The Bullwhip Effect caused by Information Distortion in a complex Supply Chain under Exponential Smoothing Forecast

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Abstract: The bullwhip effect is the phenomenon in which information on demand is transferred in the form of orders between the nodes of a supply chain tends to be distorted when it moves from downstream to upstream. In this paper, we measure the impact of bullwhip effect under Exponential Smoothing Forecast for a simple, two-stage supply chain which consists of one supplier and two retailers. And it is a simple replenishment system where a first-order autoregressive process describes the customer demand and an order-up-to inventory policy characterizes the replenishment decision. We get the influence of information distortion on the bullwhip effect through investigating the impacts of autoregressive coefficient, the lead-time, the smoothing parameter, market competition degree, and the consistency of demand volatility on the bullwhip effect by using algebraic analysis and numerical simulation. And, we also find the ways in which these parameters affect the bullwhip effect are different. Finally, we discuss some measures to mitigate the influence of information distortion on the bullwhip effect.

Key–Words: Information distortion, Bullwhip effect, Complex Supply chain, Two retailers model, Exponential Smoothing

1 Introduction

The bullwhip effect suggests that the demand variability is magnified as a customer demand signal is transformed through various stages of a serial supply chain. It is one of the most widely investigated phenomena in the modern supply chain management research. Information on demand flows have a direct impact on the production, scheduling, inventory control and delivery plans of individual members in the supply chain. Distortion in information flow is found to be a major problem.

Forrester [1] provided some of the first empirical evidence of the bullwhip effect and discusses its causes. After that, more and more researchers recognized the existence of this phenomenon in supply chains. Blanchard [2], Blinder [3], and Kahn [4] found evidence of inventory volatility which is similar to the bullwhip effect. Sterman [5] illustrated the bullwhip effect, and attributed the phenomenon to the players' irrational behavior through an experiment on the beer game. Lee et al [6,7] identified the use of demand forecasting, supply shortages, lead times, batch ordering, and price variations are the five main causes of the bullwhip effect. Since then, many researches on bullwhip effect with respect to the five main factors.

Using different forecasting methods would get different information on the demand, so it is important to investigate the influence of forecasting methods on the bullwhip effect. From past studies we could also find forecasting methods are considered as one of the most important causes because the inventory system of a supply chain is directly affected by the forecasting method in the five main factors of the bullwhip effect. Several researchers have examined the effects of simple forecasting techniques such as moving average or exponential smoothing on supply chains with autoregressive demand process. Chen et al [8,9] studied the impact of exponential smoothing forecasting technique on the bullwhip effect in a simple, two-stage supply chain with one supplier and one retailer. Then, Zhang [10] considered the impact of forecasting methods on the bullwhip effect for a simple replenishment system in which a first-order autoregressive process describes the customer demand and an order-up-to inventory policy characterizes the replenishment decision. And he also compared the impact of different forecasting methods such as the minimum mean-squared error (MMSE), exponential smoothing(ES) method and the moving average (MA) method on the bullwhip effect. Feng and Ma [11] did a Demand and Forecasting in Supply Chains Based on ARMA(1,1) Demand Process. Bayraktar et al [12] and Wright and Yuan [13] showed the impact of demand forecasting on the bullwhip effect and suggested appropriate forecast methods used in reducing the bullwhip effect through some simulation studies. Luong [14] developed an exact measure of bullwhip effect based on a replenishment model, which is similar to the one used by Chen et al.[8,9] for a two-stage supply chain with one retailer and one supplier. It was assumed that the retailer employs the base stock policy for replenishment and that demands are forecasted based on an AR(1) demand process. The behavior of the bullwhip effect was demonstrated for different values of the autoregressive coefficient and the lead time. Nepal et al [15] presented an analysis of the bullwhip effect and net-stock amplification in a three-echelon supply chain. Zotteri et al [16] analyzed the empirical demand data for fast moving consumer goods to measure the bullwhip effect. Najafi and Farahani [17] investigated the effects of various forecasting methods, such as moving average (MA), exponential smoothing (ES) and linear regression (LR) on the bullwhip effect in a four-echelon supply chain. Jaipuria, S and Mahapatra, SS[18] put forward an improved demand forecasting method to reduce bullwhip effect in supply chains. Shan et al. [19] did an empirical study of the bullwhip effect in China using data on over 1200 companies listed on the Shanghai and Shenzhen stock exchanges from 2002 to 2009. Fang and Ma [20] studied Hyperchaotic dynamic of Cournot-Bertrand duopoly game with multi-product and chaos control. Junhai Ma and Aiwen Ma [21] did a research on Supply Chain Coordination for substitutable products in competitive models.

This paper continues to study the role of forecasting in relation to the bullwhip effect, but under a new supply chain which has two retailers is different from the supply chain model usually has only one retailer in previous researches among those papers above. Here, we assume the two retailers both employ the AR(1) demand process and use the ES forecasting method. Our research not only determines an exact measure of the bullwhip effect, but also analyzes the impact of every parameter on the bullwhip effect. We have researched how every parameter affect the bullwhip effect and the numerical analysis has been given for every pattern followed by some parameters.

To quantify the increase in variability from the retailer to the supplier, we first consider a simple two-stage supply chain consists of a single supplier and two retailers. In section 2 we describe the supply chain model. In section 3, the bullwhip effect measured under the ES forecasting method is derived. In section 4, we analyze the impact of every parameter on the bullwhip effect and give the corresponding numerical analysis. Finally, we conclude in Section 5 with a discussion of the managerial insights provided by our results to mitigate the influence of information distortion on the bullwhip effect.

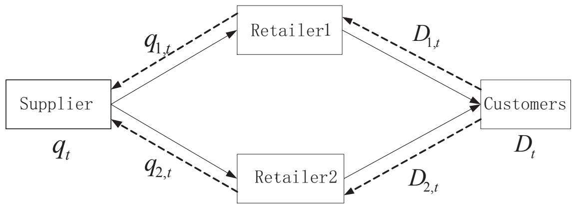

Figure 1: The supply chain model

2 Supply chain model

2.1 Replenishment model

In this research, we consider a two-stage supply chain with one supplier and two retailers as shown in Figure 1. The two retailers face customer demands and place orders to the supplier respectively. We consider that retailer1 faces an AR(1) demand model

$$D_{1,t} = \xi_1 + \theta_1 D_{1,t-1} + \varepsilon_{1,t} \tag{1}$$

where $D_{1,t}$ is the demand during period t; ξ_1 is a constant that determines the mean of the demand, $\varepsilon_{1,t}$ is a normally distributed random error with mean 0 and variance σ_1^2; and θ_1 is the first-order autocorrelation coefficient, where $-1 < \theta_1 < 1$.

For the first-order autoregressive process to be stationary, we must have

$$\begin{aligned} E(D_{1,t}) &= E(D_{1,t-1}) = \tfrac{\xi_1}{1-\theta_1} \\ Var(D_{1,t}) &= Var(D_{1,t-1}) = \tfrac{\xi_1^2}{1-\theta_1^2} \end{aligned} \tag{2}$$

Similarly, we assume retailer 2 also faces an AR(1) demand model so we have

$$\begin{aligned} D_{2,t} &= \xi_2 + \theta_2 D_{2,t-1} + \varepsilon_{2,t} \\ E(D_{2,t}) &= E(D_{2,t-1}) = \tfrac{\xi_2}{1-\theta_2} \\ Var(D_{2,t}) &= Var(D_{2,t-1}) = \tfrac{\xi_2^2}{1-\theta_2^2} \end{aligned} \tag{3}$$

2.2 Inventory policy

To supply the demand, the two retailers are assumed to follow a simple order-up-to policy in which the order-up-to level is determined to achieve a desired service level. Here, the service level is defined as the probability of meeting the demand with an on hand inventory during the lead time. In each period t, the retailer1 observes his inventory level and places an order

$q_{1,t}$ to the supplier. After the order is placed, the retailer 1 observes and fills customer demand for that period, denoted by $D_{1,t}$. Any unfilled demands are backlogged. At the beginning of period t, the retailer places an order of quantity $q_{1,t}$ to the supplier. The order quantity $q_{1,t}$ can be given as

$$q_{1,t} = S_{1,t} - S_{1,t-1} + D_{1,t-1} \tag{4}$$

where $S_{1,t}$ is the order-up-to lever of retailer 1 at period t , and it can be determined by the lead-time demand as

$$S_{1,t} = \widehat{D}_{1,t}^{L_1} + z\widehat{\sigma}_{1,t}^{L_1} \tag{5}$$

where $\widehat{D}_{1,t}^{L_1}$ is the forecast for the lead-time demand of retailer 1 which depends on the forecasting method and L_1, from Zhang[10]$\widehat{\sigma}_{1,t}^{L_1}$ remains constant over time. Hence $\widehat{\sigma}_{1,t}^{L_1}$ is the same to $\widehat{\sigma}_{1,t-1}^{L_1}$. z is the normal z score determined by the desired service level. Combining Eqs. (4) and (5), we have

$$q_{1,t} = (\widehat{D}_{1,t}^{L_1} - \widehat{D}_{1,t-1}^{L_1}) + z(\widehat{\sigma}_{1,t}^{L_1} - \widehat{\sigma}_{1,t-1}^{L_1}) + D_{1,t-1} \tag{6}$$

In this paper we consider the use of an exponential smoothing forecast to estimate $\widehat{D}_{1,t}^{L_1}$ for the demand process, first-order autoregressive process. We start by considering the exponential smoothing forecast.

2.3 Exponential smoothing forecast

The ES method is an adaptive algorithm in which one-period-ahead forecast is adjusted with a fraction of the forecasting error. Let α denote the fraction used in this process, also called the smoothing factor, then ES forecast can be written as

$$\widehat{D}_t = \alpha D_{t-1} + (1-\alpha)\widehat{D}_{t-1} \tag{7}$$

Performing recursive substitutions in the above equation, we arrive at an alternative expression for the one-period-ahead forecast:

$$\widehat{D}_t = \sum_{i=0}^{\infty} \alpha(1-\alpha)^i D_{t-i-1} \tag{8}$$

Therefore, $\widehat{D}_t$ can be interpreted as the weighted average of all past demand with exponentially declining weights. The i-period-ahead demand forecast with ES method simply extends the one-period-ahead forecast similar to the moving average (MA) case where

$$\widehat{D}_{t+i} = \widehat{D}_{t+1}, i \geq 2 \tag{9}$$

and the lead-time demand forecast $\widehat{D}_t^L$ can be expressed as

$$\widehat{D}_t^L = \sum_{i=1}^{L} \widehat{D}_{t+i} = L\widehat{D}_{t+1} \tag{10}$$

3 The measure of the bullwhip effect under the ES forecasting method

According to Zhang [10], we know the order quantity of retailer 1 at period t under the ES forecasting method is

$$\begin{aligned} q_{1,t} &= (\widehat{D}_{1,t}^{L_1} - \widehat{D}_{1,t-1}^{L_1}) + z(\widehat{\sigma}_{1,t}^{L_1} - \widehat{\sigma}_{1,t-1}^{L_1}) + D_{1,t-1} \\ &= D_{1,t} + \alpha_1 L_1 (D_{1,t} - \widehat{D}_{1,t}) \end{aligned} \tag{11}$$

where α_1 is the smoothing exponent of retailer1, and $\widehat{D}_{1,t}$ is the forecast of the demand at period t for retailer 1. The same, the order quantity of retailer 2 at period t under the ES forecasting method is

$$\begin{aligned} q_{2,t} &= (\widehat{D}_{2,t}^{L_2} - \widehat{D}_{2,t-1}^{L_2}) + z(\widehat{\sigma}_{2,t}^{L_2} - \widehat{\sigma}_{2,t-1}^{L_2}) + D_{2,t-1} \\ &= D_{2,t} + \alpha_2 L_2 (D_{2,t} - \widehat{D}_{2,t}) \end{aligned} \tag{12}$$

where α_2 is the smoothing exponent of retailer 2, and $\widehat{D}_{2,t}$ is the forecast of the demand at period t for retailer 2. Hence, total order quantity of two retailers at period t is

$$\begin{aligned} q_t &= q_{1,t} + q_{2,t} \\ &= D_{1,t} + \alpha_1 L_1 (D_{1,t} - \widehat{D}_{1,t}) \\ &\quad + D_{2,t} + \alpha_2 L_2 (D_{2,t} - \widehat{D}_{2,t}) \\ &= (1+\alpha_1 L_1) D_{1,t} - \alpha_1 L_1 \widehat{D}_{1,t} \\ &\quad + (1+\alpha_2 L_2) D_{2,t} - \alpha_2 L_2 \widehat{D}_{2,t} \end{aligned} \tag{13}$$

Taking the variance for Eq.(13), we get

$$\begin{aligned} &Var(q_t) \\ &= (1+\alpha_1 L_1)^2 Var(D_{1,t}) + (\alpha_1 L_1)^2 Var(\widehat{D}_{1,t}) \\ &+ (1+\alpha_2 L_2)^2 Var(D_{2,t}) + (1+\alpha_2 L_2)^2 Var(D_{2,t}) \\ &+ (\alpha_2 L_2)^2 Var(\widehat{D}_{2,t}) \\ &- 2\alpha_1 L_1 (1+\alpha_1 L_1) Cov(D_{1,t}, \widehat{D}_{1,t}) \\ &+ 2(1+\alpha_1 L_1)(1+\alpha_2 L_2) Cov(D_{1,t}, D_{2,t}) \\ &- 2\alpha_2 L_2 (1+\alpha_1 L_1) Cov(D_{1,t}, \widehat{D}_{2,t}) \\ &- 2\alpha_1 L_1 (1+\alpha_2 L_2) Cov(\widehat{D}_{1,t}, D_{2,t}) \\ &+ 2\alpha_1 L_1 \alpha_2 L_2 Cov(\widehat{D}_{1,t}, \widehat{D}_{2,t}) \\ &- 2\alpha_2 L_2 (1+\alpha_2 L_2) Cov(D_{2,t}, \widehat{D}_{2,t}) \end{aligned} \tag{14}$$

Proposition 1 *The variance of the total order quantity at period t under the ES forecasting method can be given as*

$$\begin{aligned} &Var(q_t) \\ &= (1 + \tfrac{1-\theta_1}{1-(1-\alpha_1)\theta_1}(2\alpha_1 L_1 + \tfrac{2\alpha_1^2 L_1^2}{2-\alpha_1})) Var(D_{1,t}) \\ &+ (1 + \tfrac{1-\theta_2}{1-(1-\alpha_2)\theta_2}(2\alpha_2 L_2 + \tfrac{2\alpha_2^2 L_2^2}{2-\alpha_2})) Var(D_{2,t}) \\ &+ (2(1+\alpha_1 L_1)(1+\alpha_2 L_2) - \tfrac{2\alpha_2^2 \theta_1 L_2 (1+\alpha_1 L_1)}{1-(1-\alpha_2)\theta_1} \\ &- \tfrac{2\alpha_1^2 \theta_2 L_1 (1+\alpha_2 L_2)}{1-(1-\alpha_1)\theta_2} \\ &+ \tfrac{2\alpha_1^2 \alpha_2^2 L_1 L_2 (1-\theta_1\theta_2(1-\alpha_1)(1-\alpha_2))}{(1-(1-\alpha_1)(1-\alpha_2))(1-(1-\alpha_1)\theta_2)(1-(1-\alpha_2)\theta_1)}) H \end{aligned} \tag{15}$$

where $H = \phi\sqrt{Var(D_{1,t})Var(D_{2,t})}$.

Proof. See the appendix.

For simplicity, Eq.(15) can be written as

$$\begin{aligned} &Var(q_t) = M_1 Var(D_{1,t}) + M_2 Var(D_{2,t}) \\ &+M_3\phi\sqrt{Var(D_{1,t})Var(D_{2,t})} \end{aligned} \tag{16}$$

where M_1 is the coefficient of $Var(D_{1,t})$, M_2 is the coefficient of $Var(D_{2,t})$, M_3 is the coefficient of $\phi\sqrt{Var(D_{1,t})Var(D_{2,t})}$. ϕ is the correlation coefficient between retailer 1 and retailer 2, representing the degree of market competition. The greater the absolute value of ϕ is, the fiercer the market competitive is. We assume two retailers face the same perfectly competitive market, and their demands present a negative correlation. So, we have $-1 \leq \phi \leq 0$.

The total demand which two retailers face is

$$D_t = D_{1,t} + D_{2,t} \tag{17}$$

The variance of D_t is determined as

$$\begin{aligned} &Var(D_t) = Var(D_{1,t} + D_{2,t}) \\ &= Var(D_{1,t}) + Var(D_{2,t}) + 2Cov(D_{1,t}, D_{2,t}) \end{aligned} \tag{18}$$

We know

$$Cov(D_{1,t}, D_{2,t}) = \phi\sqrt{Var(D_{1,t})Var(D_{2,t})} \tag{19}$$

Using Eq.(19) in Eq.(18), we can get

$$\begin{aligned} &Var(D_t) = Var(D_{1,t} + D_{2,t}) \\ &= Var(D_{1,t}) + Var(D_{2,t}) \\ &+2\phi\sqrt{Var(D_{1,t})Var(D_{2,t})} \end{aligned} \tag{20}$$

The bullwhip effect (BWE) suggests that the demand variability is magnified as a customer demand signal is transformed through various stages of a serial supply chain. The measure of bullwhip effect, which is defined as the ratio between variance of order quantity and variance of demand can be developed as follows:

$$\text{BWE} = \frac{Var(q_t)}{Var(D_t)}$$

So, from Eq.(16) and Eq.(20), the measure of the bullwhip under the ES forecasting method, can be determined as

$$\begin{aligned} &\text{BWE} = \tfrac{Var(q_t)}{Var(D_t)} \\ &= \tfrac{M_1 Var(D_{1,t}) + M_2 Var(D_{2,t}) + M_3\phi\sqrt{Var(D_{1,t})Var(D_{2,t})}}{Var(D_{1,t}) + Var(D_{2,t}) + 2\phi\sqrt{Var(D_{1,t})Var(D_{2,t})}} \\ &= \tfrac{M_1 + M_2\gamma^2 + M_3\phi\gamma}{1+\gamma+2\phi\gamma} \end{aligned} \tag{21}$$

where $\gamma = \sqrt{\frac{Var(D_{2,t})}{Var(D_{1,t})}}$, which means the consistency of demand volatility between two retailers.

4 Behavior of the bullwhip effect measure and numerical simulation

From the expression of the bullwhip effect, we know that it is a function with respect to model coefficients, lead-time of retailers, smoothing parameter, market competition degree, and the consistency of demand volatility between two retailers. In this section, algebraic analysis and numerical simulation will be done to investigate how those parameters affect the bullwhip effect.

4.1 The impact of autoregressive coefficients on the bullwhip effect

Proposition 2 *The measure of the bullwhip effect BWE processes the following properties on autoregressive coefficient* θ_1 *which is in (-1,1).*

(a) BWE is increasing when $\alpha_1 \doteq \alpha_2$ *and* $H > 1$*;*

(b) BWE is decreasing when $\alpha_1 \doteq \alpha_2$ *and* $0 < H < 1$*;*

(c) BWE is decreasing when $\alpha_1 > \alpha_2$ *and* $0 < H \leq \frac{\alpha_2}{\alpha_1}$*;*

(d) BWE is decreasing and then increasing when $\alpha_1 > \alpha_2$ *and* $\frac{\alpha_2}{\alpha_1} < H < \frac{2-\alpha_2}{2-\alpha_1}$*, the minimum value are obtained at* $\theta_1^* = \frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)}$*;*

(e) BWE is increasing when $\alpha_1 > \alpha_2$ *and* $H \geq \frac{2-\alpha_2}{2-\alpha_1}$*;*

(f) BWE is decreasing when $\alpha_1 < \alpha_2$ *and* $0 < H \leq \frac{2-\alpha_2}{2-\alpha_1}$*;*

(g) BWE is increasing and then decreasing when $\alpha_1 < \alpha_2$ *and* $\frac{2-\alpha_2}{2-\alpha_1} < H < \frac{\alpha_2}{\alpha_1}$*, the maximum value are obtained at* $\theta_1^* = \frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)}$*;*

(h) BWE is increasing when $\alpha_1 < \alpha_2$ *and* $H \geq \frac{\alpha_2}{\alpha_1}$*; where*

$$H = \frac{2\alpha_2^2 L_2(\alpha_1\alpha_2(1-L_1) - \alpha_1 - \alpha_2)\phi\gamma}{\alpha_1(1-(1-\alpha_1)(1-\alpha_2)(2\alpha_1 L_1 + \frac{2\alpha_1^2 L_1^2}{2-\alpha_1})}$$

Proof. Take the first derivative of Eq.(21) with respect to θ_1, we have

$$\frac{\partial \text{BWE}}{\partial \theta_1} = \frac{V - Y}{1+\gamma^2+2\phi\gamma}$$

where $V = \frac{2\alpha_2^2 L_2(\alpha_1\alpha_2(1-L_1)-\alpha_1-\alpha_2)\phi\gamma}{(1-(1-\alpha_1)(1-\alpha_2))(1-(1-\alpha_2)\theta_1)^2}$ and $Y = \frac{\alpha_1}{(1-(1-\alpha_1)\theta_1)^2}(2\alpha_1 L_1 + \frac{2\alpha_1^2 L_1^2}{2-\alpha_1})$.

When $\alpha_1 \doteq \alpha_2$, let $\frac{\partial \text{BWE}}{\partial \theta_1} \geq 0$ where $H > 1$. We can get $(H-1) \geq ((1-\alpha_1)H-(1-\alpha_2))\theta_1$. We at first consider the case in which $(1-\alpha_1)H-(1-\alpha_2) > 0$. In this case we have $\theta_1 < \frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)} = \frac{1}{1-\alpha_1}$. Since we know $\frac{1}{1-\alpha_1} = 1$, so we can get in the interval

$\theta_1 \in (-1, 1)$, $\frac{\partial BWE}{\partial \theta_1} \geq 0$. Hence, BWE is a increasing function. Then, we consider the case in which $(1-\alpha_1)H - (1-\alpha_2) < 0$, here $0 < H < 1$. We can get $(1-\alpha_1)H - (1-\alpha_2) > 0$. In this case we have $\theta_1 > \frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)} = \frac{1}{1-\alpha_1}$, and $\frac{1}{1-\alpha_1} = 1$, however we have $\theta_1 < 1$, so in this case α_1 is invalid. Then let $\frac{\partial BWE}{\partial \theta_1} < 0$, the same to the case in which $\frac{\partial BWE}{\partial \theta_1} \geq 0$, we can get the conclusion BWE is decreasing when $\alpha_1 \doteq \alpha_2$ and $0 < H < 1$;

Figure 2 presents the impact of θ_1 on the bullwhip effect when $\alpha_1 \doteq \alpha_2$. Those findings have a little difference from the results of other past researches. In past researches, for example, Luong [14] found the bullwhip effect is decreasing and then increasing as autoregressive coefficient magnifies and the way in which it impacts the bullwhip effect is simple and significant. And it only has one pattern to affect the bullwhip effect. However, in our paper, the way autoregressive coefficient impacts the bullwhip effect follows two patterns, not singular, and the bullwhip effect does not always decrease and then increase when the autoregressive coefficient increases.

When $\alpha_1 > \alpha_2$, it is noted that $\frac{\alpha_2}{\alpha_1} < \frac{2-\alpha_2}{2-\alpha_1} < \frac{1-\alpha_2}{1-\alpha_1}$. Let $\frac{\partial BWE}{\partial \theta_1} \geq 0$, we can get $(H-1) \geq ((1-\alpha_1)H - (1-\alpha_2))\theta_1$. Next, we consider the case in which $(1-\alpha_1)H-(1-\alpha_2) > 0$, hence $H > \frac{1-\alpha_2}{1-\alpha_1}$, in this case we can have $\theta_1 < \frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)}$. And, we have θ_1 is in the internal $(-1, 1)$; Then, we consider two cases

(1) $\frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)} \geq 1$ in that $H \geq \frac{\alpha_2}{\alpha_1}$, and because $\frac{\alpha_2}{\alpha_1} < \frac{2-\alpha_2}{2-\alpha_1}$, so we get when $H > \frac{\alpha_2}{\alpha_1}$, BWE is increasing in the interval $\theta_1 \in (-1, 1)$;

(2) $-1 < \frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)} < 1$ in that $\frac{2-\alpha_2}{2-\alpha_1} < H < \frac{\alpha_2}{\alpha_1}$, and because $\frac{\alpha_2}{\alpha_1} < \frac{2-\alpha_2}{2-\alpha_1}$, so the case is invalid; Then, we can easily know that the case in which $\frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)} \geq -1$ is invalid.

Next we consider the case in which $(1-\alpha_1)H - (1-\alpha_2) < 0$, hence $0 < H < \frac{1-\alpha_2}{1-\alpha_1}$, in this case we can have $\alpha_1 > \frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)}$. We also must consider the cases in which $\frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)} \geq 1$, $-1 < \frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)} < 1$ and $\frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)} \geq -1$ respectively, then we can get when $H \geq \frac{2-\alpha_2}{2-\alpha_1}$, in the interval (-1,1), BWE is increasing and when $\frac{\alpha_2}{\alpha_1} < H < \frac{2-\alpha_2}{2-\alpha_1}$, in the interval ($\frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)}$,1), BWE is increasing.

Then we let $\frac{\partial BWE}{\partial \theta_1} < 0$, we can get $(H-1) < ((1-\alpha_1)H-(1-\alpha_2))\theta_1$, the same to the case in which $\frac{\partial BWE}{\partial \theta_1} \geq 0$, we can get when $0 < H < \frac{\alpha_2}{\alpha_1}$, in the interval $\theta_1 \in (-1, 1)$, BWE is decreasing and when $\frac{\alpha_2}{\alpha_1} < H < \frac{2-\alpha_2}{2-\alpha_1}$, in the interval(-1,$\frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)}$),

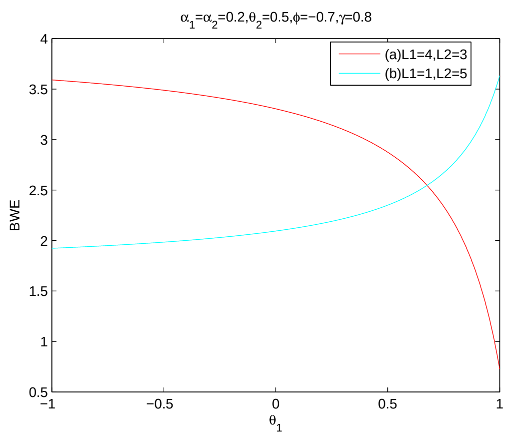

Figure 2: The impact of θ_1 on the bullwhip effect

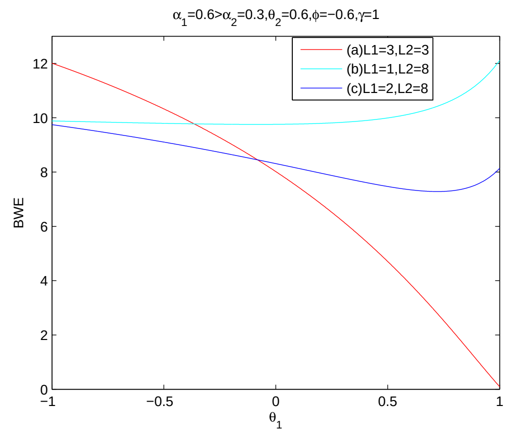

Figure 3: The impact of θ_1 on the bullwhip effect

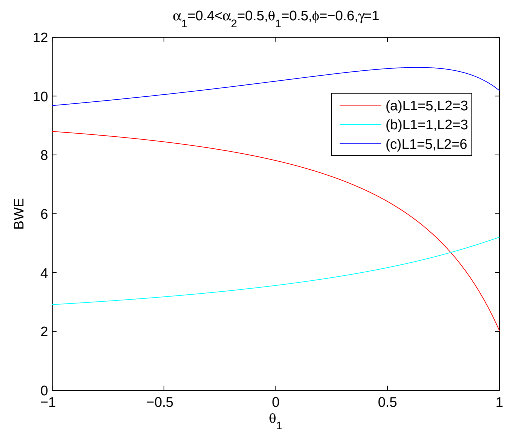

Figure 4: The impact of θ_1 on the bullwhip effect

BWE is decreasing.

So, in conclusion we can get BWE is decreasing when $\alpha_1 > \alpha_2$ and $0 < H \leq \frac{\alpha_2}{\alpha_1}$; BWE is decreasing and increasing when $\alpha_1 > \alpha_2$ and $\frac{\alpha_2}{\alpha_1} < H < \frac{2-\alpha_2}{2-\alpha_1}$; BWE is increasing when $\alpha_1 > \alpha_2$ and $H \geq \frac{2-\alpha_2}{2-\alpha_1}$.

When $\alpha_1 < \alpha_2$, the same to the case in which $\alpha_1 > \alpha_2$, we can also get BWE is decreasing when $\alpha_2 > \alpha_1$ and $0 < H \leq \frac{2-\alpha_2}{2-\alpha_1}$; BWE is increasing and decreasing when $\alpha_2 > \alpha_1$ and $\frac{2-\alpha_2}{2-\alpha_1} < H < \frac{\alpha_2}{\alpha_1}$, the maximum value is obtained at $\theta_1^* = \frac{H-1}{(1-\alpha_1)H-(1-\alpha_2)}$; BWE is increasing when $\alpha_2 > \alpha_1$ and $H \geq \frac{\alpha_2}{\alpha_1}$.

where $H = \frac{2\alpha_2^2 L_2(\alpha_1\alpha_2(1-L_1)-\alpha_1-\alpha_2)\phi\gamma}{\alpha_1(1-(1-\alpha_1)(1-\alpha_2)(2\alpha_1 L_1+\frac{2\alpha_1^2 L_1^2}{2-\alpha_1})}$.

Figure 3-4 present the impact of θ_1 on the bullwhip effect when $\alpha_1 > \alpha_2$ and $\alpha_1 < \alpha_2$ respectively. The way in which the autoregressive coefficients affect the bullwhip effect is obvious. So, we can conclude it is workable to adjust the bullwhip effect by controlling the autoregressive coefficients. □

Proposition 3 *The measure of the bullwhip effect BWE processes the following properties on autoregressive coefficient θ_2 which is in $(-1, 1)$*

(a) BWE is increasing when $\alpha_2 \doteq \alpha_2$ and $H > 1$;

(b) BWE is decreasing when $\alpha_2 \doteq \alpha_2$ and $0 < H < 1$;

(c) BWE is decreasing when $\alpha_1 > \alpha_2$ and $0 < H \leq \frac{2-\alpha_1}{2-\alpha_2}$;

(d) BWE is increasing and then decreasing when $\alpha_1 > \alpha_2$ and $\frac{2-\alpha_1}{2-\alpha_2} < H < \frac{\alpha_1}{\alpha_2}$, the maximum value are obtained at $\theta_2^* = \frac{H-1}{(1-\alpha_2)H-(1-\alpha_1)}$;

(e) BWE is increasing when $\alpha_1 > \alpha_2$ and $H \geq \frac{\alpha_1}{\alpha_2}$;

(f) BWE is decreasing when $\alpha_1 < \alpha_2$ and $0 < H \leq \frac{\alpha_1}{\alpha_2}$;

(g) BWE is decreasing and then increasing when $\alpha_1 < \alpha_2$ and $\frac{\alpha_1}{\alpha_2} < H < \frac{2-\alpha_1}{2-\alpha_2}$, the minimum value are obtained at $\theta_2^* = \frac{H-1}{(1-\alpha_1)H-(1-\alpha_1)}$;

(h) BWE is increasing when $\alpha_1 < \alpha_2$ and $H \geq \frac{2-\alpha_1}{2-\alpha_2}$; where $H = \frac{2\alpha_1^2 L_1(\alpha_1\alpha_2(1-L_2)-\alpha_1-\alpha_2)\phi\gamma}{\alpha_2(1-(1-\alpha_1)(1-\alpha_2)(2\alpha_2 L_2+\frac{2\alpha_2^2 L_2^2}{2-\alpha_2})}$.

Proof. Take the first derivative of Eq.(21) with respect to θ_2, we have

$$\frac{\partial BWE}{\partial \theta_2} = \frac{P-Z}{1+\gamma^2+2\phi\gamma}$$

where $P = \frac{2\alpha_1^2 L_1(\alpha_1\alpha_2(1-L_2)-\alpha_1-\alpha_2)\phi\gamma}{(1-(1-\alpha_1)(1-\alpha_2))(1-(1-\alpha_1)\theta_2)^2}$, $Z = \frac{\alpha_2}{(1-(1-\alpha_2)\theta_2)^2}(2\alpha_2 L_2 + \frac{2\alpha_2^2 L_2^2}{2-\alpha_2})$

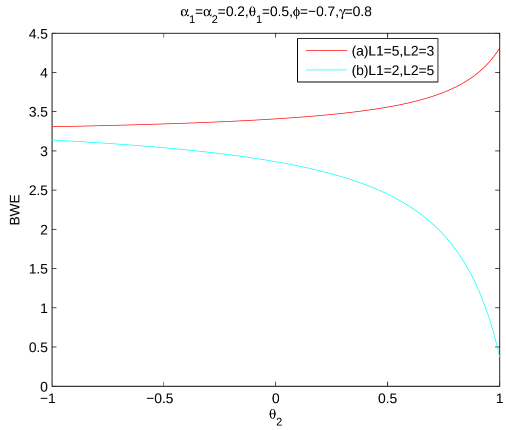

Figure 5: The impact of θ_2 on the bullwhip effect

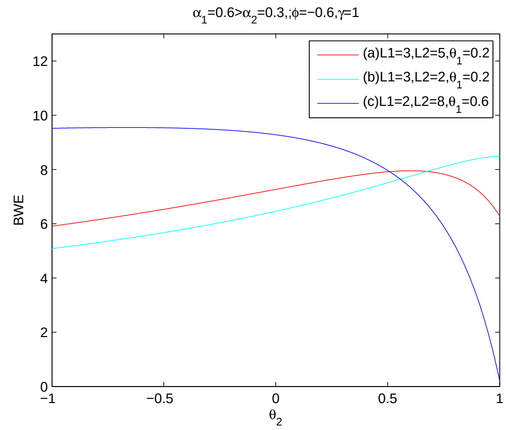

Figure 6: The impact of θ_2 on the bullwhip effect

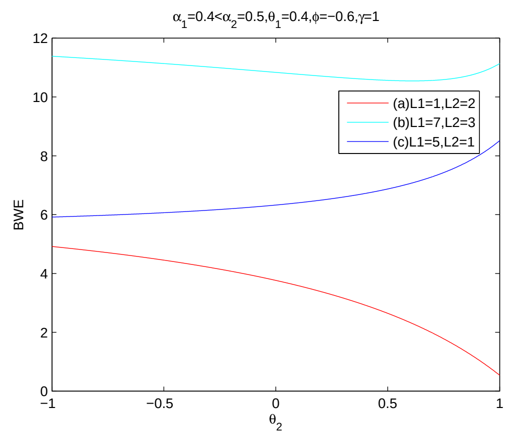

Figure 7: The impact of θ_2 on the bullwhip effect

The proof of the bullwhip effect BWE with respect to θ_2 is the same to the case in which the bullwhip effect BWE with respect to θ_1. In addition, according to the symmetry between θ_1 and θ_2. we can get the measure of the bullwhip effect BWE properties on autoregressive coefficient θ_2 in Proposition 3. $\square$

Figure 5-7 present the impact of θ_2 on the bullwhip effect when $\alpha_1 \doteq \alpha_2$, $\alpha_1 > \alpha_2$ and $\alpha_1 < \alpha_2$ respectively. θ_2 have the same significance to the bullwhip effect as θ_1.

4.2 The impact of lead-time on the bullwhip effect

Proposition 4 *The measure of the bullwhip effect BWE processes the following properties on the lead-time L_1 which is in $(0, +\infty)$.*

(a) BWE is increasing when $B \le 4\alpha_1 - 2\alpha_1^2$.

(b) BWE is decreasing and then increasing when $B > 4\alpha_1 - 2\alpha_1^2$, the minimum value is obtained at $L_1^* = \frac{B-4\alpha_1+2\alpha_1^2}{4\alpha_1^2}$, where
$$B = \phi\gamma S(2\alpha_1(1+\alpha_2 L_2) - \frac{2\alpha_1^2\theta_2(1+\alpha_2 L_2)}{(1-(1-\alpha_1)\theta_2)} - \frac{2\alpha_1\alpha_2^2\theta_1 L_2}{(1-(1-\alpha_2)\theta_1)}) + \phi\gamma S\frac{2\alpha_1^2\alpha_2^2 L_2(1-\theta_1\theta_2(1-\alpha_1)(1-\alpha_2))}{(1-(1-\alpha_1)(1-\alpha_2)(1-(1-\alpha_2)\theta_1)(1-(1-\alpha_1)\theta_2)}.$$
where $S = \frac{(1-(1-\alpha_1)\theta_1)(2-\alpha_1)}{1-\theta_1}$.

Proof. Take the first derivative of Eq. (21) with respect to L_1, we have
$$\frac{\partial BWE}{\partial L_1} = \frac{(2\alpha_1(1+\alpha_2 L_2) - \frac{2\alpha_1^2\theta_2(1+\alpha_2 L_2)}{(1-(1-\alpha_1)\theta_2)} - \frac{2\alpha_1\alpha_2^2\theta_1 L_2}{(1-(1-\alpha_2)\theta_1)})\phi\gamma}{1+\gamma^2+2\phi\gamma} + \frac{(2\alpha_1+\frac{4\alpha_1^2 L_1}{2-\alpha_1})\frac{1-\theta_1}{(1-(1-\alpha_1)\alpha_1)}}{1+\gamma^2+2\phi\gamma} + \frac{\frac{2\alpha_1^2\alpha_2^2 L_2(1-\theta_1\theta_2(1-\alpha_1)(1-\alpha_2))}{(1-(1-\alpha_1)(1-\alpha_2)(1-(1-\alpha_2)\theta_1)(1-(1-\alpha_1)\theta_2)})\phi\gamma}{1+\gamma^2+2\phi\gamma}.$$

Let $\frac{\partial BWE}{\partial L_1} \ge 0$, we can get $L_1 \ge \frac{B-4\alpha_1+2\alpha_1^2}{4\alpha_1^2}$, here $L_1 \ge 0$. Hence, we consider the case in which $\frac{B-4\alpha_1+2\alpha_1^2}{4\alpha_1^2} \le 0$, we can have $B \le 4\alpha_1 - 2\alpha_1^2$. So, we can get when $B \le 4\alpha_1 - 2\alpha_1^2$, $\frac{\partial BWE}{\partial L_1} \le 0$ in that BWE is increasing in the interval$(0, +\infty)$. Next, we consider $\frac{B-4\alpha_1+2\alpha_1^2}{4\alpha_1^2} > 0$, we can have $B > 4\alpha_1 - 2\alpha_1^2$. In conclusion, we can get BWE is increasing in the interval $(\frac{B-4\alpha_1+2\alpha_1^2}{4\alpha_1^2}, +\infty)$.

Figure 8 presents the impact of L_1 on the bullwhip effect. Here, we know that the lead-time have two different ways to influence the bullwhip effect. In some cases, Reducing the lead-time has no meaning to the lower of bullwhip effect, so managers should control the bullwhip effect according to the actual situation. And, in comparison of the autoregressive coefficient, the lead-time is easier to be controlled to lower bullwhip effect.

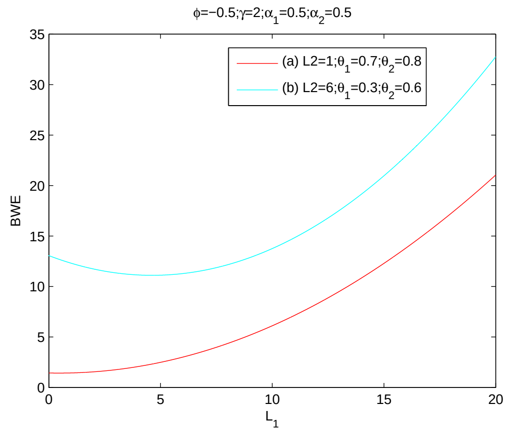

Figure 8: The impact of $Ł_1$ on the bullwhip effect

Then let $\frac{\partial BWE}{\partial L_1} < 0$, and we can have $L_1 < \frac{B-4\alpha_1+2\alpha_1^2}{4\alpha_1^2}$, here $L_1 \ge 0$. It is easily known that when $\frac{B-4\alpha_1+2\alpha_1^2}{4\alpha_1^2} \le 0$ in that $B \le 4\alpha_1 - 2\alpha_1^2$ is invalid for L_1 is in the interval $(0, +\infty)$. Next, we consider when $\frac{B-4\alpha_1+2\alpha_1^2}{4\alpha_1^2} > 0$ in that $B > 4\alpha_1 - 2\alpha_1^2$. we can get when L_1 is in the interval $(0, \frac{B-4\alpha_1+2\alpha_1^2}{4\alpha_1^2})$, $\frac{\partial BWE}{\partial L_1} < 0$ in that BWE is decreasing. $\square$

Proposition 5 *The measure of the bullwhip effect BWE processes the following properties on the lead-time L_2 which is in $(0, +\infty)$.*

(a) BWE is increasing when $C \le 4\alpha_2 - 2\alpha_2^2$.

(b) BWE is decreasing and then increasing when $C > 4\alpha_2 - 2\alpha_2^2$, the minimum value is obtained at $L_2^* = \frac{C-4\alpha_2+2\alpha_2^2}{4\alpha_2^2}$; where
$$C = \phi\gamma W(2\alpha_2(1+\alpha_1 L_1) - \frac{2\alpha_2^2\theta_1(1+\alpha_2 L_1)}{(1-(1-\alpha_2)\theta_1)} - \frac{2\alpha_2\alpha_1^2\theta_2 L_1}{(1-(1-\alpha_1)\theta_2)}) + \phi\gamma W\frac{2\alpha_2^2\alpha_1^2 L_1(1-\theta_2\theta_1(1-\alpha_1)(1-\alpha_2))}{(1-(1-\alpha_1)(1-\alpha_2)(1-(1-\alpha_1)\theta_2)(1-(1-\alpha_2)\theta_1)}.$$
where $W = \frac{(1-(1-\alpha_2)\theta_2)(2-\alpha_2)}{1-\theta_2}$.

Proof. Take the first derivative of Eq. (21) with respect to L_2, we have
$$\frac{\partial BWE}{\partial L_2} = \frac{(2\alpha_3(1+\alpha_1 L_1) - \frac{2\alpha_2^2\theta_1(1+\alpha_1 L_1)}{(1-(1-\alpha_2)\theta_1)} - \frac{2\alpha_2\alpha_1^2\theta_2 L_1}{(1-(1-\alpha_1)\theta_2)})\phi\gamma}{1+\gamma^2+2\phi\gamma} + \frac{(2\alpha_2+\frac{4\alpha_2^2 L_2}{2-\alpha_2})\frac{1-\theta_2}{(1-(1-\alpha_2)\theta_2)}}{1+\gamma^2+2\phi\gamma} + \frac{\frac{2\alpha_2^2\alpha_1^2 L_1(1-\theta_2\theta_1(1-\alpha_2)(1-\alpha_1))}{(1-(1-\alpha_2)(1-\alpha_1)(1-(1-\alpha_1)\theta_2)(1-(1-\alpha_2)\theta_1)})\phi\gamma}{1+\gamma^2+2\phi\gamma}$$

Figure 9 presents the impact of L_2 on the bullwhip effect. And those findings are similar with proposition4.

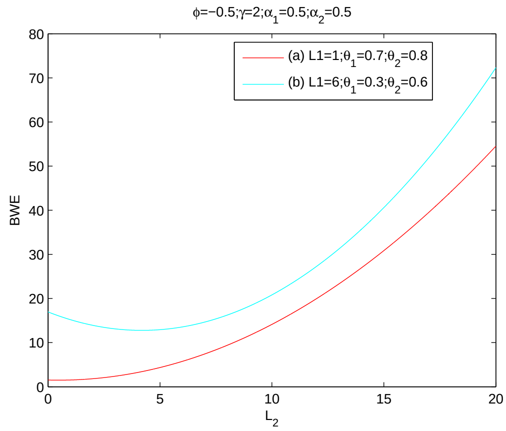

Figure 9: The impact of L_2 on the bullwhip effect

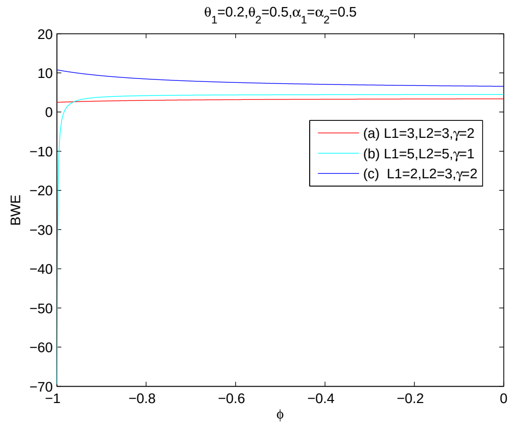

Figure 10: The impact of ϕ on the bullwhip effect

The proof of the bullwhip effect BWE with respect to L_2 is the same to the case in which the bullwhip effect BWE with respect to L_1. In addition, according to the symmetry between L_1 and L_2. □

4.3 The impact of ϕ on the bullwhip effect

Proposition 6 *The measure of the bullwhip effect BWE processes the following properties on the correlation coefficient between two retailers ϕ which is in [-1,0).*

(a) BWE is increasing when $(M_3 - 2M_2)\gamma^3 + (M_3 - 2M_1)\gamma > 0$.

(b) BWE is a fixed value when $(M_3 - 2M_2)\gamma^3 + (M_3 - 2M_1)\gamma \doteq 0$.

(c) BWE is decreasing when $(M_3 - 2M_2)\gamma^3 + (M_3 - 2M_1)\gamma < 0$.

Proof. Take the first derivative of Eq.(21) with respect to ϕ, we have

$$\frac{\partial BWE}{\partial \phi} = \frac{(M_3 - 2M_2)\gamma^3 + (M_3 - 2M_1)\gamma}{(1 + \gamma^2 + 2\phi\gamma)^2}$$

We know that $\gamma > 0$,so $(1 + \gamma^2 + 2\phi\gamma)^2 > 0$.

It is easily to known:

(1) $(M_3 - 2M_2)\gamma^3 + (M_3 - 2M_1)\gamma > 0, \frac{\partial BWE}{\partial \phi} > 0$, hence BWE is increasing;

(2) $(M_3 - 2M_2)\gamma^3 + (M_3 - 2M_1)\gamma \doteq 0, \frac{\partial BWE}{\partial \phi} \doteq 0$, hence BWE is a fixed value;

(3) $(M_3 - 2M_2)\gamma^3 + (M_3 - 2M_1)\gamma < 0, \frac{\partial BWE}{\partial \phi} < 0$, hence BWE is decreasing.

Figure 10 presents the impact of ϕ on the bullwhip effect. The bullwhip effect for ϕ is increasing or decreasing, and it will tend to be a stable value which decided by other parameters. So the way in which the degree of market competition affect the bullwhip effect is simple from managerial point of view.

4.4 The impact of γ on the bullwhip effect

Proposition 7 *The measure of the bullwhip effect BWE processes the following properties on the consistency of demand volatility between two retailers γ which is in $(0, +\infty)$.*

(a) $\lim_{\gamma \to \infty} BWE = M_2$.

(b) BWE is decreasing when $(2M_2 - M_3) \doteq 0$, $(M_2 - M_1) \doteq 0$ *and* $(2M_1 - M_3) \geq 0$.

(c) BWE is increasing when $(2M_2 - M_3) \doteq 0$, $(M_2 - M_1) \doteq 0$ *and* $(2M_1 - M_3) < 0$.

(d) BWE is increasing when $(2M_2 - M_3) \doteq 0$, $(M_2 - M_1)(2M_1 - M_3) \geq 0$.

(e) BWE is decreasing and then increasing when $(2M_2 - M_3) \doteq 0$, $(M_2 - M_1)(2M_1 - M_3) < 0$, *the minimum value is obtained at* $\gamma^* = \frac{(2M_1 - M_3)\phi}{2(M_2 - M_1)}$.

(f) BWE is increasing when $(2M_2 - M_3) < 0$, $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) \leq 0$.

(g) BWE is decreasing when $(2M_2 - M_3) > 0$, $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) \leq 0$.

(h) BWE is decreasing when $(2M_2 - M_3) > 0$, $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) > 0$, $(M_3 - 2M_1) > 0$ *and* $(M_2 - M_1) < 0$.

(i) BWE is decreasing, increasing and then decreasing when $(2M_2 - M_3) > 0$, $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) > 0$, $(M_3 - 2M_1) > 0$ *and* $(M_2 - M_1) > 0$, *the minimum value is obtained at* $\gamma^* = \frac{(M_1 - M_2) + \sqrt{(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1)}}{(2M_2 - M_3)\phi}$ *and the maximum value is obtained at* $\gamma^* = \frac{(M_1 - M_2) - \sqrt{(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1)}}{(2M_2 - M_3)\phi}$.

(j) BWE is increasing and then decreasing when $(2M_2 - M_3) > 0$, $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) > 0$, $(M_3 - 2M_1) < 0$, *the maximum value is obtained at* $\gamma^* = \frac{(M_1 - M_2) - \sqrt{(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1)}}{(2M_2 - M_3)\phi}$.

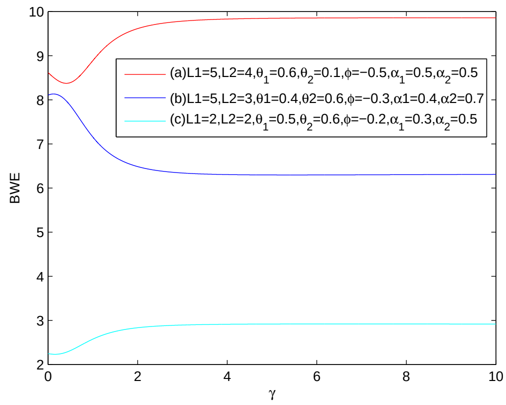

Figure 11: The impact of γ on the bullwhip effect

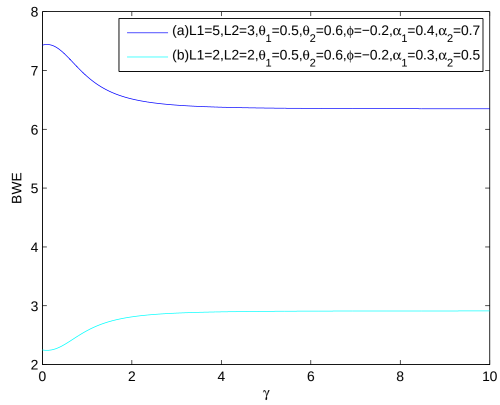

Figure 12: The impact of γ on the bullwhip effect

(k) BWE is decreasing and then increasing when $(2M_2 - M_3) < 0$, $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) > 0$, $(M_2 - M_1) > 0$ *and* $(M_3 - 2M_1) > 0$, *the minimum value is obtained at* $\gamma^* = \frac{(M_1-M_2)+\sqrt{(M_1-M_2)^2-\phi^2(2M_2-M_3)(M_3-2M_1)}}{(2M_2-M_3)\phi}$.

(l) BWE is increasing, decreasing and then increasing when $(2M_2 - M_3) < 0$, $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) > 0$, $(M_3 - 2M_1) < 0$ *and* $(M_2 - M_1) < 0$, *the maximum value is obtained at* $\gamma^* = \frac{(M_1-M_2)-\sqrt{(M_1-M_2)^2-\phi^2(2M_2-M_3)(M_3-2M_1)}}{(2M_2-M_3)\phi}$ *and the minimum value is obtained at* $\gamma^* = \frac{(M_1-M_2)+\sqrt{(M_1-M_2)^2-\phi^2(2M_2-M_3)(M_3-2M_1)}}{(2M_2-M_3)\phi}$.

(m) BWE is increasing when $(2M_2 - M_3) < 0$, $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) > 0$, $(M_3 - 2M_1) < 0$ *and* $(M_2 - M_1) > 0$.

Proof. According to L'Hospital principle, we easily know that
$\lim\limits_{\gamma\to\infty} BWE = \lim\limits_{\gamma\to\infty} \frac{M_1+M_2\gamma^2+M_3\phi\gamma}{1+\gamma^2+2\phi\gamma} = M_2$.
Then, Take the first derivative of Eq. 21 with respect to γ, we can have
$\frac{\partial BWE}{\partial\gamma} = \frac{(2M_2-M_3)\phi\gamma^2+2(M_2-M_1)\gamma+(M_3-2M_1)\phi}{(1+\gamma^2+2\phi\gamma)^2}$.
When $(2M_2 - M_3) \doteq 0$, we can have $\frac{\partial BWE}{\partial\gamma} = \frac{2(M_2-M_1)\gamma+(M_3-2M_1)\phi}{(1+\gamma^2+2\phi\gamma)^2}$.

Let $\frac{\partial BWE}{\partial\gamma} \geq 0$ in that $2(M_2 - M_1)\gamma + (M_3 - 2M_1)\phi \geq 0$. So, we can have

(1) $(M_2 - M_1) \doteq 0$, $2M_1 - M_3 \geq 0$ when γ is in the interval $(0, +\infty)$;

(2) $(M_2 - M_1) > 0$, $2M_1 - M_3 \geq 0$ when γ is in the interval $(0, +\infty)$;

(3) $(M_2 - M_1)(2M_1 - M_3) < 0$ when γ is in the interval $(\frac{(2M_1-M_3)\phi}{2(M_2-M_1)}, +\infty)$.

Let $\frac{\partial BWE}{\partial\gamma} < 0$, the same we can have:

(1) $(M_2 - M_1) \doteq 0$, $2M_1 - M_3 < 0$ when γ is in the interval $(0, +\infty)$;

(2) $(M_2 - M_1)(2M_1 - M_3) < 0$ when γ is in the interval $(0, \frac{(2M_1-M_3)\phi}{2(M_2-M_1)})$.

Then we consider $(2M_2 - M_3) \neq 0$. Solving the equation $\frac{\partial BWE}{\partial\gamma} = 0$, and we consider the case in which $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) < 0$, the equation has no zero root. Hence, when $(2M_2 - M_3) > 0$, we have $\frac{\partial BWE}{\partial\gamma} < 0$ in that BWE is decreasing when γ is in the interval $(0, +\infty)$; and when $(2M_2 - M_3) < 0$, we have $\frac{\partial BWE}{\partial\gamma} > 0$ in that BWE is increasing when γ is in the interval $(0, +\infty)$.

However, when $(M_1 - M_2)^2 - \phi^2(2M_2 - M_3)(M_3 - 2M_1) \geq 0$, the equation has two zero roots:
$\gamma_1^* = \frac{(M_1-M_2)+\sqrt{(M_1-M_2)^2-\phi^2(2M_2-M_3)(M_3-2M_1)}}{(2M_2-M_3)\phi}$;
$\gamma_2^* = \frac{(M_1-M_2)-\sqrt{(M_1-M_2)^2-\phi^2(2M_2-M_3)(M_3-2M_1)}}{(2M_2-M_3)\phi}$.

So, we can see

(1) $(2M_2 - M_3) > 0$, $(M_3 - 2M_1) > 0$ and $(M_2 - M_1) < 0$, we have $\gamma_1^* < \gamma_2^* < 0$; hence we can get When γ is in the interval $(0, +\infty)$, $\frac{\partial BWE}{\partial\gamma} < 0$ in that BWE is decreasing function with respect to γ;

(2) $(2M_2 - M_3) > 0$, $(M_3 - 2M_1) > 0$ and $(M_2 - M_1) > 0$, we have $0 < \gamma_1^* < \gamma_2^*$; hence we can get when γ is in the interval $(0, \gamma_1^*)$, $\frac{\partial BWE}{\partial\gamma} < 0$ in that BWE is decreasing; in the interval (γ_1^*, γ_2^*), $\frac{\partial BWE}{\partial\gamma} > 0$ in that BWE is increasing; in the interval $(\gamma_2^*, +\infty)$, $\frac{\partial BWE}{\partial\gamma} < 0$ in that BWE is decreasing. BWE is decreasing, increasing and then decreasing function with respect to γ;

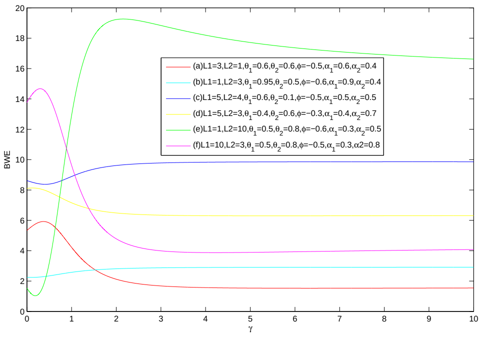

Figure 13: The impact of γ on the bullwhip effect

(3) $(2M_2 - M_3) > 0$ and $(M_3 - 2M_1) < 0$, we have $\gamma_2^* > 0$ and $0 > \gamma_1^* < 0$.

The same we can know when γ is in the interval $(0, \gamma_2^*)$, $\frac{\partial BWE}{\partial \gamma} > 0$ in that BWE is increasing; in the interval $(\gamma_2^*, +\infty)$, $\frac{\partial BWE}{\partial \gamma} < 0$ in that BWE is decreasing. BWE is increasing and then decreasing with respect to γ.

Next we can get these cases:

(1) $(2M_2 - M_3) < 0$ and $(M_3 - 2M_1) > 0$, we have $\gamma_2^* < 0$ and $\gamma_1^* > 0$; hence we can know when γ is in the interval $(0, \gamma_1^*)$, $\frac{\partial BWE}{\partial \gamma} < 0$ in that BWE is decreasing; in the interval $(\gamma_1^*, +\infty)$, $\frac{\partial BWE}{\partial \gamma} > 0$ in that BWE is increasing. So we know BWE is decreasing and then increasing with respect to γ;

(2) $(2M_2 - M_3) < 0$, $(M_3 - 2M_1) < 0$ and $(M_2 - M_1) < 0$, we have $0 < \gamma_2^* < \gamma_1^*$; hence we can get when γ is in the interval $(0, \gamma_2^*)$, $\frac{\partial BWE}{\partial \gamma} > 0$ in that BWE is increasing; in the interval (γ_2^*, γ_1^*), $\frac{\partial BWE}{\partial \gamma} < 0$ in that BWE is decreasing; in the interval $(\gamma_1^*, +\infty)$, $\frac{\partial BWE}{\partial \gamma} > 0$ in that BWE is increasing. So we know BWE is increasing, decreasing and then increasing with respect to γ;

(3) $(2M_2 - M_3) < 0$, $(M_3 - 2M_1) < 0$ and $(M_2 - M_1) > 0$, we have $\gamma_2^* < \gamma_1^* < 0$; hence we can get When γ is in the interval $(0, +\infty)$, $\frac{\partial BWE}{\partial \gamma} > 0$ in that BWE is increasing function with respect to γ.

Figure 11-13 illustrate that γ can have a significant impact on the variability and there is a very big difference between the variance of the orders in each case. The way in which γ affect the bullwhip effect is complex. And it is difficult to lower bullwhip effect through controlling the consistency of demand volatility between two retailers which is decided by the demand processes for managers. □

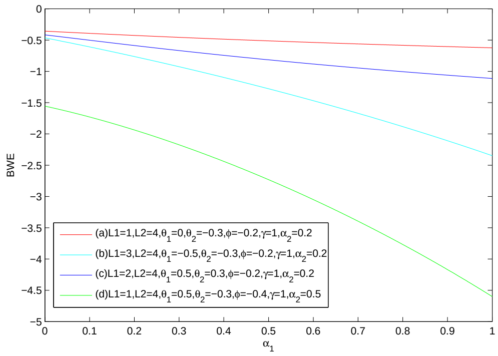

Figure 14: The impact of α_1 on the $\frac{M_3\phi\gamma}{1+\gamma^2+2\phi\gamma}$

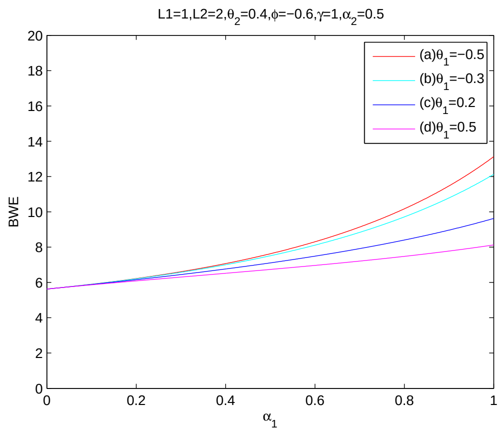

Figure 15: The impact of α_1 on the bullwhip effect with various values of θ_1.

4.5 The impact of the smoothing parameter on the bullwhip effect

Proposition 8 *The smoothing parameter is an important and complex parameter which affects the bullwhip effect.*

According to Eq.(21) we can know $BWE = \frac{Var(q_t)}{Var(D_t)} = \frac{M_1+M_2\gamma^2}{1+\gamma^2+2\phi\gamma} + \frac{M_3\phi\gamma}{1+\gamma^2+2\phi\gamma}$.
Here, we can know that $M_3 = (2(1+\alpha_1 L_1)(1+\alpha_2 L_2) - \frac{2\alpha_2^2\theta_1 L_2(1+\alpha_1 L_1)}{1-(1-\alpha_2)\theta_1} - \frac{2\alpha_1^2\theta_2 L_1(1+\alpha_2 L_2)}{1-(1-\alpha_1)\theta_2})$
$+ \frac{2\alpha_1^2\alpha_2^2 L_1 L_2(1-\theta_1\theta_2(1-\alpha_1)(1-\alpha_2))}{(1-(1-\alpha_1)(1-\alpha_2))(1-(1-\alpha_1)\theta_2)(1-(1-\alpha_2)\theta_1)}$.

It is easily know that $\frac{M_3\phi\gamma}{1+\gamma^2+2\phi\gamma}$ is decreasing when α_1 is in the interval (0,1). The maximum value of $\frac{M_3\phi\gamma}{1+\gamma^2+2\phi\gamma}$ is obtained at $\alpha_1 = 0$. We can get the maximum value is $(2(1+\alpha_2 L_2) - \frac{2\alpha_2^2\theta_1 L_2}{1-(1-\alpha_2)\theta_1})\phi\gamma < 0$. So we can have $\frac{M_3\phi\gamma}{1+\gamma^2+2\phi\gamma} < 0$.

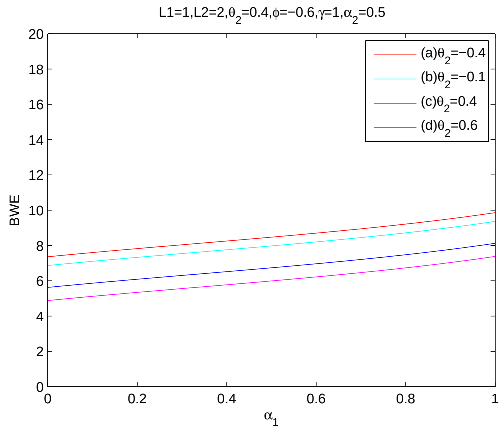

Figure 16: The impact of α_1 on the bullwhip effect with various values of θ_2.

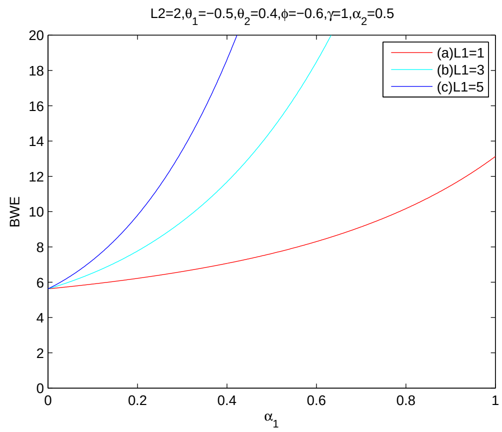

Figure 17: The impact of α_1 on the bullwhip effect with various values of Ł$_1$.

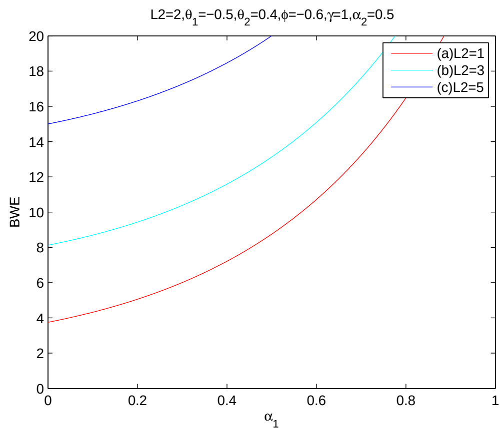

Figure 18: The impact of α_1 on the bullwhip effect with various values of Ł$_2$.

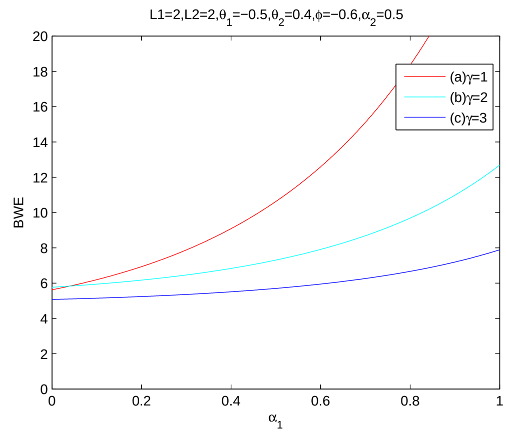

Figure 19: The impact of α_1 on the bullwhip effect with various values of γ.

Hence, $BWE = \frac{Var(q_t)}{Var(D_t)} < \frac{M_1+M_2\gamma^2}{1+\gamma^2+2\phi\gamma}$ in that

$$\frac{Var(q_t)}{Var(D_t)} < \frac{H+J}{(1+\gamma^2+2\phi\gamma)} \tag{22}$$

where $H = (1+\frac{1-\theta_1}{1-(1-\alpha_1)\theta_1}(2\alpha_1 L_1+\frac{2\alpha_1^2L_1^2}{2-\alpha_1}))$, $J = (1+\frac{1-\theta_2}{1-(1-\alpha_2)\theta_2}(2\alpha_2 L_2+\frac{2\alpha_2^2L_2^2}{2-\alpha_2}))\gamma^2$

In addition, notice that if $0 \leq \theta_1 < 1$, $\frac{1-\theta_1}{1-(1-\alpha_1)\theta_1} \leq 1$, while if $-1 < \theta_1 \leq 0$, $\frac{1-\theta_1}{1-(1-\alpha_1)\theta_1} \geq 1$. Therefore, we see that as the demand correlation θ_1 increases, the increase in variability decreases. In addition, for positively correlated demands, the increase in variability will be less than for i.i.d. demands (θ_1=0). On the other hand, for negatively correlated demands, the increase in variability will be greater than for i.i.d.demands. As θ_2 have the same significance to the supply chain model as θ_1, we can get conclusions which is the same as θ_1.

$$\frac{Var(q_t)}{Var(D_t)} < \frac{H+J}{(1+\gamma^2+2\phi\gamma)} \tag{23}$$

Figure 15-20 respectively present those characteristics accordingly. We all know the smoothing parameter have a significant impact on the bullwhip effect. The variation tendency of the bullwhip effect is not obvious when the smoothing parameter changes. But we can get the way in which the smoothing parameter affect the bullwhip effect with other parameters.

Proposition 9 *For smoothing parameter α_2, according to the symmetry between α_1 and α_2, we can get that the smoothing parameter α_2 also have the same impact on the bullwhip effect as α_1 have.*

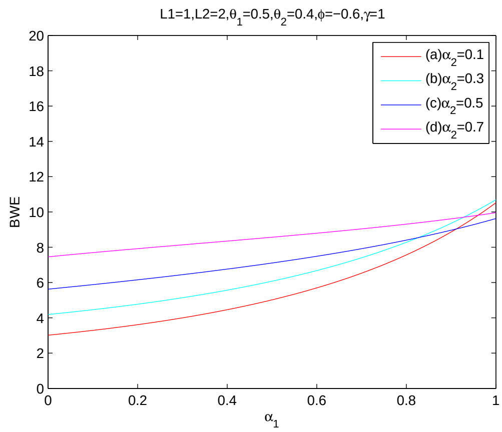

Figure 20: The impact of α_1 on the bullwhip effect with various values of α_2.

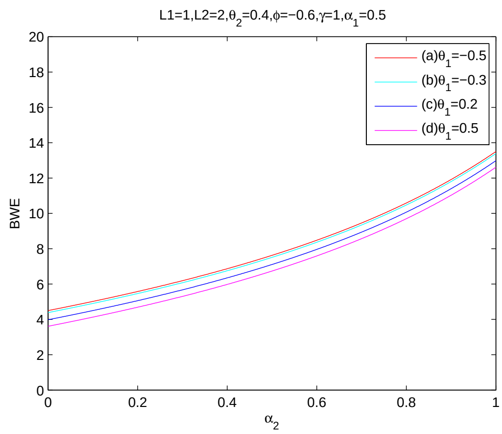

Figure 21: The impact of α_2 on the bullwhip effect with various values of θ_1.

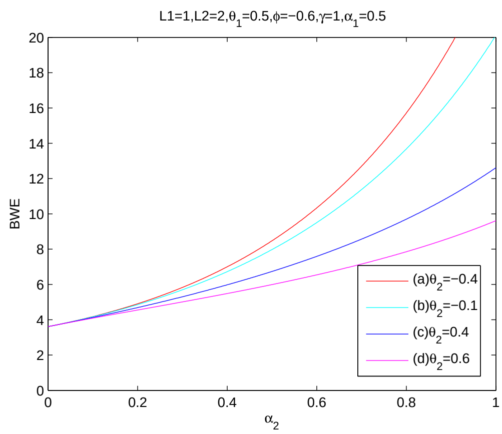

Figure 22: The impact of α_2 on the bullwhip effect with various values of θ_2.

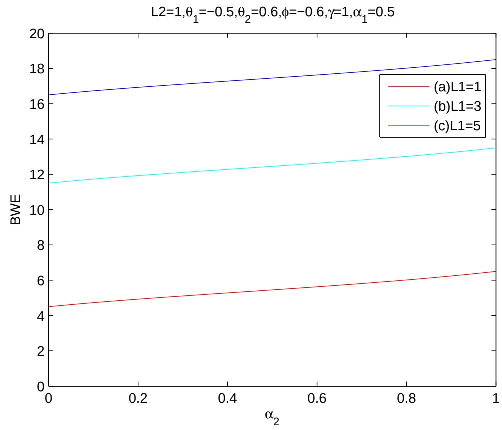

Figure 23: The impact of α_2 on the bullwhip effect with various values of $Ł_1$.

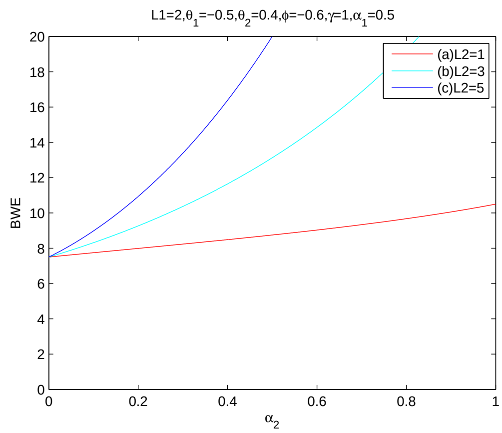

Figure 24: The impact of α_2 on the bullwhip effect with various values of $Ł_2$.

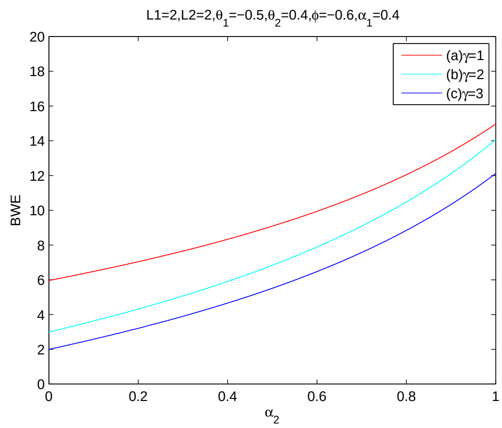

Figure 25: The impact of α_2 on the bullwhip effect with various values of γ.

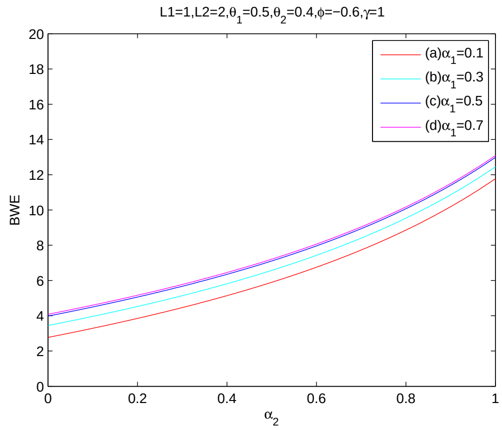

Figure 26: The impact of α_2 on the bullwhip effect with various values of α_1.

It is also easily to know that $\frac{M_3\phi\gamma}{1+\gamma^2+2\phi\gamma}$ is decreasing when α_2 is in the interval (0,1). The maximum value of $\frac{M_3\phi\gamma}{1+\gamma^2+2\phi\gamma}$ is obtained at $\alpha_2 = 0$. We can get the maximum value is $(2(1+\alpha_1 L_1) - \frac{2\alpha_1^2\theta_2 L_1}{1-(1-\alpha_1)\theta_2})\phi\gamma < 0$. So we can have $\frac{M_3\phi\gamma}{1+\gamma^2+2\phi\gamma} < 0$.

So, we can see that α_2 processes the following properties on $L1, L2, \alpha_1, \gamma, \theta_1, \theta_2$ as Figure21-26 present respectively. And we can know that the smoothing parameter α_2 also have an important impact on lowering the bullwhip effect.

5 Conclusions

We know information distortion is one of the main causes of bullwhip effect. Using different forecasting methods would transmit different information on the demand to upstream. In this research, we measure the bullwhip effect through using the Exponential Smoothing Forecasts. Then, we investigated the impact of the autoregressive coefficient, the lead time, market competition degree, the consistency of demand volatility between two retailers and the smoothing parameter on a bullwhip effect measured in a simple two-stage supply chain with one supplier and two retailers in which the demand pattern follows a autoregressive process, AR(1), and the retailer employs the order-up-to inventory police to get the influence of information distortion on the bullwhip effect. According to the analysis of the expression of the bullwhip effect by using algebraic analysis and numerical simulation, lots of interesting properties and managerial insights have been found. The autoregressive coefficient which follows more different ways to affect the bullwhip effect in our supply chains with two retailers than others with only one retailer. And those ways are decided by the magnitude relationships between parameter H, the smoothing parameter α_1 and α_2. Hence, managers can get how autoregressive coefficient affect the bullwhip effect through parameter H, the smoothing parameter α_1 and α_2 so as to control the bullwhip effect better. The bullwhip effect does not always increase when the lead time L increases. The lead time L which follows different ways to affect the bullwhip effect is a complex factor in our supply chains. Efforts to reduce bullwhip effect through lead-time reduction can be misleading, especially when managers have little knowledge of the underlying demand and are unaware of the influences when different forecasting methods are applied to predict lead-time demand.

The way of the market competition degree between two retailers in which affects the bullwhip effect is simpler. The variation tendency is clearer and the bullwhip effect is close to a current value which decided by other parameters with the increase of the market competition degree. The consistency of demand volatility between two retailers has a very complicated impact on the bullwhip effect. And it also follows different ways to influence the bullwhip effect which are decided by the sophisticated magnitude relations with other parameters. So, for managers it maybe more difficult and cost more to control the bullwhip effect by adjusting the consistency of demand volatility between two retailers.

We all know the smoothing parameter has a significant impact on the bullwhip effect. The variation tendency of the bullwhip effect is not obvious when the smoothing parameter changes. But we can get the way in which the smoothing parameter affect the bullwhip effect with other parameters. When one of other parameters is fixed, the bullwhip effect increases as the smoothing parameter increases; And when the smoothing parameter is fixed, the bullwhip effect decreases or increases as one of other parameters increases. For example, when parameter θ_1 is fixed, the bullwhip effect increases as the smoothing parameter α_1 increases; And when the smoothing parameter α_1 is fixed, the bullwhip effect decreases as parameter θ_1 increases. However, when parameter L_1 is fixed, the bullwhip effect increases as the smoothing parameter α_1 increases; And when the smoothing parameter α_1 is fixed, the bullwhip effect increases as parameter L_1 increases.

From above discussions, the influence of information distortion using different forecasting methods on bullwhip effect is complicated. For managers, they can mitigate the influence of information distortion caused by forecasting methods on the bullwhip effect through adjusting relevant parameters. Using appropriate forecasting methods has a significant impact on reducing information distortion. And we also know there certainly are other forecasting methods, and exponential smoothing may not be the “optimal” forecasting tool for the demand processes considered in this paper as Chen et al[22] said. But exponential smoothing is one of the forecasting techniques most commonly used in practice. In the further research, we can try other forecasting methods to do competition , experimental study and DOE analysis so that the research can more close to life.

Many scholars introduce Complexity, Chaos and Hopf Bifurcation into social systems to do some researches. For example, Gao and Ma [23,24] study Chaos and Hopf Bifurcation of a Finance System and Stability and Hopf Bifurcations in a Business Cycle Model with Delay; Yang and Ma [25] analysed the complexity in evolutionary game system in the real estate market.

Appendix. Proofs

Proof for Eq.(15).

According to Feng [11], we have

$$\begin{aligned} Cov(D_{1,t-1}, D_{1,t-i-1}) &= \theta_1^i Var(D_{1,t}) \\ Cov(D_{2,t-1}, D_{2,t-i-1}) &= \theta_2^i Var(D_{2,t}) \end{aligned} \tag{24}$$

After iteration computation for Eq.(1), we have

$$\begin{aligned} &D_{1,t-1} = ((1+\theta_1+\cdots+\theta_1^{i-1})\xi_1 + \theta_1^i D_{1,t-i-1} \\ &+(\varepsilon_{1,t-1}+\theta_1\varepsilon_{1,t-2}+\cdots+\theta_1^{i-1}\varepsilon_{1,t-i})) \end{aligned} \tag{25}$$

So, we can get

$$\begin{aligned} &Cov(D_{1,t-1}, D_{2,t-i-1}) \\ &= Cov((1+\theta_1+\cdots+\theta_1^{i-1})\xi_1 + \theta_1^i D_{1,t-i-1} \\ &+(\varepsilon_{1,t-1}+\theta_1\varepsilon_{1,t-2}+\cdots+\theta_1^{i-1}\varepsilon_{1,t-i}), D_{2,t-i-1}) \\ &= \theta_1^i \phi\sqrt{Var(D_{1,t})Var(D_{2,t})} \end{aligned} \tag{26}$$

and we also have

$$\begin{aligned} &Cov(D_{2,t-1}, D_{1,t-i-1}) \\ &= \theta_2^i \phi\sqrt{Var(D_{1,t})Var(D_{2,t})} \end{aligned} \tag{27}$$

Proof of Proposition 1.

Total order quantity of period t under the ES forecasting method is

$$\begin{aligned} &q_t = q_{1,t}+q_{2,t} = D_{1,t} + \alpha_1 L_1(D_{1,t}-\widehat{D}_{1,t}) + D_{2,t} \\ &+\alpha_2 L_2(D_{2,t}-\widehat{D}_{2,t}) \\ &= (1+\alpha_1 L_1)D_{1,t} - \alpha_1 L_1\widehat{D}_{1,t} \\ &+(1+\alpha_2 L_2)D_{2,t} - \alpha_2 L_2\widehat{D}_{2,t} \end{aligned} \tag{28}$$

Taking the variance for Eq.(28), we get

$$\begin{aligned} &Var(q_t) = (1+\alpha_1 L_1)^2 Var(D_{1,t}) \\ &+(\alpha_1 L_1)^2 Var(\widehat{D}_{1,t}) + (1+\alpha_2 L_2)^2 Var(D_{2,t}) \\ &+(1+\alpha_2 L_2)^2 Var(D_{2,t}) + (\alpha_2 L_2)^2 Var(\widehat{D}_{2,t}) \\ &-2\alpha_1 L_1(1+\alpha_1 L_1)Cov(D_{1,t},\widehat{D}_{1,t}) \\ &+2(1+\alpha_1 L_1)(1+\alpha_2 L_2)Cov(D_{1,t}, D_{2,t}) \\ &-2\alpha_2 L_2(1+\alpha_1 L_1)Cov(D_{1,t},\widehat{D}_{2,t}) \\ &-2\alpha_1 L_1(1+\alpha_2 L_2)Cov(\widehat{D}_{1,t}, D_{2,t}) \\ &+2\alpha_1 L_1\alpha_2 L_2 Cov(\widehat{D}_{1,t},\widehat{D}_{2,t}) \\ &-2\alpha_2 L_2(1+\alpha_2 L_2)Cov(D_{2,t},\widehat{D}_{2,t}) \end{aligned} \tag{29}$$

From Eq.(8), we have

$$\widehat{D}_{1,t} = \sum_{i=0}^{\infty}\alpha_1(1-\alpha_1)^i D_{t-i-1} \tag{30}$$

From zhang [14], we have

$$\begin{aligned} Var(\widehat{D}_{1,t}) &= \tfrac{\alpha_1(1+(1-\alpha_1)\theta_1)}{(2-\alpha_1)(1-(1-\alpha_1)\theta_1)} Var(D_{1,t}) \\ Var(\widehat{D}_{2,t}) &= \tfrac{\alpha_2(1+(1-\alpha_2)\theta_2)}{(2-\alpha_2)(1-(1-\alpha_2)\theta_2)} Var(D_{2,t}) \end{aligned} \tag{31}$$

We can get

$$\begin{aligned} Cov(D_{1,t},\widehat{D}_{1,t}) &= \tfrac{\alpha_1\theta_1}{(1-(1-\alpha_1)\theta_1)} Var(D_{1,t}) \\ Cov(D_{2,t},\widehat{D}_{2,t}) &= \tfrac{\alpha_2\theta_2}{(1-(1-\alpha_2)\theta_2)} Var(D_{2,t}) \\ Cov(D_{1,t},\widehat{D}_{2,t}) &= \tfrac{\alpha_2\theta_1\phi}{(1-(1-\alpha_2)\theta_1)} k \\ Cov(D_{2,t},\widehat{D}_{1,t}) &= \tfrac{\alpha_1\theta_2\phi}{(1-(1-\alpha_1)\theta_2)} k \end{aligned} \tag{32}$$

where $k = \sqrt{Var(D_{1,t})Var(D_{2,t})}$.

From Eq.(30), we can get

$$\begin{aligned} &Cov(\widehat{D}_{1,t},\widehat{D}_{2,t}) \\ &= Cov(a,b) \\ &= \alpha_1\alpha_2(c,d)G \\ &= \alpha_1\alpha_2(e,f)G \\ &= \tfrac{\alpha_1\alpha_2(1-\theta_1\theta_2(1-\alpha_1)(1-\alpha_2))}{(1-(1-\alpha_1)(1-\alpha_2))(1-(1-\alpha_1)\theta_2)(1-(1-\alpha_2)\theta_1)} G \end{aligned} \tag{33}$$

where

$$a = \sum_{i=0}^{\infty}\alpha_1(1-\alpha_1)^i D_{1,t-i-1}$$

$$b = \sum_{j=0}^{\infty}\alpha_2(1-\alpha_2)^j D_{1,t-j-1})$$

$$c = \sum_{i=0}^{\infty}\sum_{j=i+1}^{\infty}(1-\alpha_1)^i(1-\alpha_2)^j\theta_1^{j-i}$$

$$d = \sum_{i=0}^{\infty}\sum_{j=0}^{i}(1-\alpha_1)^i(1-\alpha_2)^j\theta_2^{i-j}$$

$$e = \sum_{i=0}^{\infty} (1-\alpha_1)^i (1-\alpha_2)^i \frac{\theta_1(1-\alpha_2)}{(1-(1-\alpha_2)\theta_1)}$$
$$f = \sum_{i=0}^{\infty} (1-\alpha_1)^i \theta_2^i \frac{(1-(\frac{1-\alpha_2}{\theta_2})^{i+1})}{1-(\frac{1-\alpha_2}{\theta_2})}$$
$$G = Cov(D_{1,t}, D_{2,t}) = \phi\sqrt{Var(D_{1,t})Var(D_{2,t})}.$$

Bring Eq.(31)-(33) into Eq.(29), then take the simplification, we can get

$$\begin{aligned}
&Var(q_t) \\
&= (1 + \tfrac{1-\theta_1}{1-(1-\alpha_1)\theta_1}(2\alpha_1 L_1 + \tfrac{2\alpha_1^2 L_1^2}{2-\alpha_1}))Var(D_{1,t}) \\
&+(1 + \tfrac{1-\theta_2}{1-(1-\alpha_2)\theta_2}(2\alpha_2 L_2 + \tfrac{2\alpha_2^2 L_2^2}{2-\alpha_2}))Var(D_{2,t}) \\
&+(2(1+\alpha_1 L_1)(1+\alpha_2 L_2) - \tfrac{2\alpha_2^2\theta_1 L_2(1+\alpha_1 L_1)}{1-(1-\alpha_2)\theta_1} \\
&- \tfrac{2\alpha_1^2\theta_2 L_1(1+\alpha_2 L_2)}{1-(1-\alpha_1)\theta_2} \\
&+ \tfrac{2\alpha_1^2\alpha_2^2 L_1 L_2(1-\theta_1\theta_2(1-\alpha_1)(1-\alpha_2))}{(1-(1-\alpha_1)(1-\alpha_2))(1-(1-\alpha_1)\theta_2)(1-(1-\alpha_2)\theta_1)})H
\end{aligned} \tag{34}$$

where $H = \phi\sqrt{Var(D_{1,t})Var(D_{2,t})}$. This completes the proof for the proposition 1.

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