using the constitutive equation for simplified Phan-Thien-Tanner [9], the analogous velocity gradient as a function of the shear stress is

$$\frac{dv}{dx} = \left(1 + \frac{2\varepsilon\lambda^2}{\eta_0^2}\tau_{yx}^2\right)\frac{\tau_{yx}}{\eta_0}, \quad (3)$$

where ε is the PTT parameter, λ is the relaxation time and η_0 is the polymer viscosity coefficient. Equating the Eqs. (2) and (3), the non-linear differential equation for the shear stress is defined by

$$\frac{d^2\tau_{yx}}{dx^2} - \frac{2\varepsilon\lambda^2\sigma B_x^2}{\eta_0^3}\tau_{yx}^3 - \frac{\sigma B_x^2}{\eta_0}\tau_{yx} = \epsilon\kappa^3\zeta E_y \frac{\sinh(\kappa x)}{\cosh(\kappa b)}, \quad (4)$$

subject to the following boundary condition in the centerline symmetry as $\tau_{yx}(x = 0) = 0$ and a compatibility condition to

ISSN: 2692-5079

find the relationship for the shear stress in the microchannel wall from the Eq. (1), that is $\left.\frac{d\tau_{yx}}{dx}\right|_{x=b} = -\sigma E_z B_x + \epsilon\kappa^2\zeta E_y$.

The energy equation is defined as

$$\rho C_p v \frac{\partial T}{\partial y} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau_{yx}\frac{du}{dx} + \sigma\left(E_y^2 + E_z^2\right) - 2\sigma E_z u B_x + \sigma v^2 B_x^2, \tag{5}$$

where ρ is the fluid density, C_p is the specific heat, T is the temperature and k is the thermal conductivity. The corresponding boundary conditions to Eq. (5) are

$$T(y=0) = T(y=L) = T_0,\ \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0,$$
$$\left.\frac{\partial T}{\partial x}\right|_{x=b} = -\frac{h_{eq}}{k}\left[T(x=b) - T_0\right]. \tag{6}$$

The equivalent heat transfer coefficient h_{eq} is:

$$h_{eq} = b^{-1}\left(\frac{b_w}{k_w L} + \frac{1}{hL}\right)^{-1}. \tag{7}$$

where k_w is the thermal conductivity of the microchannel walls.

C. Non-Dimensional model

The non-dimensional variables taking account are:

$$\bar{y} = \frac{y}{L}, \bar{x} = \frac{x}{b}, \bar{\tau}_{yx} = \frac{b\tau_{yx}}{\eta_0 v_{HS}}, \bar{v} = \frac{v}{v_{HS}}, \theta = \frac{T - T_0}{\triangle T_c}, \tag{8}$$

where $v_{HS} = -\epsilon\zeta E_y/\eta_0$ is the Helmholtz-Smoluchowski electroosmotic velocity and $\triangle T_c = \sigma_e E_y^2 Lb/k$ is the characteristic temperature change in the microchannel configuration. Therefore, Eqs. (5) and (6) can be rewritten as

$$Pe\beta\bar{v}\frac{\partial\theta}{\partial\bar{y}} = \beta^2\frac{\partial^2\theta}{\partial\bar{y}^2} + \frac{\partial^2\theta}{\partial\bar{x}^2} + Br\bar{\tau}_{yx}\frac{d\bar{v}}{d\bar{x}} + \beta\left(1+\gamma^2\right) + 2\gamma Ha^2\Phi\bar{v} + Ha^2\Psi\bar{v}^2, \tag{9}$$

$$\theta(\bar{y}=0) = \theta(\bar{y}=1) = 0,$$
$$\left.\frac{\partial\theta}{\partial\bar{x}}\right|_{\bar{x}=0} = 0,\ \left.\frac{\partial\theta}{\partial\bar{x}}\right|_{\bar{x}=1} = -\Lambda\theta(\bar{x}=1). \tag{10}$$

The non-dimensional parameters in Eqs. (9) and (10) are defined as:

$$Pe = \frac{\rho C_p v_{HS} b}{k}, \beta = \frac{b}{L}, Br = \frac{\eta_0 v_{HS}^2}{k\triangle T_c},$$
$$Ha = B_x b\sqrt{\frac{\sigma}{\eta_0}}, \gamma = \frac{E_z}{E_y}, \Phi = \frac{\epsilon\zeta}{B_x Lb\sigma},$$
$$\Psi = \frac{\epsilon^2\zeta^2}{\eta_0 Lb\sigma}, \Lambda = \frac{\frac{b}{kL}}{\frac{b_w}{k_w L} + \frac{1}{hL}} \tag{11}$$

where Pe is the Péclet number, β is ratio of the microchannel thickness to the microchannel length, Br is the Brinkman number, Ha is the Hartmann number, γ is the ratio of the applied electric field in the z-axis direction to the applied electric field in the y-axis, Φ and Ψ are parameters and finally Λ is the ratio of the thermal resistance by conduction in the fluid to the thermal resistance by conduction in the wall plus the convection in the external wall of the microchannel.

The non-dimensional velocity, $\bar{v}$, of the flow has been determined by Escandón et al. [19] in a asymptotic solution for the shear stress in powers of $Ha^2 = \alpha$, with $\alpha \ll 1$, when was introduced the expansion $\overline{\tau}_{yx} = \overline{\tau}_{yx,0} + \alpha\overline{\tau}_{yx,1} + ...$ into Eq. (4) and his corresponding boundary conditions, given the next solution for the shear stress:

α^0 :

$$\overline{\tau}_{yx,0} = -\overline{\kappa}\frac{\sinh\left(\overline{\kappa}\,\overline{x}\right)}{\cosh\left(\overline{\kappa}\right)}. \tag{12}$$

α^1 :

$$\overline{\tau}_{yx,1} = \frac{2\varepsilon De_\kappa^2\overline{\kappa}}{\cosh^3\left(\overline{\kappa}\right)}\left\{\frac{3}{4\overline{\kappa}}\left[\frac{\sinh\left(\overline{\kappa}\,\overline{x}\right)}{\overline{\kappa}} - \overline{x}\cosh\left(\overline{\kappa}\right)\right] - \frac{1}{12\overline{\kappa}}\left[\frac{\sinh\left(3\overline{\kappa}\,\overline{x}\right)}{3\overline{\kappa}} - \overline{x}\cosh\left(3\overline{\kappa}\right)\right]\right\} + \overline{x} - \frac{\sinh\left(\overline{\kappa}\,\overline{x}\right)}{\overline{\kappa}\cosh\left(\overline{\kappa}\right)} - \Omega^*\overline{x}. \tag{13}$$

Introducing the corresponding non-dimensional variables of Eq. (8) in Eq. (3), and considering the following regular expansion to the velocity $\overline{v} = \overline{v}_0 + \alpha\overline{v}_1 + ...$ and of the previous solution for the shear stress, we have to the velocity gradient in powers of α :

α^0 :

$$\frac{d\overline{v}_0}{d\overline{x}} = \overline{\tau}_{yx,0} + \frac{2\varepsilon De_\kappa^2}{\overline{\kappa}^2}\overline{\tau}_{yx,0}^3. \tag{14}$$

α^1 :

$$\frac{d\overline{v}_1}{d\overline{x}} = \overline{\tau}_{yx,1} + \frac{6\varepsilon De_\kappa^2}{\overline{\kappa}^2}\overline{\tau}_{yx,0}^2\overline{\tau}_{yx,1}. \tag{15}$$

Both Eqs.(14) and (15) are integrated once and by using the no-slip boundary condition we obtain the velocity profile of the flow in the microchannel. The dimensionless parameters in Eqs. (12)-(15) are defined as:

$$\varepsilon De_\kappa^2 = \varepsilon\left(\lambda v_{HS}\kappa\right)^2, \bar{\kappa} = \kappa b, \Omega^* = \frac{E_z}{B_x v_{HS}}, \tag{16}$$

where De_κ is the Deborah number, εDe_κ^2 is the viscoelastic parameter, $\bar{\kappa}$ is the electrokinetic parameter and Ω^* is a parameter. We identify that the product $\Omega^* Ha^2$ is the ratio of magnetic forces to electroosmotic forces.

The order of magnitude of the changes in temperature through the y, x coordinates are $\Delta T_y \sim \sigma E_y^2 L^2/k$ and $\Delta T_x \sim \sigma E_x^2 b^2/k$, respectively, as can be obtained from Eq.(5). Hence, the dominant temperature changes in the microchannel are in the longitudinal direction, that is $(\Delta T_y/\Delta T_x) \sim (L/b)^2 \gg 1$; thus, in a first approximation the fluid temperature can be calculated as a function of the axial coordinate, that is $T = T(y)$ or $\theta = \theta(\bar{y})$. Integrating the Eq. (17) in the transverse direction and considering the corresponding boundary conditions of Eq. (10), the energy equation is transformed as

$$\beta^2\frac{d^2\theta}{d\bar{x}^2} - Pe\beta k_1\frac{d\theta}{d\bar{x}} - \Lambda\theta = -Brk_3 - \beta\left(1+\gamma^2\right) - Ha^2\left(2\gamma\Phi k_1 + \Psi k_2\right), \tag{17}$$

ISSN: 2692-5079

where the constants

$$k_1 = \int_{\bar{x}=0}^{\bar{x}=1} \bar{v} d\bar{x}, k_2 = \int_{\bar{x}=0}^{\bar{x}=1} \bar{v}^2 d\bar{x},$$
$$k_3 = \int_{\bar{x}=0}^{\bar{x}=1} \left(\bar{\tau}_{yx} \frac{d\bar{v}}{d\bar{x}} \right) d\bar{x}, \tag{18}$$

For typical electroosmotic flows the Brinkmann number is $Br \ll 1$ and the combined electroosmotic and magnetohydrodynamic effect is given trough of $k_3 \sim 10^1 - 10^2$, therefore de viscous dissipation can be neglected in the thermal analysis, $Brk_3 \ll 1$. Additionally, the parameter $\Psi \ll 1$ and $k_2 \sim 10^0 - 10^1$, then the product associated to magnetohydrodynamic effects $\Psi k_2 \ll 1$, also can be neglected. Considering the boundary conditions given by Eq. (10) at the inlet and outlet of the microchannel, the solution of the non-homogeneous ordinary differential equation, Eq. (17) is

$$\theta(\bar{y}) = \frac{\beta\left(1+\gamma^2\right) + 2\gamma\Phi k_1 Ha^2}{\Lambda} \times \left[\frac{1-\exp(\omega)}{\exp(\omega)-\exp(\pi)} \exp(\pi\bar{y}) + \frac{\exp(\pi)-1}{\exp(\omega)-\exp(\pi)} \exp(\omega\bar{y}) + 1 \right]. \tag{19}$$

In the previous equation π and ω are defined as

$$\pi = \frac{Pek_1}{2\beta} + \sqrt{\left(\frac{Pek_1}{2\beta}\right)^2 + \frac{\Lambda}{\beta^2}}, \tag{20}$$

$$\omega = \frac{Pek_1}{2\beta} - \sqrt{\left(\frac{Pek_1}{2\beta}\right)^2 + \frac{\Lambda}{\beta^2}}, \tag{21}$$

D. Results and discussion

In Figures 2 and 3, the dimensionless temperature distributions are shown as function of the dimensionless axial coordinate. In order to compare the temperature distributions of the combined EO+MHD effects in the fluid flow, was introduced the case of purely EO flow when $E_z \to 0$ and $B_x \to 0$, resulting in $\gamma = 0$, $Ha \ll 1$, $\Omega^* = O(1)$ and $\Phi = O(1)$.

Figure 2 shows the dimensionless temperature distributions along the length of the microchannel with different values of the ratio of the applied electric field E_z to the applied electric field E_y, $\gamma(= 0.25, 0.5, 0.75, 1)$. The other fixed parameters are $\beta = 0.01, \Phi = 0.5, Pe = 0.05, \Lambda = 0.001, \bar{\kappa} = 100, \varepsilon De_\kappa^2 = 1, \Omega^* = 2\text{x}10^6$ and $Ha = 0.001$. In the cases with $\gamma > 0$, the ratio of magnetic forces to electroosmotic forces is $\Omega^* Ha^2 = 2$, and are a combination of electroosmotic and magnetohydrodynamic flows. The value of $\gamma = 1$ represents the condition with $E_z = E_y$, in which the Joule heating increases doubling the magnitude of the temperature distribution respect to $\gamma = 0$ (case of purely electroosmotic flow). Reducing the electric field E_z below or equal of 50% of value of E_y, that is $\gamma \leq 0.5$, the combined electroosmotic and magnetohydrodynamic flows can be achieve temperature distributions similar or below of an purely electroosmotic flow due to the increases of the convection heat transfer via k_1 in the product Pek_1 in Eqs. (20) and (21). But, to maintained the same value of $\Omega^* Ha^2 = 2$ and the corresponding fluid flow conditions, the magnetic field B_x must be diminished and the microchannel height should to be increased.

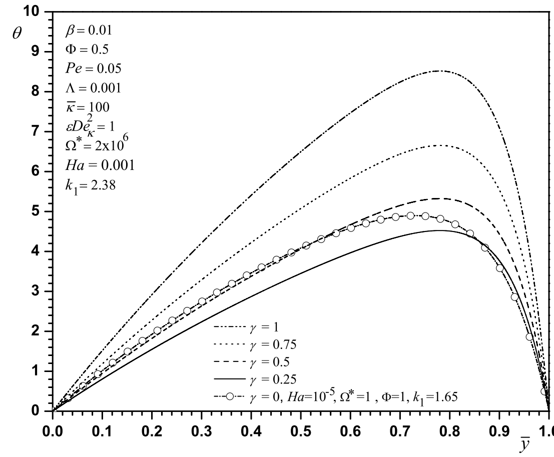

Fig. 2. Dimensionless temperature distribution as function of the dimensionless axial coordinate in combined electroosmotic and magnetohydrodynamic flows with different values of the parameter γ.

In Figure 3, any applied electric field E_z interacting with the magnetic field B_x to generate a Lorentz force, produce an increase of the temperature in the fluid (as shown in Figure 2). For the fluid flow conditions in in this figure, a value of $\gamma = 0.5$ and increasing values of the Hartmann number, diminish the temperature distributions by convection heat transfer effects. Thereby, is possible obtain an increase of the fluid flow by the interaction of the electroosmotic and magnetohydrodynamic flows without increasing the temperature too much, under the following conditions $\gamma \leq 0.5$ and $\Omega^* Ha^2 > 1$, i.e. high Hartmann numbers. Values of $\Omega^* Ha^2 < 1$ only produce an excess of Joule heating, without significantly increasing the fluid flow, via the constant k_1.

III. Conclusion

In this study, we have examined the effects of heat transfer to find the temperature distribution through of a Non-Newtonian fluid flow. An analysis of the Joule effects on the electroosmotic and magnetohydrodynamic flow of viscoelastic fluids in parallel flat plate microchannels, with the simplified Phan-Thien-Tanner model has been made. The undesirable Joule heating in the microchannels fluid flow can be reduced by the combination of electroosmotic and magnetohydrodynamic forces, increasing the fluid flow without increase the fluid temperature when $\gamma \leq 0.5$ and $\Omega^* Ha^2 > 1$. Therefore, this study contributes to the understanding of the different coupled transport mechanisms for the design of microfluidic systems.

Acknowledgment

This work was supported by research grants 169718 SEP-CONACYT and 20131319 SIP-IPN in Mexico.

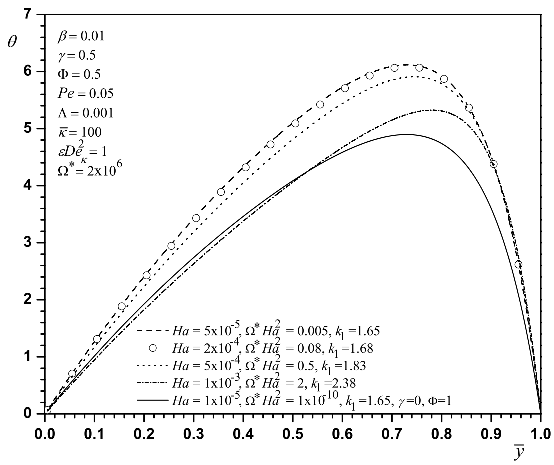

Fig. 3. Dimensionless temperature distribution as function of the dimensionless axial coordinate in combined electroosmotic and magnetohydrodynamic flows with different values of the Hartman number.

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