On Some Comparisons of Multistep Methods and Their Applications to Solve Ordinary Differential Equations

IMANOVA MEHRIBAN[1,3], VAGIF IBRAHIMOV[2,3]
[1]Science Development Foundation,
Baku,
AZERBAIJAN

[2]Chief of the Department of Computational Mathematics,
Baku State University,
Baku,
AZERBAIJAN

[3]Institute of Control Systems,
Azerbaijan National Academy of Sciences,
Baku,
AZERBAIJAN

Abstract: - The significance of Multistep Methods with constant coefficients and their application in addressing various Natural Science issues is universally acknowledged. Dahlquist conducted foundational research on these methods. Building on this, this text outlines certain developments in these theories and their use in solving Ordinary Differential, Volterra Integral, and Volterra Integro-Differential Equations. Advanced (forward-jumping) methods are examined, with a comparison made between the outcomes of these methods and those established by Dahlquist. Additionally, the study focuses on advanced second derivative multistep methods, demonstrating that the stable variants of these advanced methods yield greater accuracy. Furthermore, the research identifies the maximum achievable degree for the advanced methods. The constructed methods have been utilized to tackle model problems, and the resulting findings are presented here for illustration.

Key-Words: - Initial-value problem, Ordinary Differential Equation, The Volterra Integro-Differential Equation, Stability and Degree, Multistep Multiderivative Methods, Maximum volume for the degree.

Received: July 29, 2024. Revised: November 29, 2024. Accepted: December 19, 2024. Published: February 26, 2025.

1 Introduction

During the Middle Ages, researchers often found themselves captivated by the movements of celestial bodies, framing their inquiries within the context of an initial-value problem associated with Ordinary Differential Equations (ODEs). This exploration was not merely academic; it was an endeavor to understand the cosmos, reflecting humanity's quest for knowledge. As mathematicians began to seek solutions, they turned to various numerical methods that could approximate these elusive paths. Among these, the power series technique emerged as a prominent choice, allowing scholars to express solutions in terms of infinite series. However, it was Leonid Euler who critically assessed the limitations of these early approaches. He noted that while power series offered a mechanism for calculation, they often fell short in terms of convergence and accuracy for complex celestial problems. In response, Euler introduced a novel methodology that sought to enhance the reliability of these calculations, paving the way for more sophisticated analyses. The journey did not end there; it was the Adams-Moulton and Adams-Bashforth methods that truly revolutionized numerical techniques for solving ODEs. By employing these predictor-corrector methods, scholars were able to achieve greater precision and stability in their computations, illuminating the paths of celestial bodies with newfound clarity. Thus, the evolution of numerical methods during this period reflected both the challenges faced and the ingenuity of mathematicians striving to understand the universe, [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]:

$$y'(x) = f(x, y),\ y(x_0) = y_0,\ x_0 \le x \le X. \qquad (1)$$

Here, suppose that, this problem has the unit continuous solution $y(x)$, which is defined in the segment $[x_0, X]$. A continuous totality of arguments function $f(x, y)$ has been defined in some closed set, which has the continuous partial derivatives to some p, inclusively.

As previously mentioned, the objective of this study is to determine the numerical solution for problem (1). At this stage, the precise value of the solution at point x_i is represented by $y(x_i)$, while the corresponding approximate values are indicated as $y_i (i = 0,1,2,..,N)$. The mesh points x_i (where $i = 0,1,.,N$) are defined such that $x_{i+1} = x_i + h$ (for $i = 0,1,..N-1$). Here, h is a constant known as the step size, which segments the interval$[x_0, X]$ into N equal parts. We will let f denote the function values $f(x_i, y_i)$ (for $i = 0,1,..,N$).

It's important to understand that Euler's approach can be derived from Adams's technique as a specific instance. Experts focused on creating and utilizing numerical methods for solving problem (1) have broadly generalized all established numerical techniques. This process led to the development of various methods, including the following:

$$\sum_{i=0}^{k} \alpha_i y_{n+i} = h \sum_{i=0}^{k} \beta_i f_{n+i}, n = 0,1,2,..,N-k, \alpha_k \ne 0, \qquad (2)$$

when utilizing this approach to address certain issues of the kind (1), the question of the method's convergence comes into play. The study referenced in [2] explores this issue and demonstrates that for the method (2) to converge, the roots of the polynomial must meet specific criteria

$$\rho(\lambda) \equiv \alpha_k \lambda^k + \alpha_{k-1} \lambda^{k-1} + \ldots + \alpha_1 \lambda + \alpha_k \qquad (3)$$

The condition requires that the locations are situated within the unit circle, where no roots are repeated on its boundary. This principle is referred to as the dispersion concept. In reference [8], this principle is recognized as a criterion for the stability of method (2). It has been demonstrated that when $\beta_k = 0$ and method (2) maintains stability, then the relationship $p \le k$ holds for the $k \le 10$, where p signifies the p accuracy level of method (2).

It's important to mention that method (2) has been analyzed by numerous researchers, but its in-depth examination was conducted by Dahlquist. In his investigation of method (2), Dahlquist utilized the notions of stability and accuracy, which he defined in specific terms.

Definition 1. The approach (2) is termed stable if the roots of the polynomial $\rho(\lambda)$ are situated on the unit circle, and there are no repeated roots along its boundary.

Definition 2. An integer p is referred to as the degree of the method (2) if the subsequent asymptotic equalities hold true.,

$$\sum_{i=0}^{k} (\alpha_i y(x + ih) - h\beta_i y'(x + ih)) = O(h^{p+1}), h \to 0. \qquad (4)$$

He has outlined the inherent constraints that affect the coefficients of the method detailed in (2).

A. The values α_j and β_j (where $j = 0,1,2,..,k-m; i = 0,1,..,k$) are defined as real numbers, with the stipulation that $\alpha_k \ne 0$.
B. The characteristic polynomials
$$\rho(\lambda) \equiv \sum_{i=0}^{k-m} \alpha_i \lambda^i; \delta(\lambda) \equiv \sum_{i=0}^{k} \beta_i \lambda^i$$
are free of common factors, apart from constants
C. The conditions $\rho(1) = 0; \rho'(1) = \delta(1)$ are satisfied.

Theorem 1. Should method (2) exhibit stability and possess a degree of p, it follows that:

$$p \le 2[k/2] + 2 \qquad (5)$$

and for every k, there exist stable methods categorized as type (2) that correspond to the maximum degree p_{max}.

Based on the relationships in (5), it can be inferred that the extent of the technique outlined in (2) is limited. To enhance the precision of the numerical approaches, the study in [10] has suggested implementing the method described below:

$$\sum_{i=0}^{k-m} \alpha_i y_{n+i} = h \sum_{i=0}^{k} \beta_i f_{n+i}, (n = 0,1,..,N-k, m < 0). \qquad (6)$$

Here, suppose that $|\beta_{k-m+1}| + \ldots + |\beta_k| \ne 0$.

Method (6) is typically referred to as the advanced or forward-jumping technique. The categories of methods (2) and (6) differ significantly from one another. The following paragraph will focus on the exploration of advanced methodologies.

§1. The exploration of sophisticated techniques

The sophisticated techniques were developed at the start of the 20th century by Kowell, [11], [12], [13]. As a result, some researchers refer to method (6) as the Kowell method. The comprehensive analysis of method (6) is detailed in work (10). Utilizing the criteria $m > 0$ and $|\beta_{k-m+1}|+\ldots+|\beta_k| \neq 0$, it has been established that the category of method (6) is distinct and autonomous from method (2). Consequently, essential conditions have been identified which constrain the coefficients of method (6), and they can be articulated as follows:

A! The coefficients λ_j, β_i *(where* $j = 0,1,\ldots,k-m; i = 0,1,\ldots,k)$ *) represent certain real numbers, with the condition that* $\lambda_{k-m} \neq 0$ is not equal to zero.

B! The polinomials $\bar{\rho}(\lambda) \equiv \sum_{i=0}^{k-m} \alpha_i \lambda^i; \delta(\lambda) \equiv \sum_{i=0}^{k} \beta_i \lambda^i$.

C! $\bar{\rho}(1) = 0$ and $\bar{\rho}'(1) = 0 = \delta(1)$ take place.

Haven't common factor different from constant.

It has been observed that renowned scientists such as Laplace, Steklov, Klero, Kowell, and others have developed advanced methodologies. In reference [10], they introduced a concert method with a degree of $p = 5$ for $k - 3$. Based on Dahlquist's findings, it can be concluded that within the Multistep Method class (2), there are no stable methods with a degree of $p = k + 2$ when k takes the value of $2v - 1$ (an odd number). A straightforward comparison between methods in classes (2) and (6) demonstrates that stable methods of type (6) offer greater accuracy than those of type (2). To further assess the accuracy of methods (2) and (6), we will examine the following theorem.

Theorem 2. Should the approach (6) demonstrate stability and possess a degree of p, the subsequent occurs:

$$p \leq k + m + 1 (k \geq 3m).$$

It can be inferred that method (6) presents a more favorable option. There are instances where applied problems require the utilization of more precise methods for their resolution. Thus, this discussion focuses on developing a more accurate numerical technique to address the problem (1). To achieve this, Euler introduced an approach that involves calculating subsequent terms of the Taylor series. For the design of a more precise numerical method for addressing the problem (1), Dahlquist suggests an alternate technique such as [11], [12], [13], [14], [15], [16], [17], [18], [19]:

$$\sum_{i=0}^{k} \alpha_i y_{n+i} = h \sum_{i=0}^{k} \beta_i f_{n+i} + h^2 \sum_{i=0}^{k} \gamma_i g_{n+i}; n = 0,1,\ldots N-k; \alpha_{k-m} \neq 0. \quad (7)$$

The function $g(x, y)$ is defined in the following manner: $g(x, y) = f_x'(x, y) + f_y'(x, y) f(x, y)$.

Dahlquist fundamentally investigated this method and proved some theorems. The main result of the named work can be presented as follows.

Theorem 3. If we assume that the technique described in (7) is stable and possesses a degree of p, then

$$p \leq 2k + 2 if |\beta_k| + |\gamma_k| \neq 0 \text{ and } p = 2k, \beta_k = \gamma_k = 0. \quad (8)$$

For each k, there exists a stable method of type (7) with a difference of $p = 2k + 2$.

To enhance the precision of method (7), a modified version of method (7) has been introduced in [12] as follows:

$$\sum_{i=0}^{k-m} \alpha_i y_{n+i} = h \sum_{i=0}^{l} \beta_i f_{n+i} + h^2 \sum_{i=0}^{s} \gamma_i g_{n+i}; n = 0,1,\ldots N-k; \alpha_{k-m} \neq 0, m > 0. \quad (9)$$

The technique outlined in method (7) can be derived from method (9) as a specific instance, as demonstrated in [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39].

The approach was thoroughly examined in the study referenced as [12]. It is important to mention that method (9), in the scenario where $max(l, s) \leq k - m$, was analyzed by Dahlquist. Consequently, we will focus on evaluating method (9) when $max(l, s) > k - m$. To proceed, let us define what constitutes the highest order of accuracy for method (9) under the condition where $max(l, s) > k - m$.

Theorem 4. If technique (9) possesses a degree of p and is stable when $max(l, s) > k - m$, it follows that there exist stable methods of type (9) that are characterized by the maximum degree p_{max}.

It is important to observe that if in method (7) the coefficients β_i (where $i = 0,1,..,k$) meet the criterion $\beta_i = 0$ (for $= 0,1,..,k$), the interpretation of stability is significantly altered. In such a scenario, the concept of stability can be articulated as follows:

Definition 3. Method (7) is termed stable when $\beta_i = 0$ *for* $i = 0,1,..,k$, *provided that the roots of the polynomial* $\rho(\lambda) \equiv \sum_{i=0}^{k} \alpha_i \lambda^i$ are situated within the unit circle, and that there are no multiple roots present on its boundary, apart from the double root $\lambda = 1$.

To exemplify the findings mentioned earlier, we can examine the methods categorized as (7) and (9).

$$y_{n+3} = (y_{n+2} + y_{n+1} + y_n)/3 + h(10781 f_{n+3} + 22707 f_{n+2} + 16659 f_{n+1} + +4285 f_n)/27216 + h^2(-2099 g_{n+3} + 7227 g_{n+1} + 979 g_n)/45360 (p = 9). \quad (10)$$

$$y_{n+2} = (416 y_{n+1} - 103 y_n)/313 + h(157 f_{n+3} + 11233 f_{n+2} + 8521 f_{n+1} - -2830 f_n)/25353 + h^2(-11 g_{n+3} + 630 g_{n+2} + 1557 g_{n+1} - 92 g_n)/8451 (p = 9). \quad (11)$$

$$y_{n+1} = y_n + h(1985 f_{n+3} + 12015 f_{n+2} + 142255 f_{n+1} + +34465 f_n)/90720 + h^2(-163 g_{n+3} + 2421 g_{n+2} + 7659 g_{n+1} + 1283 g_n)/30240 (p = 8). \quad (12)$$

Through straightforward comparison, it can be acknowledged that the characteristics of these techniques are influenced by the previously derived results of the law. Comparable findings can be found in the literature.

2 Numerical Results

To illustrate the new approach for obtaining more precise results regarding construction, we can apply method type (2) to tackle a straightforward problem: $y' = cosx$, with the condition $y(0) = 0$, where $0 \leq x \leq 1$. The exact solution is known to be $y(x) = sinx$. For this particular example, we will employ the Simpson method as follows:

$$y_{n+1} = y_n + h(f_n + 4f_{n+1} + f_{n+2})/3 \quad (13)$$

the subsequent Simpson method, utilizing a step-size of $h \coloneqq h/2$:

$$y_{n+2} = y_n + h(f_n + 4f_{n+1/2} + f_{n+2})/3 \quad (14)$$

The outcomes obtained from these techniques have been organized in Table 1 (Appendix).

3 Conclusion

In this section, we will compare results derived from Multistep Methods that incorporate constant coefficients through the first and second derivatives of the solution to the problem (1). It has been demonstrated that certain classes of methods leverage the first and second derivatives of the solution to this problem. This category of methods generalizes the established Multistep Second Derivative Methods with constant coefficients. A thorough comparison of methods employing the first derivative of the solution to the problem (1) is provided. Moreover, it has been established that the newly formulated class of methods, classified as type (6), represents a distinct category. A straightforward comparison reveals that these Multistep Methods, similar to advanced approaches, possess both benefits and drawbacks. Specifically, the advanced methods require prior knowledge of the sought values at forthcoming points to define y_(n+k-m). It is noteworthy that the predictor-corrector method can be employed to tackle this challenge, thereby extending the stability region for stable advanced methods. This discourse elaborates on how Multistep Methods can effectively address initial-value problems for Ordinary Differential Equations, while also applying to other scenarios, such as initial-value issues for Volterra Integral Differential Equations. To confirm this, it suffices to express method (2) in an alternative format:

$$\sum_{i=0}^{k} \alpha_i y_{n+i} = h \sum_{i=0}^{k} \beta_i y'_{n+i}, n = 0,1,\ldots,N-k, \alpha_k \neq 0.$$

Acknowledgments:

The authors express their gratitude to Academician T. Aliyev and Academician A. Abbasov for their valuable insights and guidance.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work the authors used Grammy in order to correct syntactic and semantic inaccuracies in the text. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

References:

[1] A. Iserles, S.P. Norset, Two-step methods and Bi-orthogonality, *Math. of Compu.t,* no.180, 1987, p.543-552.

[2] G. Dahlquist, Convergence and stability in the numerical integration of ordinary differential equations, *Math.Scand*., 1956, No 4, 33-53.

[3] E. Hairer, C. Lubuch, G. Wanner, *Geometric numerical integration,* Second Edition, Springer (2004),p 644.

[4] D.O. Awoyemi, S.J. Kayode, An optimal order continuous multistep algorithm for initial value of special second order differential equations. *Journal of the Nigeria Association of Mathematical Physics*, vol.6, (2002) 285-292.

[5] I. Babushka, E. Vitasek, M. Prager, *Numerical processes for solving differential equations,* Mir 1969, p.368.

[6] L. Skvortsov, Explicit two-step Runge-Kutta methods, *Math. modeling,* 21,2009, p.54–65.

[7] M.R. Shura-Bura, Error estimates for numerical integration of ordinary differential equations, *Prikl.matem. and mech.,* 1952, № 5, p.575-588, (Russian).

[8] N.S. Bakhvalov, Some remarks on the question of numerical intefraction of differential equation by the finite-difference method, *Academy of Science report,* USSA, N3, 1955, 805-808 p., (Russian).

[9] J. Butcher, A modified multistep method for the numerical integration of 530 535 540 545 550 ordinary differential equations, *J. Assoc. Comput. Math.,* 12, 1965, p. 124–135.

[10] P. Henrici, *Discrete variable methods in ODE,* John Wiley and Sons, Inc., New York. London, 1962.

[11] V.R. Ibrahimov, A relationship between order and degree for a stable formula with advanced nodes, *Computational Mathematics and Mathematical Physics (USSR)*, 30, 1990, p.1045–1056.

[12] G. Dahlquist, *Stability and Error bounds in the numerical integration of ordinary differential equations,* Uppsala, Almqvist and Wiksells boktr.-1959-No.130-P.5-92.

[13] Oluwaseun Adeyeye, Zurni Omar, New uniform order eight hybrid third derivative block method for solving second order initial value problems, *Far East Journal of Mathematical Sciences (FJMS)*, 2016 Pushpa Publishing House, Allahabad, India Published Online: October 2016, Vol. 100, Number 9, 2016, pp.1515-1531

[14] Faruk Muritala, Abdul Azeez K. Jimoh, Muideen Ogunniran, Abdulmalik A. Oyedeji, J.O. Lawal, K -Step Block Hybrid Method For Numerical approximation Of Fourth-Order Ordinary differential Equations, Authorea. May 11, 2023, [Online]. https://www.researchgate.net/publication/370696656_k_-STEP_BLOCK_HYBRID_METHOD_FOR_NUMERICAL_APPROXIMATION_OF_FOURTH-ORDER_ORDINARY_DIFFERENTIAL_EQUATIONS (Accessed Date: December 14, 2024).

[15] Z. Omar, M.F .Alkasassbeh, Generalized one – step third derivative implicit hybrid block method for the direct solution of second order ordinary differential equation, *Applied Mathematical Sciences,* vol.10, 2016, no 9, p. 417-430.

[16] Adewale James, A. OlaideAdesanya, Sunday Joshua, Continuous block method for the solution of second order initial value problems of ordinary differential equation, *International Journal of Pure and Applied Mathematics*, Vol. 83 No. 3 2013, 405-416.

[17] Jimevwo G. Oghonyon, Solomon Adebola Okunuga, Sheila A. Bishop, A 5-step block predictor and 4-step corrector methods for solving general second order ordinary differential equations, *Global Journal of Pure and Applied Mathematics,* ISSN: 0973-1768, Vol. 11, Number 5 (2015), pp. 3847-3862.

[18] Chenglian Liu, Chieh-Wen Hsu, T. E. Simos, Ch. Tsitouras, Phase fitted, six-step method for solving x=f(t,x), *Math. Meth. Appl. Sci.,* 2019, 42, p.3942–3949.

[19] Faieza Samat, Eddie Shahril Ismail, A two-step modified explicit hybrid method with step-size dependent parameter for oscillatory problems, *Hindawi Journal of Mathematics,* Vol. 2020, Article ID 5108482, p. 1-7.

[20] C. Liu, C.W. Hsu, Ch. Tsitouras, T.E. Simos, Hybrid numerov type methods with coefficients trained to perform better on classical orbits, *Springer Link,* Vol. 42, 24 May 2019, p. 2119–2134.

[21] Y.A. Yahaya, A.M. Sagir, M. Tech, An order five implicit 3-step block method for solving ordinary differential equations, *The Pacific*

Journal of Science and Technology, Vol. 14. Number 1. May 2013 (Spring), p. 176-183.

[22] G.Yu. Mehdiyeva, V.R. Ibrahimov, M.N. Imanova, Application of a second derivative multi-step method to numerical solution of Volterra integral equation of second kind, Pakistan *Journal of Statistics and Operation Research,* 28.03.2012, pp.245-258, https://doi.org/10.18187/pjsor.v8i2.294 .

[23] O.A. Akinfenwa, B. Akinnukawe, S.B. Mudasiru, A family of continuous third derivative block methods for solving stiff systems of first order ordinary differential equations, *Journal of the Nigerian Mathematical Society,* Vol. 34, Issue 2, August 2015, pp.160-168.

[24] T.E. Simos, Optimzing a hybrid two-step method numerical solution of the Schrodinger equation and related problems with respect to phase-lag, *Hindawi Publishing Corporation Journal of Applied Mathematics,* Vol. 2012, Article ID 420387, p. 17.

[25] Awari Y. Sani, Derivation and application of six-point linear multistep numerical method for solution of second order initial value problems, *IOSR Journal of Mathematics (IOSR-JM)*, e-ISSN: 2278-5728,p-ISSN: 2319-765X, Vol. 7, Issue 2 (Jul. - Aug. 2013), pp.23-29.

[26] Mufutau Ajani, Ali Shokri, Omole Ezekiel, A one point third derivative hybrid multistep technique for solving second order oscillatory and periodic problems, *Hindawi Journal of Mathematics*, Vol. 2023, Article ID 2343215, p.12.

[27] Kamoh Nathaniel, Kumleng Geoffrey, Sunday Joshua, Continuous one step linear multi-step hybrid block method for the solution of first order linear and nonlinear initial value problem of ordinary differential equations, *Recent Developments in the Solution of Nonlinear Differential Equations,* September 2021, p. 1-9.

[28] T.Kamoh Aboiyar, A. Kimbir, Continuous multistep methods for Volterra integro-differential equational equations of the second order, *Science World Journal*, Vol. 12(No 3) 2017, pp.1-14.

[29] G. Mehdiyeva, V. Ibrahimov, M. Imanova, Some refinement of the notion of symmetry for the Volterra integral equations and the construction of symmetrical methods to solve them, *Journal of Computational and Applied Mathematics,* 2016/11/1, pp.1-9.

[30] D.A. Juraev, “Solution of the ill-posed Cauchy problem for matrix factorizations of the Helmholtz equation on the plane”, *Global and Stochastic Analysis.,* 8:3 (2021), 1–17.

[31] D.O. Awoyemi, Algorithmic collocation methods for the direct solution of special and general fourth order initial value problems in ordinary differential equations. Inter. *Journal of Computer Math*. Vol.6, (2001) 271-218.

[32] R. Cortell, Application of the fourth-order Runge-Kutta method for the solution of high-order general initial value problems, *Computers and Structures*, vol.49, no. 5,pp. (1993) 897-900.

[33] T.A. Anake, D.O. Awoyemi, A.O. Adesanya, A one-step method for the solution of general second order ordinary differential equations. *Inter Journal of Science and Technology,* 2(4); (2012) 159-163.

[34] S.N. Jator, A sixth order linear multistep method for the direct solution of y=f(x,y,y’). *Inter. Journal of Pure and Applied Mathematics,* 40(1), (2007) 457-472.

[35] J.D. Bulnes, D.A. Juraev, J.L. Bonilla, M.A.I. Travassos, “Exact decoupling of a coupled system of two stationary Schrödinger equations”, *Stochastic Modelling & Computational Sciences,* 3:1 (2023), 23–28.

[36] D.A. Juraev, “The Cauchy problem for matrix factorization of the Helmholtz equation in a multidimensional unbounded domain”, *Boletim da Sociedade Paranaense de Matematica,* 41 (2023), 1–18.

[37] J.D. Lambert, Computational Methods in ODEs. John Wiley & Sons, New York (1973).

[38] S.N. Jator, J. Li, A self-starting linear multistep methods for the direct solution of general second order initial value problems. *Intern. Journal of Computer Math.* 86(5), (2009) 817-836.

[39] U.A. Mohammed, Six-step Block Method for the Solution for Fourth Order Ordinary Differential Equation. *The Pacific Journal of Science and Technology*, vol. 10(1), (2010) 259-265.

APPENDIX

Table 1. Results for the step size h:=0.1

Step size	Variable x	Error for the Simpson's method	Error for the method (14)
h=0.1	0.2	0.11E-06	0.69E-08
	0.3	0.10E-06	0.10E-07
	0.4	0.21E-06	0.13E-07
	0.5	0.21E-06	0.16E-07
	0.6	0.31E-06	0.19E-07
	0.7	0.30E-06	0.22E-07
	0.8	0.39E-06	0.24E-07
	0.9	0.38E-06	0.27E-07
	1.0	0.46E-06	0.29E-07

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Mehriban Imanova investigated the relation between the Multistep Methods of type (2) and (6). She also illustrated some results received here by the model problem.
- Vagif Ibrahimov investigated methods (7) and (9) and has constructed methods (10), (11) and (12).

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

This research received partial funding from the Science Development Foundation under the auspices of the President of the Republic of Azerbaijan, with grant number AEF-MCG-2022-1(42)-12/4/1-M-4.

Conflict to interest

The authors clarify that there is no confusion regarding any potential conflict of interest among them. We affirm that all the techniques outlined in this paper are solely our own.