An Algorithm for Storage Tanks Deflection Identification Based on Pattern Search

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Abstract: By analyzing the function relationship between the retained volume of oil and numerical readings of oil probe in storage tanks with two spherical crowns, a mathematical model is given for identifying the horizontal and vertical deflection angles. According this model, a Matlab program, which is based on Hooke-Jeeves Pattern Search Algorithm (HJPSA), is implemented for precisely establishing the deflection angles. A set of data about the volume of oil pumped out, the numerical readings of the oil probe and the retained oil, which is obtained by experiment, indicates that the storage tanks may occurs the horizontal and vertical deflections 6.69 degrees and 13.33 degrees respectively. Moreover, for the retained oil in the storage tanks, a recalibrated table is shown at the end of the paper.

Key–Words: Deflection identification, Least Square Method, Hooke-Jeeves Pattern Search Algorithm

1 Introduction

The storage tanks, which is a horizontal type cylinder metal cans with two spherical crowns at both ends, is widely used in storing oil and other liquids at airports and gas stations, etc. Zhan, Duan and Peng concern on the storage tanks with no crowns at both ends, analyze the relationship among the retained volume, oil level height and deflection parameters in some different ways and obtain some models to recalibrate the tank capacity table [1, 2, 3]. Zhao study the retained oil, probe numerical readings and deflection parameters of the storage tanks. An approximate calculating method is given to establish the tank capacity table for the given deflection parameters [4]. Unfortunately, when the horizontal and vertical deflections are indeterminated, this method is useless to estimate the deflection parameters.

In this paper, according to the idea of numerical integral [5], the storage tanks are subdivided into many frustums. The shadow area of any circular cross section is studied in detail, a general expression of it is represented. And so, the volume of any frustum could be evaluated, and the volume of the storage tanks is the cumulative sum of all the frustums. For now, a more accurate mathematical model is proposed, which is more efficient for Matlab programming. An objective function based on the Least Square Method is established for precisely evaluating the deflection parameters of the storage tanks.

According to HJPSA, the minimum of the function and the corresponding deflection parameters that the storage tanks occurs could be evaluated precisely. Fortunately, by the model in this paper, a recalibrated table for the retained oil in the storage tanks will be obtained much easily.

2 A Model to Calculate the Retained Oil

When the storage tanks with two spherical crowns at both ends occurs no deflection, a front view with a space rectangular coordinates is given in Figure 1. Let the oil probe be the axis z , the bottom plane be the plane xOy (the axis x is not marked), L_1, L_2 be the left and right boundaries of cylinder part, y_l , y_r be the left and right boundaries of the entire storage tanks, and O_1, O_2 be the centers of the two spherical crowns, respectively. In the storage tanks, h is the oil level height (numerical readings of the oil probe at this moment), R is the radius of the circular cross section of the cylinder part.

Using the planes paralleled to plane xOz , the storage tanks is subdivided into many frustums. A circular cross section at any $y \in [y_l, y_r]$ is shown in Figure 2. Let r_y , h_y be the radius and oil height respectively. The shadow area in the circular cross sec-

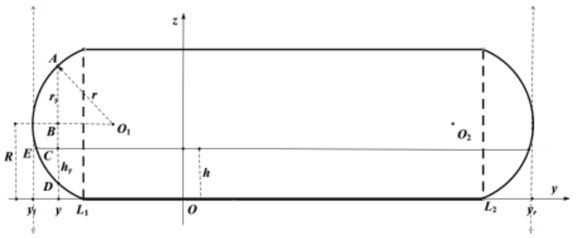

Figure 1: Front view of the storage tanks

tion of the tanks is denoted by $S(r_y, h_y)$, then

$$\begin{aligned} S(r_y, h_y) &= 2\int_0^{h_y} \sqrt{2zr_y - z^2}dz \\ &= \frac{\pi r_y^2}{2} + (h_y - r_y)\sqrt{2r_y h_y - h_y^2} \\ &\quad + r_y^2 \arcsin \frac{h_y - r_y}{r_y}. \end{aligned} \tag{1}$$

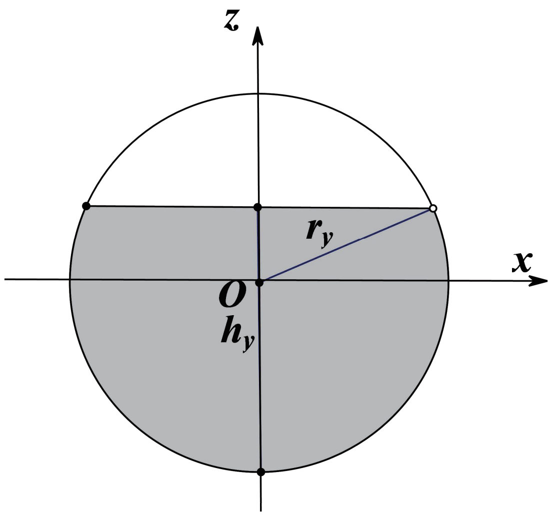

Figure 2: A circular cross section at any $y \in [y_l, y_r]$

When the oil level height is h , the retained volume in the tanks is denoted by $V(h)$, then

$$V(h) = \int_{y_l}^{y_r} S(r_y, h_y)dy, \tag{2}$$

in which r_y and h_y are both the functions of y.

According to the geometrical relations in the spherical crown which are shown in Figure 1, the length of AB is denoted by r_y, then

$$r_y = \begin{cases} \sqrt{2r(y - y_l) - (y - y_l)^2}, & y \in [y_l, L_1] \\ R, & y \in [L_1, L_2] \\ \sqrt{2r(y_r - y) - (y_r - y)^2}, & y \in [L_2, y_r] \end{cases} \tag{3}$$

The oil level height in the circular cross section is denoted by $h_y = CD$, then

(i) For $y \in [y_l, L_1]$ or $y \in [L_2, y_r]$,

$$h_y = \begin{cases} c + r_y, & r_y \geq |c| \\ 2r_y, & r_y < |c| \text{ and } c > 0 \\ 0, & r_y < |c| \text{ and } c \leq 0 \end{cases} \tag{4}$$

in which $c = h - R$.

(ii) For $y \in [L_1, L_2]$,

$$h_y = h. \tag{5}$$

Obviously, if you put (1), (3), (4), (5) into (2), it is much more complicated to work out the value of the definite integral, which may not be represented to an analytical expression. According to the definition of the definite integral, (2) can be rewritten into an infinite sum form, namely

$$V(h) = \sum_{i=1}^{\infty} S(r_{y_i}, h_{y_i})\Delta y_i. \tag{6}$$

For the numerical integral, $V(h)$ could be finitely subdivided into n equal parts, then will be proximately represented by the following formula,

$$V(h) \approx \frac{y_r - y_l}{n} \sum_{i=1}^{n} S(r_{y_i}, h_{y_i}) \tag{7}$$

According to the formulas (1), (3), (4), (5), (7), a Matlab program for calculating $V(h)$ will be easily implemented. Assume that $y_l = -3$, $y_r = 7$, $L_1 = -2$, $L_2 = 6$, $R = 1.5$, $n = 10000$, using the Matlab program, a function relationship between the retained oil $V(h)$ and the oil level height h can be drawn, see Figure 3. All the discrete points (the readings of displayed height and corresponding displayed volume which are given by the oil probe in Appendix 1) are on the curve $V(h)$ precisely. It indicates that these data points are gained at the moment that the tanks occurs no deflection.

The storage tanks are likely to occur horizontal or vertical deflection after using a period of time. Suppose that the tanks occurs a horizontal deflection angle α only. A space rectangular coordinates is established just like Figure 1, see Figure 4. For now, the oil height h at the oil probe is coincided to the readings as before, and the oil plane $z = h - y\tan\alpha$ is not paralleled to the plane xOy . We can subdivide it into many frustums as it did before, and the circular cross section is shown in Figure 2 as well.

All the expressions above can be retained except h_y, since the oil plane expression is not the same as it is before. And we can still discuss h_y as it did before.

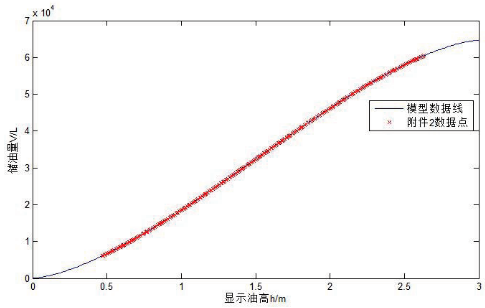

Figure 3: The function relationship between $V(h)$ and h

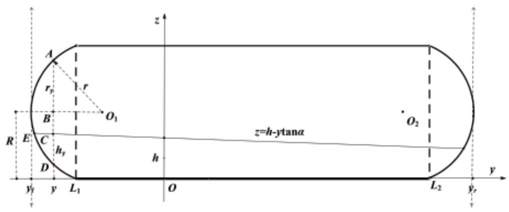

Figure 4: A front view when the tanks occurs a horizontal deflection angle α

(i) For $y \in [y_l, L_1]$ or $y \in [L_2, y_r]$,

$$h_y = \begin{cases} c' + r_y, & r_y \geq |c'| \\ 2r_y, & r_y < |c'| \text{ and } c' > 0 \\ 0, & r_y < |c'| \text{ and } c' \leq 0 \end{cases} \tag{8}$$

in which $c' = h - y \tan \alpha - R$.

(ii) For $y \in [L_1, L_2]$,

$$h_y = \begin{cases} h - y \tan \alpha, & h - y \tan \alpha \geq 0 \\ 0, & h - y \tan \alpha < 0 \end{cases} \tag{9}$$

Considering the storage tanks also occurs a vertical deflection angle β, the circular cross section at the oil probe is shown in Figure 5. Let the oil height $h = CD$, the readings of the oil probe $h_d = AB$, and $\beta = \angle BOD$. Then the following equation must hold:

$$h = R + (h_a - R) \cos \beta. \tag{10}$$

If you put (8), (9) and (10) into (1), (2), (3) and (7), a formula $V = V(h_d, \alpha, \beta)$ will be established. Nevertheless, the calculation is so complex that you could not work out an analyzing expression. Even so, it will be easily to do some adjustments to the Matlab program by (8), (9) and (10).

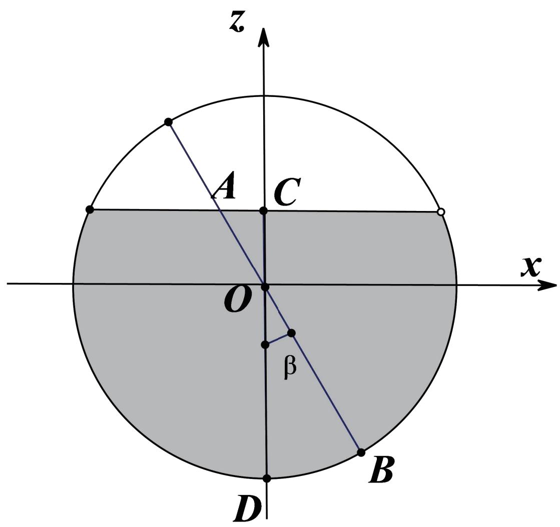

Figure 5: A circular cross section at the oil probe

3 An Approach for Accurately Establishing the Deflection Angles α and β

Suppose α and β are constant. Then using the equation $V = V(h_d, \alpha, \beta)$, a sequence $(V_1, V_2, \cdots, V_N)$ which is corresponding to the sequence of oil level height $(h_{d_1}, h_{d_2}, \cdots, h_{d_N})$, can be evaluated by the Matlab program.

Suppose $\Delta V_i = V_{i+1} - V_i, \Delta h_i = h_{d_{i+1}} - h_{d_i}, (i = 1, 2, \cdots, N-1)$, then a sequence of 2-tuples will be written as follow,

$$(\Delta h_1, \Delta V_1), (\Delta h_2, \Delta V_2), \cdots, (\Delta h_{N-1}, \Delta V_{N-1}). \tag{11}$$

Suppose that $(\Delta h_1, \Delta V_1^*), (\Delta h_2, \Delta V_2^*), \cdots, (\Delta h_{N-1}, \Delta V_{N-1}^*)$ is another sequence of 2-tuples from the experiment. Naturally, we look forward to finding out the value of α and β , making the two sequences fully closed to each other. The Least Square Method tells us how to do [6]. An objective function is given as follow,

$$F(\alpha, \beta) = \sum_{i=1}^{N-1} (\Delta V_i - \Delta V_i^*)^2. \tag{12}$$

The value of $F(\alpha, \beta)$ must be positive and the less the better. The next work is to determine the parameters α and β , which could make the value of $F(\alpha, \beta)$ to be the least.

Since $F(\alpha, \beta)$ has not analytical expression, in order to obtain the optimal solution of the function, the direct search methods will be efficient for this purpose. The Hooke-Jeeves Pattern Search Algorithm which is proposed by Hooke and Jeeves in 1961, is one of the direct search methods [6, 7]. Many Optimizations eventually turn into a question of searching the maximal(or minimal) value of an objective function with multi parameters. [8, 9, 10, 11, 12, 13, 14] show some examples of solving practical problems based on pattern search method.

Let $x_0 = [\alpha_0, \beta_0]$ be the initial base point. Then $x_k = [\alpha_k, \beta_k]$ is the base point after k steps move. For the kth axial move, let $e_1 = [1, 0]$ and $e_2 = [0, 1]$ be the two directions, $\delta_k (> 0)$ be the step length, y be the recent base point. Then $y = y + \delta_k e_j$ [$y = y - \delta_k e_j$) indicate that the base point will move δ_k to a new base point along the positive (negative) direction of e_j . For the combined pattern move, $y = 2x_{k+1} - x_k$ means the base point moving double speed down to a new base point. Let $\sigma (\in (0, 1))$ be the contractive factor and $\varepsilon (> 0)$ be the maximum margin of error between the base point and the real point. Then the circulation of the algorithm is terminated while $\delta_k < \varepsilon$.

The implementation of HJPSA is formalized as follows:

Hooke-Jeeves Pattern Search Algorithm

1. Obtain initial base point x_0 . Determine the step length δ_0 , the contractive factor σ and the max error ε.
2. Determine a new base point. Let $y = x_k$, $j = 1$.
3. The axial move. Move the base point along the positive direction of e_j . If $F(y+\delta_k e_j) < F(y)$, then let $y = y - \delta_k e_j$, go to 4. If not, go to 5.
4. Move the base point along the negative direction of e_j . If $F(y - \delta_k e_j) < F(y)$, then let $y = y + \delta_k e_j$, go to 5. If not, go to 5 immediately.
5. If $j < 2$, let $j = j + 1$, return to 3. If not, let $x_{k+1} = y$, go to 6.
6. The kth combined pattern move. If $F(x_{k+1}) < F(x_k)$, do combined pattern move from the base point x_{k+1} along the accelerative direction, namely, $y = 2x_{k+1} - x_k$, $\delta_{k+1} = \delta_k$, $k = k + 1$, $j = 1$, return to 3. If not, go to 7.
7. Check if the termination criterion is satisfied. If $\delta_k < \varepsilon$, terminate, return the approximate optimal solution. If not, go to 8.
8. Contract the step length. If $x_{k+1} = x_k$, let $\delta_{k+1} = \sigma\delta_k$, $k = k + 1$, return to 2. If not, let $x_{k+1} = x_k$, $\delta_{k+1} = \delta_k$, $k = k + 1$, return to 2.

Let $\alpha_0 = 2^\circ$, $\beta_0 = 4^\circ$, the initial step length $\delta_0 = 1^\circ$, the contractive factor $\sigma = 0.1$, and the max error $\varepsilon = 0.1$. Then by HJPSA, an approximate optimal solution $(\alpha, \beta) = (6.69^\circ, 13.33^\circ)$ will be worked out according to the data in Appendix 1. A searching approach is given in Table 1.

Table 1: An approach for (α, β) by HJPSA

(α,β)	$F(\alpha,\beta)$	(α,β)	$F(\alpha,\beta)$
(2,4)	6359	(6.9,12.1)	258.03
(3,5)	4117.96	(6.9,12.3)	257.89
(5,7)	1137.61	(6.8,12.6)	249.26
(8,8)	716.74	(6.8,12.9)	249.15
(8,8)	716.74	(6.7,13.3)	245.82
(7,9)	279.67	(6.7,13.3)	245.82
(7,11)	273.09	(6.7,13.3)	245.82
(7,12)	271.94	(6.69,13.33)	245.78
(7,12)	271.94	(6.69,13.33)	245.78
(7,12)	271.94	(6.69,13.33)	245.78

Assume that

$$e_i = |\Delta V_i - \Delta V_i^*|, e_{\max} = \max\{e_i\}, \quad (13)$$

$$\varepsilon_i = \frac{|\Delta V_i - \Delta V_i^*|}{\Delta V_i^*} \times 100\%, \varepsilon_{\max} = \max\{\varepsilon_i\}, \quad (14)$$

for all $i = 1, 2, \cdots, N-1$. If the storage tanks occurs horizontal and vertical deflection angles $\alpha = 6.69^\circ$ and $\beta = 13.33^\circ$ respectively, then by (13), (14), we obtain that $e_{\max} = 3.04$, $\varepsilon_{\max} = 4.27\%$. Using the Matlab program again to evaluate the retained oil, a recalibrated table for the retained oil with the intervals of 10cm will be easily worked out, set Table 2.

Table 2: A recalibrated table for the retained oil with the intervals of 10cm

Probe Reading(cm)	Retained Oil (L)	Probe Reading(cm)	Retained Oil (L)
10	354.51	160	33047.95
20	1062.06	170	35857.84
30	2215.33	180	38648.58
40	3692.52	190	41404.71
50	5420.62	200	44110.51
60	7357.84	210	46749.76
70	9473.07	220	49305.59
80	11740.75	230	51760.23
90	14138.75	240	54094.56
100	16647.15	250	56287.66
110	19247.56	260	58315.83
120	21922.72	270	60151.09
130	24656.1	280	61757.86
140	27431.78	290	63084.01
150	30234.18	300	64018.23

4 Conclusion

Compared with the model given in [4], the mathematical model to evaluate the retained oil in the storage tanks in this paper can be easily implemented through Matlab programming. By HJPSA, the horizontal and vertical deflection angles that the storage tanks occurs will be accurately established. It is useful to recalibrate the tanks capacity table. Table 2 shows a recalibrated table for the retained oil with the intervals of 10cm. Indeed, by the Matlab program, an arbitrary precision capacity table for the storage tanks could be worked out. If someone wants to develop a management system of oil level measurement for the storage tanks, this model is much valuable to him.

Appendix 1

A set of experiment data from some storage tanks. We record down the oil pumped out, the displayed height and corresponding displayed volume of the oil probe respectively.

NO	Oil Pumped Out/L	Displayed Height /mm	Displayed Volume/L
201	60	2632.23	60448.88
202	149.09	2624.3	60311.43
203	68.45	2620.67	60248.03
204	199.27	2610.29	60065.11
205	70.05	2606.61	59999.69
206	136.36	2599.59	59874.06
207	232.74	2587.6	59657.02
208	107.97	2582.05	59555.51
209	49.24	2579.57	59509.94
210	80.65	2575.44	59433.77
211	120.29	2569.46	59322.85
212	108.24	2564.12	59223.17
213	83.46	2559.83	59142.66
214	229.93	2548.47	58927.69
215	181.7	2539.63	58758.61
216	238.52	2528.01	58534.01
217	131.79	2521.63	58409.58
218	238.33	2510.23	58185.31
219	42.92	2508.17	58144.52
220	171.34	2500.07	57983.36
221	212.34	2490.06	57782.53
222	92.38	2485.73	57695.08
223	243.85	2474.4	57464.67
224	206.69	2464.77	57267.02
225	224.5	2454.51	57054.65
226	169.26	2446.77	56893.24
227	220.09	2436.85	56684.86
228	117.54	2431.55	56572.86
229	93.44	2427.32	56483.12
230	114.46	2422.2	56374.11
231	174.69	2414.35	56206.14
232	232.77	2404.05	55984.22
233	110.86	2399.15	55878.05
234	138.59	2393.12	55746.87
235	242.21	2382.5	55514.45
236	186.43	2374.35	55334.9
237	275.38	2362.44	55070.68
238	92.65	2358.4	54980.57
239	239.28	2348.13	54750.4
240	206.68	2339.37	54552.85
241	104.63	2334.88	54451.17
242	158.8	2328.13	54297.75
243	142.43	2322.14	54161.06
244	189.17	2314.14	53977.71
245	238.95	2304.14	53747.27
246	73.58	2301.09	53676.72
247	245.27	2290.87	53439.37
248	251.78	2280.46	53196.16
249	134.59	2274.92	53066.14
250	153.02	2268.61	52917.56
251	188.26	2260.89	52735.08
252	220.97	2251.88	52521.14
253	229.97	2242.46	52296.37
254	237.73	2232.88	52066.65
255	144.04	2226.99	51924.85
256	158.25	2220.7	51772.96
257	287.87	2209.13	51492.32
258	192.77	2201.4	51303.95
259	262.67	2190.91	51047.2
260	121.96	2186.14	50930.04
261	208.47	2177.92	50727.53
262	198.58	2170.04	50532.69
263	297.29	2158.4	50243.62
264	72.41	2155.54	50172.37
265	178.4	2148.54	49997.62
266	184.06	2141.32	49816.82
267	74.38	2138.42	49744.05
268	285.23	2127.37	49465.96
269	279.15	2116.53	49191.94
270	166.23	2110.14	49029.86
271	254.41	2100.32	48779.99
272	89.64	2096.84	48691.21
273	214.75	2088.64	48481.56
274	120.77	2084.03	48363.42
275	168.81	2077.58	48197.79

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NO	Oil Pumped Out/L	Displayed Height /mm	Displayed Volume/L
276	272.92	2067.14	47928.87
277	103.92	2063.17	47826.34
278	131.94	2058.14	47696.24
279	181.74	2051.3	47518.96
280	142.11	2045.92	47379.23
281	264.62	2035.94	47119.35
282	313.84	2024.06	46808.88
283	96.66	2020.47	46714.83
284	116.16	2016.11	46600.46
285	239.12	2007.08	46363.09
286	154.5	2001.33	46211.59
287	314.56	1989.59	45901.47
288	316.03	1977.87	45590.8
289	226.29	1969.43	45366.43
290	285.9	1958.83	45083.88
291	163.75	1952.81	44923.05
292	224.69	1944.49	44700.35
293	321.69	1932.64	44382.33
294	205.69	1925.05	44178.12
295	309.66	1913.71	43872.31
296	249.73	1904.51	43623.6
297	186.43	1897.67	43438.33
298	231.42	1889.27	43210.41
299	297.79	1878.4	42914.84
300	109.19	1874.41	42806.16
301	162.87	1868.46	42643.93
302	328.52	1856.54	42318.32
303	166.13	1850.51	42153.3
304	237.66	1841.92	41917.87
305	303.97	1830.91	41615.55
306	330.11	1818.96	41286.7
307	235.69	1810.42	41051.26
308	86.15	1807.34	40966.26
309	66.88	1804.98	40901.09
310	225.69	1796.8	40675.04
311	213.49	1789.13	40462.79
312	323.39	1777.51	40140.74
313	263.59	1768.05	39878.12
314	240.84	1759.42	39638.21
315	201.31	1752.19	39437
316	128.8	1747.59	39308.87
317	324.38	1735.97	38984.85
318	206.56	1728.57	38778.25
319	64.47	1726.26	38713.71
320	251.13	1717.3	38463.22
321	201.55	1710.1	38261.74
322	72.35	1707.51	38189.22
323	305.18	1696.61	37883.79

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NO	Oil Pumped Out/L	Displayed Height /mm	Displayed Volume/L
324	148.15	1691.31	37735.15
325	120.83	1687.01	37614.49
326	88.98	1683.87	37526.35
327	142.33	1678.77	37383.13
328	121.05	1674.48	37262.6
329	240.4	1665.96	37023.08
330	75.05	1663.3	36948.26
331	133.08	1658.55	36814.61
332	134.22	1653.73	36678.94
333	303.81	1642.93	36374.74
334	181.64	1636.48	36192.94
335	268.23	1626.92	35923.33
336	226.45	1618.89	35696.73
337	276.49	1609.06	35419.17
338	87.44	1605.92	35330.47
339	331.77	1594.13	34997.3
340	293.8	1583.65	34700.98
341	72.05	1581.14	34629.99
342	187.88	1574.48	34441.58
343	148.35	1569.22	34292.75
344	233.46	1560.92	34057.83
345	120.98	1556.62	33936.1
346	220.76	1548.82	33715.25
347	224.72	1540.79	33487.84
348	224.46	1532.79	33261.23
349	183.46	1526.3	33077.38
350	65.53	1523.95	33010.8
351	200.46	1516.81	32808.51
352	170.84	1510.73	32636.23
353	86.76	1507.65	32548.96
354	187.61	1501.06	32362.23
355	181.73	1494.55	32177.83
356	210.27	1487.03	31964.75
357	282.54	1476.98	31680
358	253.04	1467.97	31424.74
359	300.69	1457.25	31121.1
360	70.11	1454.73	31049.73
361	118.47	1450.53	30930.79
362	185.56	1443.93	30743.92
363	323.97	1432.35	30416.13
364	277.01	1422.45	30136.01
365	181.51	1415.93	29951.59
366	149.46	1410.6	29800.87
367	72.2	1408.01	29727.64
368	262.95	1398.6	29461.68
369	197.9	1391.53	29261.94
370	111.01	1387.53	29148.98
371	174.85	1381.26	28971.96

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NO	Oil Pumped Out/L	Displayed Height /mm	Displayed Volume/L
372	103.94	1377.57	28867.81
373	263.82	1368.05	28599.24
374	157.63	1362.37	28439.09
375	317.84	1350.95	28117.3
376	59.23	1348.78	28056.19
377	286.14	1338.49	27766.55
378	229.92	1330.22	27533.96
379	204.09	1322.84	27326.55
380	235.19	1314.33	27087.57
381	255.78	1305.06	26827.49
382	80.7	1302.12	26745.05
383	297.81	1291.34	26443.03
384	58.55	1289.2	26383.11
385	134.84	1284.26	26244.87
386	105.09	1280.46	26138.58
387	309.59	1269.19	25823.64
388	74.59	1266.51	25748.82
389	213.87	1258.7	25530.91
390	228.59	1250.33	25297.63
391	232.85	1241.82	25060.73
392	291.52	1231.17	24764.66
393	69.58	1228.61	24693.57
394	274.83	1218.44	24411.4
395	199.23	1211.15	24209.41
396	240.5	1202.21	23962.04
397	114.57	1198.09	23848.16
398	199.32	1190.65	23642.72
399	243.63	1181.63	23394
400	312.12	1170.06	23075.57
401	172.18	1163.62	22898.62
402	77.28	1160.77	22820.38
403	68.22	1158.19	22749.6
404	222.45	1149.9	22522.38
405	267.6	1139.94	22249.89
406	238.23	1131.02	22006.33
407	143.68	1125.57	21857.74
408	303.6	1114.1	21545.6
409	191.38	1106.9	21350.06
410	304.75	1095.36	21037.34
411	71.71	1092.63	20963.48
412	108.7	1088.56	20853.45
413	254.23	1078.81	20590.32
414	293.8	1067.66	20290.19
415	255.77	1057.84	20026.55
416	129.62	1052.88	19893.65
417	91.24	1049.36	19799.43
418	273.31	1038.89	19519.72
419	252.4	1029.04	19257.31

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NO	Oil Pumped Out/L	Displayed Height /mm	Displayed Volume/L
420	122.34	1024.33	19132.09
421	109.36	1020.05	19018.44
422	134.35	1014.84	18880.29
423	265.55	1004.54	18607.78
424	261.97	994.32	18338.22
425	196.85	986.62	18135.68
426	195.02	978.9	17933.11
427	122.92	974.04	17805.83
428	226.9	965.05	17570.93
429	252.54	955.08	17311.22
430	159.7	948.67	17144.71
431	162.52	942.21	16977.27
432	166.93	935.55	16805.03
433	118.14	930.76	16681.41
434	218.17	922	16455.87
435	273.32	910.97	16172.9
436	271.65	899.88	15889.56
437	232.94	890.41	15648.56
438	112.97	885.81	15531.81
439	216.63	876.91	15306.54
440	80.75	873.59	15222.71
441	78.26	870.36	15141.26
442	94.48	866.45	15042.81
443	84.35	862.99	14955.81
444	189.36	855.14	14758.91
445	64.76	852.41	14690.59
446	244	842.18	14435.27
447	221.72	832.95	14205.87
448	267.55	821.69	13927.29
449	162.77	814.77	13756.79
450	200.98	806.22	13546.88
451	254.71	795.32	13280.49
452	171.33	787.93	13100.67
453	112.21	783.11	12983.73
454	271.64	771.35	12699.59
455	54.02	768.99	12642.77
456	121.5	763.72	12516.14
457	267.19	751.99	12235.51
458	128.44	746.34	12100.96
459	114.5	741.25	11980.09
460	72.75	738.04	11904.03
461	249.97	726.89	11640.87
462	74.75	723.53	11561.89
463	119.01	718.21	11437.14
464	242.18	707.24	11181.08
465	153.85	700.24	11018.54
466	179.45	692.09	10830.14
467	169.32	684.25	10649.77

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NO	Oil Pumped Out/L	Displayed Height /mm	Displayed Volume/L
468	196.36	675.27	10444.23
469	48.46	672.98	10391.99
470	157.16	665.7	10226.44
471	48.8	663.38	10173.84
472	219.44	653.11	9941.94
473	113.93	647.71	9820.62
474	220.43	637.24	9586.64
475	93.66	632.78	9487.47
476	163.22	624.97	9314.53
477	233.2	613.59	9064.22
478	49.86	611.11	9009.94
479	51.84	608.57	8954.45
480	47.19	606.36	8906.24
481	178.87	597.55	8714.85
482	161.73	589.56	8542.35
483	62.71	586.42	8474.84
484	199.66	576.46	8261.77
485	162.83	568.28	8087.99
486	51.86	565.58	8030.88
487	53.85	562.89	7974.1
488	66.32	559.53	7903.35
489	189.63	549.64	7696.21
490	58.51	546.71	7635.17
491	83.87	542.34	7544.4
492	84.74	537.9	7452.52
493	60.32	534.85	7389.6
494	199.47	524.27	7172.62
495	168.82	515.29	6990.02
496	174.06	505.95	6801.65
497	156.82	497.44	6631.41
498	131.25	490.28	6489.23
499	94.42	485.02	6385.39
500	156.18	476.4	6216.36
501	56.73	473.22	6154.36
502	60.81	469.74	6086.74

Appendix 2

1. A Matlab function for calculating the shadow area of the circular cross section.

```
function S=Srh(r,h)
%% A function for calculating the oil
%% area S of the circular cross section
%% with the radius r and oil level
%% height h.
%% ----------------------------------
if r==0 || h==0
    S=0;
else
    S=pi*r^2/2+r^2*asin((h-r)/r)+(h-r)*sqrt(2*r*h-h^2);
end
```

2. A Matlab function for calculating the retained oil while the tanks occurs a horizontal deflection angle α and the oil probe reading is h. If $\alpha = 0$, then $V = V(h)$.

```
function [V,ry,hy,Sy]=Volume(yl,yr,h,alpha)
%% A function for calculating the
%% retained oil V while the tanks
%% occurs a horizontal deflection
%% angle alpha and the oil probe
%% reading is h. If alpha=0, then
%% V=V(h).
%% ----------------------------------
%% [yl,yr], the left and right
%%          boundaries of the
%%          entire storage tanks
%% alpha, horizontal deflection angle
%% h, the readings of the oil probe
%% ry, a row of recording the radius
%%     of the circular cross sections
%% hy, a row of recording the oil level
%%     height of the circular cross
%%     sections
%% Sy, a row of recording the oil area
%%     of the circular cross sections
%% ----------------------------------
n=10000; % the number of the
         % frustums
dy=(yr-yl)/n; % the height of the
              % frustums
y=linspace(yl,yr,n+1);
R=1.5;   % the radius of the circular
         % cross section of the cylinder
         % part
H=1.0;   % H=L1-yl or H=L2-yr.
L1=-2.0; % the left boundary of the
         % cylinder part
L2=6.0;  % the right boundary of the
         % cylinder part
r=(H^2+R^2)/(2*H);
ry=zeros(1,n+1);
hy=zeros(1,n+1);
for i=1:n+1
    if y(i)<=L1
        ry(i)=sqrt(2*r*(y(i)-yl)-(y(i)-yl)^2);
        c=h-y(i)*tan(alpha)-R;
        if ry(i)>=abs(c)
            hy(i)=h-y(i)*tan(alpha)-R+ry(i);
        else
            if c<=0
                hy(i)=0;
            else
                hy(i)=2*ry(i);
            end
        end
```

```
    elseif y(i)>=L2
        ry(i)=sqrt(2*r*(yr-y(i))-(yr-y(i))^2);
        c=h-y(i)*tan(alpha)-R;
        if ry(i)>=abs(c)
            hy(i)=h-y(i)*tan(alpha)-R+ry(i);
        else
            if c<=0
                hy(i)=0;
            else
                hy(i)=2*ry(i);
            end
        end
    else
        ry(i)=R;
        hy(i)=h-y(i)*tan(alpha);
        if hy(i)>=2*R;
            hy(i)=2*R;
        elseif hy(i)<=0
            hy(i)=0;
        end
    end
end
Sy=zeros(1,n+1);
for i=1:n
        Sy(i)=Srh(ry(i),hy(i));
end
V=dy*sum(Sy);
```

3. A Matlab function for evaluating the value of $F(\alpha, \beta)$. Moreover, the function has some other outputs which are also very important for us.

```
function [Vtotal, DeltaV, rtol, maxtol, minS] = mainfun(h, OutV, alpha, beta)
%% A Matlab function for evaluating the
%% value of F(alpha, beta). Moreover,
%% the function has some other outputs
%% which are also very important for us.
%% -----------------------------------
%% alpha, the horizontal deflection
%%        angle
%% beta, the vertical deflection angle
%% h, a column about the oil level
%%    height
%% OutV, a column about the volume of
%%       the oil pumped out
%% Vtotal, a column of the retained
%%         volume corresponding with
%%         the oil level height h
%% DeltaV, a column about the change of
%%         the retained oil when the oil
%%          level height changed
%% rtol, the relative error,
%%       (DeltaV-OutV)/OutV*100%
%% maxtol, the maximum of the relative
%%         error
%% minS, the function value of
%%       F(alpha, beta)
%% -----------------------------------
m=length(h);
Vtotal=zeros(1,m);
DeltaV=zeros(1,m)';
rtol=zeros(1,m)';
yl=-3;
yr=7;
alpha=alpha/180;
beta=beta/180;
R=1.5;
h=R+(h./1000-R).*cos(beta);
for i=1:m
    Vtotal(i)=Volume(yl,yr,h(i),alpha);
end
Vtotal=Vtotal'*1000;
DeltaV(2:m)=Vtotal(1:m-1)-Vtotal(2:m);
rtol(2:m)=(DeltaV(2:m)-OutV(2:m))./OutV(2:m).*100;
minS=sum((DeltaV(2:m)-OutV(2:m)).^2);
maxtol=max(abs(rtol));
```

4. A Matlab function with the ideal of Hooke-Jeeves Pattern Search Algorithm. It will give a searching trace of α and β, and record down the trace of the corresponding value of $F(\alpha, \beta)$.

```
function [xstar,traceMinS]= patternsearchmethod(h,OutV,x0)
%% A Matlab function with the ideal of
%% Hooke-Jeeves Pattern Search
%% Algorithm.It will give a searching
%% trace of alpha and beta, and record
%% down the trace of the corresponding
%% value of F(alpha, beta).
%% -----------------------------------
%% h, a column about the oil level
%%    height
%% OutV, a column about the volume of
%%       the oil pumped out
%% x0, the first component is alpha,
%%     the second component is beta.
%%     The initial base point
%% xstar, the searching trace of alpha
%%        and beta
%% traceMinS, the trace of the
%%            corresponding value
%%            of F(alpha, beta).
%% -----------------------------------
delta=zeros(100,1);  % a column about
                     %the change of the
                     %step length
delta(1)=1;
sigma=0.1;    % the contractive factor
epsilon=0.1;  % the maximum margin of
              % error
X=zeros(100,2);
X(1,:)=x0;
traceMinS=zeros(100,1);
[Vtotal,DeltaV,rtol,maxtol,minS] =mainfun(h,OutV,X(1,1),X(1,2));
traceMinS(1)=minS;
E=[1,0;0,1];
```

```
k=1;
while k<=100
    y=X(k,:);
    fy=traceMinS(k);
    j=1;
    while j<=2
        t=y+delta(k)*E(j,:);
        [Vtotal,DeltaV,rtol,maxtol,minS] =
            mainfun(h,OutV,t(1),t(2));
        if minS<fy
            fy=minS;
            y=t;
        else
            t=y-delta(k)*E(j,:);
            [Vtotal,DeltaV,rtol,maxtol,minS
                ] =mainfun(h,OutV,t(1),t(2)
                );
            if minS<fy
                fy=minS;
                y=t;
            end
        end
        if j<2
            j=j+1;
        else
             X(k+1,:)=y;
             traceMinS(k+1)=fy;
             if traceMinS(k+1)<traceMinS(k)
                 y=2*X(k+1,:)-X(k,:);
                 delta(k+1)=delta(k);
                 k=k+1;
                 j=1;
             else
                 if delta(k)<epsilon
                     xstar=X;
                     return;
                 else
                     if X(k+1,:)==X(k,:)
                         delta(k+1) =sigma*
                             delta(k);
                         k=k+1;
                         break;
                     else
                         X(k+1,:)=X(k,:);
                         delta(k+1)=delta(k
                             );
                         k=k+1;
                         break;
                     end
                 end
             end
        end
    end
end
xstar=X;
```

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