Performance Evaluation in Macroeconomics based on DEA Malmquist Index with a New Approach for the Efficiency Evaluation in a Two-Stage Process

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Abstract: - The goal of this work is to know and evaluate the macroeconomic development progress dynamics based on the Malmquist index by also giving a new and as explicit as possible approach to the evaluative reasoning of the DMUs' overall performance. The calculation of the DEA (Data envelopment analysis) Malmquist index and of its composing components in coordination with the DMUs' ranking positions opens up perspectives for a broad comprehensive and evaluative study. Rankings and the Malmquist index are, each in its-self, a performance test. To coordinate these two tests in as organic as possible way, two factors, the so called Z_1 and Z_2 "statistical" indicators are used in this work. They are expressed as functions (given by the formula, by further calculating the geometric mean of their values) in the evaluation of the overall performance. In this work, the effects of each variable factor are analyzed. A new approach, studied as a two-stage process, is given here. The inserted intermediate variable factors are not simply dealt with as "exogenous" variables fixed in the approach of the two-stage process, but they are considered as included in the total system of the studied variables set (input-output). To interpret the advantages and disadvantages that support the macroeconomic development dynamics, along with the indication of the best practices, the factors that affect the inefficient DMUs are studied. The study covers the 2015-2020 period of time and includes 19 European region countries.

Key-Words: - moderated efficiency, Cluster classification, ranking positioning, "statistical" indicators, Malmquist index, two-stage process.

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1 Introduction

The generating capacity of any analysis to support with studies the encouragement of the production productivity growth, especially in the conditions of an economic globalization with added and converged multi-polar tendencies, remains today a requisite of high perception sensitivity.

DEA (Data Envelopment Analysis) models are widely applied over the necessary operational and managerial requirements generated by the businesses in different sectors or by certain institutions or organization for a developing activity that calls for self-evaluation.

The change growth for the economic productivity indicator is related to the evaluation of the criteria for the different variable factors that bring changes to the production technology and itself the value of this index which indicates the change of the productivity growth. The selection of the input-output variable criteria in the study is based on the experiences of the studies made in the macroeconomics area, regardless of the approaches used. Here, in selecting the variable factors, is used the calculation of the correlations among them, defining this way the main necessary and required resources, the so called inputs and outputs. The defined factors in inputs and outputs are: Gross national expenditure (current US$) (GNE), Foreign direct investment (FDI), Imports of goods and services (current US$) (IGS), GDP per capita (constant 2015 US$), GNI per capita, Atlas method (current US$), Exports of goods and services (BoP, current US$) (EGS), and the Revenue, excluding grants (% of GDP), which in the work is the intermediate variable factor in the two-stage process.

The Nobel prize winner, Krugman, states that the "Productivity isn't everything, but in the long run it is almost everything", [1]. The contemporary in-depth studies in the theory of economy served to me as an added encouragement to conduct the analysis of this study.

In the economic studies is also emphasized that the regional Trade, [1], or that of each country, has an important impact in the economic development progress. The interdependencies in the economic relations bring substantial impacts, which were particularly noted in the case of the Western Balkans countries (hereinafter, only the countries in the study will be referred to) as compared to the other EU member countries. In the evaluation of the overall performance, the Western Balkans countries show a lower performance than the other countries. It is noted that the index of the change of production productivity growth for the countries in study, considered year after year, shows also the DMUs' advantages and disadvantages over the effects of the variable factors impact in the dynamics of the macroeconomic development progress. Visible deviations from the general tendencies of the impact that characterize the 'n' DMUs are noted during the examination of the impact of each input variable. Since, using the two basic DEA models (CCR and VRS), the distinctions of the efficiency change factor or of the technological factor cannot be clearly defined (in the Malmquist productivity index) for the DMUs with the efficiency value equal to 1 (one) during a period of time 't', the evaluation of the super efficiency is made for these units.

Applying the distance value of the efficiency value from the unit value by using $\|d\|= \sum_{i=1}^{t}(1 - Ef_i^t)$ (with which the inefficiencies are also evaluated), was noted that the general average of this indicator has the value of $\|d\|$= 0.366. Of the 19 countries in the study, 73% have a smaller value than the average of this indicator. In the Cluster 1 (C1) grouping, are included the countries that individually have the value of the distance indicator from the unit value ($\|d\|$) greater than the general average value of this indicator (the average of this indicator for these countries is 1.003), in the Cluster 2 (C2) grouping are included the countries which individually have the $\|d\|$ value smaller than the general average value and evaluated as best practice countries (for these countries, $\|d\|$=0.031), in the Cluster 3 (C3) grouping are included the other countries which individually have the $\|d\|$ value smaller than the general average value (for these countries, $\|d\|$= 0.236). The Western Balkans countries (as a specific geographic location) belong to the Cluster 1.

In this work, the new approach (the moderated efficiency) as a two-stage process that offers a more visible tableau for the DMUs' ranking is given. This approach will also serve for the study of the effects of the specific variable factors impact that may be used as inputs or outputs. The evaluation of the overall performance is done by using the "statistical" indicators that are expressed as Z_1 and Z_2 functions, where Z_1 includes an independent variable ($\frac{R_i}{n}$, of the ranking positioning), while the Z_2 function includes the parameter of the Malmquist index values, which are explained in more details in the methodology framework.

The following hypotheses testing are set forth for verification in the study:

Hypothesis 2: There are visible differences that can be explained through the performance evaluation levels based on the values for the period.

Hypothesis 3: There are strong impacts that advantage or disadvantage the conditionalities in the growth index of the productivity change.

Hypothesis 4: There are differentiated impacts in the final evaluation of the performance ranking as compared to the given rankings based on the models.

The study brings contributions to the "Connoisseur-study-analysis" by offering a new and explicit approach to performance evaluation, studying the productivity change index and its components, and continuing with scientific analysis and discussions as follows:

- Pursuing the DEA models approach based on the stated objectives for the conduct of the study analysis, more relief spaces are created for as comprehensible as possible knowledge of the macroeconomic development dynamics under the influence of the given factors.
- Pursuing and defining the evaluative gravity of the impact effects that show the variable factors against the efficiency values, the establishment of a wider tableau for the performance evaluation is enabled.
- The definition of the Cluster groupings in the Malmquist index evaluation is made based on the "gravity" of the distance value ($\|d\|$) of the efficiency value from the unit value that each DMU has and on the specific inclusion of the DMUs according to a given geographical positioning.
- The final ranking of the DMUs' performance evaluation is given based on the two correlated criteria: a) the results gained from the application of the Malmquist index; b) the ranking positioning defined based on the application of DEA models in the efficiency values. This also serves as an encouragement for new operational and managerial research.

- The truthfulness of the hypothesis will give a meaningful evaluation of the progress of the countries' macroeconomic development. It also evaluates the reciprocal impacts in the efficiency value of a grouping against another grouping based on the Cluster classification.
- The year by year evaluations during the 2015-2020 period of time help in finding out the progress recurrence rhythm for each factor depending on the progress of the macroeconomic development dynamics.

2 Literature Review

The evaluation, measurement, and definition of the productivity growth change and the technological factor change, and the examination of the effects of differentiated impacts of the variable factors (input/output) call for the knowledge of the theoretical and applicative studies. Along with the econometric models, [2], [3] for the evaluation of the economic development progress, which models are related to the application of the statistical methods, with simulations for the evaluative finding, with preliminary assumptions, and with the application of the multiple linear regress model, the DEA method stands as a non-parametric multi-applicative approach. The appropriate search and selection of the theoretical study models is done depending on the objectives that are applied to best identify any role factor that advantages in economic growth and progress. The DEA method for the performance evaluation with the efficiency measurement is initially based on the two base models, [4], [5] and in continuation with its extensions that served to the analysis of this study.

The study of economic growth, the comprehensible progress of technological change, and the search for the influences of different factors are initially conducted with econometric methods, [6]. Solow built the generalized function of production and the model on long-run economic growth by proposing the total factor productivity (TFP). The function, examined for its peak points related to the national income and the productivity growth ratio by examining as variables the capital investment and the labor, served as encouragement for further studies in the theory of economy.

The Malmquist index set forth by [7], is a twofold index that is connected with two different time points that can serve to the comparisons and is also based on the concept of the production function for the study of the maximum possible production. This retaken concept is subsequently brought by [8], by also inserting Farell's technical efficiency concept, [9], to define the Malmquist productivity index. The evaluation of the productivity change by DEA is first used by [10], [11], [12], through the calculation of a unique Malmquist index by the geometric mean of the two indexes of [8]. [13], developed Cost Malmquist Type productivity index. [14], applied a new approach to the Malmquist index in the evaluation of inefficiencies. The use of the optimistic and pessimistic data envelopment analysis simultaneously in a new approach for the evaluation of the Malmquist productivity index is in [15]. The study of the agriculture productivity evaluation over 93 developed and developing countries during the 1980-2000 period of time is given in [16]. The use of the Malmquist productivity is also seen in the performance of the digital economy, [12], [17]. The level of productivity growth in the East Asia countries during the 1990-2019 period of time by using the oriented from the output Malmquist index is given in [18]. The study of the total factor productivity and agriculture management based on the DEA Malmquist model is treated in [11]. In [19], the orientation from the output Malmquist index is used to explore the level and the nature of the productivity growth for 34 Tunis production industries during the 2002-2016 period of time. An approach to the evaluation of the overall performance with efficiency measurements is given in [20], [21], [22], [23], [24], [25]. The Malmquist productivity index finds extensive applications in different areas of the economy, business, services, [26], [27], [28], [29], [30] etc.

The evaluation of efficiency in a two-stage process is conducted and applied in many studies with predefined objectives, [31]. An application to non-life insurance in Taiwan, by analyzing the efficiency in an analysis of the two-stage process, is given by [32], while [33], gives the use of the two-stage process to measure the efficiency under the condition of constant returns to scale. The two-stage process model in the game approach and Efficiency decomposition is seen in [34].

The theoretical literature review is valued not only for the behavior of ordinary application approaches but also to establish wider horizons for scientific application and discussion. Selection of the needs for the examination of the incoming resources (inputs) that can be selected for a possible set of production where the T: $R_+^m \to R_+^s$ is also done using the DEA base rules that are related with the number of the DMUs and the general sum of the variable criteria (input-output), [35].

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This study examines with scientific analysis and discussion the effects of the macroeconomic dynamics and the recurrence rhythm for the progress increase of each component. On these bases, the respective methodology is built. The selected set of variables that are used in the application is created by first examining their correlative relations and then the experiences from the macroeconomic studies.

3 Methodology

3.1 Malmquist Index

From the evaluation study of the macroeconomic development progress in the European region countries, discernible differences that are related both to the dimensions of the variable indicators per capita and to the harmonization between the inputs and outputs are noted. Nevertheless, each country of the region must define and know not only the self-evaluation but also the racing evaluation for the change index of productivity growth. One of the widely used models is the Malmquist DEA model.

Let us bring in a simplified manner based on the distance function concept for the evaluation of the technical efficiency of 5 homogeneous decision-making units which operate with two inputs and one output (Figure 1).

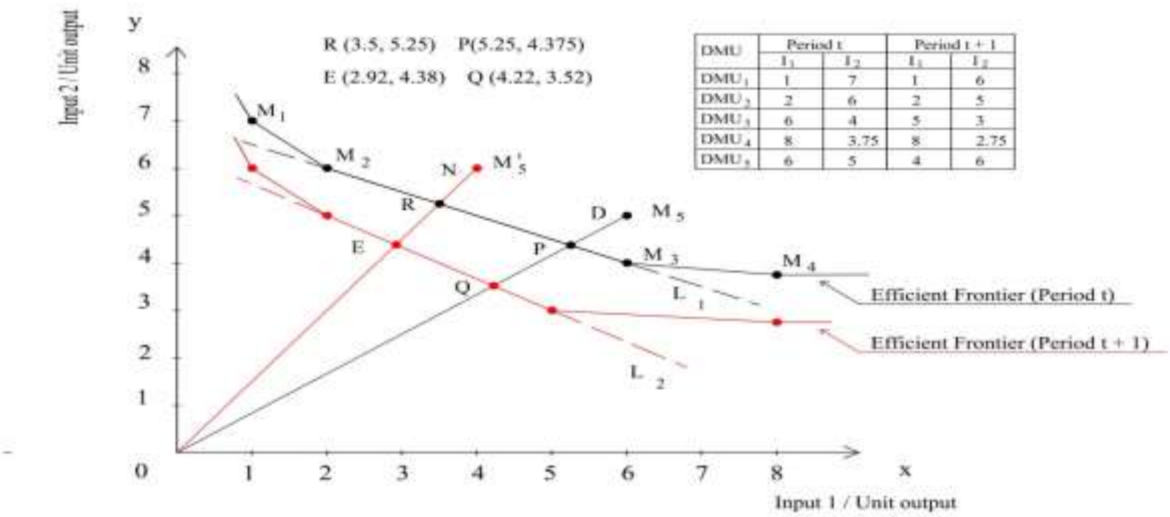
Let's examine the evaluations for this technical efficiency in the 't' and 't+1' periods in the version of the efficient frontier shift. The efficient frontiers may symbolically be noted as f^t(x, y) and f^{t+1}(x, y). The input variables will incur changes in value wherefrom the distance

of each DMU from the two efficient frontiers and the technical efficiency may be evaluated.

DMU_5 is inefficient. It is located, both during the 't' period and during the 't+1' period, out of the efficient frontier. The input-technical efficiency of the DMU_5 with the given input values against the efficient frontier of the 't' period can be expressed as $\delta(\frac{OP}{OD})$, (where in general we have $0 < \delta(\frac{OP}{OD}) \leq 1$) and $\delta(\frac{OR}{ON})$ with the data of the (t+1) period of time, but against the efficient frontier of the 't' period of time with the condition that the efficient frontier is not shifted. So, the productivity change can be given as $(\frac{OR}{ON} : \frac{OP}{OD})$ or we can say that the productivity index as related to the 't' period of time for one DMU_0 is given by $MI_0^t = \delta_0^t\,(x_0^{t+1}, y_0^{t+1}) / \delta_0^t\,(x_0^t, y_0^t)$.

Examining the DMU_0 against the efficient frontier of the 't+1' period of time with the data from the 't' period of time is $\frac{OQ}{OD}$ and with data from the 't+1' period of time, but against the 't+1' efficient frontier, this ratio of the productivity change is given by $(\frac{OE}{ON}: \frac{OQ}{OD})$. Here we can say that $MI_0^{t+1} = \delta_0^{t+1}\,(x_0^{t+1}, y_0^{t+1}) / \delta_0^{t+1}\,(x_0^t, y_0^t)$.

[8], used the two indexes of productivity change. [11] used their geometric mean, giving this way the Malmquist productivity index $MI_0 = [(\frac{OR}{ON} : \frac{OP}{OD}) \text{ x } (\frac{OE}{ON}: \frac{OQ}{OD})]^{1/2}$.



DMU	Period t		Period t + 1	
	I_1	I_2	I_1	I_2
DMU_1	1	7	1	6
DMU_2	2	6	2	5
DMU_3	6	4	5	3
DMU_4	8	3.75	8	2.75
DMU_5	6	5	4	6

Fig. 1: Measuring Productivity change. The Malmquist index and efficiency

Source: [36] and Processing by the author

Using the data of the example from the above figure, the value of this index can be defined:
OD = $\sqrt{61} \approx 7.81$; OP ≈ 6.83; OQ ≈ 5.495; ON ≈ 7.21; OR ≈ 6.31; OE ≈ 5.27 and

by making the above calculations, the MI_0 has a value of 1.01923. This shows that the growth of the DMU_5 unit productivity has progressed.
The Malmquist Index, with a simple algebra processing (multiplied and divided within the root with the same factor for the DMU_0), can be written:

$$MI_0^{I-C} = \frac{Ef_0^{t+1}(x_0^{t+1}, y_0^{t+1})}{Ef_0^{t}(x_0^{t}, y_0^{t})} \sqrt{\frac{Ef_0^{t}(x_0^{t+1}, y_0^{t+1}) \cdot Ef_0^{t}(x_0^{t}, y_0^{t})}{Ef_0^{t+1}(x_0^{t+1}, y_0^{t+1}) \cdot Ef_0^{t+1}(x_0^{t}, y_0^{t})}} \quad (1)$$

The first factor is the efficiency change according to the periods for the respective efficient frontier, the second factor is the technological change factor, [14], [36], [37].

The Malmquist Index may also be expressed as the production of three factors:

$$MI_0 = \frac{(VRS)\_Ef_0^{t+1}(x_0^{t+1}, y_0^{t+1})}{Ef_0^{t}(x_0^{t}, y_0^{t})} \text{ x } \frac{SE\_Ef_0^{t+1}(x_0^{t+1}, y_0^{t+1})}{SE\_Ef_0^{t}(x_0^{t}, y_0^{t})} \text{ x } \sqrt{\frac{Ef_0^{t}(x_0^{t+1}, y_0^{t+1}) \cdot Ef_0^{t}(x_0^{t}, y_0^{t})}{Ef_0^{t+1}(x_0^{t+1}, y_0^{t+1}) \cdot Ef_0^{t+1}(x_0^{t}, y_0^{t})}} \quad (2)$$

Here, the first factor shows the ratio of the pure technical efficiency change, the second factor shows the ratio of the scale efficiency change, and the third factor is the technological change factor.

With the models [M.1], [M.2], [M.3], [M.4] the respective efficiencies are evaluated as follows, [36], [38]:

The efficiency $Ef_0^t(x_0^t, y_0^t) = \theta_0^*$ is evaluated with the optimal value of θ_0 as it is obtained by the model:

Min θ_0
Subject to:
$\sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta_0 x_{ij_0}^t$, i =1,2,…,m;
$\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{rj_0}^t$, r=1,2,…,s
$\lambda_j \ge 0$, j=1,2,…,n
[M.1]

For the evaluation of efficiency $Ef_0^t(x_0^{t+1}, y_0^{t+1})$ is used the following model:

Min h_0
Subject to:
$\sum_{j=1}^{n} \lambda_j x_{ij}^t \le h_0 x_{ij_0}^{t+1}$, i =1,2,…,m;
$\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{rj_0}^{t+1}$, r=1,2,…,s;
$\lambda_j \ge 0$, j=1,2,…,n
[M.2]

For the evaluation of efficiency $Ef_0^{t+1}(x_0^t, y_0^t)$ is used the following model:

Min ω_0
Subject to:
$\sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le \omega_0 x_{ij_0}^t$, i =1,2,…,m;
$\sum_{j=1}^{n} \lambda_j y_{ij}^{t+1} \ge y_{rj_0}^t$, r=1,2,…,s;
$\lambda_j \ge 0$, j=1,2,…,n
[M.3]

For the evaluation of efficiency $Ef_0^{t+1}(x_0^{t+1}, y_0^{t+1})$ is used the following model:

Min f_0
Subject to:
$\sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le f_0 x_{ij_0}^{t+1}$, i =1,2,…,m;
$\sum_{j=1}^{n} \lambda_j y_{ij}^{t+1} \ge y_{rj_0}^{t+1}$, r=1,2,…,s;
$\lambda_j \ge 0$, j=1,2,…,n
[M.4]

The evaluation for the change of the production productivity growth total index is examined based on the values that are obtained about the distance from the unit, where three given possibilities are shown:

a) MI > 1, the unit is evaluated with progress in the increase of the change of the total index of productivity
b) MI < 1, the unit is evaluated with regress in the increase of the change of the total index of productivity
c) MI = 1, the unit is evaluated with stationary standing in the change of the total index of productivity.

The two composing components may be examined separately, to judge the biggest impacting role of each of them in the Malmquist index value.

This work includes a more meaningful analysis for the study of the macroeconomic development and reasoning over the evaluation of the overall performance during the period ‘t’ by also examining the rhythm of the change's recurrent progress.

The evaluation of the overall performance based on the calculation of the two indicators, of the Malmquist index and performance ranking according to the efficiency measurement, is done by applying the indicator called 'norm' evaluating indicator (entered by the author). This indicator is given by the formula $\|E_0\| = \sqrt{Z_1 \cdot Z_2}$, where Z_1 = 1 – exp [- (1 - $\frac{R_i}{n}$)], R_i is in the ranking positioning of each DMU$_0$, 'n' is the number of DMUs and Z_2 = 1 – exp (- MI), where MI is the value of the Malmquist index for each DMU$_0$ from the set of 'n' DMUs (the values of the parameters $\frac{R_i}{n}$ and MI are values obtained from the relative technical efficiencies for each DMU).

Let us take an illustrating example for the evaluation of the overall performance of the DMUs:

If the DMU$_1$ has the Malmquist index, MI=1.1 and it is in the first place in the ranking positioning of the relative technical efficiency value, while a DMU$_2$ has the Malmquist index value MI=1.2 and in the ranking positioning of the relative technical efficiency value is in the fifth place, then based on the calculations, for the DMU$_1$ we have $\|E_1\|$= 3.95290 and for the DMU$_2$ we have $\|E_2\|$= 4.08873. This shows that when judging on the overall performance evaluation, one can say that DMU$_2$ has a better overall performance then the DMU$_1$ (regardless the fact that either the Malmquist index or the ranking positioning of the DMU$_0$ may be mentioned separately).

In the analysis of the study given for each DMU, to reason on the dynamics of its progress from year to year, the rhythm of the 'progress' is also evaluated as given by the formula $\alpha_p = \frac{\sum_{i=1}^{t} (\ln y^t - \ln y^{t-1})}{t-1} \cdot 100\%$, where 'y' can as well be the efficiency value (y = Ef) for the efficiency 'progress' rhythm as it can be y = MI for the 'progress' rhythm of the Malmquist index (MI). (To avoid the asymmetry of the negative values that may result, the 'progress' rhythm will not be evaluated by using the ratio $\frac{y^t - y^{t-1}}{y^{t-1}}$). In addition, the variance for the relative technical efficiency during the period of time 't' is evaluated, according to Fisher [3], with the variance formula ($S^2 = \frac{\sum_{i=1}^{t}(x_i - \bar{x}_i)^2}{t-1}$). ($x_i$ is the value of the relative technical efficiency for each year for one DMU$_0$ or x_i may as well be the value of the Malmquist indexes during the period 't' also for one DMU$_0$).

3.2 Evaluation of the Impacting Effects of Each Variable Factor in the Efficiency Value – A New Approach for a Two-Stage Process

The examination of the impacting effects of each variable factor on the efficiency value is done by composing the input-output connections grouping and by using a two-stage process as it is described below. To make this evaluation as detailed as possible, the examination of the two-stage process in this study is made together with the application of the Malmquist index study to best know the macroeconomic development dynamics for a "Connoisseur-study-analysis". The evaluation approach as a two-stage process may be made when it is necessary to loosen the rigorous rule in DEA where the number of the DMUs is not bigger than the triple of the inputs and outputs total sum.

The Malmquist index evaluates the total index of the productivity growth change for each DMU, but when it is also based on the study of knowing the impacting effects, the advantages and disadvantages in the macroeconomic dynamics will be better evaluated.

The use of the two-stage process is found in many applications of the DEA study. This work has a different objective, not only the DMUs ranking according to the efficiency value, but also examining and knowing the impacting effects of the variable factors. To the intermediate variable factor (the inserted factor) that is defined with a double 'role', once as output and once as input, the 'gravity' of the effects of its impact may be calculated by evaluating each stage independently. For this, firstly, the models [M.5.a] and [M.5.b] are used by examining each stage independently, [39]:

[M.5.a] (output- oriented):

$$\text{Max: } \omega_0 + \varepsilon \left[\sum_{i=1}^{m} s_i^- + \sum_{d=1}^{D} s_d^+\right]$$

$$\text{Subject to: } \sum_{j=1}^{n} \lambda_j x_{ij}^{①} + s_i^- = x_{ij_0}^{①}$$

$$\sum_{j=1}^{n} \lambda_j z_{ij}^{①} - s_d^+ = \omega_0 z_{dj_0}$$

[M.5.b] (input- oriented):

$$\text{Min: } h_0 - \varepsilon\left[\sum_{d=1}^{D} s_d^- + \sum_{r=1}^{s} s_r^+\right]$$

$$\text{Subject to: } \sum_{j=1}^{n} \lambda_j z_{dj}^{①} + s_d^- = h_0 z_{dj_0}^{①}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{①} - s_i^+ = y_{rj_0}^{①}$$

$$i = 1,2,\dots, m;\quad d = 1,2,\dots,D;\quad r = 1,\dots,s;\quad \lambda_j \geq 0;\quad j = 1,2,\dots,n$$

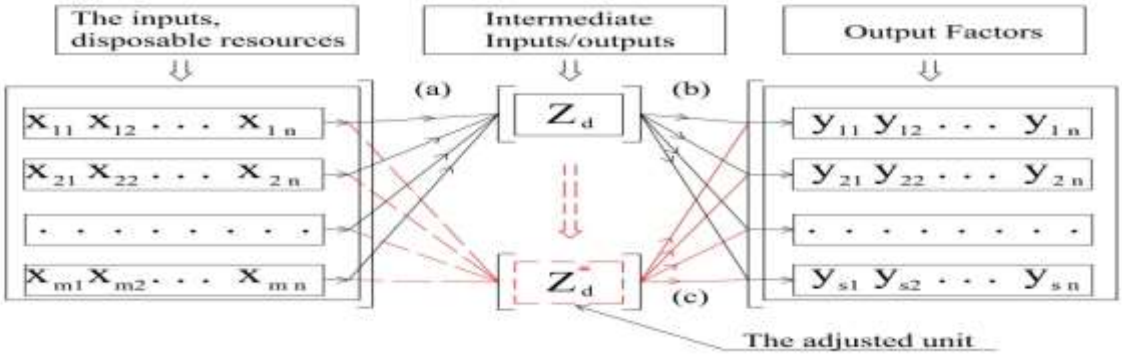


Fig. 2: Measuring efficiency of two stage model
Source: Created by the author

Using the obtained results from the [M.5.a] model, the respective efficient frontier is defined and the efficient and inefficient DMUs are identified.

For the DMUs that are evaluated as inefficient, using the target projections, [36], [39] is made possible that all DMUs are positioned in the respective efficient frontier (Figure 2).

Using the obtained data for the intermediate variable Z_d the set of the adjusted values is defined for the Z_d from the target projections for each DMU. In the second step, the [M.6] model is used (with the adjusted data).

Min: $\theta_0 - \varepsilon[\sum_{d=1}^{D} s_d^- + \sum_{r=1}^{s} s_r^+]$

Subject to: $\sum_{j=1}^{n} \lambda_j z_{dj}^{②} + s_d^- = \theta_0 z_{dj_0}^{②}$

$\sum_{j=1}^{n} \lambda_j y_{rj}^{②} - s_r^+ = y_{rj_0}^{②}$

d =1,2,…,D; r = 1,…,s; j = 1,2,…,n; $\lambda_j \geq 0$

[M.6]

The evaluations for the first stage efficiencies valued with the models [M.5.a] and [M.5.b] may be conceptualized as managerial efficiencies for the respective groupings ($I^{①}$(input) _ $Z^{①}$(output)) and ($Z^{①}$(input) _ $Y^{①}$(output)) that also study the 'gravity' of the revenue per capita (the intermediate variable Z_d) separately for the inputs grouping and for the outputs grouping (by calculating afterwards the differences $(\theta_0 - \omega_0)$ dhe $(\theta_0 - h_0)$). The efficiency θ_0 obtained by the second stage may be conceptualized as the efficiency of 'direction', where the evaluation according to the revenue per capita can be considered as the joint direction (where the revenue per capita is conceptualized as input and as output). The obtained efficiency is called "moderated" efficiency (with adjusted data). The values of the overall efficiency (moderated) enable a performance ranking that can be later examined together with the Malmquist index.

Along with the presented approach as a two-stage process and the obtained results, the evaluation of the impact effects that each variable indicator shows is afterward used by defining this way the "gravity" of each input (output) factor with the other variable factors of the inputs (outputs), which can be given by the formulas g$Ef_0(I_i)$ = $\frac{Ef_0(I_i)}{\sum_{i=1}^{m} Ef_0(I_i)}$ for each input and g$Ef_0(O_r)$ = $\frac{Ef_0(O_r)}{\sum_{r=1}^{s} Ef_0(O_r)}$ for each output.

For the gravity evaluation of each variable factor in the process of the inputs and outputs transformation, let us see the Figure 3.a and the Figure 3.b where the respective efficiencies according to the direction of each input (output) variable factor are evaluated as follows:

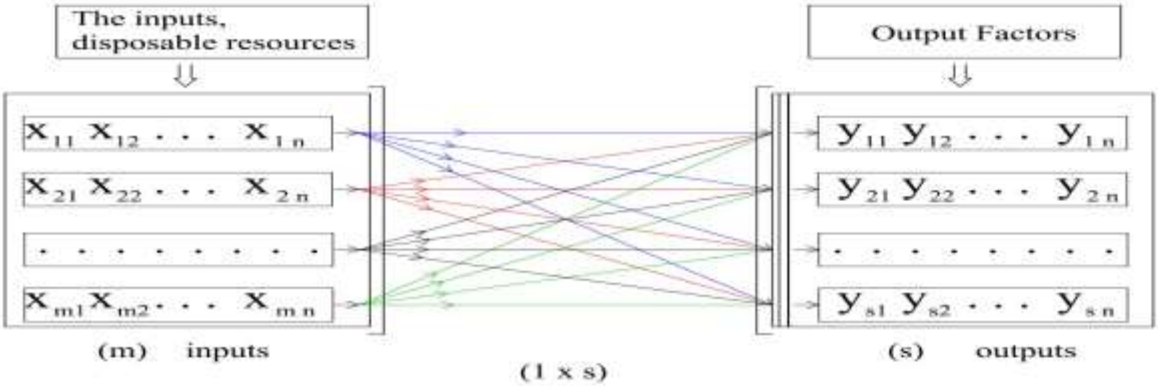


Fig. 3.a: Measuring efficiency, one input item and s output items
Source: Created by the author

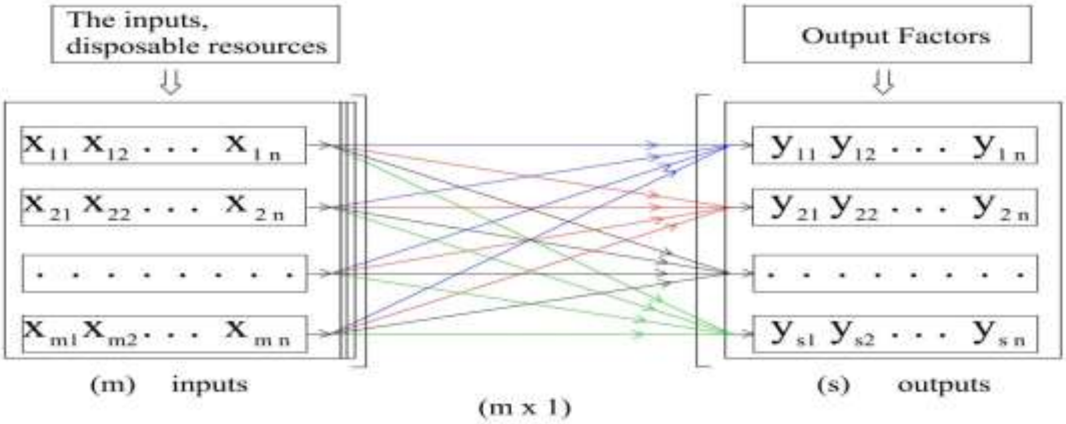


Fig. 3.b: Measuring efficiency, m input items and one output item
Source: Created by the author

For the evaluation of the respective efficiencies, the models [M.7.a] and [M.7.b] are used [5], [39]

[M.7.a] (input-oriented):

$$\begin{aligned} &\text{Min } \theta_0 \\ &\text{Subject to: } \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_0 x_{i0} \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0} \\ &\lambda_j \geq 0 \end{aligned}$$

[M.7.b] (output-oriented):

$$\begin{aligned} &\text{Max } \varphi_0 \\ &\text{Subject to: } \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{i0} \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi_0 y_{r0} \\ &\lambda_j \geq 0 \end{aligned}$$

i = 1, 2,…,m; r = 1,2,…, s; j= 1,2,…,n

4 Numerical Application

The DEA Malmquist index is used in this work to study the progress of the macroeconomic development dynamics of the 19 European region countries. The study is expanded in the 2015-2020 period of time.

The composing components of the ‘n’ set of DMUs are European Union member and non-member countries. The selection of the variable factors as incoming and outgoing (inputs-outputs) is based on the stated objectives and hypotheses (introduction) to better explore the progress of the changes in the increase of the total index of production productivity and the ranking for performance evaluation.

The ‘n’ set of DMUs consists of the following countries: 1. Albania (ALB), 2. North Macedonia (MKD), 3. Serbia (SRB), 4. Montenegro (MNE), 5. Bosnia and Herzegovina (BIH), 6. Croatia (HRV), 7. Cyprus (CYP). 8. Czech Republic (CZE), 9. Estonia (EST), 10. Greece (GRC), 11. Hungary (HUN), 12. Latvia (LVA), 13. Lithuania (LTU), 14.

Malta (MLT), 15. Poland (POL), 16. Romania (ROU), 17. Slovak Republic (SVK), 18. Slovenia (SVN), 19. Bulgaria (BGR).

The Western Balkans countries are non-members of the European Union.

The variable indicators (input-output) were evaluated as closely connected with the gravity they showed and in their correlative relations. These variable indicators as inputs (I) are: Gross national expenditure (current US$) ($I_1$), Foreign direct investment (I_2), Imports of goods and services (current US$) ($I_3$), and as outputs (O) are: GDP per capita (constant 2015 US$) ($O_1$), GNI per capita, Atlas method (current US$) ($O_2$), Exports of goods and services (BoP, current US$) (O3). In addition, the Revenue, excluding grants (% of GDP), is in this work the intermediate variable factor in the two-stage process which, in the first stage (3x1) is output and in the second stage (1x3) is input (which is also included in the totality of the variable factors and is studied for the gravity of the impact when it is taken as output and as input).

The input-output data (I_1; I_3; O_1; O_2; O_3; Population, total; Revenue, excluding grants (% of GDP)) are taken from the Databases of the World Development Indicators, [40]. For Montenegro, the Revenue data are taken from the International Monetary Fund (IMF) (it is taken as a percentage of the GDP and then calculated in US$ per capita as for any other country, the data is public, [41], Foreign direct investment (is taken from the UNCTADSTAT database (Inward, US dollars at current prices per capita, Stock, the data is public, [42]).

For the normalization of the data in DEA, the data are calculated per capita (as the population differs from country to country).

In Table 1, the coefficients of the correlative relations of the variable factors are given (Pearson test). It is noted from Table 1 that the vector of the correlation coefficients' average values according to each column is:

V_ρ= (0.83, 0.63, 0.87, 0.91, 0.87, 0.88, 0.76). The average value of this vector's coordinates is: $\bar{\rho}_i$ = 0.821742.

In addition, Table 1 shows that the average value of the correlation coefficients between the whole group of the incoming factors (inputs) with each of the outgoing factors (outputs – GDP, GNI, export, revenue) pertains to the segment [0.79; 0.9], the correlation coefficient between the import and export is ρ = 0.9964. These parameters talk of strong correlative relations of the inputs grouping with each of the outgoing output factors. Table 2 has the statistical data of the values for each variable according to the three groupings: the grouping for 19 countries considered altogether, the grouping for the European Union non-member countries (the Western Balkans), and the other grouping where the European Union member countries.

Table 3 shows the results obtained from the application of the Malmquist index for the 2015-2020 period of time (by calculating the average value from the results that are obtained for each year, t=1,2,3,4,5,6). Denominations C1, C2, C3 represent the groupings according to Cluster classifications.

S_E^2 dhe S_{MI}^2 are the respective variances of the relative technical efficiency and of the Malmquist index, while α_p^E and α_p^{MI} are respectively the values of the "progress" rhythm for the efficiency and the Malmquist index.

From Table 3 is noted that 5 countries (which belong to Cluster C2), during the entire period, have the technical efficiency value equal to 1 (CCR model). Two countries, Hungary and Slovakia (which also belong to Cluster C2) are evaluated with the efficiency value 1 for some years (for some consecutive years) and with the general average value, respectively, 0.965 and 0.999. For that reason, the countries with a technical efficiency value equal to 1 for the entire period and the two countries that have the average technical efficiency value of 0.965 and 0.999, but that also have an efficiency value equal to 1 for consecutive years, are included in the C2 grouping. Therefore, for this grouping, the

super-efficiencies for the countries that have a relative technical efficiency equal to 1 are evaluated based on the CCR model. The evaluation of the super-efficiencies is done based on the model, [43], as none of the values from the input output variables is close to zero.

Table 4 shows the evaluation of the Malmquist index and its components by calculating their values based on the evaluation of the super-efficiencies. This is done to increase the differentiating power in the performance evaluation between the countries from Cluster 2, the countries that are denominated as the best practices.

Table 1. The matrix of the correlative relations between the variable values (input/output) for the countries in the study (2015-2020)

	GNE	FDI	IGS	GDP	GNI	EGS	Revenue	TRADE
GNE	1							
FDI	0.46734	1						
IGS	0.81257	0.66162	1					
GDP	0.97064	0.63185	0.81682	1				
GNI	0.97750	0.59805	0.80048	0.99815	1			
EGS	0.83314	0.63358	0.99648	0.83526	0.82076	1		
Revenue	0.94381	0.59772	0.75323	0.97048	0.96929	0.76583	1	
TRADE	0.82389	0.64775	0.99907	0.82704	0.81164	0.99917	0.76039	1

Table 2. Variable values per capita

Country	Indicator	GNE	FDI	IGS	GDP	GNI	EGS	Revenue	TRADE
For 19 Countries	Min.	5523	2401	2095	4280	4725	1382	1065	3477
	Max	24277	496704	38786	26496	26352	39302	9821	78089
	Average	14370	56879	10486	13991	14638	10557	5137	21043
	Dev.ST	1549	6029	1161	807	1288	487	305	2332
WB	Min.	5523	2401	2095	4280	4725	1382	1065	3477
	Max	9685	8313	4968	6972	7877	3222	2888	8060
	Average	7085	4178	3559	5533	5978	2599	1962	6158
	Dev.ST	841	635	541	323	575	487	140	1010
Other Countries	Min.	8485	3239	4902	7672	8350	4581	2586	9483
	Max	24277	496704	38786	26496	26352	39302	9821	78089
	Average	16972	75700	12960	17012	17730	13399	6271	26359
	Dev.ST	1802	7956	1383	980	1542	1448	363	2804

Table 3. The average values of the Malmquist index and its composing components during the period 't' (2015-2020)

Cluster	DMU	CCR Technical efficiency			Malmquist index decomposition				Malmquist index		
		avg.	S_E^2	$\alpha_p^E(\%)$	EC	PEC	SEC	TEC	avg.	S_{MI}^2	$\alpha_p^{MI}(\%)$
C1	ALB	0.883	0.003	3.311	1.035	1.000	0.883	0.977	1.011	0.008	4.082
C1	MKD	0.804	1E-04	-0.356	0.997	1.000	0.804	0.993	0.990	0.002	1.585
C1	SRB	0.863	9E-04	-1.092	0.989	0.992	0.879	0.991	0.980	0.004	1.33
C1	MNE	0.790	6E-04	-1.519	0.986	0.981	0.932	0.983	0.966	0.007	1.009
C1	BIH	0.824	8E-04	1.876	1.019	1.019	0.852	0.979	0.998	0.003	1.083
C3	HRV	0.980	5E-05	-0.121	0.999	0.997	0.990	0.995	0.994	8E-04	0.226
C2	CYP	1.000	0	0.000	1.000	1.000	1.000	1.000	1.000	0	0
C3	CZE	0.979	8E-05	-0.286	0.997	0.999	0.997	1.003	1.000	2E-04	0.139
C3	EST	0.942	3E-04	-1.001	0.990	0.990	0.996	0.992	0.982	0.001	-0.058
C2	GRC	1.000	0	0.000	1.000	1.000	1.000	1.000	1.000	0	0
C2	HUN	0.965	0.001	-1.383	0.986	0.999	0.970	1.001	0.987	4E-04	-0.363
C3	LVA	0.947	2E-04	0.540	1.006	1.003	0.992	0.991	0.997	0.003	1.308
C3	LTU	0.944	0.001	1.636	1.017	1.014	0.990	0.989	1.006	0.002	1.675
C2	MLT	1.000	0	0.000	1.000	1.000	1.000	1.000	1.000	0	0
C2	POL	1.000	0	0.000	1.000	1.000	1.000	0.999	0.999	2E-06	0
C3	ROU	0.957	5E-04	0.295	1.003	1.000	0.966	0.985	0.988	0.002	1.545
C2	SVK	0.999	3E-06	0.000	1.000	1.000	0.999	0.992	0.992	2E-04	-0.785
C2	SVN	1.000	0	0.000	1.000	1.000	1.000	1.000	1.000	0	0
C3	BGR	0.956	4E-04	-0.391	0.996	1.000	0.956	1.004	1.000	8E-04	-0.689
Average		0.939	5E-04	0.079	1.0011	0.9997	0.958	0.9935	0.9946	0.002	0.636

EC -Efficiency change; PEC -Pure Efficiency Change; SEC- Scale Efficiency Change; TEC- Technological Change; MI- Malmquist index; Average Value -avg.

Table 4. The average values of the Malmquist index with its composing components based on the Cluster 2 grouping super-efficiencies during the 2015-2020 period of time

DMU	EC	PEC	SEC	TEC	MI
CYP	0.995	1.000	0.996	1.002	0.997
GRC	0.976	1.007	0.968	1.015	0.991
HUN	0.998	0.995	1.003	0.995	0.993
MLT	1.001	1.000	1.002	0.999	1.000
POL	1.034	1.012	1.022	0.984	1.017
SVK	1.000	1.000	1.000	0.992	0.992
SVN	0.986	1.000	0.987	1.011	0.997
Average	0.9986	1.002	0.997	0.9997	0.998

From the above Table 3 is noted that the countries that have a technical efficiency value smaller than the average value, which is 0.939, are the Western Balkans countries (Cluster C1) where $\overline{Ef}_{c1} = 0.8328 < 0.939$. The average efficiency of the European Union countries is 0.9769, while the average efficiency for Cluster 3 is $\overline{Ef}_{c3} = 0.96$. It is noted that the difference in the efficiency value between the C1 Cluster grouping and the C3 Cluster grouping is -12.4%.

The average variance S_E^2 for the technical efficiency values for the 19 countries is 5E-04, while it is smaller for the C1 grouping countries (1E-04).

The rhythm of the "progress" flow for the efficiency values shows that 42% of the DMUs have the average rhythmic value of the progress negative. Of the 5 Western Balkans countries, 3 countries have negative values of the "progress" rhythm (or 60% of them), while for the European Union countries, only 35.7% of them have negative values of the "progress" rhythm.

It is noted that 42% of the countries in the study have a Malmquist index value (Table 3) bigger or equal to one and 58% of the countries have it less than one. The Malmquist index values are from the segment [0.966; 1.011], the average of the Malmquist index for the 19 countries is 0.9946.

The C1 Cluster grouping countries have an average value of the Malmquist index of 0.989, which means smaller than the general average. When considering the values of the Malmquist index, in general, the biggest impact on its increase comes from the first factor of the efficiency change. Even in the Malmquist index which is composed of three components is noted that the biggest impact in the increase of the Malmquist index comes from the first factor. This scene is not the same for each grouping considered separately. So, the average variance for the Malmquist index (S_{MI}^2) is 0.002. For the C1 Cluster grouping, it is 0.0048, for the C2 Cluster grouping it is 0.0148, for the C3 Cluster grouping it is 0.00685. This shows that the closest to stability in the dynamics of macroeconomic development for the Malmquist index is the C1 Cluster grouping. The evaluation of the progress flow rhythm in percentage as an average is 0.636. In Table 4 which pertains to the separate evaluation of the C2 Cluster grouping according to super-efficiencies, the average value of the Malmquist index is from the segment [0.991, 1.017]. It is the technological factor that has the biggest role in this grouping. To judge about specific countries, the respective values of the Malmquist index and its components are examined. The gained results for the "moderated" efficiency according to the two-stage process are given in Table 5. In addition, Table 5 gives the evaluations of the impact effects of each variable as related to its inputs and outputs grouping and as evaluated from the 3x3 relation of this grouping. In the "moderated" efficiency evaluation based on the new approach of the two-stage process evaluated for each year, no more than two countries are evaluated with the efficiency value equal to 1, while in the evaluation based on the CCR model the number of the countries with the efficiency value equal to 1 varied from 5 to 7. It was noted that the coefficient of the correlative relation (Pearson test) between two rankings (CCR model and the two-stage process) in an average for the entire period, is bigger than 0.8 (which shows a strong correlative relation between the two rankings). Of the 19 countries in the study, only Greece was noted a visible deviation between the two rankings, which shows the impact the change of the variable factors number has (when this number can be increased). According to the CCR model, 73.6% of the countries have an efficiency value greater than the general average value while, when using the evaluation of the two-stage process efficiency (the new approach), only 52.6% of the countries have a bigger efficiency than the average. This shows that the distinguishing power in the efficiency evaluation is bigger in the new approach. The effects of the inserted new factor impact that operates as input or output, as appropriate, for the respective grouping of the inputs or outputs are also evaluated in the new approach.

These results are given in the Table 5, Figure 4. a and Figure 4.b.

In Table 5, d_1 (d_1= Ef- Ef_1) shows the difference of the first stage efficiency value (independent) from the efficiency that is valued as "moderated" efficiency obtained from the two-stage process (3x1 grouping, where the revenue is output), d_2 (d_2= Ef- Ef_2) shows the difference between the efficiency values of the second stage (independent) from the efficiency that is evaluated as "moderated" efficiency obtained from the two-stage process (1x3 grouping, where the revenue is input).

The revenue that is once used as output and once used as input (in the two-stage process) showed that the differences from the "moderated" efficiency of each stage (valued independently) to the general average values are respectively 0.095 when the revenue is used as output and 0.060 when it is used as input. This shows that the separating "gap" from the moderated efficiency values is not the same. Especially for the Western Balkans countries in the study, the average of d_2= - 0.3354 and the average of d_1= 0.0582, which shows that the effect of the revenue in the Western Balkans as compared to the average of the 19 countries has a much weaker effectiveness when it is used as output and it is obviously distinguished from when it is used as input. This impact effect is also shown in the Figure 4.a and Figure 4.b.

Figure 4.a shows the effects of the impact weight in percentage towards the efficiency value of each variable factor against the totality of the inputs (or the totality of the outputs). (a.1) shows the average values of the weight of the impact effects in percentage for the 19 countries as a whole, while (a.2) shows these average values only for the Western Balkans countries.

Table 5. The effects of the variable factors' impact

Cluster	DMU	Two-stage DEA			In the 3x3 grouping					
		Ef	d_1	d_2	I_1	I_2	I_3	O_1	O_2	O_3
C1	ALB	0.732	0.289	-0.259	35.19	18.86	45.95	35.70	38.06	26.24
C1	MKD	0.705	0.223	-0.256	34.28	26.15	39.57	34.08	34.86	31.06
C1	SRB	0.739	-0.025	0.1025	37.59	16.77	45.64	34.21	34.86	30.93
C1	MNE	0.671	0.008	0.0391	41.86	12.71	45.43	34.57	38.36	27.07
C1	BIH	0.69	0.027	0.038	34.85	23.63	41.52	35.07	36.63	28.30
C3	HRV	0.825	0.026	0.1599	35.54	18.53	45.93	33.44	35.11	31.45
C2	CYP	1	0.000	0.3118	51.32	0.67	48.01	34.51	34.51	30.98
C3	CZE	0.885	0.205	-0.003	35.81	20.74	43.45	32.79	32.80	34.41
C3	EST	0.874	0.136	0.079	38.26	16.76	44.98	32.67	33.73	33.60
C2	GRC	0.82	-0.180	0.2944	26.92	36.54	36.54	35.40	35.40	29.20
C2	HUN	0.908	0.058	0.1668	35.63	23.38	40.99	33.01	32.89	34.10
C3	LVA	0.831	-0.005	0.175	35.06	22.51	42.43	33.00	34.60	32.40
C3	LTU	0.863	0.261	-0.098	31.96	29.74	38.30	32.88	33.51	33.61
C2	MLT	1	0.183	0	51.59	1.82	46.59	32.10	31.07	36.83
C2	POL	0.853	0.094	0.088	32.64	24.80	42.56	33.82	33.39	32.79
C3	ROU	0.79	0.193	-0.047	33.24	22.27	44.49	33.67	35.14	31.19
C2	SVK	0.904	0.092	0.1106	34.57	27.56	37.87	32.70	32.46	34.84
C2	SVN	0.949	0.052	0.2223	30.11	34.97	34.92	33.33	33.34	33.33
C3	BGR	0.837	0.160	0.0167	37.06	16.35	46.59	32.03	33.88	34.09
Average		0.836	0.095	0.060	36.50	20.78	42.72	33.63	34.45	31.92

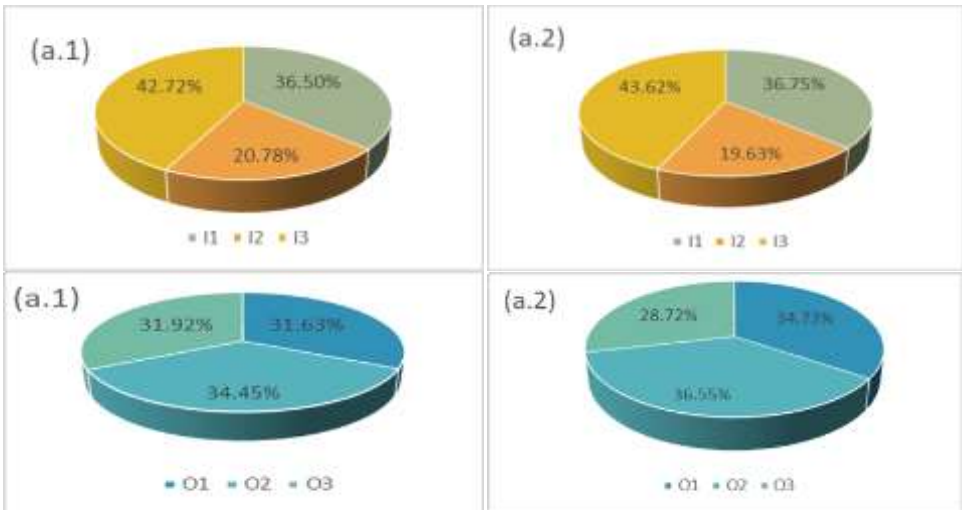


Fig. 4.a: Effects of the impact weight for each input (output) variable factor in the 3x3 grouping ((a.1) for the 19 countries and (a.2) for the Western Balkans)

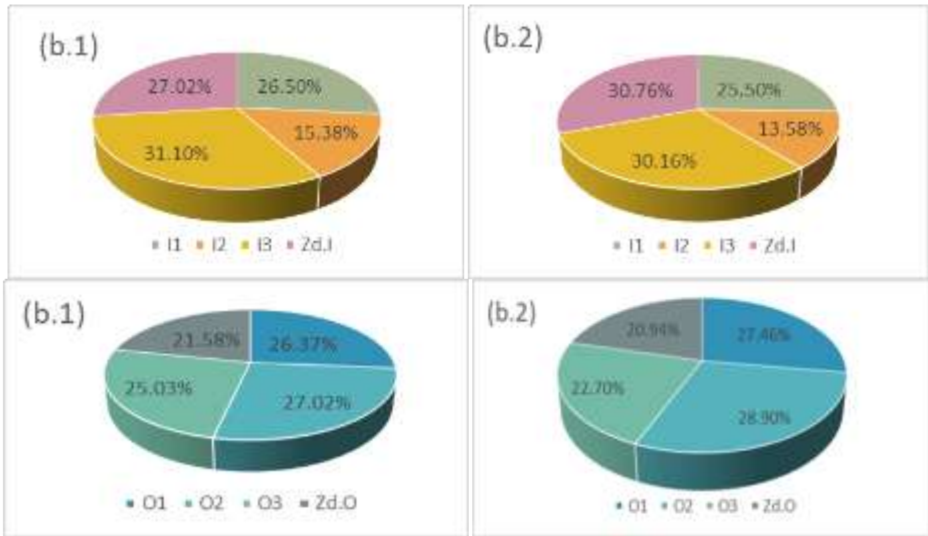


Fig. 4.b: The effects of the impact weight of each included input (output) variable factor and the intermediate factor in the totality of the variable factors ((b.1) for the 19 countries and (b.2) for the Western Balkans

Figure 4.b. shows the weight of the impact effects in the efficiency value when the intermediate factor (Zd) (the revenue per capita) is included in the whole of the variable factors and operates once as output and once as input.

In the evaluation of the impact of the technical efficiency value that is evaluated according to the specific groupings for the variable factors (one input with s outputs and m inputs with one output) is noted that the import with 42.72% has the biggest impact in inputs, while for the Western Balkans countries it is 43.62%. Meanwhile, of the outputs, it is the export with 31.92% that has the weakest impact, more emphasized this weak impact is in the Western

Balkans countries with 28.72%. In the Table 6 is given the ranking for the evaluation of the general performance according to the “norm” evaluating indicator and according to the Cluster groupings by also evaluating the overall performance (OP) within each Cluster grouping separately.

Table 6. Evaluation of the overall performance according to the Cluster groupings and “norm” evaluating criteria

Cluster	DMU	Rank CRS	Z_1	Z_2	$\|\|E_0\|\|$	Evaluation of OP	
						a	b
C1	ALB	15	0.550671	0.636145	0.591867	1	15
C1	MKD	18	0.181269	0.628423	0.337511	4	18
C1	SRB	16	0.451188	0.624689	0.530898	2	16
C1	MNE	19	0.000000	0.619398	0.000000	5	19
C1	BIH	17	0.329680	0.631384	0.456240	3	17
C2	CYP	3	0.348561	0.631015	0.468985	4	7
C2	GRC	3	0.575627	0.628795	0.601624	1	2.5
C2	HUN	9	0.133122	0.629536	0.289491	6	8
C2	MLT	3	0.248523	0.632121	0.396354	5	14
C2	POL	3	0.435282	0.638322	0.527115	3	2.5
C2	SVK	6	0.000000	0.629166	0.000000	7	9
C2	SVN	3	0.510458	0.631015	0.567544	2	12
C3	HRV	7	0.575627	0.629907	0.602156	1	13
C3	CZE	8	0.510452	0.632121	0.568038	2	2.5
C3	EST	14	0.000000	0.625439	0.000000	7	5
C3	LVA	12	0.248523	0.631015	0.396007	5	10
C3	LTU	13	0.133122	0.634321	0.290589	6	6
C3	ROU	10	0.435282	0.627679	0.522702	3	2.5
C3	BGR	11	0.348561	0.632121	0.469396	4	11

a – within the Cluster grouping in itself; b – for all countries, with the evaluation of the norm criteria according to the general ranking and to the Malmquist index evaluated in Table 3.

The overall performance is evaluated with the "norm" indicator set forth in the Methodology, $\|E_0\|= \sqrt{Z_1 \cdot Z_2}$, where $Z_1 = 1 - \exp [- (1 - \frac{R_i}{n})]$, R_i is the positioning in the ranking of each DMU_0 and $Z_2 = 1 - \exp (- MI)$, where MI is the Malquist index value for each DMU_0 from the 'n' set of DMUs.

The Western Balkans countries are ranked in the 15th, 16th, 17th, 18th, 19th positions in the evaluation of the overall performance, while their ranking within the C1 Cluster is also done. Similarly, the C2 Cluster and C3 Cluster are also considered. The obtained results, which are given using tables and figures, create an extensive comprehensible tableau for the evaluation of the overall performance in the progress dynamics of the region's and each country's specifically macroeconomic development based on the evaluation of the DEA Malmquist index correlated with the ranking positioning.

5 Discussions and Conclusions

The results of this study gained by the use of the DEA method, evaluate the dynamics of the macroeconomic development of the 19 countries from the European region during the 2015-2020 period of time.

In this study, the DEA Malmquist index model is applied for the evaluation of the growth change of the total production productivity along with its composing components evaluated step by step during the period 't' (t=1,2,3,4,5,6). For each step of the period, the ranking for the performance evaluation, judged based on the efficiency value, is defined.

Along with the presentation of the results and their examination shown in the tables, more detailed analyses are also given in this study. But, together with the examination of the obtained results, two respective "statistical" indicators (proposed by the author) are used to judge about the evaluation of the overall performance: $z_1 = 1 - \frac{1}{\exp(1-\frac{R_i}{n})}$ (has the scaled parameter $\frac{R_i}{n}$) and $z_2 = 1 - \frac{1}{\exp(MI)}$ (has the Malmquist index values as parameter – its discreet values). These indicators are based both in the ranking positioning and in the values of the Malmquist index according to the DMUs, evaluating subsequently the geometric mean of these two indexes $\|E_0\| = \sqrt{z_1 \cdot z_2}$ (the values of the parameters $\frac{R_i}{n}$ and MI are values obtained from the relative technical efficiencies for each DMU).

This is done to better define the judging about the overall performance evaluation for each DMU (one DMU as it is showed from the results, e.g. DMU_1 has the Malmquist index value MI=1.011, (that is the highest value) but, in the final ranking, this DMU is positioned in the 15th place).

The evaluation of the overall performance is also given by the Cluster groupings separately.

In the study, the new approach that is based on the two-stage process concept (the so-called "moderated" efficiency) is applied. This, along with the application of the calculation models for the impact effects in the efficiency values of each variable factor, takes, based on the obtained results, more comprehensive and explanatory reasoning.

In the study response is given for the formulated hypothesis and the presented objectives reaching this way in the conclusion that there are expressed impacts in the rankings of the productivity change and the regional parts in the DMUs' set taken into consideration. The average value of the productivity change index as a period for all 19 countries is 0.995, while for the EU member countries, it is 0.9769 and for the EU non-member countries (the Western Balkans in the study) is 0.9022. The stated hypothesis: Are there visible differences that can be explained through the performance evaluation levels over the following steps of the period? – By not mentioning the visible ranking positioning, we can set aside the "variances" (S_E^2 dhe S_{MI}^2), where the EU member countries have the average value of the variance (0.00029) smaller than the average value of the variance for the EU non-member countries (0.00108) and smaller than the general average of the 19 countries (0.0005). So, the oscillations of the efficiency value are much more visible for the EU non-member countries while the EU member countries show better stability and a more increased efficiency value. Also, the growth "progress" rhythm for the EU countries is 0.058, while for the EU non-member countries, 60% of them have a negative "progress" rhythm as compared to the 28% of the EU countries that have a negative "progress" rhythm. Visible differences are noted for the variance, for the growth "progress" rhythm, and for the Malmquist index between the regions.

From the examination of the impacting effects of the variable factors over the efficiency values, is noted that in the EU non-member countries, there is an unsound "harmonization" of the variable values in the relations of the incoming factors (inputs) with the outgoing factors (outputs). So, output 3 (export) in the EU non-member countries has a gravity of 28.72% in the total of the impacting effects that the outputs show, while in the EU member countries, this gravity is 33.06% where the general average for the impact effects of this factor is 31.92%. As far as the ratio of the first input (expenses) with the first output (GDP), this ratio for the EU non-member countries (Western Balkans countries) shows a value that is bigger than 1 (1.28), while for the EU member countries, this ratio has a value that is smaller than 1 (0.91625). In general, the Trade, as compared to the GDP of the EU non-member countries, is about 1.5 times smaller than that of the EU member countries taken into consideration in the study. The same can also be said for other visible differentiations in the shown effects of the impact; as such we can mention the revenue per capita (used in the two-stage process).

The EU member countries have resulted in more advantageous and better harmonization of the variable factors. They are also more advantageous in the operations of the Trade that is related to a lower technology in the operations of transforming the inputs into outputs.

This study may serve as an alternative for the application of the Malmquist index in the evaluation of the production productivity change increase, evaluated together with the ranking performance for the evaluation of overall performance by using an indicator, the "norm" $\|E_0\|$ indicator. This study may also serve as an encouragement for further operational and managerial research in the evaluation of the DMUs overall performance.

References:

[1] P. Krugman, *The Age of Diminished Expectations*, Third Edition, The MIT Press, 1997, ISBN: 9780262611343.

[2] A. Charnes, W. W. Cooper, Chance-Constrained Programming, *Management Science*, Vol. 6, No. 1 (1959), 73-79, https://doi.org/10.1287/mnsc.6.1.73.

[3] Y.Dodge, *The Concise Encyclopedia of Statistics*, Springer Science + Business Media, 2008, ISBN: 978-0-387-32833-1.

[4] R. D. Banker, A. Charnes, W. W. Cooper, Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis, *Management Science*, Vol.30, No. 9, (1984), 1078- 1092.

[5] A. Charnes, W.W.Cooper, E.Rhodes, Measuring the efficiency of decision making units, *European Journal of Operational Research*, 2 (1978), 429-444.

[6] Robert M. Solow, Technical Change and the Aggregate Production Function, The Review of Economics and Statistics, Vol. 39, No. 3 (1957), 312-320, http://www.jstor.org/stable/1926047.

[7] S. Malmquist, Index numbers and indifference surfaces, *Trabajos de Estadistica*, Vol. 4, (1953), 209–242, https://doi.org/10.1007/BF03006863.

[8] Douglas W. Caves, Laurits R. Christensen, and W. Erwin Diewert, The economic theory of Index Numbes and the measurement of Input, Output, and productivity, *Econometrica*, Vol.50, No. 6, (1982), 1393-1414, https://doi.org/10.2307/1913388.

[9] M.J, Farrel, The Measurement of productive efficiency, J. R. Statist. Soc: Ser, A, Vol.120, No. 3, (1957), 253-290. https://doi.org/10.2307/2343100.

[10] Rolf Färe, Shawna Grosskopf, Malmquist Productivity indexes and Fisher Ideal indexes, *The Economic Journal*, Vol. 102, Issue 410, (1992), 158-160, https://doi.org/10.2307/2234861.

[11] B. Wan, E. Zhou, Research of Total Factor Productivity and Agricultural Management Based on Malmquist-DEA Modeling, Mathematical Problems in Engineering, Vol. 2021, 8 pages https://doi.org/10.1155/2021/2828061.

[12] X. Gong, A Study on the Performance of Digital Economy Companies based on DEA-malmquist Model, Advances in Economics, Business and Management Research, Vol, 656, *Proceedings of the 2022 2nd International Conference on Enterprise Management and Economic Development (ICEMED 2022)*.

[13] N. Maniadakis, E.Thanassoulis, Assessing productivity changes in UK hospitals reflecting technology and Input prices, Applied Economics, Vol.32, No.12 (2000), 1575-89, DOI: 10.1080/000368400418970.

[14] R. M. Thrall, Measures in DEA with an Application to the Malmquist Index, Journal of Productivity Analysis, Vol.13.No.2 (2000), 125-137, [Online]. http://www.jstor.org/stable/41770010.

[15] Y-M. Wang, Y-X. Lan, Measuring Malmquist productivity index: A new approach based on double frontiers data envelopment analysis, Mathematical and Computer Modelling, Vol. 54, No.11-12, (2011), 2760-2771, https://doi.org/10.1016/j.mcm.2011.06.064

[16] T. J. Coelli, D. S. Prasada Rao, Total factor productivity growth in agriculture: a

Malmquist index analysis of 93 countries, 1980-2000, (2005), https://doi.org/10.1111/j.0169-5150.2004.00018.x.

[17] D. Akbarian, Overall profit Malmquist productivity index under data uncertainty, Financ Innov, Vol. 6, No. 6, (2020). https://doi.org/10.1186/s40854-020-0170-0.

[18] F. Abdelkaoui, M. Bouzidi, Malmquist Indexes of Productivity Change in the East Asian Countries, South Asian Res J Bus Manag, Vol.3, No. 4 (2021), DOI: 10.36346/sarjbm.2021.v03i04.001.

[19] H. Zrelli, A. H. Alsharif, I. Tlili, Malmquist Indexes of Productivity Change in Tunisian Manufacturing Industries, *Sustainability*, Vol. *12*, No. 4, (2020) 1367, https://doi.org/10.3390/su12041367.

[20] Blerta (Kristo) Nazarko, Ditila Ekmekçiu, An Approach to the Evaluation of the Overall Performance with Efficiency Measurements by Means of an Efficiency Evaluation Chain Using DEA and Fuzzy DEA, *Mathematics and Statistics*, Vol. 10, No. 5, 1089 - 1104, (2022). DOI: 10.13189/ms.2022.100519.

[21] V. Charles, J. Aparicio, J. Zhu, The curse of dimensionality of decision-making units: A simple approach to increase the discriminatory power of data envelopment analysis, *European Journal of Operational Research*, Vol. 279, No.3, (2019) 929-940, https://doi.org/10.1016/j.ejor.2019.06.025.

[22] M. Mehdiloozada, I. Roshdib, Analyzing the concept of super-efficiency in data envelopment analysis: A directional distance function approach, [Online]. https://arxiv.org/ftp/arxiv/papers/1407/1407.2599.pdf (Accessed Date: February 25, 2024).

[23] M.Khodabakhshi, K. Aryavash, Ranking all units in data envelopment analysis, *Applied Mathematics Letters*, Vol.25, No.12 (2012), 2066-2070, https://doi.org/10.1016/j.aml.2012.04.019.

[24] A. Bazleh, P. Gholami, F. Soleymani, A new approach using data envelopment analysis for ranking classification algorithms, *Journal of Mathematics and Statistics*, Vol. 7, No.4, (2011), 282-288.

[25] A. G. Abri, G. R. Jahanshahloo, F. H. Lotfi, N. Shoja, M. F. Jelodar, A new method for ranking non-extreme efficient units in data envelopment analysis, *Optimization Letters*, Vol.7, (2013), 309-324, https://doi.org/10.1007/s11590-011-0420-1.

[26] A. Firsova, G. Chernyshova, Efficiency Analysis of Regional Innovation Development Based on DEA Malmquist Index, *Information*, Vol. 11, No. 6 (2020), 229, https://doi.org/10.3390/info11060294.

[27] G. Avagyan, Q. Vardanyan, G. Petrosyan, M. Navasardyan, A. Margaryan, The Malmquist productivity Index and its analysis on the example of the RA, *Sciences of Europe*, No. 82, (2021).

[28] A. Trakakis, M. Nektarios, S. Tziaferi, P. Prezerakos, Total productivity change of Health Centers in Greece in 2016–2018: a Malmquist index data envelopment analysis application for the primary health system of Greece, *Cost Eff Resour Alloc*, Vol.19, No. 72 (2021), https://doi.org/10.1186/s12962-021-00326-z.

[29] C.Kao, Malmquist productivity index based on common-weights DEA: The case of Taiwan forests after reorganization, *Omega*, Vol.38. No.6 (2010), 484-491, DOI: 10.1016/j.omega.2009.12.005.

[30] S. Munisamy, B. Arabi, Eco-efficiency change in power plants: using a slacks-based measure for the meta-frontier Malmquist–Luenberger productivity index, *Journal of Cleaner Production*, Vol.105, (2015), 218-232, https://doi.org/10.1016/j.jclepro.2014.12.081.

[31] M. Flegl, C. A. Jiménez-Bandala, I. Sánchez-Juárez, E. Matus, Analysis of production and investment efficiency in the Mexican food industry: Application of Two-stage DEA, *Czech Journal of Food Sciences*, Vol.40, No.2, (2022), https://doi.org/10.17221/172/2021-CJFS.

[32] C.Kao, Sh-N.Hwang, Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan, *European Journal of Operational Research*, Vol. 185, (2008), 418-429, doi: 10.1016/j.ejor.2006.11.041.

[33] Y.Chen, L.Liang, J.Zhu, Equivalence in Two-stage DEA approaches, *European Journal of Operational Research*, Vol.193, No.2, (2007), 600-604, doi: 10.1016/j.ejor.2007.11.040.

[34] [34] L.Liang,w.D.Cook, J.zhu, DEA Models for two-stage Processes:Game Approach and efficiency Decomposition, *Naval Research*

Logistics, Vol.55, (2008), DOI: 10.1002/nav.20308.

[35] H. Morita, N. K. Avkiran, Selecting inputs and outputs in Data Envelopment Analysis by designing statistical experiments, *Journal of the Operations Research Society of Japan*, Vol.52, No.2, (2009), 163-173.

[36] E. Thanassoulis, Introduction to the Theory and Application of Data Envelopment Analysis: A Foundation text with Integrated Software, *Springer; Softcover reprint of the original* 1st ed. 2001 edition (March 1, 2013), ISBN-10: 1461355389, ISBN-13: 978-1461355380.

[37] Rolf Färe, Shawna Grosskopf, Mary Norris, Zhongyang Zhang, Productivity growth, technical progress, and efficiency change in industrialized countries, *The American Economic Review*, (1994), Vol. 84, No.1, 66-83.

[38] J. Zhu, Quantitative Models for performance evaluation and benchmarking.Data envelopment Analysis with spreadsheets, third Editions, USA: Springer, International Series in Operations Research &Management Science, 2014. DOI: 10 1007/978-3-319-06647-9.

[39] W. W. Cooper, L.M.Seiford, K.Tone, Data Envelopment analysis, A comprehensive text with models , applications, references, and DEA-Solver software, Kluwer academic publishers, (2002), eBook ISBN:0-306-47541-3

[40] Data Bank, World Development Indicators, [Online]. https://databank.worldbank.org/source/world-development-indicators (Accessed Date: September 5, 2022).

[41] World Revenue Longitudinal Data, [Online]. https://data.imf.org/?sk=77413F1D-1525-450A-A23A-47AEED40FE78 (Accessed Date: September 5, 2022).

[42] UNCTAD. STAT. https://unctadstat.unctad.org/wds/TableViewer/tableView.aspx?ReportId=96740 (Accessed Date: September 5, 2022).

[43] P. Andersen, N.C. Petersen, A procedure for ranking efficient units in data envelopment analysis, Management Science, Vol. 39, No.10, (1993), 1261–1264, https://doi.org/10.1287/mnsc.39.10.1261.

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