Effect of Mass per Unit Length on freely vibrating Simply Supported Rayleigh Beam

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Abstract: - In this paper, free vibration characteristics of a uniform Rayleigh beam are studied using the differential transform method. The procedure entails transforming the partial differential equation governing the motion of the beam under consideration and the associated boundary conditions. The transformation yields a set of difference equations. Some simple algebraic operations are performed on the resulting difference equations to determine any ith natural frequency and the closed-form series function for any ith mode shape. Finally, one problem is presented to illustrate the implementation of the present method and analyse the effect of mass per length on the natural frequencies of the beam.

Key-Words: - Differential transform, free vibration, Harmonic motion, Natural frequency, Mode shape, Rayleigh beam, Mass per unit length

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1 Introduction

Vibration of elastic bodies, because of its real-life applications, has been studied by quite a number of scholars. Many authors have considered the forced vibrations of elastic bodies such as beams and plates. A study on the influence of a moving load with variable velocity on the dynamic response of a simply supported Euler-Bernoulli beam was undertaken by Awodola [1]. The beam was assumed to be resting on a uniform foundation and excited by a load moving with variable velocity. Oni and Omolofe [2] investigated the transverse vibration of a prismatic Rayleigh beam using generalized finite integral transform and modified Struble's asymptotic method. The effects of boundary conditions, slenderness ratio and elastic foundation on which the beam rests were analysed. Auciello and Lippiello [3] investigated the dynamic response of a column partially immersed in water, using the Rayleigh beam theory to model the column. Golas [4] worked on the influence of the rotary inertia on the eigenvalues of composite beams. It was found that the influence of the rotary inertia is over ten times smaller than the influence of shear deformations. It was therefore suggested that rotary inertia might be neglected.

Rajesh and Kumar [5] carried out a study on free vibration behaviour of some viscoelastic sandwich beams based on the Euler-Bernoulli beam model at different end classical conditions. The viscoelastic sandwich beams considered had aluminium and mild steel as face material and the core material was modelled using neoprene rubber. The study reveals that higher natural frequencies are associated with mild steel used as face material compared with when aluminium is used. It was further shown that the natural frequencies reduce when neoprene rubber was used as the core material. Usman, Ogunsan, Okusaga and Solanke [6] studied the influence of damping coefficient on an Euler-Bernoulli beam excited by distributed load using the finite Fourier sine transform and finite difference method. It was reported that an increase in the speed of the load causes a decrease in the amplitude of the beam's deflection in the presence of damping coefficient. Contrarily, it was found that the amplitude of the beam's deflection decreases as the speed of the load increases when damping is neglected.

Usman, *et al.* [7] used the series solution method to obtain the response Euler-Bernoulli beam under the excitation of a concentrated moving load. Jimoh and Ajoge [8] employed Galerkin's method and the integral transform techniques to study the influence of rotatory inertia and axial force on the vibration characteristics of non-uniform beam. It was assumed that the beam resting on Pasternak foundation was harmonically excited by moving loads with varying magnitude. Jimoh and Ajoge [9] considered the influence of rotatory inertial and damping coefficient on the dynamic response of a uniform Rayleigh beam traversed moving loads of constant magnitude. The authors used the Fourier Sine and Laplace Integral Transformations. It was found that the beam's amplitude of displacement decreases due to increase in the values of rotatory inertia and damping coefficient of the beam. The effects of shear modulus, foundation modulus and axial force on the beam's amplitude of deflection were also investigated.

The effect of variable prestress and foundation constants on the natural frequencies of a simply supported Rayleigh beam subjected to distributed loads was analysed by Andi and Wilson [10]. The generalized Galerkin's and modified Struble's asymptotic methods were applied to solve the vibration problem. The study reported that both the natural and modified frequencies increase when the values of prestress increase. Another finding of the study has it that resonance is reached earlier for lower values of prestress and lower values of the foundation constant. The problem of dynamic behaviour of two-steps nanobeam modelled using the Rayleigh beam theory was studied by Hossain and Lellep [11]. They analysed the influence of rotatory inertia on the dynamic characteristics of the system. The study shows that the effect of rotatory inertia is highly significant in the nanobeam and its influence rises with the increase of mode of frequency. Omolofe and Adara [12] in a study applied Galerkin's residual method and Struble's asymptotic technique in conjunction with Duhamel's integral transform to analyse the response of a beam under the action compressive axial force and moving masses.

Differential transform method (DTM) used in this paper has been proved to be highly effective in solving both ordinary and partial differential equations. Research works in which the method has been successfully applied to solve problems in solid mechanics and computational fluid mechanics. These include the work of Opanuga, Adesanya, Okagbue and Agboola [13] where DTM was used to obtain semi-analytical solutions of the equations governing the entropy generation of radiative Magnetohydrodynamic mixed convection Casson fluid. In another work by Opanuga *et al.* [14], DTM was used to solve the velocity and energy equations associated with the entropy generation of unsteady hydromagnetic Couette flow through vertical microchannel. Agboola *et al.* [15] used DTM to study the entropy generation of a steady natural convection flow between two vertical parallel micro-channels with Hall effect.

The aim of this work is to analyse numerically the vibration characteristics of a uniform Rayleigh beam considering simply supported end conditions. The effect of mass per unit length on the non-dimensional frequencies of the freely vibrating beam is also explored. It is pertinent to note that the effects of rotary inertia and shear deformation are neglected in the Euler-Bernoulli beam theory, which make the theory applicable to an analysis of long and slender beams only. On the other hand, the Rayleigh beam theory takes cognizance of the effect of rotary inertia, while the Timoshenko beam theory, which is applicable to short and thick beams, considers the effects of both rotary inertia and shear deformation. In real life engineering application, Rayleigh beams are used to model spinning beam. [16].

2 Problem Formulation

The partial differential equation governing the free vibration of a uniform Rayleigh beam is given by

$$EI\frac{\partial^4 V(x,t)}{\partial x^4}+\mu\frac{\partial^2 V(x,t)}{\partial t^2}-\mu b\frac{\partial^4 V(x,t)}{\partial x^2\partial t^2}=0 \tag{1}$$

Here, E is the Young's modulus, I is the moment of inertia of the cross section of the beam, μ is the mass per unit length of the beam, b is the rotatory inertia of the beam, $V(x,t)$ is the transverse displacement of the beam at point x and time t.

The beam is assumed to be simply-supported at both ends. Thus, the associated boundary conditions at the two ends of the beam, that is at $x=0$ and $x=L$ (L being the length of the beam) for the partial differential equation in (1) are as follows:

$$V(0,t)=0\,, \tag{2}$$

$$\frac{\partial^2 V(0,t)}{\partial x^2}=0\,, \tag{3}$$

$$V(L,t)=0\,, \tag{4}$$

and

$$\frac{\partial^2 V(L,t)}{\partial x^2}=0\,. \tag{5}$$

The initial conditions are given by

$$V(x,0)=0, \quad \frac{\partial^2 V(x,0)}{\partial t^2}=0. \tag{6}$$

3 Problem Solution

3.1 Description of the Method of Solution

The differential transform method, which is based on the Taylor series expansion, can be used to obtain analytical solutions of differential equations. To apply this method, certain transformation rules are used to transform the governing equation of motion and the associated boundary conditions of the problem under consideration. This process will yield a system of algebraic equations in terms of the differential transforms of the original functions. Solving this resulting system of algebraic equations yields the desired solution of the transformed problem. The differential transform method is described as follows:

Let x be any point in a domain D. Also suppose $F(x)$ is analytic in domain D. Then a power series whose center is x_0 can be used to represent the function. The differential transform of the function $f(x)$ is given by

$$\bar{F}(k)=\frac{1}{k!}\left(\frac{d^k F(x)}{dx^k}\right)\Bigg|_{x=x_0}, \tag{7}$$

where $F(x)$ is the function to be transformed and $\bar{F}(k)$ called the transformed function is the new function obtained after the transformation. The inverse transformation is defined as

$$F(x)=\sum_{k=0}^{\infty}(x-x_0)^k \bar{F}(k). \tag{8}$$

To express $F(x)$ by a finite series, Eqs. (7) and (8) are combined to get the series

$$F(x)=\sum_{k=0}^{m}\frac{(x-x_0)^k}{k!}\left(\frac{d^k F(x)}{dx^k}\right)_{x=x_0}. \tag{9}$$

It shoulde be noted that the value of m depends largely on the convergence of the natural frequencies. Some of the theorems that are useful when transforming the governing differential equation are provided as follows.

Theorem 1:

If $F(x)=G(x)\pm H(x)$, then $\bar{F}(r)=\bar{G}(r)\pm\bar{H}(r)$.

Theorem 2:

If $F(x)=\lambda G(x)$, then $\bar{F}(r)=\lambda\bar{G}(r)$.

Theorem 3:

If, $F(x)=G(x)H(x)$, then $\bar{F}(r)=\sum_{s=0}^{r}\bar{G}(s)\bar{H}(r-s)$.

Theorem 4:

If $F(x)=\frac{d^n G(x)}{dx^n}$, then $\bar{F}(r)=\frac{(r+n)!}{r!}\bar{G}(r+n)$.

Theorem 5:

If $F(x)=x^n$, then $\bar{F}(r)=\delta(r-n)=\begin{cases}0 & if \quad r\neq n\\ 1 & if \quad r=n\end{cases}$.

The basic DTM theorems that are used for transforming boundary conditions, which are found applicable in this paper are as follows:

Theorem 6:

If $F(0)=0$, then $\bar{F}(0)=0$.

Theorem 7:

If $\frac{dF}{dx}(0)=0$, then $\bar{F}(1)=0$.

Theorem 8:

If $F(1)=0$, then $\sum_{k=0}^{\infty}\bar{F}(k)=0$.

Theorem 9:

If $\frac{dF}{dx}(1)=0$, then $\sum_{k=0}^{\infty}k\bar{F}(k)=0$.

Theorem 10:

If $\frac{d^2F}{dx^2}(1)=0$, then $\sum_{k=0}^{\infty}k(k-1)\bar{F}(k)=0$.

3.2 Using the DTM to Analyze the Free Vibration Problem of Rayleigh Beam

To obtain the solution of the differential equation (1) subject to the given conditions, a sinusoidal variation of $V(x,t)$ is assumed and consequently the function is approximated as

$$V(x,t)=W(x)e^{i\Omega t}, \tag{10}$$

where $W(x)$ is the modal deflection and Ω is the circular natural frequency of the harmonic function of time.

Using equation (10), then equations (1)-(5) can be expressed as follows:

$$EI\frac{d^4W(x)}{dx^4}-\mu\Omega^2W(x)+\mu b\Omega^2\frac{d^2W(x)}{dx^2}=0, \quad 0<x<L \tag{11}$$

$$W(0)=0, \tag{12}$$

$$\frac{d^2W(0)}{dx^2}=0, \tag{13}$$

$$W(L)=0, \tag{14}$$

$$\frac{d^2W(L)}{dx^2}=0. \tag{15}$$

Let us introduce the dimensionless quantities as follows

$$\xi=\frac{x}{L},\quad w(\xi)=\frac{W(x)}{L} \tag{16}$$

The governing equation (11) can then be written in the following dimensionless form:

$$\frac{d^4w(\xi)}{d\xi^4}+\beta\Omega^2\frac{d^2w(\xi)}{d\xi^2}-\mathrm{M}\Omega^2w(\xi)=0 \tag{17}$$

where the dimensionless coefficients are given by

$$\mathrm{M}=\frac{\mu L^4}{EI},\ \beta=\frac{\mu bL^2}{EI}. \tag{18}$$

The boundary conditions in equations (12)–(15) have the following dimensionless form:

$$w(0)=0, \tag{19}$$

$$\frac{d^2w(0)}{d\xi^2}=0, \tag{20}$$

$$w(1)=0, \tag{21}$$

$$\frac{d^2w(1)}{d\xi^2}=0. \tag{22}$$

Taking the differential transform of equation (11) in accordance with the Theorems 1-4, one obtains

$$\begin{aligned}&(r+1)(r+2)(r+3)(r+3)\bar{W}(r+4)\\&+(r+1)(r+2)\beta\Omega^2\bar{W}(r+2)-\mathrm{M}\Omega^2\bar{W}(r)=0\end{aligned}. \tag{23}$$

The following recursive equation can be obtained from equation (23):

$$\bar{W}(r+4)=\frac{\mathrm{M}\bar{W}(r)-(r+1)(r+2)\beta\bar{W}(r+2)}{(r+1)(r+2)(r+3)(r+4)}\Omega^2 \tag{24}$$

Now applying appropriate transformation theorems 6-8 and 10, the boundary conditions (19)-(22) become

$$\bar{W}(0)=0, \tag{25}$$

$$\bar{W}(2)=0, \tag{26}$$

$$\sum_{r=0}^{m}\bar{W}(r)=0, \tag{27}$$

and

$$\sum_{r=0}^{m}r(r-1)\bar{W}(r)=0. \tag{28}$$

Let us define

$$\bar{W}(1)=c_1, \tag{29}$$

$$\bar{W}(3)=c_2, \tag{30}$$

as the unknown parameters.

Substituting equations (29) and (30) into equation (24), we have

For $r=0$:

$$\bar{W}(4)=0 \tag{31}$$

For $r=1$:

$$\bar{W}(5)=\frac{\mathrm{M}c_1-6\beta c_2}{5!}\Omega^2 \tag{32}$$

By following the same recursive procedure, $\bar{W}(6)$ up to $\bar{W}(m)$ can be evaluated; where m is to be determined by the convergence of natural frequencies.

Substituting $\bar{W}(j)$, for $j=0,1,\ldots,m$ into equations (27) and (28), we obtain a system of two algebraic equations which can be put in the matrix form as follows:

$$\begin{bmatrix} f_{11}^{[m]}(\Omega) & f_{12}^{[m]}(\Omega) \\ f_{21}^{[m]}(\Omega) & f_{22}^{[m]}(\Omega) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (33)$$

where $f_{11}^{[m]}(\Omega)$, $f_{12}^{[m]}(\Omega)$, $f_{21}^{[m]}(\Omega)$ and $f_{22}^{[m]}(\Omega)$ are polynomials of Ω.

For the nontrivial solutions of equation (33), the determinant of the coefficient matrix is set to zero. Thus, we have

$$\begin{vmatrix} f_{11}^{[m]}(\Omega) & f_{12}^{[m]}(\Omega) \\ f_{21}^{[m]}(\Omega) & f_{22}^{[m]}(\Omega) \end{vmatrix} = 0 \quad (34)$$

The dimensionless natural frequencies are then calcuated by solving equation (34). $\Omega_j^{[m]}$ is the estimated jth dimensionless natural frequency that corresponds to m. The value of m is decided by the following convergence criterion:

$$|\Omega_j^{[m]} - \Omega_j^{[m-1]}| \le \varepsilon, \quad (35)$$

where $\Omega_j^{[m-1]}$ is the jth estimated dimensionless natural frequency corresponding to $m-1$ and ε is a predefined small value.

3.3 Verification and Case Study

Setting $M = \beta = 1$ and the first dimensionless natural frequency and mode shape for demonstration, the computations and results corresponding to $m = 16$ are described as follows:

Substituting Equations (25), (26), (29) and (30) and $r = 0$ into Eq. (24), we have

$$\bar{W}(4) = 0 \quad (36)$$

Substituting Eqs. (25), (26), (29)-(31) and $r = 1$ into Eq. (24), we have

$$\bar{W}(5) = \frac{1}{5!}(c_1 - 6c_2)\Omega^2. \quad (37)$$

Substituting Equations (25), (26), (29)-(31) and $r = 2$ into Eq. (24), we have

$$\bar{W}(6) = 0. \quad (38)$$

Substituting Equations (25), (26), (29)-(32) and $r = 3$ into Eq. (24), we have

$$\bar{W}(7) = \frac{1}{7!}\Omega^4\left[(1+\Omega^2)c_1 - 6(2+\Omega^2)c_2\right] \quad (39)$$

Following the same recursive procedure, we calculate up to the 20th term $\bar{W}(16)$. Substituting $\bar{W}(j),\ j = 0,1,2,\ldots,16$ into Eqs. (27) and (28) and using Eq. (34), we have the frequency equation as follows

$$\begin{aligned} &1.126964276\times10^{-9}\Omega^{12} - 1.700514259\times10^{-7}\Omega^{10} \\ &+0.00001858512347\Omega^{8} - 0.001325165135\Omega^{6} \quad (40) \\ &+0.05481481481\Omega^{4} - 1.066666667\Omega^{2} + 6 = 0. \end{aligned}$$

Solving Eq. (34), we have the first two roots

$$\Omega_1^{[20]} = 2.9936 \quad (41)$$

$$\Omega_2^{[20]} = 5.9019 \quad (42)$$

When $j = 15$, by the same method we obtain

$$\Omega_1^{[19]} = 2.9936 \quad (43)$$

From Eqs. (35) and (37), we have

$$|\Omega_1^{[20]} - \Omega_1^{[19]}| = 0 < \varepsilon, \quad (44)$$

which implies convergence.

So, $\Omega_1 = 2.9936$ is taken as the first dimensionless natural frequency.

Substituting $\Omega_1 = 2.9936$ into $V(j),\ j = 0,1,\ldots,16$ and using $V(\xi) = \sum_{j=0}^{16}\xi^j V(j)$, we obtain the closed form series solution of the first mode shape.

$$\begin{aligned} V_1(\xi) = &\ \xi - 1.644936963\xi^3 + 0.8117470647\xi i^5 \\ &- 0.1907536301\xi^7 + 0.02614819400\xi^9 \\ &- 0.002346120939\xi^{11} + 0.001484318771\xi^{13} \\ &- 0.000006976044909\xi^{15} \end{aligned} \quad (45)$$

Following the same routine demonstrated above, one can determine the other natural frequencies and their associated mode shapes. As the number of terms, denoted by m increases, the first five non-dimensional natural frequencies Ω_1 up to Ω_5 of the Rayleigh beam converge to 2.993593837, 6.205088447, 9.372169410, 12.52676961, 15.72132054. The predefined value of ε used to monitor the convergence of the natural frequencies is $\varepsilon = 0.0001$. In the example considered, the first five natural frequencies converge quickly one by one without missing any frequency. The natural frequencies are then used to determine their corresponding mode shapes. The first three mode shapes of the freely vibrating beam with the given configuration are shown in Figs. 2, 3 and 4. The combination of all the first three mode shapes is given in Fig. 5.

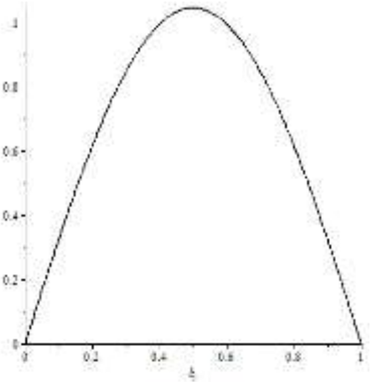

Fig. 1: First mode shape

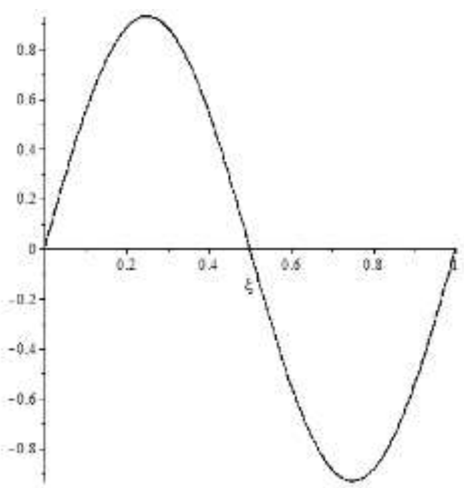

Fig. 2: Second mode shape

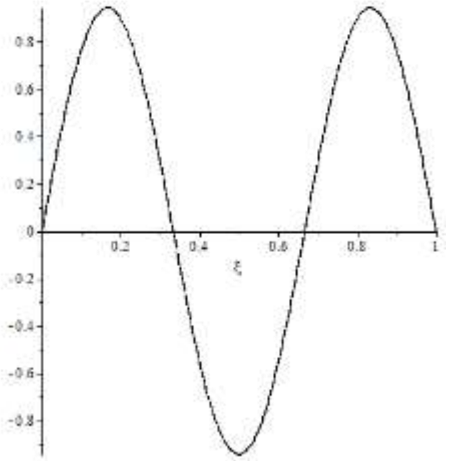

Fig. 3: Third mode shape

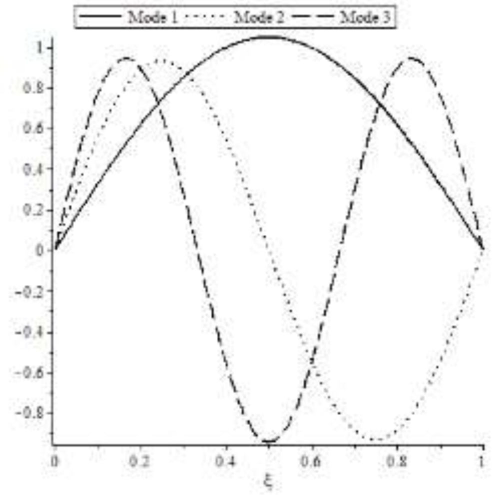

Fig. 4: The first, second and third mode shapes

Table 1 shows the effect of mass per unit length of the beam on the first five natural frequencies of the simply supported Rayleigh beam. The results reveal that the non-dimensional frequencies calculated become smaller with the increase in the mass per unit length of the beam. The implication of this is that there will be a decrease in natural frequencies of excitation of the beam if there is an increase in the beam's mass per unit length.

Table 1. Effect of Mass per Unit length of the beam on Non-dimensional frequencies of Simply-Supported Rayleigh Beam ($\beta = 1$)

	M			
	1	5	10	15
Ω_1	2.9940	2.5595	2.2141	1.9791
Ω_2	6.2051	5.9195	5.6124	5.3487
Ω_3	9.3722	9.1702	8.9352	8.7174
Ω_4	12.5268	12.3703	12.1864	12.0089
Ω_5	15.7213	15.5955	15.4428	15.2943

4 Conclusion

The differential transform method has been used to the closed form series solution of the freely vibrating uniform Rayleigh beam. As earlier noted, solving the problem using DTM involves three main steps. The first step is to transform the equation governing the motion as well as the boundary conditions into a system of algebraic difference equations. The second step entails solving the set of algebraic difference equations in step one. Finally, the solution of the transformed problem is inverted using the inverse differential transform to determine the natural frequencies and their associated closed form series solution of the mode shape. The influence of mass per unit length on the natural frequencies of a simply supported Rayleigh beam freely vibrating has also been investigated. The governing differential equation is solved and the dimensionless natural frequencies for various values of the mass per unit length of the beam were obtained and presented in table. It was found that the natural frequency of the beam decreases with increase in the value of the mass per unit length. It is recommended that the further studies on the influence of rotary inertia on the vibration characteristics of freely vibrating Rayleigh beam should be carried out.

Nomenclature

E	Young's modulus,
I	Moment of inertia of the cross section of the beam
μ	Mass per unit length of the beam,
b	Rotatory inertia of the beam
x	spatial location along the beam
T	time
$V(x,t)$	Transverse displacement of the beam at point x and time t.
L	length/span of the beam
Ω	Circular natural frequency of harmonic function of time
$W(x)$	Modal deflection of the beam
ξ	Non-dimensional parameter of the spatial location along the beam
$w(\xi)$	Non-dimensional parameter of the modal deflection of the beam
M	Non-dimensional parameter of the mass per unit lenght of the beam
β	Non-dimensional rotary inertia

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

-Olasunmbo O. Agboola carried out the formulation of the problem and performed the numerical caclculations.

-Talib Eh. Elaikh contributed to the interpretation and discussion of the results.

-Jimevwo G. Oghonyon and Olajide Ibikunle contributed to the analysis of the results and to the writing of the manuscript.

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