Displacement of Fluids in Permeable Media

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Abstract: Viscoelastic (Maxwellian) slow, immiscible liquid-liquid displacement in a permeable medium is considered. The necessary and sufficient criteria for stability are that the displacing fluid is denser and less mobile than the displaced fluid. The instability criteria and critical wave length are found to be the same as those for ordinary viscous liquid-liquid displacements in permeable media.

Keywords: Immiscible Displacements, Maxwellian Viscoelastic Fluids, Porous Medium

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1 Introduction

Immiscible fluid displacement in porous media is fundamental for many environmental processes, including infiltration of water in soils, groundwater remediation, enhanced recovery of hydrocarbons and carbon geosequestration. Microstructural heterogeneity, in particular of particle sizes, can significantly impact immiscible displacement. For instance, it may led to unstable flow and preferential displacement patterns. The displacement process involving two immiscible fluids is of considerable importance in ground water hydrology and reservoir engineering. The nature of immiscible flow in a porous medium is different from that in the Hele-Shaw cell. Unlike the Hele-Shaw cell, where the interfacial tension acts at a single interface (Homsy [1]), the capillary forces in a porous medium act on a multitude of microscopic interfaces, giving rise to a single dispersed interface at the macroscopic level (Lake [2], Yortsos and Hickernell [3]). Additionally, the mobility of individual phases within the pore space is governed by the wettability properties of the porous medium (Bear [4], Scheidegger [5]). Therefore, the dynamics of immiscible displacements in porous media is represented by Darcy's law for each phase with an associated relative permeability function (Lake [2]). In the case in which the injected fluid is of higher mobility than the resident fluid, the displacement becomes unstable and results in macroscopic viscous fingers. These fingers are different from the microscopic fractal structure that occurs in the limit of vanishing capillary number and infinite viscosity ratio, modeled, respectively, by invasion percolation and diffusion-limited aggregation (Sahimi [6]). The regime of geological fluid flow, which falls between these limiting cases, e.g., with viscosity ratio ∼100−102∼100−102 and capillary number ∼100−103∼100−103, can be represented by the Darcy equation based on local averages (Blunt and King [7], Yortsos et al. [8].

A theoretical and experimental investigation of the occurrence of macroscopic instabilities in the displacement of one viscous liquid by another immiscible one through a uniform porous medium was made by Chuoke et al. [9]. The theoretical description of the instability of fluid displacements in permeable media is immensely complex due to gross inhomogeneities of porous media. However, a quantitative prediction of finger-spacing is possible in a porous medium known to be macroscopically homogeneous and isotropic throughout. The fluid displacement between closely spaced parallel plates is two-dimensional, involves only microscopic fluid-fluid interface and has been shown to be mathematically analogous to two-dimensional flow in a porous medium, by Saffman and Taylor [10]. The displacements in a porous medium are three-dimensional and the macroscopic interface represents many moving microscopic fluid-fluid interfaces. The results for the parallel-plate system can be considered a specialization of those for porous media, as the formal mechanics of theory are the same.

Certain assumptions and limitations of Chuoke et al. [9] theory should be mentioned. In analogy with the actual interfacial tension, the assumption of `effective interfacial tension` across the macroscopic interface has been made. No good agreement has been found between calculated and observed finger spacing between parallel plates, as well as transparent glass powder pack experiments. When pure water was injected to displace oil from preferentially water-wet porous medium containing connate water, it was found that the fingering observations could not test the theory quantitatively. Scheideggar [11] studied the stability of displacement fronts in porous media. Payatakes et al. [12] studied oil ganglion dynamics during immiscible displacements. Ekwere and Donald [13] studied the onset of instability during two-phase immiscible displacements in porous media. They extended the work of Chuoke et al. [9] by means of a stability analysis, a universal dimensionless scaling group and its critical value for predicting the onset of instability during immiscible displacement in porous media. Stability analysis of immiscible displacement problems has been carried out among others, by Cruz and Spanos [14], Maloy et al. [15], Lenormand et al. [16], Hilfer and Oren [17] and Rao et al. [18].

In many reservoirs, the oils naturally occurring beneath the surface of the earth are found to exhibit some non-Newtonian behavior (Allen and Boger [19]). The consideration of the viscoelastic nature of fluid to be displaced is closer to field reservoirs and therefore to primary oil recovery processes. Oldroyd [20] proposed a theoretical model for a class of viscoelastic fluids. Since viscoelastic fluids play an important role in polymers and electrochemical industry, the studies of waves and stability in different viscoelastic fluid dynamical configuration has been carried out by several researchers in the past. The nature of instability and some factors may have different effects on viscoelastic fluids as compared to the Newtonian fluids. For example, Bhatia and Steiner [21] have considered the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that the rotation has a destabilizing influence, for a certain numerical range, in contrast to the stabilizing effect on Newtonian fluid. In another study, Bhatia and Steiner [22] have studied the problem of thermal instability of a viscoelastic fluid in hydromagnetics and have found that the magnetic field has the stabilizing influence on Maxwell fluid just as in the case of Newtonian fluid. The thermosolutal instability in a Maxwellian viscoelastic fluid in porous medium to include the Hall effect has been considered by Sharma and Kumar [23]. In another study, Kumar and Singh [24] have considered the instability of the plane interface between two viscoelastic (Maxwellian) superposed fluids in porous medium in the presence of uniform rotation and variable magnetic field. For stable density stratification, the system is found to be stable for disturbances of all wave numbers and the magnetic field stabilizes the potentially unstable stratification for small wave-length perturbations which are otherwise unstable. Kazachkov [25] has studied the development and analysis of the mathematical model for mixing and heat transfer in the two-fluid turbulent heterogeneous jet of mutually immiscible liquids.

Keeping in mind the immiscible displacements process for primary oil recovery in field reservoirs and the viscoelastic nature of the fluid to be displaced (e.g. oil), a theoretical study has been made of the immiscible displacement of viscoelastic (Maxwellian) fluid by another viscoelastic fluid of similar nature in permeable media.

2 Formulation of the Problem and Basic Equations

Here we consider the configuration of two Maxwellian viscoelastic fluids, labeled 1 and 2, each of infinite extent, with a plane macroscopic interface moving slowly through a uniform permeable medium with velocity W, normal to the interface. A viscoelastic fluid 1 displaces another viscoelastic fluid 2 with velocity W in the positive $z-$ direction, which is normal to the plane, macroscopic interface between the two. The vertically upward direction is chosen as the $z'-$ axis of a fixed system of co-ordinates. If fluid 1 is displacing fluid 2, W is a positive quantity. The z-component of the gravitational acceleration is $-g cos(zz')$. The porous medium is assumed to be homogeneous and isotropic.

The Maxwellian viscoelastic fluid is described by the constitutive relations:

$$\Gamma_{ij} = -p\delta_{ij} + \tau_{ij} \ ,$$
$$\left(1 + \lambda \frac{d}{dt}\right)\tau_{ij} = 2\mu e_{ij} \ ,$$
$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}\frac{\partial v_j}{\partial x_i}\right), \qquad (1)$$

where Γ_{ij}, τ_{ij}, e_{ij}, δ_{ij}, μ, λ, p, v_i, x_i and $\frac{d}{dt}$ denote, respectively the stress tensor, the shear stress tensor, the rate-of-strain tensor, the Kronecker delta, the viscosity, the stress relaxation time, the isotropic pressure, the velocity vector, the position vector and the convective derivative.

For flows through porous media, the Brinkman and viscous derivative terms are very small in magnitude as compared to the Darcian term, which is retained here. Using the volume averaging procedure, a derivation of the equations of motion and continuity for viscoelastic fluids and the dominance of Darcian term as the resistance term has been shown by Slattery ([26], [27]). The linearized macroscopic equations of motion and continuity for incompressible, Maxwellian viscoelastic fluid, a set for each fluid, are

$$\vec{V} = -grad\left[\frac{(1+\lambda T)}{\frac{\rho}{\varepsilon}(T+\lambda T^2)+\frac{\mu}{k'}}\{p + \rho g z\, cos(zz')\} + Wz\right], \qquad (2)$$

or

$$\vec{V} = -grad\, \chi \ , \qquad (3)$$

where

$$\chi = \left[\frac{(1+\lambda T)}{\frac{\rho}{\varepsilon}(T+\lambda T^2)+\frac{\mu}{k'}}\{p + \rho g z\, cos(zz')\} + Wz\right]$$

and

$$div\, \vec{V} = -\nabla^2\chi = 0 \ , \qquad (4)$$

where ρ, p, μ, g, ε, k' and $\vec{V}(u, v, w)$ with the subscripts 1 and 2 distinguishing the two fluids, stand for density, pressure, viscosity, acceleration due to gravity, medium porosity, effective medium permeability and perturbation in velocity, respectively and $T = \frac{\partial}{\partial t}$.

The above equations describe the viscoelastic fluid motion in a coordinate system moving with velocity W in which the unperturbed interface is at rest. Equations (2) and (3) possess the integrals

$$p_2 = \left\{ \frac{\frac{\rho_2}{\varepsilon}(T + \lambda T^2) + \frac{\mu_2}{k_2'}}{(1 + \lambda T)} \right\} (\chi_2 - Wz) - \rho_2 gz\, cos(zz') + P_2(t) \, , \quad (5)$$

$$p_1 = \left\{ \frac{\frac{\rho_1}{\varepsilon}(T + \lambda T^2) + \frac{\mu_1}{k_1'}}{(1 + \lambda T)} \right\} (\chi_1 - Wz) - \rho_1 gz\, cos(zz') + P_1(t) \, , \quad (6)$$

where $P_2(t)$ and $P_1(t)$ are arbitrary functions of time.

3 Criteria for Stability and Discussion

Let us take $\zeta(x, y, t)$ to be the arbitrary deformation of the macroscopic interface and assume its Fourier decomposition of the form

$$\zeta = \varepsilon_1 e^{i(k_x x + k_y y) + nt} \, , \quad (7)$$

where k_x, k_y are wave numbers along x, y directions; $\vec{k} = \hat{\imath} k_x + \hat{\jmath} k_y$ is the resultant wave vector of magnitude $k = (k_x^2 + k_y^2)^{1/2}$, n is in general complex and $\varepsilon_1 = \varepsilon_1(\overline{k})$.

The kinematic conditions to be satisfied at the interface $z = \zeta$ are

$$\frac{\partial \zeta}{\partial t} = \left(-\frac{\partial \chi_1}{\partial z}\right)_{z=\zeta} = \left(-\frac{\partial \chi_2}{\partial z}\right)_{z=\zeta} \, . \quad (8)$$

Since χ_1 and χ_2 must satisfy (8) and since the z-components of the perturbation velocity must vanish at $z \to \mp\infty$, the solutions of (4) are

$$\chi_1 = -\frac{n}{k} \varepsilon_1 e^{i(k_x x + k_y y) + nt + kz} \, , \quad (9)$$

for fluid 1, and

$$\chi_2 = +\frac{n}{k} \varepsilon_1 e^{i(k_x x + k_y y) + nt - kz} \, , \quad (10)$$

for fluid 2. Here $k\zeta$ is assumed small under the first-order theory.

At each point of the macroscopic interface there is conceived to be a pressure discontinuity consisting of two types of terms, i.e.,

$$(p_1 - p_2)_{z=\zeta} = T^*(c_1 + c_2) + P_c(t) \, , \quad (11)$$

where $P_c(t)$ is independent of curvature of the macroscopic interface but may be a function of time and is related to the capillary pressure drops across the microscopic fluid-fluid interfaces underlying the macroscopic interface, whereas T^* is an effective interfacial tension and c_1, c_2 are the signed principal curvatures of the macroscopic interface, to be taken as negative when the respective center of curvature falls in the domain of fluid 2.

Using (5) and (6) in relation (11), we obtain

$$\left\{ \frac{\frac{\rho_2}{\varepsilon}(T + \lambda T^2) + \frac{\mu_2}{k_2'}}{(1 + \lambda T)} \right\} (\chi_2 - Wz)_{z=\zeta} - \left\{ \frac{\frac{\rho_1}{\varepsilon}(T + \lambda T^2) + \frac{\mu_1}{k_1'}}{(1 + \lambda T)} \right\} (\chi_1 - Wz)_{z=\zeta} - [(\rho_2 - \rho_1) g\, cos(zz')]\zeta + P_2(t) - P_1(t) + P_c(t) - T^*\left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial x^2}\right) = 0, \quad (12)$$

where

$$c_1 \sim -\frac{\partial^2 \zeta}{\partial x^2} \ and\ c_2 \sim -\frac{\partial^2 \zeta}{\partial y^2}.$$

Using (7), (9) and (10), (12) yields

$$P_1(t) - P_2(t) = P_c(t) \, , \quad (13)$$

and the characteristic equation

$$\left[\frac{1}{\varepsilon}(\rho_1+\rho_2)\lambda\right]n^3 + \left[\frac{1}{\varepsilon}(\rho_1+\rho_2) - \frac{k\lambda}{\varepsilon}(\rho_2-\rho_1)W\right]n^2 + \left[\left(\frac{\mu_2}{k_2'}+\frac{\mu_1}{k_1'}\right) - (\rho_2-\rho_1)\frac{kW}{\varepsilon} + k^3T^*\lambda - (\rho_2-\rho_1)g\cos(zz')k\lambda\right]n - \left[\left(\frac{\mu_2}{k_2'}-\frac{\mu_1}{k_1'}\right)W + (\rho_2-\rho_1)g\cos(zz') - k^2T^*\right]k = 0\,. \quad (14)$$

Equation (14) determines n as a function of wave number k and yields the kinematics of early growth.

For $k > 0$, it is evident from equation (14) that the necessary and sufficient criterion for instability, i.e., for n to be positive, is given by

$$\left(\frac{\mu_2}{k_2'}-\frac{\mu_1}{k_1'}\right)W + (\rho_2-\rho_1)g\cos(zz') - k^2T^* > 0\ . \quad (15)$$

Now, introducing the volumetric velocity U, we may say that instability will occur for all velocities $U > U_c$, where U_c is a critical velocity defined by

$$\left(\frac{\mu_2}{k_2'}-\frac{\mu_1}{k_1'}\right)U_c + (\rho_2-\rho_1)g\cos(zz') = 0\quad , \quad (16)$$

provided the perturbation contains wavelength $\lambda\left(=\frac{2\pi}{k}\right)$ greater than a critical wavelength λ_c , defined by

$$\lambda_c = 2\pi\left[\frac{T^*}{\left(\frac{\mu_2}{k_2'}-\frac{\mu_1}{k_1'}\right)(U-U_c)}\right]^{1/2}\ . \quad (17)$$

Equation (14) admits of no positive root if the constant term and the coefficients of n, n^2 and n^3 are all positive. The stability criteria (i.e. conditions to check fingering phenomena on the macroscopic scale), which are of fundamental importance in oil recovery processes, are then

$$\rho_1 > \rho_2 \ and\ \frac{k_1'}{\mu_1} < \frac{k_2'}{\mu_2}\ . \quad (18)$$

Physically, the Necessary and Sufficient Criteria for Stability are:

(i) the displacing fluid is denser than the fluid to be displaced,

and (ii) the less mobile fluid displaces the more mobile one.

The criteria for instability and critical wavelength for the case of Maxwellian viscoelastic liquid-liquid displacements in permeable media remain the same as those for ordinary viscous liquid-liquid displacements in permeable media.

4 Conclusions

The flow of viscous incompressible fluids in the presence of porous bodies seems to have generated a great deal of interest among researchers, because of its applications in numerous Scientific and Industrial fields like Lubrication of Porous Bearings, Ground Water Hydrology, Petroleum Industries, Industrial filteration and Agricultural Engineering etc. Stability analysis of immiscible displacement problems has been carried out by different researchers in the past and in particular, a theoretical and experimental investigation of the stability of two slow, immiscible, viscous liquid-liquid displacements in porous media has been given by Chouke et al. [9]. Since in many reservoirs, the oils naturally occurring beneath the surface of the earth are found to exhibit some non-Newtonian behavior and keeping in mind the immiscible displacements process for primary oil recovery in field reservoirs and the viscoelastic nature of the fluids, a theoretical study of the generalization of Chouke et al.'s work has been made in the present paper by considering slow, immiscible viscoelastic (Maxwellian) liquid-liquid displacements in permeable medium.

The necessary and sufficient stability conditions which are of fundamental importance in oil recovery processes are obtained and are that the displacing fluid is denser and less mobile than the displaced fluid. The instability criteria and critical wave length are found to be the same as those for ordinary viscous liquid-liquid displacements in permeable media.

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