Maintenance Policy with Lifetime Reduction Based on Uncertain Renewal Process

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Abstract: This paper focus on a (N, T) preventive maintenance (PM) policy with lifetime reduction discount rate. In our consideration, the lifetime of component is assumed to be an uncertain variable due to the absence of historical operational data, an uncertain (N, T) PM model with the lifetime reduction in a certain proportion is proposed based on uncertain renewal process. Accordingly, the uncertain model, which minimizes the expected maintenance cost rate, is formulated to find the optimal replacement cycle length T^* and the number N^* of preventive maintenance. Finally, a numerical example with sensitivity analysis of parameters is provided to illustrate the proposed model, the results imply that the parameters of cost and lifetime can significantly affect the optimal solutions N^* and T^*, which can provide a useful reference and guidance for aircraft maintenance decision.

Key-Words: - (N, T) preventive maintenance policy, Minimal repair, Lifetime reduction, Uncertain renewal process.

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1 Introduction

Preventive maintenance (PM) mainly refers to the maintenance method of carrying out a series of maintenance on the premise that the mechanical equipment has no failure or damage, and all activities are carried out to prevent functional failure and keep it in the specified state through systematic inspection, maintenance and replacement of products. Therefore, it is usually used in engineering areas to avoid the consequences of failure endangering safety, affecting the completion of tasks or causing major economic losses. Barlow and Hunter, [19], first proposed preventive maintenance policy to reduce maintenance costs, together with risks and losses caused by accidents. Kulshrestha considered the reliability of preventive maintenance, and studied the system with preventive maintenance under different distributions, [9]. Beichelt, [10], gave the optimum preventive maintenance of systems, which were age replacement policy and minimal repair policy, when the system may have two kinds of faults.

Over time, maintenance issues are attracting more and more attention. Boland and Proschan, [18], proposed to replace or repair regularly at a fixed multiple of the predetermined time T, and minimal repair if the item failed. And in [21], Nakagawa presented the minimal repair strategy in the age replacement policy. If the components failed before T, minimal repair was carried out. When the components failed in the fixed cycle T, we carried out the preventive replacement. A repairable item system, performance of which was measured by a service level, was introduced by De Haas and Verrijdt, [12], for the maintenance support of a flet of aircraft. In [22], Makis and Cheng considered a repair/replacement problem for a single unit system with random repair cost. When the component failed, we decided whether to replace or repair it by observing the repair cost. Park et al., [17], developed a renewable minimal repair-replacement warranty policy and proposed an optimal maintenance model after the warranty expires. In [20], Gopalan proposed an inverse optimization model, and performed all required aircraft maintenance activities with a stipulated periodicity. Safaei et al., [11], proposed a policy to determine whether a repairable system should be repaired or replaced with a new system when it failed in the optimization of maintenance policy.

As mentioned in the previous literatures, the lifetimes of the components were usually assumed as a stochastic variable. As we know, only when there are enough samples can the knowledge of probability theory be applied. In this paper, the components we focus on are made of new materials, and have few or even no historical operating data of their lifetime. In this case, we usually invite some domain experts to provide belief degree of lifetime to estimate its distribution. However, human often tends to overestimate unlikely events (Tversky and Kahneman in [13]), which make the variance of belief degree much greater than that of the frequency, and the probability theory is no longer suitable. To deal with this kind of problem, uncertainty theory was founded by Liu, [1], then refined by Liu, [3], which has become a branch of axiomatic mathematics.

At present, the uncertainty theory based on Professor Liu has been further developed and applied in various fields of practical production practice. Liu in [2] provided an independent and comprehensive uncertain programming theory to solve modeling optimization problems in uncertain environments. In [24], Liu and Ha proved the expected values of monotone functions with uncertain variables and gave some useful expressions for the expected values of functions with uncertain variables. In the same year, Liu, [3], defined an uncertain renewal process, in which events occurred continuously and independently of one another in uncertain times. Next, Yao and Li, [14], proposed an uncertain alternating renewal process, which its alternating arrival time was an uncertain variable.

Then, several researchers applied the uncertainty renewal process in maintenance areas. Yao and Dan, [15], assumed that the lifetime of component was an uncertain variable based on the age replacement policy involving random age, and found out the optimal time to replace the component. Zhang and Guo, [5], discussed the optimal age replacement policy for the lifetime of components in different uncertainty distributions. Next year, Zhang and Guo in [6] proposed an uncertain block replacement model without replacement in case of failure, and the existence conditions of the optimal replacement time were given. Yao, [16], gave an uncertain block replacement policy, assuming that the lifetime of the components was an uncertain variable. Recently, based on the uncertainty theory, an uncertain (N, T) block replacement policy for aircraft structures with corrosion damage was studied by Zhang and Li, [7]. Zhang and Li, [8], proposed an extended uncertain stochastic renewal reward theorem, and established an uncertain random programming model with the objective of minimizing the expected maintenance cost rate.

In this paper, we consider a (N, T) preventive maintenance policy with lifetime reduction discount rate. The policy means that a component is carried out preventive maintenance at the fixed time $KT(K = 1, 2, \ldots, N)$ symmetrically, and after each imperfect preventive maintenance, the lifetime of component will be reduced by $\beta(0 \le \beta \le 1)$, where β is a specific number. We only carry out minimal repairs between preventive maintenance KT and $(K + 1)T(K = 1, 2, \ldots, N)$, then this component is replaced by a new one at the time $NT(N = 1, 2, \ldots)$. Assuming that the lifetime of new component is an uncertain variable due to no historical operation data, we will develop an uncertain (N, T) preventive maintenance model that minimizes the expected maintenance cost based on the uncertain renewal process, where the renewal cycle length T and the number N of preventive maintenance are decision variables.

The rest of this paper is organized as follows. In Section 2, we make the problem description. In Section 3 we build a uncertain (N, T) preventive maintenance model in uncertain circumstance, and analyze its optimal solution. A numerical example with parameter sensitivity analysis is provided, which illustrates the usefulness of the proposed model in Section 4. Finally, the Section 5 presents some conclusions.

2 Problem description

In this section, we consider a uncertain (N, T) preventive maintenance policy with lifetime reduction discount rate, where the lifetime of component is regarded as an uncertain variable. The component needs to carry out preventive maintenance at the fixed time $KT(K = 1, 2, \ldots, N)$. After each imperfect preventive maintenance, the lifetime of component will deduce by x units. Then, preventive replacement is made at the Nth preventive maintenance. Here, we assume that the failure rate is reduced to 0 after minimal repair, each PM is an imperfect preventive maintenance and the time of minimal repair and preventive maintenance are negligible. The preventive maintenance policy we proposed can be illustrated using the notations in Fig.1.

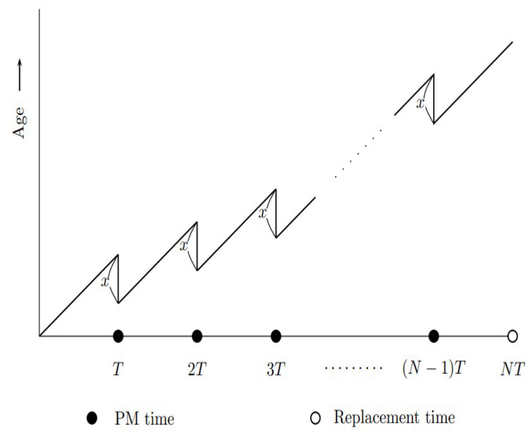


Fig.1: Preventive maintenance policy based on lifetime discount rate ($x = (1 - \beta)\xi$)

In order to better establish the model, some notations are made as follows:

T : the replacement cycle length of preventive maintenance;

N : the number of preventive maintenance;

ξ_k : the lifetime of components in the $kth(k =$

$1, 2, \ldots, N)$ preventive maintenance cycle, is an uncertain variable;

β : the percentage of component lifetime reduction after each preventive maintenance$(0 \leq \beta \leq 1)$, then the lifetime of the component after the kth preventive maintenance is $\xi_k = \beta^{k-1}\xi_1 (K = 1, 2, \ldots, N)$;

$\Phi(t)$: the uncertainty distribution function of structural component lifetime ξ_1 in the first preventive maintenance cycle;

$M(k)$: the number of minimal repair from the $(k-1)th$ to the kth preventive maintenance, is an uncertain renewal process;

c_1 : cost of minimal repair of components before time T;

c_2 : cost of preventive maintenance of components at time T;

c_3 : replacement cost of components at time NT, $c_3 > c_2$.

3 Mathematical formulation

According to the above notations, the total cost of the whole preventive renewal cycle can be expressed as

$$TC(N, T, \xi) = c_1 \sum_{k=1}^{N} M(k) + c_2(N-1) + c_3 \quad (1)$$

Then, based on the above analysis, the objective function is

$$C(N, T, \xi) = \frac{c_1 \sum\limits_{k=1}^{N} M(k) + c_2 (N-1) + c_3}{NT} \quad (2)$$

where $c_1 \sum\limits_{k=1}^{N} M(k)$ is total minimal repair cost, $c_2 (N-1)$ is total cost of preventive maintenance, c_3 is the cost of planned replacement at time NT.

Recall ξ is an uncertain variable, the cost rate function $C(N, T, \xi)$ is an uncertain variable as well, so it cannot be directly minimized with respect to N and T, and we may minimize its expected value:

$$C(N; T) = \min_{N,T} E[C(N, T, \xi)] \quad (3)$$

That is, the optimal N, T should be solved by the following model:

$$\min_{N,T} \frac{c_1 E(\sum\limits_{k=1}^{N} M(k)) + c_2(N-1) + c_3}{NT} \quad (4)$$

In order to solve the model (4), we present the following lemmas:

Lemma 3.1 [23] Let f and g be comonotonic functions. Then for any uncertain variables ξ, we have

$$E[f(\xi) + g(\xi)] = E[f(\xi)] + E[g(\xi)] \quad (5)$$

And then, the Lemma 3.1 can be extended to a limited number.

Lemma 3.2 [1] Let ξ be an uncertain variable with uncertainty distribution $\Phi(x)$. Then

$$E(\xi) = \int_0^{+\infty} (1 - \Phi(x))dx - \int_{-\infty}^{0} \Phi(x)dx \quad (6)$$

Lemma 3.3 [3] Let $M(k)$ be a renewal process with iid positive uncertain interarrival times $\xi_1, \xi_2, \xi_3, \ldots$. If Φ_k is the common regular uncertainty distribution of those interarrival times, then $M(k)$ has an uncertainty distribution

$$\gamma_k(x) = 1 - \Phi_k(\frac{T}{\lfloor x \rfloor + 1}), \ \forall x \geq 0 \quad (7)$$

where $\lfloor x \rfloor$ represents the maximal integer less than or equal to x.

Since $M(k)(k = 1, 2, \ldots, N)$ is respectively a subtractive function of $\xi_k (k = 1, 2, \ldots, N)$, according to Lemma 3.1, we have:

$$E[\sum_{k=1}^{N} M(k)] = \sum_{k=1}^{N} E[M(k)] \quad (8)$$

From Lemma 3.2 and Lemma 3.3:

$$E[M(k)] = \int_0^{\infty} \Phi_k(\frac{T}{\lfloor x \rfloor + 1})dx \quad (9)$$

For $\xi_k = \beta^{k-1}\xi_1$, it has

$$\begin{aligned} \Phi_k(x) &= \mathcal{M}(\xi_k \leq x) \\ &= \mathcal{M}\left(\beta^{k-1}\xi_1 \leq x\right) \\ &= \mathcal{M}\left(\xi_1 \leq \beta^{1-k}x\right) \\ &= \Phi\left(\frac{x}{\beta^{k-1}}\right), \quad k > 1 \end{aligned} \quad (10)$$

then,

$$\begin{aligned} E[M(k)] &= \int_0^{\infty} \Phi_k(\frac{T}{\lfloor x \rfloor + 1})dx \\ &= \int_0^{\infty} \Phi(\frac{T}{\lfloor x\beta^{1-k} \rfloor + 1})dx \end{aligned} \quad (11)$$

Substitute $E[M(k)]$ into (4), the model is transformed into:

$$\begin{aligned}C(N;T) &= \min_{N,T} E\left[C\left(N,T,\xi\right)\right] \\ &= \min_{N,T} \frac{c_1 E(\sum_{k=1}^{N} M(k)) + c_2(N-1)+c_3}{NT} \\ &= \min_{N,T} \frac{c_1 \sum_{k=1}^{N} \int_0^{\infty} \Phi(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1})dx + c_2(N-1)+c_3}{NT}\end{aligned} \tag{12}$$

Next, we discuss the optimal solution of the model:

For specified $T > 0$ and β, we seek an optimum replacement number $N^*(1 \le N^* \le \infty)$ that minimizes $C(N,T)$. From the inequality $C(N+1;T) \ge C(N;T)$, that is

$$\begin{aligned}&\frac{c_1 \sum_{k=1}^{N+1} \int_0^{\infty} \Phi(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1})dx + c_2 N+c_3}{(N+1)\,T} \\ &\ge \frac{c_1 \sum_{k=1}^{N} \int_0^{\infty} \Phi(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1})dx + c_2(N-1)+c_3}{NT}\end{aligned} \tag{13}$$

We have,

$$L\left(N;T\right) \ge \frac{(c_3-c_2)}{c_1} \qquad (N=1,2,\cdots) \tag{14}$$

where,

$$\begin{aligned}L\left(N;T\right) &= N\int_0^{\infty} \Phi\left(\frac{T}{\lfloor x\beta^{-N}\rfloor+1}\right)dx \\ &\quad - \sum_{k=1}^{N}\int_0^{\infty} \Phi\left(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1}\right)dx \\ &= \sum_{k=1}^{N}\int_0^{\infty}\left[\Phi\left(\frac{T}{\lfloor x\beta^{-N}\rfloor+1}\right) - \Phi\left(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1}\right)\right]dx \\ &= \sum_{k=1}^{N}\int_0^{\infty}\Phi\left[\left(\frac{T}{\lfloor x\beta^{-N}\rfloor+1}\right) - \left(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1}\right)\right]dx\end{aligned} \tag{15}$$

In addition, we have

$$\begin{aligned}&L\left(N+1;T\right) - L\left(N;T\right) \\ &= \sum_{k=1}^{N+1}\int_0^{\infty}\Phi\left[\left(\frac{T}{\lfloor x\beta^{-N-1}\rfloor+1}\right) - \left(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1}\right)\right]dx \\ &\quad - \sum_{k=1}^{N}\int_0^{\infty}\Phi\left[\left(\frac{T}{\lfloor x\beta^{-N}\rfloor+1}\right) - \left(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1}\right)\right]dx \\ &= (N+1)\int_0^{\infty}\Phi\left[\left(\frac{T}{\lfloor x\beta^{-N-1}\rfloor+1}\right) - \left(\frac{T}{\lfloor x\beta^{-N}\rfloor+1}\right)\right]dx\end{aligned} \tag{16}$$

It can be seen that the above formula (16) is always greater than 0. So $L(N;T)$, when T and β are specified, is a constant increasing function with respect to N.

Therefore, we have the following optimum policy:

(i) If $L\left(\infty;T\right) \equiv \lim_{N\to\infty} L\left(N;T\right) > (c_3-c_2)/c_1$, then there exists a finite and unique minimum N^* that satisfies (14).

(ii) If $L\left(\infty;T\right) \le (c_3-c_2)/c_1$, then $N^* = \infty$, and the expected cost rate is

$$\begin{aligned}C\left(\infty;T\right) &= \lim_{N\to\infty} C\left(N;T\right) \\ &= \lim_{N\to\infty} \frac{c_1 \sum_{k=1}^{N} \int_0^{\infty} \Phi(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1})dx + c_2(N-1)+c_3}{NT} \\ &= c_1 \lim_{N\to\infty} \frac{\sum_{k=1}^{N} \int_0^{\infty} \Phi(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1})dx}{NT} + \frac{c_2}{T}.\end{aligned} \tag{17}$$

4 Numerical example

In this section, we apply our policy to the maintenance of a new kind of aircraft parts, such as cargo bulkhead, which is made by a new developed aluminum alloy. The data of the lifetime are unavailable, and as a result, the lifetime is regarded as a uncertain variable with regular uncertainty distribution. We invite some experts in the field of cargo bulkhead to provide possible lifetime and belief degrees. According to the experts' belief, we find that the lifetime of cargo bulkhead follows a normal uncertain distribution, [1]. And then the uncertainty distribution function of the lifetime ξ_1 is:

$$\Phi\left(x\right) = \left(1+\exp\left(\frac{\pi\left(e-x\right)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad x \in \Re \tag{18}$$

Then,

$$
\begin{aligned}
&C(N;T)\\
&=\min_{N,T}\frac{c_1\sum_{k=1}^{N}\int_0^{\infty}\Phi\left(\frac{T}{\lfloor x\beta^{1-k}\rfloor+1}\right)dx+c_2(N-1)+c_3}{NT}\\
&=\min_{N,T}\frac{c_1\sum_{k=1}^{N}\int_0^{\infty}\left(1+\exp\left(\frac{\pi\left(e-\frac{T}{\lfloor x\beta^{1-k}\rfloor+1}\right)}{\sqrt{3}\sigma}\right)\right)^{-1}dx}{NT}\\
&\quad+\frac{c_2(N-1)+c_3}{NT}
\end{aligned}
\tag{19}
$$

According to the principle of least squares given by Liu in [4], we obtain the estimated values of unknown parameters of the uncertainty distribution: $\hat{e}=10$, $\hat{\sigma}=\sqrt{3}$. Let the minimal repair cost of component $c_1=\$1.5$, cost of preventive maintenance $c_2=\$5$, and the cost of planned replacement $c_3=\$25$. The optimal solutions of the number N^* and replacement cycle length T^* of preventive maintenance are obtained for minimizing the expected cost function $C(N;T)$, and shown in Fig.2.

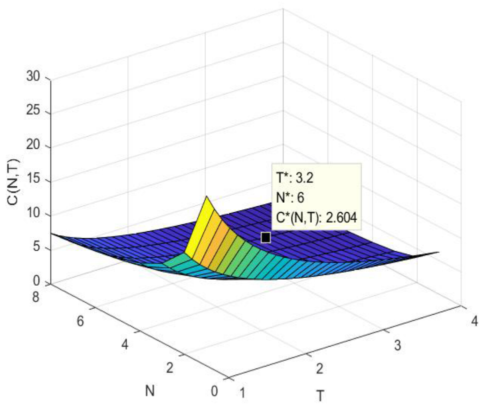
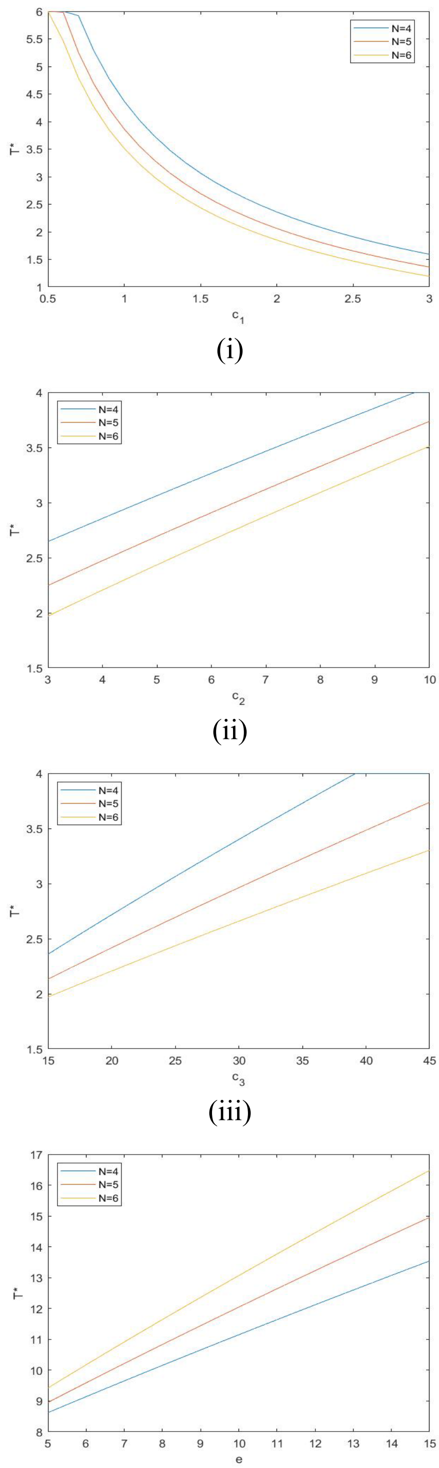


Fig.2: Optimal replacement cycle length T^* and the number N^* of preventive maintenance

It can be found from the Fig.2 that the objective function is a convex function, that is, there is an optimal solution. We can obtain the minimal cost $C^*(N;T)$ when $N^*=6$ and $T^*=3.2$. So, the aircraft cargo bulkhead shoud be subject to preventive maintenance every 3.2 years, and can be planned to be replaced at the 6th preventive maintenance. Then, the minimal preventive maintenance cost is calculated to be \$2.604.

The sensitivity analysis of parameters in our model are given in Fig.3.



(i)

(ii)

(iii)

(iv)

Fig.3: Optimal replacement cycle length T^* for increasing values of (i) c_1, (ii) c_2, (iii) c_3, (iv) e

Analyzing from Fig.3 (i) to (iii), we can see that the optimal value is significantly sensitive with respect to cost factor. Fig.3 (i) shows that the optimal replacement cycle length T^* is decreasing with the growth of the cost of minimal repair c_1, and decreases more slowly with an increasing value of c_1, which indicates that the replacement cycle length should be shortened when the cost of minimal repair is too high. Fig.3 (ii), (iii) indicate that the optimal replacement cycle length T^* is increasing with the cost of preventive maintenance c_2 increases, and when the replacement cost c_3 is large enough, T^* will increase, too. It can be interpreted that the replacement cycle length should be extended properly to reduce the cost rate if the cost of preventive maintenance c_2 and the replacement cost c_3 are too much in a replacement cycle. From Fig.3 (iv), the optimal replacement cycle length T^* increases when the e becomes large.

Meanwhile, in Fig.3 (i) to (iii), the optimal replacement cycle length T^* is inversely proportional to the number of preventive maintenance N^*, that is, when the number N^* is increasing, the optimal replacement cycle length T^* is on a downtrend. However, in Fig.3 (iv), the optimal replacement cycle length T^* is proportional to the number of preventive maintenance N^*.

5 Conclusion

This paper studied the preventive maintenance policy with lifetime reduction discount rate based on the uncertain renewal process. A kind of component which was made by a new developed composite material, so there was no historical data to obtain the probability distribution of the lifetime, the stochastic method was not applicable for our research yet. Thus, the lifetime of component was regarded as an uncertain variable, and considering that the lifetime of component would reduce by a fixed unit after each imperfect preventive maintenance. Next, we took the preventive maintenance cycle T and number N as the decision variables, the minimal expected cost rate as the objective function. Then, a preventive maintenance model with the lifetime reduction discount rate was established. Finally, a numerical example was given and the sensitivity analysis of the parameters were made. The results showed that the optimal preventive replacement cycle length T^* would decrease with the increase of minimal repair cost of component, and increase with the increase of preventive maintenance cost and replacement cost.

In the future works, we may focus on the following issues:

(i) In this work, the time of minimal repair and preventive maintenance are negligible. Next, we can consider the situation that its time can not be ignored in the future.

(ii) Also, in this work, we assumed that the lifetime of component follows an uncertain distribution due to no available lifetime data. Furthermore, when the operational data is fully obtained, some random lifetime distributions for the original system lifetime can be considered, such as weibull or normal distribution.

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Contribution of individual authors to the creation of a scientific article

Chunxiao Zhang proposed the idea of the model and checked the correctness of the manuscript.
Xiaona Liu established the mathematical model and gave the existence conditions of the solution.
Xinwang Li gave an numerical example and wrote the article.
Yizhou Bai checked the logic of the article and the coherence of the language.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.