Rogue waves of the Hirota equation in terms of quasi-rational solutions depending on multi-parameters

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Abstract:Quasi-rational solutions to the Hirota equation are given. We con- struct explicit expressions of these solutions for the first orders. As a byproduct, we get quasi-rational solutions to the focusing NLS equation and also rational solutions to the mKdV equation. We study the patterns of these configurations in the (x, t) plane.

Key-Words: Hirota equation - quasi-rational solutions.

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1. Introduction

We consider the Hirota (H) equation in the following normalization

$$iu_t + \alpha(u_{xx} + 2|u|^2 u) - i\beta(u_{xxx} + 6|u|^2 u_x) = 0, \quad (1)$$

with as usual the subscript meaning the partial derivatives and α, β real numbers.
Hirota proposed this equation in 1973 in a slightly different formulation

$$iu_t + \rho_1 u_{xx} + \delta_1 |u|^2 u + i\sigma_1 u_{xxx} + 3i\alpha_1 |u|^2 u_x = 0, \quad (2)$$

with α_1, ρ_1, σ_1, δ_1 real numbers satisfying $\alpha_1\rho_1 = \sigma_1\delta_1$ and built a kind of soliton in [1].
This last equation can be rewritten as (1) by choosing $\alpha_1 = -2\beta$, $\delta_1 = 2\alpha$, $\rho_1 = \alpha$, $\sigma_1 = -\beta$.
Maccari introduced in 1998 [2]a two dimensional extension of this equation. This equation is used to identify many kinds of nonlinear phenomena in the fields of physics, in particular in optical fibers. It describes the evolution of the slowly varying amplitude of a nonlinear train in weakly nonlinear systems. Hirota equation is an integrable equation which has a number of physical applications, such as the propagation of optical pulses [3] or in plasma physics, [4].
A lot of methods to solve this equation have been presented such as general projective Riccati equation method [5], Darboux transformation [6, 7] or the trace method [8].
Here, we try to construct other types of solutions to the Hirota equation, belonging to the AKNS hierarchy, in order to get as particular case some solutions to the NLS and mKdV equations. We construct quasi-rational solutions for the first orders.

2. Quasi-rational solutions of order 1 to the Hirota equation

Theorem 2.1 *The function $v(x,t)$ defined by*

$$v(x,t) = \left(1 - 4\,\frac{1 + 4\,i\alpha t}{1 + (2\,x + 12\,\beta t)^2 + 16\,\alpha^2 t^2}\right) e^{2\,iat} \quad (3)$$

is a solution to the Hirota equation (1)

$$iu_t + \alpha(u_{xx} + 2|u|^2 u) - i\beta(u_{xxx} + 6|u|^2 u_x) = 0.$$

Proof: Just replace the expression of the solution given by (3) and check that (1) is satisfied.

We give another type of solution with one real parameter a_1.

Theorem 2.2 *The function $v(x,t)$ defined by*

$$v(x,t) = \left(1 - 4\,\frac{1 + i\,(4\,\alpha t - 24\,a_1)}{1 + (2\,x + 12\,\beta t)^2 + (4\,\alpha t - 24\,a_1)^2}\right) \times e^{i(2\,\alpha t - 6\,a_1)} \quad (4)$$

is a solution to the Hirota equation (1)

$$iu_t + \alpha(u_{xx} + 2|u|^2 u) - i\beta(u_{xxx} + 6|u|^2 u_x) = 0,$$

with a_1 arbitrary real parameter.

Proof: It is sufficient to replace the expression of the solution given by (4) and check that (1) is satisfied.

Remark 2.1 *It is possible to add other real parameters, but this does not radically change the structure of the solution as in the case of a parameter a_1. Therefore, we do not mention these parameters in the following.*

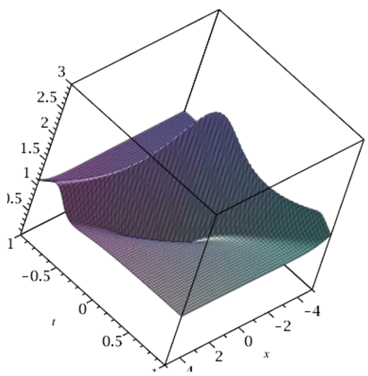

Figure 1. Solution of order 1 to (1), $a_1 = 0$; with $\alpha = \beta = 1$.

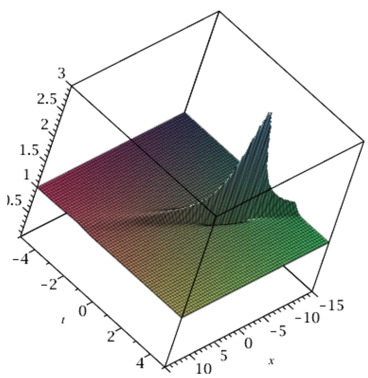

Figure 2. Solution of order 1 to (1), $a_1 = 0.2$; with $\alpha = \beta = 1$.

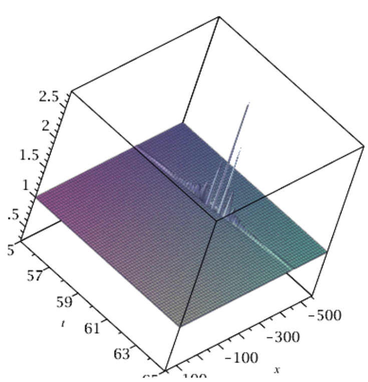

Figure 3. Solution of order 1 to (1), $a_1 = 10$; with $\alpha = \beta = 1$.

Remark 2.2 *If we choose, $\alpha = 1$, $\beta = 0$ and the other parameters equal to 0, we recover the classical Peregrine breather, solution to the NLS equation*

$$iu_t + u_{xx} + 2|u|^2 u = 0, \qquad (5)$$

$$v(x,t) = \left(1 - 4\,\frac{1 + 4\,it}{1 + 4x^2 + 16\,t^2}\right) e^{2\,it}. \quad (6)$$

This solution was first found by Peregrine [9].
It was rediscovered by the present author by using degeneracy of Riemann theta functions in [10].

Remark 2.3 *If we choose, $\alpha = 0$, $\beta = 1$ and the parameters equal to 0, we recover a non singular rational solution to the mKdV equation*

$$u_t = u_{xxx} + 6|u|^2 u_x \qquad (7)$$

$$v(x,t) = \left(1 - \frac{4}{1 + (2x + 12t)^2}\right). \quad (8)$$

This solution is different from those constructed by the author in [11].

3. Quasi-rational solutions of order 2 to the Hirota equation

Theorem 3.1 *The function $v(x,t)$ defined by*

$$v(x,t) = \left(1 - 12\,\frac{n(x,t)}{d(x,t)}\right) e^{2\,i\alpha\,t} \qquad (9)$$

with
$n(x,t) = (2\,x + 12\,\beta\,t)^4 + 6\,(16\,\alpha^2 t^2 + 1)(2\,x + 12\,\beta\,t)^2 + 192\,\beta\,t(2\,x + 12\,\beta\,t) + 1280\,\alpha^4 t^4 + 288\,\alpha^2 t^2 - 3 + i(4\,\alpha\,t(2\,x +$

$12\,\beta\,t)^4 + 2\,(64\,\alpha^3 t^3 - 12\,\alpha\,t)(2\,x + 12\,\beta\,t)^2 + 768\,\alpha\,t^2\beta\,(2\,x + 12\,\beta\,t) + 1024\,\alpha^5 t^5 + 128\,\alpha^3 t^3 - 60\,\alpha\,t)$

and

$d(x,t) = ((2\,x + 12\,\beta\,t)^2 + 16\,\alpha^2 t^2 + 1)^3 - 192\,\beta\,t(2\,x + 12\,\beta\,t)^3 - 24\,(16\,\alpha^2 t^2 - 1)(2\,x + 12\,\beta\,t)^2 + 576\,(16\,\alpha^2 t^2 + 1)\beta\,t(2\,x + 12\,\beta\,t) + 6144\,\alpha^4 t^4 + 8 + 1536\,\alpha^2 t^2 + 9216\,\beta^2 t^2$

is a solution to the Hirota equation (1).

Proof: By replacing the expression of the given solution with (9), we check that the relation (1) is satisfied.

We can give also the solution depending on 2 real parameters.
The function $v(x,t)$ defined by

Theorem 3.2

$$v(x,t) = \left(1 - 12\,\frac{n(x,t)}{d(x,t)}\right) \mathrm{e}^{i(2\,\alpha\,t - 6\,a_1)} \qquad (10)$$

with

$n(x,t) = (2\,x + 12\,\beta\,t + 60\,b_1)^4 + 6\,((4\,\alpha\,t - 24\,a_1)^2 + 1)(2\,x + 12\,\beta\,t + 60\,b_1)^2 - 24\,(-8\,\beta\,t - 80\,b_1)(2\,x + 12\,\beta\,t + 60\,b_1) + 5\,(4\,\alpha\,t - 24\,a_1)^4 + 18\,(4\,\alpha\,t - 24\,a_1)^2 - 384\,a_1(4\,\alpha\,t - 24\,a_1) - 3 + i((4\,\alpha\,t - 24\,a_1)(2\,x + 12\,\beta\,t + 60\,b_1)^4 + 2\,((4\,\alpha\,t - 24\,a_1)^3 - 12\,\alpha\,t + 168\,a_1)(2\,x + 12\,\beta\,t + 60\,b_1)^2 - 24\,(4\,\alpha\,t - 24\,a_1)(-8\,\beta\,t - 80\,b_1)(2\,x + 12\,\beta\,t + 60\,b_1) + (4\,\alpha\,t - 24\,a_1)^5 + 2\,(4\,\alpha\,t - 24\,a_1)^3 - 192\,a_1(4\,\alpha\,t - 24\,a_1)^2 - 60\,\alpha\,t + 552\,a_1)$

and

$d(x,t) = ((2\,x + 12\,\beta\,t + 60\,b_1)^2 + (4\,\alpha\,t - 24\,a_1)^2 + 1)^3 + 24\,(-8\,\beta\,t - 80\,b_1)(2\,x + 12\,\beta\,t + 60\,b_1)^3 - 24\,((4\,\alpha\,t - 24\,a_1)^2 - 48\,a_1(4\,\alpha\,t - 24\,a_1) - 1)(2\,x + 12\,\beta\,t + 60\,b_1)^2 - 72\,((4\,\alpha\,t - 24\,a_1)^2 + 1)(-8\,\beta\,t - 80\,b_1)(2\,x + 12\,\beta\,t + 60\,b_1) + 24\,(4\,\alpha\,t - 24\,a_1)^4 - 384\,a_1(4\,\alpha\,t - 24\,a_1)^3 + 96\,(4\,\alpha\,t - 24\,a_1)^2 - 3456\,a_1(4\,\alpha\,t - 24\,a_1) + 36864\,{a_1}^2 + 144\,(-8\,\beta\,t - 80\,b_1)^2 + 8$

is a solution to the Hirota equation (1).

Proof: By replacing the expression of the solution given by (10), we check that the relation (1) is satisfied.

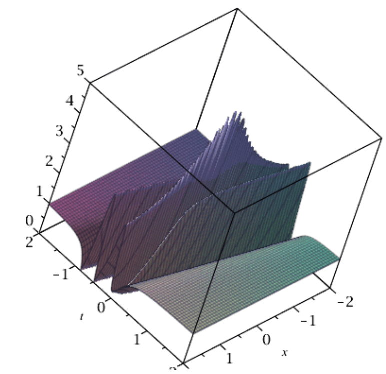

Figure 4. Solution of order 1 to (1); $a_1 = 0$, $b_1 = 0$ with $\alpha = \beta = 1$.

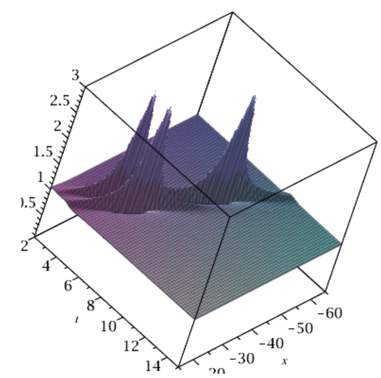

Figure 5. Solution of order 1 to (1); $a_1 = 1$, $b_1 = 0$ with $\alpha = \beta = 1$.

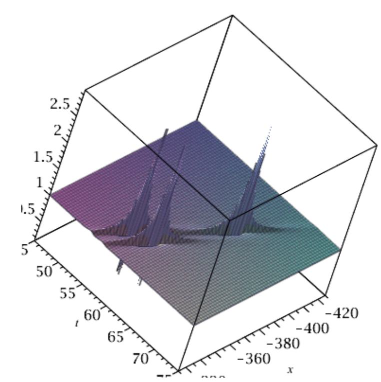

Figure 6. Solution of order 1 to (1), $a_1 = 10$, $b_1 = 0$ with $\alpha = \beta = 1$.

We observe a very fast evolution of the structure of the solution according to the value of th parameter a_1 toward three peaks.

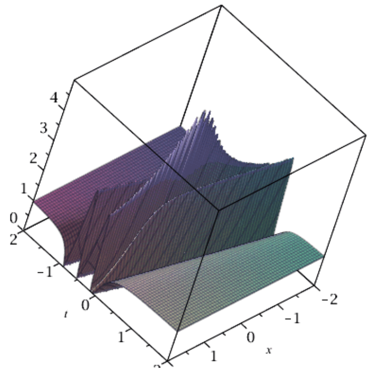

Figure 7. Solution of order 1 to (1), $a_1 = 0$, $b_1 = 0.01$ with $\alpha = \beta = 8$.

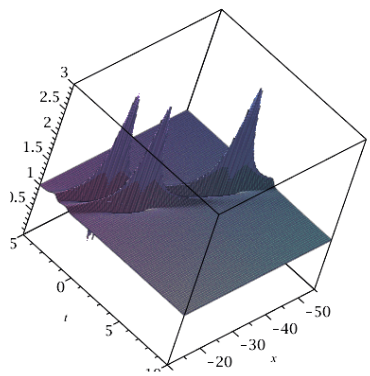

Figure 8. Solution of order 1 to (1), $a_1 = 0$, $b_1 = 0.05$ with $\alpha = \beta = 8$.

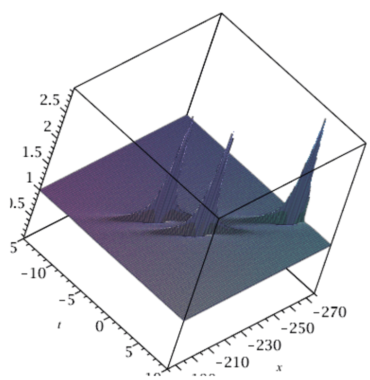

Figure 9. Solution of order 1 to (1), $a_1 = 0$, $b_1 = 1$ with $\alpha = \beta = 8$.

The evolution previously observed is the same as in the case of the parameter a_1.

Remark 3.1 *If we choose,* $\alpha = 1$, $\beta = 0$ *and the parameters equal to* 0, *we recover solutions to the NLS equation*

$$iu_t + u_{xx} + 2|u|^2 u = 0, \qquad (11)$$

$$u_k(x,t) = (1 - 12\,\frac{n(x,t)}{d(x,t)})\mathrm{e}^{2\,it} \qquad (12)$$

with
$n(x,t) = 16\,x^4 + 24\,(16\,t^2+1)x^2 - 3 + 1280\,t^4 + 288\,t^2 + i(64\,tx^4 + 8\,(64\,t^3 - 12\,t)x^2 + 1024\,t^5 + 128\,t^3 - 60\,t)$
and
$d(x,t) = (4\,x^2 + 16\,t^2 + 1)^3 + 8 - 96\,(16\,t^2 - 1)x^2 + 6144\,t^4 + 1536\,t^2$.
We recover the solutions given by the author in [10].

Remark 3.2 *If we choose,* $\alpha = 0$, $\beta = 1$ *and the parameters equal to* 0, *we recover rational solutions to the mKdV equation*

$$u_t = u_{xxx} + 6|u|^2 u_x \qquad (13)$$

$$u(x,t) = (1 - 12\,\frac{n(x,t)}{d(x,t)}) \qquad (14)$$

with
$n(x,t) = (2\,x + 12\,t)^4 + 6\,)2\,x + 12\,t)^2 + 192\,t(2\,x + 12\,t) - 3$
and $v(x,t) = ((2\,x + 12\,t)^2 + 1)^3 - 192$
$,t(2\,x + 12\,t)^3 + 24\,(2\,x + 12\,t)^2$
$+ 576\,t(2\,x + 12\,t) + 8 + 9216\,t^2$.
These solutions are different from these presented in [12].

4. Quasi-rational solutions of order 3 to the Hirota equation

Theorem 4.1 *The function $v(x,t)$ defined by*

$$v(x,t) = \left(1 - 24\,\frac{n(x,t)}{d(x,t)}\right) e^{2\,i\alpha\,t} \quad (15)$$

with

$n(x,t) = (2\,x+12\,\beta\,t)^{10}+15\,(1+16\,\alpha^2t^2)(2\,x+12\,\beta\,t)^8 + (210 - 960\,\alpha^2t^2 + 12800\,\alpha^4t^4)(2\,x+12\,\beta\,t)^6 + (92160\,\alpha^2t^3\beta+5760\,\beta\,t)(2\,x+12\,\beta\,t)^5 + (7200\,\alpha^2t^2-38400\,\alpha^4t^4+286720\,\alpha^6t^6 - 450 + 345600\,\beta^2t^2)(2\,x+12\,\beta\,t)^4 + (4915200\,\alpha^4t^5\beta - 57600\,\beta\,t + 1843200\,\alpha^2t^3\beta)(2\,x+12\,\beta\,t)^3 + (1728000\,\alpha^4t^4 + 1720320\,\alpha^6t^6 +2949120\,t^8\alpha^8-43200\,\alpha^2t^2(5+256\,\beta^2t^2) - 675 - 691200\,\beta^2t^2)(2\,x+12\,\beta\,t)^2 + (55050240\,\alpha^6t^7\beta + 1382400\,\alpha^2t^3\beta - 86400\,\beta\,t - 22118400\,\beta^3t^3 - 7372800\,\alpha^4t^5\beta)(2\,x+12\,\beta\,t) + 8970240\,\alpha^6t^6 + 32440320\,t^8\alpha^8 + 11534336\,t^{10}\alpha^{10} + 675 + 115200\,\alpha^4t^4(-17 + 1792\,\beta^2t^2) + 10800\,\alpha^2t^2(-3 +1024\,\beta^2t^2)+1728000\,\beta^2t^2+i((-60\,\alpha\,t + 320\,\alpha^3t^3)(2\,x+12\,\beta\,t)^8 + (-600\,\alpha\,t -8960\,\alpha^3t^3+10240\,\alpha^5t^5)(2\,x+12\,\beta\,t)^6 + (122880\,\alpha^3t^4\beta - 23040\,\alpha\,t^2\beta)(2\,x+12\,\beta\,t)^5 + (-28800\,\alpha^3t^3 - 215040\,\alpha^5t^5 + 163840\,\alpha^7t^7 + 1800\,\alpha\,t(-3 + 768\,\beta^2t^2))(2\,x+12\,\beta\,t)^4 - 14254080\,\alpha^7t^7 + (3932160\,\alpha^5t^6\beta - 230400\,\alpha\,t^2\beta + 2457600\,\alpha^3t^4\beta)(2\,x +12\,\beta\,t)^3+(1751040\,\alpha^5t^5-983040\,\alpha^7t^7 +1310720\,t^9\alpha^9-57600\,\alpha^3t^3(7+256\,\beta^2t^2) +2700\,\alpha\,t(7+1024\,\beta^2t^2))(2\,x+12\,\beta\,t)^2 + 6553600\,t^9\alpha^9 + (31457280\,\alpha^7t^8\beta + 43200\,\alpha\,t(-24\,\beta\,t - 2048\,\beta^3t^3) -5529600\,\alpha^3t^4\beta-41287680\,\alpha^5t^6\beta)(2\,x + 12\,\beta\,t) + 4194304\,t^{11}\alpha^{11} + 4\,\alpha\,t(2\,x +12\,\beta\,t)^{10}+92160\,\alpha^5t^5(-107+1792\,\beta^2t^2)- 14400\,\alpha^3t^3(11+5120\,\beta^2t^2)-2700\,\alpha\,t(-7+ 3584\,\beta^2t^2))$

and

$d(x,t) = 2024+(-1920\,\alpha^2t^2+120)(2\,x+ 12\,\beta\,t)^8+(7680\,\alpha^2t^2-61440\,\alpha^4t^4+2320+ 138240\,\beta^2t^2)(2\,x+12\,\beta\,t)^6+(1474560\,\alpha^4t^5\beta- 552960\,\alpha^2t^3\beta+51840\,\beta\,t)(2\,x+12\,\beta\,t)^5+ (-368640\,\alpha^4t^4+3360+3840\,\alpha^2t^2(56+ 8640\,\beta^2t^2)+2073600\,\beta^2t^2)(2\,x+12\,\beta\,t)^4+ (31457280\,\alpha^6t^7\beta+16588800\,\alpha^2t^3\beta+88473600\,\alpha^4t^5\beta- 345600\,\beta\,t+44236800\,\beta^3t^3)(2\,x+12\,\beta\,t)^3+ (55050240\,\alpha^6t^6+15728640\,t^8\alpha^8-61440\,\alpha^4t^4(-326+ 2880\,\beta^2t^2)+7680\,\alpha^2t^2(-76+8640\,\beta^2t^2)+ 12144 - 6220800\,\beta^2t^2)(2\,x+12\,\beta\,t)^2 + (188743680\,\alpha^8t^9\beta + 47185920\,\alpha^6t^7\beta - 66355200\,\alpha^4t^5\beta+1036800\,\alpha^2t^2(40\,\beta\,t- 2048\,\beta^3t^3)+648000\,\beta\,t-132710400\,\beta^3t^3)(2\,x+ 12\,\beta\,t)+22809600\,\beta^2t^2+243793920\,t^8\alpha^8+ 125829120\,t^{10}\alpha^{10} + 2123366400\,\beta^4t^4 - 960\,\beta\,t(2\,x+12\,\beta\,t)^9+327680\,\alpha^6t^6(191+ 4032\,\beta^2t^2)+61440\,\alpha^4t^4(599+8640\,\beta^2t^2)+ 384\,\alpha^2t^2(3881+777600\,\beta^2t^2)+(1+(2\,x+ 12\,\beta\,t)^2 + 16\,\alpha^2t^2)^6$

is a solution to the Hirota equation (1).

Proof: Check that the relation (1) is satisfied when we replace the expression of the solution defined by (15).

We can give also the solutions to the Hirota equation depending on 4 real parameters. Because of the length of the expression, we only give it in the appendix.
We give patterns of the modulus of the solutions in function of the parameters.

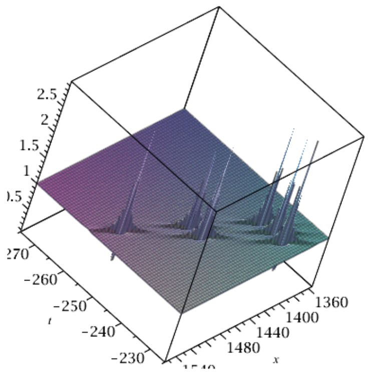

Figure 10. Solution of order 3 to (1), $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$ with $\alpha = \beta = 1$.

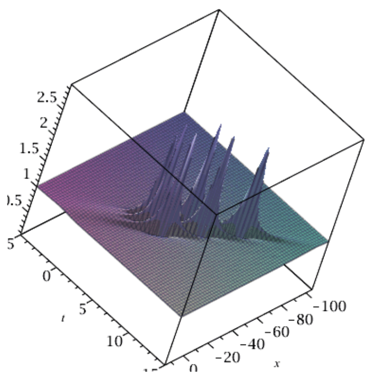

Figure 11. Solution of order 3 to (1), $a_1 = 1$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$ with $\alpha = \beta = 1$.

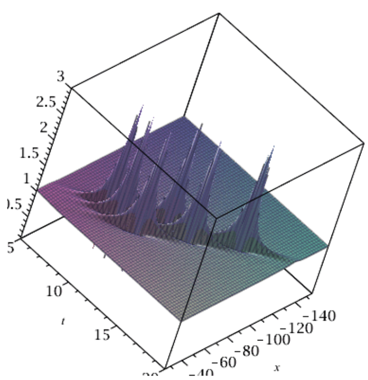

Figure 12. Solution of order 3 to (1), to the right $a_1 = 2$, $b_1 = 0$, $a_2 = 10$, $b_2 = 0$ with $\alpha = \beta = 1$.

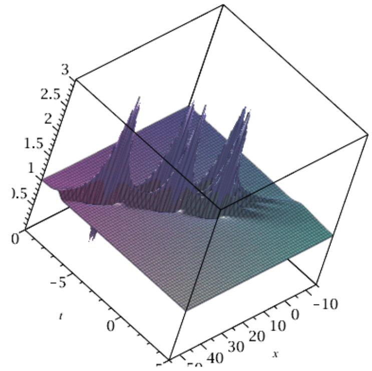

Figure 13. Solution of order 3 to (1), $a_1 = 0$, $b_1 = 0$, $a_2 = 0.1$, $b_2 = 0$ with $\alpha = \beta = 1$.

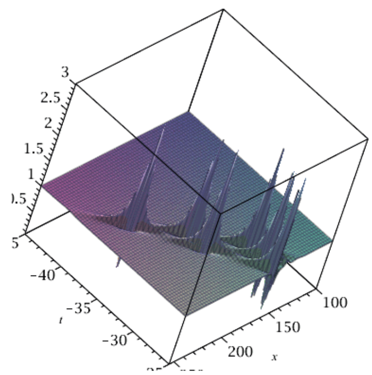

Figure 14. Solution of order 3 to (1), $a_1 = 0$, $b_1 = 10$, $a_2 = 1$, $b_2 = 0$ with $\alpha = \beta = 1$.

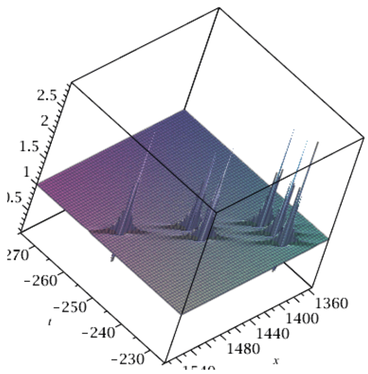

Figure 15. Solution of order 3 to (1), $a_1 = 10$, $b_1 = 0$, $a_2 = 8$, $b_2 = 0$ with $\alpha = \beta = 1$.

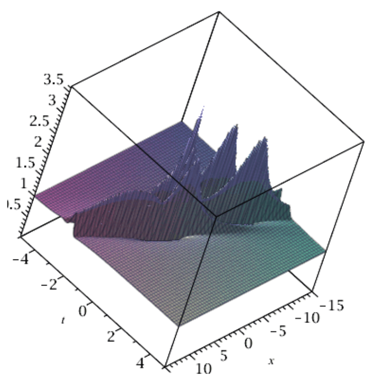

Figure 16. Solution of order 3 to (1), $a_1 = 0$, $b_1 = 0.1$, $a_2 = 0$, $b_2 = 0$ with $\alpha = \beta = 1$.

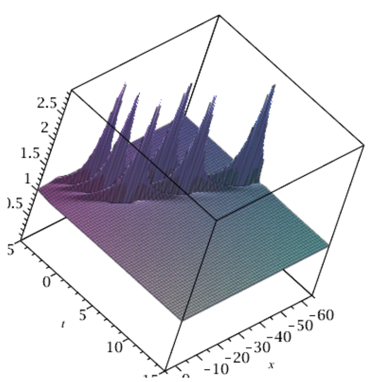

Figure 17. Solution of order 3 to (1), $a_1 = 0$, $b_1 = 1$, $a_2 = 0$, $b_2 = 0$ with $\alpha = \beta = 1$.

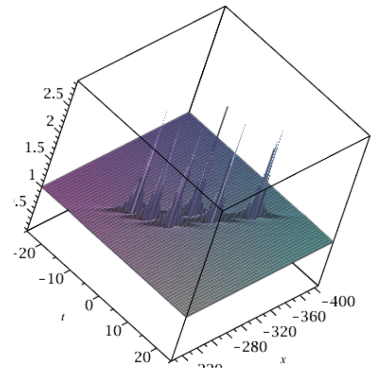

Figure 18. Solution of order 3 to (1), $a_1 = 2$, $b_1 = 10$, $a_2 = 0$, $b_2 = 0$ with $\alpha = \beta = 1$.

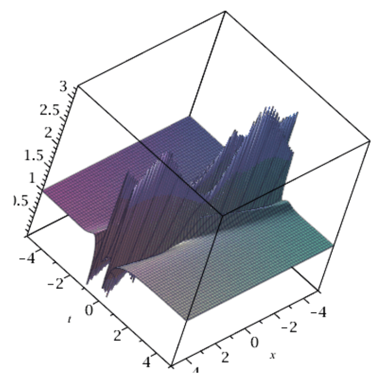

Figure 19. Solution of order 3 to (1), $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0.01$ with $\alpha = \beta = 1$.

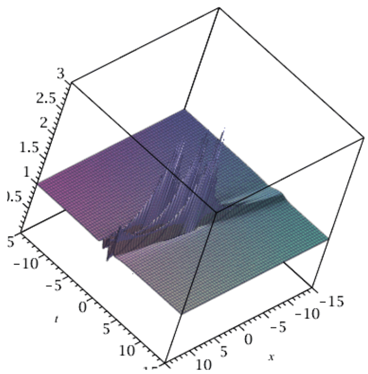

Figure 20. Solution of order 3 to (1), $a_1 = 0$, $b_1 = 10$, $a_2 = 0$, $b_2 = 0.1$ with $\alpha = \beta = 1$.

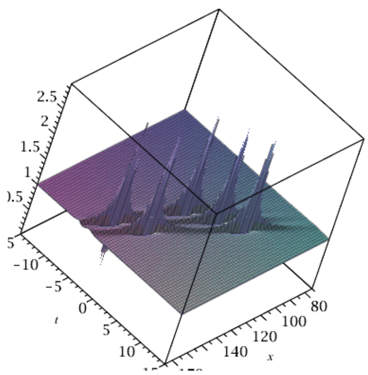

Figure 21. Solution of order 3 to (1), $a_1 = 10$, $b_1 = 0$, $a_2 = 0$, $b_2 = 1$ with $\alpha = \beta = 1$.

In these examples, we note the appearance of triangles with six peaks.

Remark 4.1 *If we choose, $\alpha = 1$, $\beta = 0$ and the parameters equal to 0, we recover solutions to the NLS equation*

$$iu_t + u_{xx} + 2|u|^2u = 0, \qquad (16)$$

$$u_k(x,t) = (1 - 24\,\frac{n(x,t)}{d(x,t)})\mathrm{e}^{2\,i\alpha\,t} \qquad (17)$$

with
$n(x,t) = 16\,x^4 + 24\,((4\,t - 24000)^2 + 1)x^2 + 9215999997 + 5\,(4\,t - 24000)^4 + 18\,(4\,t - 24000)^2 - 1536000\,t + i(16\,(4\,t - 24000)x^4 + 8\,((4\,t - 24000)^3 - 12\,t + 168000)x^2 + (4\,t - 24000)^5 + 2\,(4\,t - 24000)^3 - 192000\,(4\,t - 24000)^2 - 60\,t + 552000)$
and
$d(x,t) = (4\,x^2 + (4\,t - 24000)^2 + 1)^3 + 119808000008 - 96\,((4\,t - 24000)^2 - 192000\,t + 1151999999)x^2 + 24\,(4\,t - 24000)^4 - 384000\,(4\,t - 24000)^3 + 96\,(4\,t - 24000)^2 - 13824000\,t.$
*We recover the solutions constructed in [**?**].*

Remark 4.2 *If we choose, $\alpha = 0$, $\beta = 1$ and the parameters equal to 0, we recover rational solutions to the mKdV equation*

$$u_t = u_{xxx} + 6|u|^2u_x \qquad (18)$$

$$u_k(x,t) = (1 - 24\,\frac{n(x,t)}{d(x,t)})\mathrm{e}^{2\,i\alpha\,t} \qquad (19)$$

with
$n(x,t) = (2\,x + 12\,t)^4 + 6\,(2\,x + 12\,t)^2 + 192\,t(2\,x + 12\,t) - 3$
and $d(x,t) = ((2\,x + 12\,t)^2 + 1)^3 - 192\,t(2\,x + 12\,t)^3 + 24\,(2\,x + 12\,t)^2 + 576\,t(2\,x + 12\,t) + 8 + 9216\,t^2$.
These solutions are different from these given in [11, 12].

5. Conclusion

Quasi-rational solutions to the Hirota equation have been given for the first orders. As a byproduct, we have recover quasi-rational solutions to the NLS equation and rational solutions to the mKdV equation.
We can mention some other recent works about this equation. For example, in [13], by using the inverse scattering transform an explicit soliton solution formula for the Hirota equation has been

constructed. This formula allows to get, as a particular case, the N-soliton solution, the breather solution and, most relevantly, a new class of solutions called multi-pole soliton solutions.
By using the Hirota direct method, lump-soliton to the HIrota equation were constructed in [14].
In [15], the mixed localized wave solutions of the Hirota equation have been constructed through the modified Darboux transformation. One of them is the mixed 1-breather and 1-rogue wave solution and the other two are the mixed 1-breather and 2-rogue wave solution, and the mixed 2-breather and 1-rogue wave solution. These mixed localized wave solutions are presented graphically by choosing proper parameters and their dynamic behavior is briefly studied.

It will relevant to describe a more general formulation of the the solutions to the Hirota equation in terms of wronskians or Fredholm determinants as in the works [16, 17, 18] for NLS, KP or KdV equations.

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Appendix

Solution of order 3 to the Hirota equation depending on 4 real parameters :
The function $v(x,t)$ defined by

$$v(x,t) = \left(1 - 24\,\frac{n(x,t)}{d(x,t)}\right) \mathrm{e}^{2\,i\alpha\,t} \qquad (20)$$

with
$n(x,t) = 675 + 15\,(1 + (4\,\alpha\,t - 24\,a_1 + 120\,a_2)^2)(2\,x + 12\,\beta\,t + 60\,b_1 - 280\,b_2)^8 +$
$(210 - 60\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^2 + 50\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^4 + 480\,(4\,\alpha\,t - 24\,a_1 +$
$120\,a_2)(16\,a_1 - 160\,a_2))(2\,x + 12\,\beta\,t + 60\,b_1 - 280\,b_2)^6 + (-720\,(4\,\alpha\,t - 24\,a_1 +$
$120\,a_2)^2(-8\,\beta\,t - 80\,b_1 + 560\,b_2) + 5760\,\beta\,t - 11520\,b_1 + 564480\,b_2)(2\,x + 12\,\beta\,t +$
$60\,b_1 - 280\,b_2)^5 + (450\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^2 - 150\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^4 +$
$70\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^6 + 1200\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^3(16\,a_1 - 160\,a_2) - 450 +$
$5400\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)^2 - 1800\,(16\,a_1 - 160\,a_2)^2 + 3600\,(4\,\alpha\,t - 24\,a_1 +$
$120\,a_2)(16\,a_1 - 224\,a_2))(2\,x + 12\,\beta\,t + 60\,b_1 - 280\,b_2)^4 + (-2400\,(4\,\alpha\,t - 24\,a_1 +$
$120\,a_2)^4(-8\,\beta\,t - 80\,b_1 + 560\,b_2) + 28800\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)(-8\,\beta\,t - 80\,b_1 +$
$560\,b_2)(16\,a_1 - 160\,a_2) - 57600\,\beta\,t - 806400\,b_1 + 7257600\,b_2 - 7200\,(4\,\alpha\,t - 24\,a_1 +$
$120\,a_2)^2(-16\,\beta\,t - 128\,b_1 + 672\,b_2))(2\,x + 12\,\beta\,t + 60\,b_1 - 280\,b_2)^3 + (6750\,(4\,\alpha\,t -$
$24\,a_1 + 120\,a_2)^4 + 420\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^6 + 45\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^8 -$
$2700\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^2(5 + 4\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)^2 - 12\,(16\,a_1 - 160\,a_2)^2) -$
$675 - 10800\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)^2 - 10800\,(16\,a_1 - 160\,a_2)^2 + 21600\,(4\,\alpha\,t -$
$24\,a_1 + 120\,a_2)(32\,a_1 - 384\,a_2) - 7200\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^3(32\,a_1 - 128\,a_2))(2\,x +$
$12\,\beta\,t + 60\,b_1 - 280\,b_2)^2 + (-1680\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^6(-8\,\beta\,t - 80\,b_1 + 560\,b_2) -$
$28800\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^3(-8\,\beta\,t - 80\,b_1 + 560\,b_2)(16\,a_1 - 160\,a_2) - 10800\,(4\,\alpha\,t -$
$24\,a_1 + 120\,a_2)^2(-8\,\beta\,t - 272\,b_1 + 3248\,b_2) - 86400\,\beta\,t - 1209600\,b_1 + 10886400\,b_2 +$
$43200\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)^3 + 43200\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)(16\,a_1 - 160\,a_2)^2 +$
$3600\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^4(-8\,\beta\,t + 80\,b_1 - 1680\,b_2) - 86400\,(4\,\alpha\,t - 24\,a_1 +$
$120\,a_2)((-8\,\beta\,t - 80\,b_1 + 560\,b_2)(16\,a_1 - 160\,a_2) + (16\,a_1 - 160\,a_2)(32\,b_1 - 448\,b_2) -$
$64\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)a_2))(2\,x + 12\,\beta\,t + 60\,b_1 - 280\,b_2) - 720\,(4\,\alpha\,t - 24\,a_1 +$
$120\,a_2)^7(16\,a_1 - 160\,a_2) + 450\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^4(-17 + 28\,(-8\,\beta\,t - 80\,b_1 +$
$560\,b_2)^2 + 12\,(16\,a_1 - 160\,a_2)^2) - 3600\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^3(48\,a_1 - 1376\,a_2) -$
$720\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^5(272\,a_1 - 3168\,a_2) + 10800\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)(-16\,a_1 +$
$224\,a_2 + 4\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)^2(16\,a_1 - 160\,a_2) + 4\,(16\,a_1 - 160\,a_2)^3) +$
$675\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^2(-3 + 16\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)^2 + 16\,(16\,a_1 -$
$160\,a_2)^2 - 128\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)(32\,b_1 - 448\,b_2) - 8192\,(16\,a_1 - 160\,a_2)a_2) -$
$86400\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)(32\,b_1 - 448\,b_2) - 11059200\,(16\,a_1 - 160\,a_2)a_2 +$
$(2\,x + 12\,\beta\,t + 60\,b_1 - 280\,b_2)^{10} + 2190\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^6 + 27000\,(-8\,\beta\,t -$
$80\,b_1 + 560\,b_2)^2 + 91800\,(16\,a_1 - 160\,a_2)^2 + 495\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^8 + 11\,(4\,\alpha\,t -$
$24\,a_1 + 120\,a_2)^{10} + 86400\,(32\,b_1 - 448\,b_2)^2 + 353894400\,{a_2}^2 + i(-21600\,(-8\,\beta\,t -$
$80\,b_1 + 560\,b_2)^2(16\,a_1 - 160\,a_2) + (4\,\alpha\,t - 24\,a_1 + 120\,a_2)(2\,x + 12\,\beta\,t + 60\,b_1 -$
$280\,b_2)^{10} + (-60\,\alpha\,t + 840\,a_1 - 6600\,a_2 + 5\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^3)(2\,x + 12\,\beta\,t +$
$60\,b_1 - 280\,b_2)^8 + (-600\,\alpha\,t - 240\,a_1 + 58800\,a_2 - 140\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^3 +$
$10\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^5 + 240\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^2(16\,a_1 - 160\,a_2))(2\,x +$
$12\,\beta\,t + 60\,b_1 - 280\,b_2)^6 + (-240\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^3(-8\,\beta\,t - 80\,b_1 + 560\,b_2) -$
$1440\,(-8\,\beta\,t - 80\,b_1 + 560\,b_2)(16\,a_1 - 160\,a_2) + 720\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)(-8\,\beta\,t -$
$176\,b_1 + 1904\,b_2))(2\,x + 12\,\beta\,t + 60\,b_1 - 280\,b_2)^5 + (-450\,(4\,\alpha\,t - 24\,a_1 + 120\,a_2)^3 -$

$$
\begin{aligned}
&210\,(4\,\alpha\,t-24\,a_1+120\,a_2)^5+10\,(4\,\alpha\,t-24\,a_1+120\,a_2)^7+300\,(4\,\alpha\,t-24\,a_1+\\
&120\,a_2)^4(16\,a_1-160\,a_2)+450\,(4\,\alpha\,t-24\,a_1+120\,a_2)(-3+12\,(-8\,\beta\,t-80\,b_1+\\
&560\,b_2)^2-4\,(16\,a_1-160\,a_2)^2)-14400\,a_1+259200\,a_2+1800\,(4\,\alpha\,t-24\,a_1+\\
&120\,a_2)^2(16\,a_1-224\,a_2))(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^4+(-480\,(4\,\alpha\,t-24\,a_1+\\
&120\,a_2)^5(-8\,\beta\,t-80\,b_1+560\,b_2)+14400\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2(-8\,\beta\,t-80\,b_1+\\
&560\,b_2)(16\,a_1-160\,a_2)+7200\,(4\,\alpha\,t-24\,a_1+120\,a_2)(-8\,\beta\,t-48\,b_1+112\,b_2)-\\
&2400\,(4\,\alpha\,t-24\,a_1+120\,a_2)^3(-16\,\beta\,t-128\,b_1+672\,b_2)-14400\,(-8\,\beta\,t-80\,b_1+\\
&560\,b_2)(16\,a_1-160\,a_2)-14400\,(16\,a_1-160\,a_2)(32\,b_1-448\,b_2)+921600\,(-8\,\beta\,t-\\
&80\,b_1+560\,b_2)a_2)(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^3+(1710\,(4\,\alpha\,t-24\,a_1+120\,a_2)^5-\\
&60\,(4\,\alpha\,t-24\,a_1+120\,a_2)^7+5\,(4\,\alpha\,t-24\,a_1+120\,a_2)^9-900\,(4\,\alpha\,t-24\,a_1+\\
&120\,a_2)^3(7+4\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2-12\,(16\,a_1-160\,a_2)^2)+675\,(4\,\alpha\,t-24\,a_1+\\
&120\,a_2)(7+16\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2+16\,(16\,a_1-160\,a_2)^2)-345600\,a_1+\\
&4492800\,a_2-21600\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2(16\,a_1-160\,a_2)-21600\,(16\,a_1-\\
&160\,a_2)^3+691200\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2a_2-1800\,(4\,\alpha\,t-24\,a_1+120\,a_2)^4(64\,a_1-\\
&448\,a_2))(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^2+(-240\,(4\,\alpha\,t-24\,a_1+120\,a_2)^7(-8\,\beta\,t-\\
&80\,b_1+560\,b_2)-7200\,(4\,\alpha\,t-24\,a_1+120\,a_2)^4(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-\\
&160\,a_2)+10800\,(4\,\alpha\,t-24\,a_1+120\,a_2)(-24\,\beta\,t-400\,b_1+3920\,b_2+4\,(-8\,\beta\,t-\\
&80\,b_1+560\,b_2)^3+4\,(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)^2)+3600\,(4\,\alpha\,t-\\
&24\,a_1+120\,a_2)^3(-24\,\beta\,t-176\,b_1+784\,b_2)+720\,(4\,\alpha\,t-24\,a_1+120\,a_2)^5(-56\,\beta\,t-\\
&400\,b_1+1680\,b_2)+21600\,(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)+43200\,(16\,a_1-\\
&160\,a_2)(32\,b_1-448\,b_2)-2764800\,(-8\,\beta\,t-80\,b_1+560\,b_2)a_2-43200\,(4\,\alpha\,t-\\
&24\,a_1+120\,a_2)^2((-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)+(16\,a_1-160\,a_2)(32\,b_1-\\
&448\,b_2)-64\,(-8\,\beta\,t-80\,b_1+560\,b_2)a_2))(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)-90\,(4\,\alpha\,t-\\
&24\,a_1+120\,a_2)^8(16\,a_1-160\,a_2)+90\,(4\,\alpha\,t-24\,a_1+120\,a_2)^5(-107+28\,(-8\,\beta\,t-\\
&80\,b_1+560\,b_2)^2+12\,(16\,a_1-160\,a_2)^2)+5400\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2(176\,a_1-\\
&2464\,a_2+4\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2(16\,a_1-160\,a_2)+4\,(16\,a_1-160\,a_2)^3)-\\
&120\,(4\,\alpha\,t-24\,a_1+120\,a_2)^6(80\,a_1-1248\,a_2)+900\,(4\,\alpha\,t-24\,a_1+120\,a_2)^4(464\,a_1-\\
&4000\,a_2)-225\,(4\,\alpha\,t-24\,a_1+120\,a_2)^3(11+80\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2+80\,(16\,a_1-\\
&160\,a_2)^2+128\,(-8\,\beta\,t-80\,b_1+560\,b_2)(32\,b_1-448\,b_2)+8192\,(16\,a_1-160\,a_2)a_2)-\\
&675\,(4\,\alpha\,t-24\,a_1+120\,a_2)(-7+56\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2+88\,(16\,a_1-160\,a_2)^2-\\
&128\,(-8\,\beta\,t-80\,b_1+560\,b_2)(32\,b_1-448\,b_2)-128\,(32\,b_1-448\,b_2)^2-524288\,{a_2}^2)+\\
&5529600\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2a_2-5529600\,(16\,a_1-160\,a_2)^2a_2-172800\,(-8\,\beta\,t-\\
&80\,b_1+560\,b_2)(16\,a_1-160\,a_2)(32\,b_1-448\,b_2)+64800\,(16\,a_1-160\,a_2)^3-870\,(4\,\alpha\,t-\\
&24\,a_1+120\,a_2)^7+25\,(4\,\alpha\,t-24\,a_1+120\,a_2)^9+(4\,\alpha\,t-24\,a_1+120\,a_2)^{11}-151200\,a_1+\\
&1857600\,a_2)
\end{aligned}
$$

and

$$
\begin{aligned}
&d(x,t)=2024-777600\,(-8\,\beta\,t-80\,b_1+560\,b_2)(32\,b_1-448\,b_2)-82944000\,(16\,a_1-\\
&160\,a_2)a_2+(-120\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2+360\,(4\,\alpha\,t-24\,a_1+120\,a_2)(16\,a_1-\\
&160\,a_2)+120)(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^8+(480\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2-\\
&240\,(4\,\alpha\,t-24\,a_1+120\,a_2)^4+960\,(4\,\alpha\,t-24\,a_1+120\,a_2)^3(16\,a_1-160\,a_2)+2320+\\
&2160\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2+5040\,(16\,a_1-160\,a_2)^2-1440\,(4\,\alpha\,t-24\,a_1+\\
&120\,a_2)(64\,a_1-960\,a_2))(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^6+(-720\,(4\,\alpha\,t-24\,a_1+\\
&120\,a_2)^4(-8\,\beta\,t-80\,b_1+560\,b_2)-17280\,(4\,\alpha\,t-24\,a_1+120\,a_2)(-8\,\beta\,t-80\,b_1+\\
&560\,b_2)(16\,a_1-160\,a_2)+4320\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2(-8\,\beta\,t-176\,b_1+1904\,b_2)+\\
&51840\,\beta\,t+103680\,b_1+2177280\,b_2)(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^5+(-1440\,(4\,\alpha\,t-\\
&24\,a_1+120\,a_2)^4+720\,(4\,\alpha\,t-24\,a_1+120\,a_2)^5(16\,a_1-160\,a_2)+240\,(4\,\alpha\,t-24\,a_1+
\end{aligned}
$$

$120\,a_2)^2(56+135\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2-45\,(16\,a_1-160\,a_2)^2)+32400\,(4\,\alpha\,t-24\,a_1+120\,a_2)(16\,a_1-288\,a_2)+7200\,(4\,\alpha\,t-24\,a_1+120\,a_2)^3(48\,a_1-544\,a_2)+3360+32400\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2-54000\,(16\,a_1-160\,a_2)^2+86400\,(-8\,\beta\,t-80\,b_1+560\,b_2)(32\,b_1-448\,b_2)+5529600\,(16\,a_1-160\,a_2)a_2)(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^4+(-960\,(4\,\alpha\,t-24\,a_1+120\,a_2)^6(-8\,\beta\,t-80\,b_1+560\,b_2)+57600\,(4\,\alpha\,t-24\,a_1+120\,a_2)^3(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)-43200\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2(-24\,\beta\,t-272\,b_1+2128\,b_2)-7200\,(4\,\alpha\,t-24\,a_1+120\,a_2)^4(-48\,\beta\,t-448\,b_1+2912\,b_2)-345600\,\beta\,t-5529600\,b_1+53222400\,b_2-86400\,(-8\,\beta\,t-80\,b_1+560\,b_2)^3-86400\,(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)^2+172800\,(4\,\alpha\,t-24\,a_1+120\,a_2)((-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)-(16\,a_1-160\,a_2)(32\,b_1-448\,b_2)+64\,(-8\,\beta\,t-80\,b_1+560\,b_2)a_2))(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^3+(13440\,(4\,\alpha\,t-24\,a_1+120\,a_2)^6+240\,(4\,\alpha\,t-24\,a_1+120\,a_2)^8-240\,(4\,\alpha\,t-24\,a_1+120\,a_2)^4(-326+45\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2-135\,(16\,a_1-160\,a_2)^2)+480\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2(-76+135\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2+1215\,(16\,a_1-160\,a_2)^2)-129600\,(4\,\alpha\,t-24\,a_1+120\,a_2)^3(32\,a_1-256\,a_2)-12960\,(4\,\alpha\,t-24\,a_1+120\,a_2)^5(32\,a_1-256\,a_2)-64800\,(4\,\alpha\,t-24\,a_1+120\,a_2)(-96\,a_1+1280\,a_2+4\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2(16\,a_1-160\,a_2)+4\,(16\,a_1-160\,a_2)^3)+12144-97200\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2+32400\,(16\,a_1-160\,a_2)^2+518400\,(32\,b_1-448\,b_2)^2-33177600\,(16\,a_1-160\,a_2)a_2+2123366400\,{a_2}^2)(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^2+(-360\,(4\,\alpha\,t-24\,a_1+120\,a_2)^8(-8\,\beta\,t-80\,b_1+560\,b_2)-17280\,(4\,\alpha\,t-24\,a_1+120\,a_2)^5(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)-1440\,(4\,\alpha\,t-24\,a_1+120\,a_2)^6(-8\,\beta\,t-240\,b_1+2800\,b_2)+32400\,(4\,\alpha\,t-24\,a_1+120\,a_2)^4(-8\,\beta\,t+112\,b_1-2128\,b_2)+64800\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2(40\,\beta\,t+752\,b_1-7728\,b_2+4\,(-8\,\beta\,t-80\,b_1+560\,b_2)^3+4\,(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)^2)-777600\,(4\,\alpha\,t-24\,a_1+120\,a_2)((-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)+2\,(16\,a_1-160\,a_2)(32\,b_1-448\,b_2)-128\,(-8\,\beta\,t-80\,b_1+560\,b_2)a_2)-172800\,(4\,\alpha\,t-24\,a_1+120\,a_2)^3(3\,(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)+(16\,a_1-160\,a_2)(32\,b_1-448\,b_2)-64\,(-8\,\beta\,t-80\,b_1+560\,b_2)a_2)+648000\,\beta\,t+8553600\,b_1-74390400\,b_2+259200\,(-8\,\beta\,t-80\,b_1+560\,b_2)^3+1296000\,(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)^2-1036800\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2(32\,b_1-448\,b_2)+1036800\,(16\,a_1-160\,a_2)^2(32\,b_1-448\,b_2)-132710400\,(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)a_2)(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)+120\,(-8\,\beta\,t-80\,b_1+560\,b_2)(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^9+1440\,(32\,b_1-448\,b_2)(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^7-120\,(4\,\alpha\,t-24\,a_1+120\,a_2)^9(16\,a_1-160\,a_2)+80\,(4\,\alpha\,t-24\,a_1+120\,a_2)^6(191+63\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2+27\,(16\,a_1-160\,a_2)^2)-2160\,(4\,\alpha\,t-24\,a_1+120\,a_2)^5(240\,a_1-4576\,a_2)+21600\,(4\,\alpha\,t-24\,a_1+120\,a_2)^3(-368\,a_1+3488\,a_2+4\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2(16\,a_1-160\,a_2)+4\,(16\,a_1-160\,a_2)^3)-1440\,(4\,\alpha\,t-24\,a_1+120\,a_2)^7(80\,a_1-864\,a_2)+240\,(4\,\alpha\,t-24\,a_1+120\,a_2)^4(599+135\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2-225\,(16\,a_1-160\,a_2)^2-360\,(-8\,\beta\,t-80\,b_1+560\,b_2)(32\,b_1-448\,b_2)-23040\,(16\,a_1-160\,a_2)a_2)-16200\,(4\,\alpha\,t-24\,a_1+120\,a_2)(496\,a_1-6240\,a_2+80\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2(16\,a_1-160\,a_2)+16\,(16\,a_1-160\,a_2)^3+128\,(-8\,\beta\,t-80\,b_1+560\,b_2)(16\,a_1-160\,a_2)(32\,b_1-448\,b_2)-4096\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2a_2+4096\,(16\,a_1-160\,a_2)^2a_2)+24\,(4\,\alpha\,t-24\,a_1+120\,a_2)^2(3881+12150\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2+28350\,(16\,a_1-160\,a_2)^2+21600\,(-8\,\beta\,t-80\,b_1+560\,b_2)(32\,b_1-448\,b_2)+21600\,(32\,b_1-448\,b_2)^2+88473600\,{a_2}^2)+1036800\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2(16\,a_1-160\,a_2)^2+356400\,(-8\,\beta\,t-80\,b_1+560\,b_2)^2+874800\,(16\,a_1-160\,a_2)^2+3720\,(4\,\alpha\,t-24\,a_1+120\,a_2)^8+120\,(4\,\alpha\,t-24\,a_1+120\,a_2)^{10}+518400\,(32\,b_1-448\,b_2)^2+2123366400\,{a_2}^2+(1+(2\,x+12\,\beta\,t+60\,b_1-280\,b_2)^2+(4\,\alpha\,t-24\,a_1+120\,a_2)^2)^6+518400\,(-8\,\beta\,t-80\,b_1+560\,b_2)^4+518400\,(16\,a_1-160\,a_2)^4$

is a solution to the Hirota equation (1).