The Design of Continuous Sampling Plan G-TF-CSP

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Abstract: The purpose of this paper is to design the G-TF-CSP for the concept of a three-level continuous sampling plan, derive and test the accuracy of the performance measure formulas, namely, the average fraction inspected (*AFI*), the average outgoing quality (*AOQ*) and the average fraction of the total produced accepted on sampling basis ($Pa(p)$). The plan is defined, the sampling frequency at level 1 is $f_1 = 1/r$, the sampling frequency at level 2 is $f_2 = 1/(r–1)$, the sampling frequency at level 3 is $f_3 = 1/(r+1)$ when r is the sampling interval ($r = 3$) and the number of conforming units to be found in the sampling inspection at level 1 is k and $k = i$ when i is the clearance number ($i = 20$, 40 and 50), the maximum allowable number of non-conforming units at level 2 or 3 (m) are 2 and 3, and the probability of a unit produced by the process being nonconforming (p) are 0.005, 0.008 and 0.01. The derivation of the performance measure formulas is based on the Markov Chain. The accuracy of the performance measure formulas have been tested by extensive simulations for all sets of parameter values and p.

Key-Words: continuous sampling plan, markov chain, average fraction inspected, average outgoing quality, average fraction of the total produced accepted on sampling basis

1 Introduction

Continuous Sampling Plans or CSPs are a method of checking products in the production process that is produced one unit at a time, continuously according to the production line and used for inspecting each product unit on a production line. The result of the inspection is either accept or reject for a process. CSPs alternate between two phases of inspection, i.e. 100% inspection and sampling inspection and there are two types of CSPs: single-level continuous sampling plans and multi-level continuous sampling plans. At present, there are many types of CSPs, which was the first continuous sampling plan that was developed in 1943 by Dodge [4] and it is well known as the continuous sampling plan type 1 or CSP-1. The procedure of CSP-1 is the simplest and most commonly used type of single-level continuous sampling plan and CSP-1 has been developed as other plans such as CSP-2 and CSP-3 by Dodge and Torrey [5], CSP-M by Lieberman and Solomon [2], MLP-T-2 by Kandaswamy and Govindaraju [1], the general MLP-T-2 by Balamurali and Kalyanasundaram [12], MCSP-C by Balamurali and Subramani [11], MCSP-2-C by Guayjarernpanishk and Mayureesawan [8] and so on.

Balamurali and Govindaraju [10] developed a new two-level continuous sampling plan, namely the Modified MLP-T-2. The procedure of this plan is when the first i consecutive conforming unit is found immediately after commencing the 100% inspection then switch to the sampling inspection at level 2 (f_2). Otherwise the 100% inspection is continued until any run of i successive conforming units are found and then switch to the sampling inspection at level 1 (f_1, $f_1 > f_2$). The number of units inspected of the Modified MLP-T-2 is decreased if the first i consecutive conforming units are found immediately after commencing the 100% inspection.

Guayjarernpanishk [7] modified a fractional sampling plan, namely CSP-F-L, which reduces the number of units inspected of the Modified MLP-T-2 and starts with sampling inspection at level 1 (f_1). If the first k consecutive units are found clear of nonconforming units, and then switch to sampling inspection at level 2 (f_2, $f_2 < f_1$). Otherwise, switch to 100% inspection of units in the order of production. During the 100% inspection, if the first i consecutive units are found clear of nonconforming units, discontinue 100% inspection and switch to sampling inspection at level 2. Otherwise, continue 100% inspection until i successive units are found clear of nonconforming units, then proceed to sampling inspection at level 1. When a nonconforming unit is found at level 2, immediately revert to the sampling inspection at level 1.

Guayjarernpanishk and Mayureesawan [9] developed CSP-F-L and the resultant plan is designated as MCSP-F-L, which starts with sampling inspection at level 1 of the units the same as CSP-F-L. The difference between the two plans is in the phases of sampling inspection at level 2 by MCSP-F-L, which is characterized by a maximum allowable number of inspected units (*l*) to decide when to switch from the phases of sampling inspection at level 2 to the phases of sampling inspection at level 1. During the inspection at level 2, if there are no nonconforming units, the inspection is continued until l sampled units have been inspected before switching to sampling inspection at level 1.

In this paper, we designed a new three-level continuous sampling plan which is called G-TF-CSP. The purpose of the design G-TF-CSP is for the sampling inspection $f_1 = 1/r$ to be given first priority for starting inspection and extending the sampling inspection phase that adds two sampling inspections $f_2 = 1/(r–1)$ and $f_3 = 1/(r+1)$, which start sampling inspections at level 2 and 3 before starting 100% inspection phase. This new continuous sampling plan will help reduce the number of products that are inspected by inspectors. Moreover, this plan is suitable for good quality production lines.

The main objectives of this paper are to design the G-TF-CSP, to give details of operating procedures and performance measure formulas, such as the average fraction inspected (*AFI*), the average outgoing quality (*AOQ*), the average outgoing quality limit (*AOQL*), and the average fraction of the total production accepted under sampling basis (*Pa*(*p*)), and to summarize the results of tests of the accuracy of the formulas for performance measures *AFI*, *AOQ*, and *Pa*(*p*) by comparison of the values computed from the formulas with values obtained through simulations.

2 Materials and Methods

2.1 The Operating Procedure of the G-TF-CSP

The G-TF-CSP uses four parameters for inspection of the units being produced on the production line, namely four positive integers *i*, *k*, *m* and *r* which are defined by:

i = the clearance number,

k = the number of conforming units to be found in the sampling inspection level 1,

m = the maximum allowable number of non-conforming units at level 2 or 3,

r = the sampling interval.

Where

f_1 = the sampling fraction at inspection level 1 or $f_1 = 1/r$,

f_2 = the sampling fraction at inspection level 2 or $f_2 = 1/(r–1)$,

f_3 = the sampling fraction at inspection level 3 or $f_3 = 1/(r+1)$.

The flow diagram showing the procedure for inspection of the G-TF-CSP is given in Figure 1.

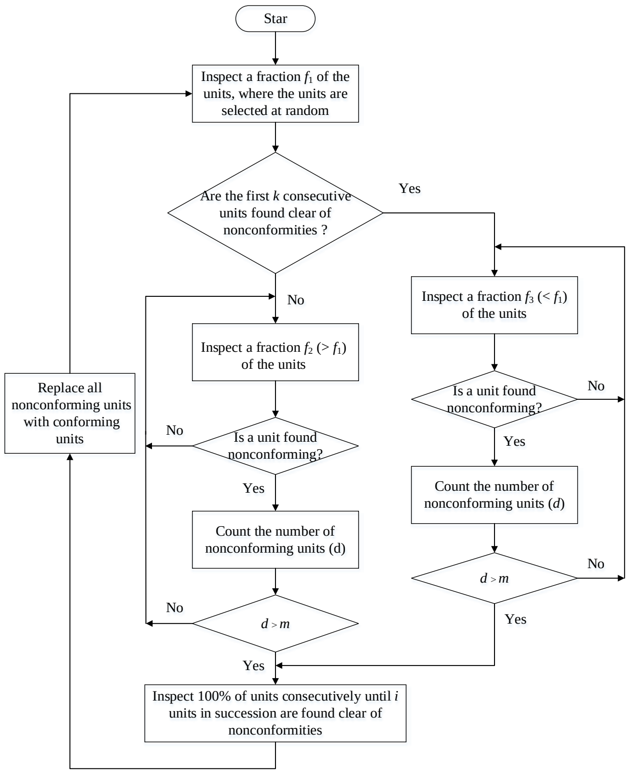

Fig. 1. Flow diagram of the procedure for inspection using the G-TF-CSP.

The performance measure formulas of the G-TF-CSP are derived using a Markov Chain model, assuming that the production process is under statistical control.

2.2 The G-TF-CSP Procedure as a Markov Chain

Let $[X_t]$ (t = 1, 2, …) denote a discrete-parameter Markov Chain with finite state space (S_j), j = 1, …, $3k+6m+i+7$. The states of the process are defined, in the same way as Roberts [14] and Lasater [3], as follows:

$S_{3g+1} = f_1 N_{g+1}$ (g = 0, 1, 2, …, k–1).

= Sampling inspection at level 1 is in effect and the *g* units submitted for inspection were all found to be conforming except the last unit, which was not selected for inspection.

S_{3g+2}= f_1In$_{g+1}$ (g = 0, 1, 2, …, k–1).
= Sampling inspection at level 1 is in effect and the g+1 units submitted for inspection were all found to be conforming.

S_{3g+3}= f_1Id$_{g+1}$ (g = 0, 1, 2, …, k–1).
= Sampling inspection at level 1 is in effect, the g+1 units submitted for inspection and only unit g+1 was found to be nonconforming.

$S_{3k+3l+1}$ = f_2Id$_{l+1}$ (l = 0, 1, 2, …, m).
=Sampling inspection at level 2 is in effect and the l+1 units submitted for inspection were all found to be nonconforming.

$S_{3k+3l+2}$ = f_2N$_l$ (l = 0, 1, 2, …, m).
=Sampling inspection at level 2 is in effect, the last unit was not inspected and the number of nonconforming units found during this sampling phase is l.

$S_{3k+3l+3}$ = f_2In$_l$ (l = 0, 1, 2, …, m).
=Sampling inspection at level 2 is in effect, the last unit submitted for inspection and found to be conforming and the number of nonconforming units found during this sampling phase is l.

$S_{3k+3m+3l+4}$ = f_3Id$_{l+1}$ (l = 0, 1, 2, …, m).
= Sampling inspection at level 3 is in effect and the l+1 units submitted for inspection were all found to be nonconforming.

$S_{3k+3m+3l+5}$ = f_3N$_l$ (l = 0, 1, 2, …, m).
= Sampling inspection at level 3 is in effect, the last unit was not inspected and the number of nonconforming units found during this sampling phase is l.

$S_{3k+3m+3l+6}$ = f_3In$_l$ (l = 0, 1, 2, …, m).
= Sampling inspection at level 3 is in effect, the last unit submitted for inspection and found to be conforming and the number of nonconforming units found during this sampling phase is l.

$S_{3k+6m+7}$ = A$_0$.
= Non-conforming unit is found on 100% inspection.

$S_{3k+6m+j+7}$ = A$_j$ (j = 1, 2, …, i).
= j consecutive conforming units found during 100% inspection.

The set of (3k+6m+i+7) states defined above completely describe the mutually exclusive phases of inspection for the G-TF-CSP procedure. The one-step transition probability matrix for the process is given in Table 1 and a flow chart showing the description of the process by means of states and transitions is given in Figure 2. The transition probability matrix reveals that the process is a discrete-parameter, finite, recurrent, irreducible, aperiodic (DFRIA) Markov Chain (see Karlin [13] and Lasater [3]).

Table 1. One-step transition probability matrix of the G-TF-CSP.

	f_1N_1	f_1In_1	f_1Id_1	…	f_1N_k	f_1In_k	f_1Id_k	f_2N_0	f_2In_0	f_2Id_1	…	f_2N_m	f_2In_m	f_2Id_{m+1}	f_3N_0	f_3In_0	f_3Id_1	…	f_3N_m	f_3In_m	f_3Id_{m+1}	A_0	A_1	…	A_i
f_1N_1	$1-f_1$	f_1q	f_1p	…							…							…						…	
f_1In_1				…							…							…						…	
f_1Id_1				…				$1-f_2$	f_2q	f_2p	…							…						…	
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
f_1N_k				…	$1-f_1$	f_1q	f_1p				…							…						…	
f_1In_k				…							…				$1-f_3$	f_3q	f_3p	…						…	
f_1Id_k				…				$1-f_2$	f_2q	f_2p	…							…						…	
f_2N_0				…				$1-f_2$	f_2q	f_2p	…							…						…	
f_2In_0				…				$1-f_2$	f_2q	f_2p	…							…						…	
f_2Id_1				…							…							…						…	
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
f_2N_m				…							…	$1-f_2$	f_2q	f_2p				…						…	
f_2In_m				…							…	$1-f_2$	f_2q	f_2p				…						…	
f_2Id_{m+1}				…							…							…				p	q	…	
f_3N_0				…							…				$1-f_3$	f_3q	f_3p	…						…	
f_3In_0				…							…				$1-f_3$	f_3q	f_3p	…						…	
f_3Id_1				…							…							…						…	
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
f_3N_m				…							…							…	$1-f_3$	f_3q	f_3p			…	
f_3In_m				…							…							…	$1-f_3$	f_3q	f_3p			…	
f_3Id_{m+1}				…							…							…				p	q	…	
A_0				…							…							…				p	q	…	
A_1				…							…							…				p		…	
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
A_i	$1-f_1$	f_1q	f_1p	…							…							…						…	

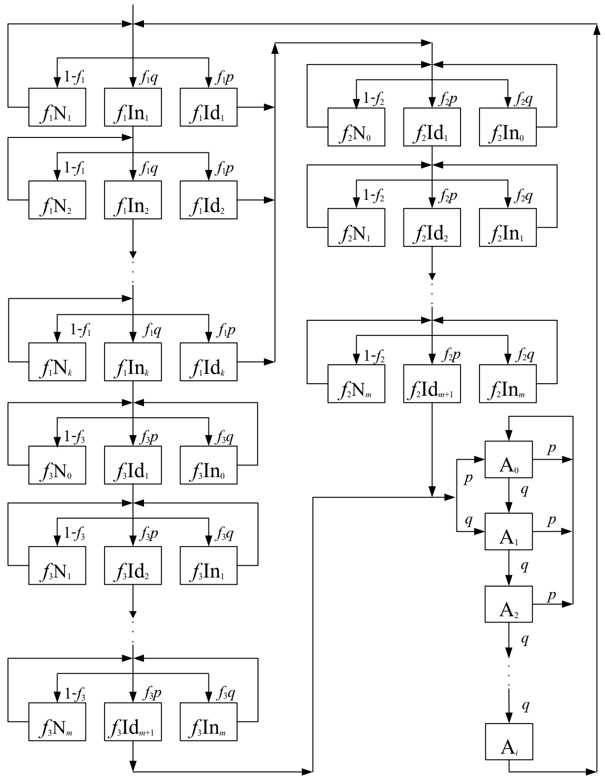

Fig. 2. Flow chart showing states and transitions of the G-TF-CSP procedure.

2.3 Test of the Accuracy of Performance Measures for the G-TF-CSP

For testing the accuracy of the performance measure formulas that are defined for the G-TF-CSP, the results from the formulas were compared with the values obtained from extensive simulations. Three different levels were examined for the probability p of nonconforming units produced on the line 0.005, 0.008 and 0.01. For each p, values of i = 20, 40 and 50, values of $k = i$, values of r = 3, and values of m = 2 and 3. For each set of values of p, i, k, r and m, a simulation was carried out to compute the fraction of units inspected, the fraction of outgoing nonconforming units and the fraction of the total produced accepted on sampling basis. The simulation was repeated on 500 different product lines and the values of the average fraction inspected (*AFI*), the average outgoing quality (*AOQ*) and the average fraction of the total produced accepted on sampling basis ($Pa(p)$) were calculated and then compared with the values of *AFI*, *AOQ* and $Pa(p)$ computed from the formulas.

When %dif_*AFI*, %dif_*AOQ* and %dif_*Pa* were defined by

$$\%\text{dif\_}AFI = \left|\frac{(AFI\text{\_F}) - (AFI\text{\_S})}{AFI\text{\_S}}\right| \times 100 \le 2, \quad (1)$$

$$\%\text{dif\_}AOQ = \left|\frac{(AOQ\text{\_F}) - (AOQ\text{\_S})}{AOQ\text{\_S}}\right| \times 100 \le 2, \quad (2)$$

$$\%\text{dif\_}Pa = \left|\frac{(Pa(p)\text{\_F}) - (Pa(p)\text{\_S})}{Pa(p)\text{\_S}}\right| \times 100 \le 2, \quad (3)$$

where

*AFI*_F was the *AFI* value from the formula of the G-TF-CSP,

AFI _S was the *AFI* value from the simulation of the G-TF-CSP,

*AOQ*_F was the *AOQ* value from the formula of the G-TF-CSP,

*AOQ*_S was the *AOQ* value from the simulation of the G-TF-CSP,

$Pa(p)$_F was the $Pa(p)$ value from the formula of the G-TF-CSP,

$Pa(p)$_S was the $Pa(p)$ value from the simulation of the G-TF-CSP.

The *AFI*, *AOQ* and $Pa(p)$ formulas are accepted as the accurate formulas if %dif_*AFI*, %dif_*AOQ* and %dif_*Pa* were less than or equal to 2. The accuracy of the formulas was then compared for each set of values of p, i, k, r and m and the results are presented in the next section.

3 Results

3.1 The Performance Measures of the G-TF-CSP

Letting p be the probability of a unit produced by the process being nonconforming, the following performance measures may be obtained:

The average number of units inspected in a 100% screening sequence following the finding of a nonconforming unit, u:

$$u = \frac{1-q^i}{pq^i}. \quad (4)$$

The average number of units passed under the sampling inspection, v:

$$v = \frac{1}{f_1 f_2 f_3 p}\begin{bmatrix} f_1 f_2 (m+1) q^k \\ + f_1 f_3 (m+1)(1-q^k) \\ + f_2 f_3 (1-q^k) \end{bmatrix}. \quad (5)$$

The average fraction inspected, *AFI*:

$$AFI = \frac{f_1 f_2 f_3}{D}[1 + (m+1)q^i - q^{i+k}]. \quad (6)$$

The average outgoing quality, *AOQ*:

$$AOQ = \frac{pq^i}{D}\begin{bmatrix} f_1 f_2 (1-f_3)(m+1) q^k \\ + f_1 f_3 (1-f_2)(m+1)(1-q^k) \\ + f_2 f_3 (1-f_1)(1-q^k) \end{bmatrix}. \quad (7)$$

The average outgoing quality limit, *AOQL*:

$$AOQL = \max_{\text{all } p} AOQ \, . \qquad (8)$$

The average fraction of the total produced accepted on sampling basis, *Pa*(*p*):

$$Pa(p) = \frac{q^i}{D}\begin{bmatrix} f_1 f_2 (m+1) q^k \\ + f_1 f_3 (m+1)(1-q^k) \\ + f_2 f_3 (1-q^k) \end{bmatrix}. \qquad (9)$$

Where

$$D = \begin{matrix} f_1 f_2 f_3 (1-q^i) + f_1 f_2 (m+1) q^i q^k \\ + f_1 f_3 (m+1) q^i (1-q^k) + f_2 f_3 q^i (1-q^k) \end{matrix}$$

3.2 The Accuracy Performance Measures for the G-TF-CSP

The difference of the *AFI* values from the simulations and the *AFI* values from the formula (%dif_*AFI*), the difference of the *AOQ* values from the simulations and the *AOQ* values from the formula (%dif_*AOQ*) and the difference of the *Pa*(*p*) values from the simulations and the *Pa*(*p*) values from the formula (%dif_*Pa*) for each set of *p*, *i*, *k* and *r* values are shown in Table 2 and Table 3 for *m* = 2 and 3, respectively. It was found that the %dif_*AFI*, %dif_*AOQ* and %dif_*Pa* values were less than 2 for all sets of *p*, *i*, *k*, *r* and *m* values. So the simulations signified that the *AFI*, the *AOQ* and the *Pa*(*p*) formulas are accurate.

Table 2. The %dif_*AFI*, %dif_*AOQ* and %difI_ *Pa* values of the G-TF-CSP when *r* = 3 and *m* = 2

p	*i* = *k*	*AFI*_F	*AFI*_S	%dif_*AFI*	*AOQ*_F	*AOQ*_S	%dif_*AOQ*	*Pa*(*p*)_F	*Pa*(*p*)_S	%dif_ *Pa*
0.005	20	0.2708	0.2705	0.113	0.0036	0.0036	1.454	0.9911	0.9908	0.033
	40	0.2915	0.2910	0.177	0.0035	0.0035	1.352	0.9810	0.9806	0.036
	50	0.3018	0.2980	1.262	0.0035	0.0035	0.051	0.9755	0.9756	0.016
0.008	20	0.2833	0.2834	0.033	0.0057	0.0058	1.319	0.9851	0.9849	0.023
	40	0.3162	0.3212	1.560	0.0055	0.0055	0.627	0.9672	0.9677	0.051
	50	0.3326	0.3364	1.156	0.0053	0.0053	0.151	0.9570	0.9576	0.061
0.01	20	0.2916	0.2863	1.833	0.0071	0.0070	1.028	0.9809	0.9809	0.001
	40	0.3326	0.3305	0.650	0.0067	0.0066	0.855	0.9570	0.9586	0.171
	50	0.3530	0.3463	1.932	0.0065	0.0065	0.164	0.9431	0.9466	0.370

Table 3. The %dif_*AFI*, %dif_*AOQ* and %difI_ *Pa* values of the G-TF-CSP when *r* = 3 and *m* = 3

p	*i* = *k*	*AFI*_F	*AFI*_S	%dif_*AFI*	*AOQ*_F	*AOQ*_S	%dif_*AOQ*	*Pa*(*p*)_F	*Pa*(*p*)_S	%dif_ *Pa*
0.005	20	0.2688	0.2680	0.291	0.0037	0.0037	0.015	0.9933	0.9933	0.001
	40	0.2876	0.2860	0.532	0.0036	0.0036	0.347	0.9855	0.9856	0.008
	50	0.2969	0.2929	1.363	0.0035	0.0035	0.533	0.9812	0.9819	0.069
0.008	20	0.2801	0.2790	0.388	0.0058	0.0057	0.666	0.9887	0.9892	0.046
	40	0.3101	0.3139	1.204	0.0055	0.0055	0.160	0.9747	0.9750	0.028
	50	0.3251	0.3299	1.462	0.0054	0.0055	1.392	0.9667	0.9660	0.067
0.01	20	0.2877	0.2827	1.767	0.0071	0.0070	1.996	0.9855	0.9856	0.015
	40	0.3252	0.3228	0.745	0.0067	0.0067	1.464	0.9667	0.9679	0.131
	50	0.3439	0.3385	1.605	0.0066	0.0064	1.908	0.9555	0.9579	0.249

4 Conclusion

In this paper, the G-TF-CSP has been proposed as the sampling inspection level 1 to be given first priority for starting inspection and extending the sampling inspection phase by adding sampling inspection level 2 and 3 before starting 100% inspection phase. The formulas of the G-TF-CSP have been derived for performance measures such as the values of the average fraction inspected (*AFI*), the average outgoing quality (*AOQ*), the average outgoing quality limit (*AOQL*) and the average fraction of the total produced accepted on sampling basis (*Pa*(*p*)).

The validity of the *AFI*, *AOQ*, and *Pa*(*p*) for the G-TF-CSP has been tested by extensive simulations over wide and representative ranges of values of the five parameters (*p*, *i*, *k*, *m* and *r*), where *p* is the probability of a unit produced by the process being non-conforming, *i* is the clearance number, *k* is the number of conforming units to be found in the sampling inspection level 1, *m* is the maximum allowable number of non-conforming units at level 2 or 3 and *r* is the sampling interval where $f_1 = 1/r$, $f_2 = 1/(r-1)$, and $f_3 = 1/(r+1)$ are the specified sampling frequencies at level 1, 2, and 3, respectively. The percentage difference between the

AFI, *AOQ*, and *Pa*(*p*) values from the formula and the *AFI*, *AOQ*, and *Pa*(*p*) values from the simulations were found to agree within 2%in all simulations.

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Appendix:
Glossary of symbols

S_n = the n^{th} state of the process,

$P(S_n)$ = the steady-state probability for the state S_n,

p_{in} = the probability that the process transits from state S_i to S_n in one step.

Derivation of Performance Measures of the G-TF-CSP

The formulations of the G-TF-CSP using the Markov Chain development is similar to Stephens [6]. Let $[X_t]$ (t = 1, 2, …)denote a discrete-parameter Markov Chain with finite state space (S_n), n = 1,…, $3k+6m+i+7$. The states of the process are defined, in a way similar to that of Roberts [14].

These steady-state probabilities $P(S_n)$satisfy the following conditions:

$$P(S_n) \geq 0 \text{ for } n = 1, 2,\dots, 3k+6m+i+7, \qquad (10)$$

$$P(S_n) = \sum_{x=1}^{3k+6m+i+7} P(S)\, p_{xn} \text{ for } n = 1,2,\dots, k+6m+i+7, \qquad (11)$$

$$\sum_{\text{all}\, n} P(S_n) = 1. \qquad (12)$$

From conditions (11) and (12), where g = 1, 2,…, k, l = 0, 1, 2,…, m, and j = 1, 2, …, i. We acquire the following:

$$P(f_1 N_g) = \frac{1-f_1}{f_1} q^{g-1} P(A_i), \qquad (13)$$

$$P(f_1 In_g) = q^g P(A_i), \qquad (14)$$

$$P(f_1 Id_g) = pq^{g-1} P(A_i), \qquad (15)$$

$$P(f_2 N_l) = \frac{1-f_2}{f_2 p}(1-q^k) P(A_i), \qquad (16)$$

$$P(f_2 In_l) = \frac{q}{p}(1-q^k) P(A_i), \qquad (17)$$

$$P(f_2 Id_{l+1}) = (1-q^k)P(A_i), \quad (18)$$

$$P(f_3 N_l) = \frac{1-f_3}{f_3 p} q^k P(A_i), \quad (19)$$

$$P(f_3 In_l) = \frac{q^{k+1}}{p} P(A_i), \quad (20)$$

$$P(f_3 Id_{l+1}) = q^k P(A_i), \quad (21)$$

$$P(A_0) = p\begin{bmatrix} P(f_2 Id_{m+1}) + P(f_3 Id_{m+1}) \\ +\sum_{j=0}^{i-1} P(A_j) \end{bmatrix}, \quad (22)$$

$$P(A_j) = q^j [P(f_2 Id_{m+1}) + P(f_3 Id_{m+1}) + P(A_0)], \quad (23)$$

$$\begin{bmatrix} \sum_{g=1}^{k} [P(f_1 N_g) + P(f_1 In_g) + P(f_1 Id_g)] \\ +\sum_{l=0}^{m} [P(f_2 N_l) + P(f_2 In_l) + P(f_2 Id_{l+1})] \\ +\sum_{l=0}^{m} [P(f_3 N_l) + P(f_3 In_l) + P(f_3 Id_{l+1})] \\ +\sum_{j=0}^{i} P(A_j) \end{bmatrix} = 1. \quad (24)$$

By equations (13) to (21), (24) can be written (22) as

$$P(A_0) = \frac{f_1 f_2 f_3 p(1-q^i)}{D},$$

where

$$D = f_1 f_2 f_3 (1-q^i) + f_1 f_2 (m+1) q^i q^k + f_1 f_3 (m+1) q^i (1-q^k) + f_2 f_3 q^i (1-q^k)$$

The steady-state probabilities can be written as follows:

$$P(A_j) = \frac{f_1 f_2 f_3 p q^j}{D}; \ j = 1, 2, \ldots, i,$$

$$P(f_1 N_g) = \frac{(1-f_1) f_2 f_3 p q^{i+g-1}}{D}; \ g = 1, 2, \ldots, k,$$

$$P(f_1 In_g) = \frac{f_1 f_2 f_3 p q^{i+g}}{D}; \ g = 1, 2, \ldots, k,$$

$$P(f_1 Id_g) = \frac{f_1 f_2 f_3 p^2 q^{i+g-1}}{D}; \ g = 1, 2, \ldots, k,$$

$$P(f_2 N_l) = \frac{(1-f_2) f_1 f_3 q^i (1-q^k)}{D}; \ l = 0, 1, \ldots, m,$$

$$P(f_2 In_l) = \frac{f_1 f_2 f_3 q^{i+1} (1-q^k)}{D}; \ l = 0, 1, \ldots, m,$$

$$P(f_2 Id_{l+1}) = \frac{f_1 f_2 f_3 p q^i (1-q^k)}{D}; \ l = 0, 1, \ldots, m,$$

$$P(f_3 N_l) = \frac{f_1 f_2 (1-f_3) q^{i+k}}{D}; \ l = 0, 1, \ldots, m,$$

$$P(f_3 In_l) = \frac{f_1 f_2 f_3 q^{i+k+1}}{D}; \ l = 0, 1, \ldots, m,$$

$$P(f_3 Id_{l+1}) = \frac{f_1 f_2 f_3 p q^{i+k}}{D}; \ l = 0, 1, \ldots, m.$$

Then

$$u = \frac{\sum_{j=0}^{i} P(A_j)}{P(f_2 Id_{m+1}) + P(f_3 Id_{m+1})}$$

$$v = \frac{\begin{bmatrix} \sum_{g=1}^{k} [P(f_1 N_g) + P(f_1 In_g) + P(f_1 Id_g)] \\ +\sum_{l=0}^{m} [P(f_2 N_l) + P(f_2 In_l) + P(f_2 Id_{l+1})] \\ +\sum_{l=0}^{m} [P(f_3 N_l) + P(f_3 In_l) + P(f_3 Id_{l+1})] \end{bmatrix}}{P(f_2 Id_{m+1}) + P(f_3 Id_{m+1})}$$

$$AFI = 1 - \sum_{g=1}^{k} P(f_1 N_n) - \sum_{l=0}^{m} P(f_2 N_l) - \sum_{l=0}^{m} P(f_3 N_l)$$

$$AOQ = \begin{bmatrix} p\sum_{g=1}^{k} P(f_1 N_n) + p\sum_{l=0}^{m} P(f_2 N_l) \\ + p\sum_{l=0}^{m} P(f_3 N_l) \end{bmatrix}$$

$$Pa(p) = 1 - \sum_{j=0}^{i} P(A_j).$$

By simplifying the above equations, we can get the performance measures of the G-TF-CSP which are given in equations (6) to (9).