Nonlinear Modeling of Helical Gear Pair with Friction Force and Frictional Torque

QINGLIN CHANG, LI HOU, ZHIJUN SUN, YONGQIAO WEI, YUNXIA YOU
School of Manufacturing Science and Engineering
Sichuan University
No.24, South Section 1, Yihuan Road, 610065 Chengdu
CHINA
(chang_qinglin@126.com)

Abstract: **-** Helical gear transmission is widely used in various industrial departments. The dynamic characteristic of helical gear system is very important and attracts scholar’s attention all around the world. A lot of dynamic models have been put forward. But the models consisted time-varying friction force and frictional torque are very few. So a dynamic model contained time-varying friction force and frictional torque is proposed by a new algorithm which is developed by Chinmaya Kar. Then the dimensionless method was used, and the dimensionless dynamic model was summarized. At last, through the numerical method, the nonlinear characteristics in particular parameters were studied.

Key-Words: - Time-varying frictional force; contact line; helical gear; nonlinear characteristic.

1 Introduction

Gear transmission system is widely used in every industry field for all over the world, so the gear pair is one kind of key component which is very important. As a common transmission system, there are three forms when the axis is paralleled, spur gear transmission system, helical gear transmission system and double-helical gear transmission system. Because alternate meshing of single-tooth and double-tooth is existed in the spur gear transmission system, there is a larger vibration and noise motivated when the contact ratio is not an integer and the meshing stiffness changes large [1]. There isn’t alternate meshing of single-tooth and double-tooth and large meshing stiffness changing in helical gear, leading to rotate smoother than spur gear system.

Some disadvantages are existed in the helical gear system due to the helix angle,, such as dynamic load at axial direction, time-varying length of contact line, complicated time-varying friction force and frictional torque and so on [2-3]. For the double-helical gear system, there are complicated time-varying friction force and frictional torque too, but the dynamic load at axial direction was counteracted internal. From the studies in literature [4-5], the time-varying friction force and frictional torque is an important internal excitation for gear system dynamic characteristic. The researches aimed at the relationship between time-varying friction force and frictional torque and dynamic characteristic in spur gear system are rich [6-9], but the research on helical gear system is seldom for the authors’ knowledge. However, the time-varying friction force and frictional torque is a very important excitation source for helical gear system, so the research on time-varying friction force and frictional torque and its dynamical effect is also very important.

Fortunately, a lot of works which would be good examples have been done by other researchers, and this paper could be carried out smoothly. Wesley Blankenship.G and Singh.R [10] researched the dynamic meshing force, dynamic meshing stiffness and transfer matrix of helical gear system, and studied the backlash nonlinear characteristic under different modal parameter. Yi Guoa and Robert G. Parker [11] studied the nonlinear dynamic characteristics of spur planetary gear. Walha.L [12] et al studied the nonlinear dynamic response of helical gear system consisted mass eccentricity and two stage clutch. Wei Yiduo [13-14] did in-depth research on friction-nonlinear characteristic with spur gear system, but the helical gear is not involved. MA Hui and ZHU Lisha [15] et al analyzed the deflection load influence on helical gear, and the dynamic characteristic was revealed by the model of analysis and coupling analysis. WANG Qing and ZHANG Yidu [16] deduced the coupling dynamic model of two stage helical gear system and the differential equation have obtained by centralized parameter method. N. Leiba,b, S. Nacivet B and F. Thouverez [17] took experimental and numerical study method to research the gear-shift system. Song Xiaoguang, Cui Li and Zheng Jianrong [18] researched the dynamic chaos-nonlinear characteristic of flexible shift-helical gear system taking backlash in circular tooth, radial clearance of bearing and unbalanced force into consideration. TANG Jinyuan and CHEN Siyu [19-20] put forward a improved nonlinear dynamic model for spur gear pair, and the backlash-nonlinear characteristic was researched. WANG Cheng and WANG Feng [21-23] established a kinetic model for double-helical gear system, and the dynamic characteristic was researched elementary, but the time-varying parameters was not taken into consideration. LI Wenliang [24] studied the bending effect of the friction force with helical gear. WEI Jing and SUN Wei [25] et al studied the backlash-nonlinear characteristic also. For more works like [25-35] does, lots of method and tools could be used in the studies.

Particularly, the research did in literature [2-3] are more valuable to study the nonlinear dynamic effect of friction force and frictional torque in antecedent researches. The author of Refs [2-3] researched the time-varying length of contact line, friction force and frictional torque through some necessary simplification, and the transmission error was neglected in the paper.

In this paper, the formulas to calculate the frictional force and frictional torque in Refs [2-3] were used directly, just a little modified by taking the transmission error into consideration. Then refer to the method of Refs [25], the vibration model consisted of 8 degrees (6 displacement degrees and 2 rotation degrees) was established. To numerical calculation facility, the dimensionless kinetic equation of the helical gear system with transmission error also obtained. Then by taking the numerical method, the nonlinear characteristics in particular parameters was studied.

2 Time-varying Parameters

2.1 Transmission Error Taken into Account in the Helical Gear System

In the literature [2-3], the author neglected the transmission error in helical gear system, and the time-varying length of contact line, frictional force and frictional torque were calculated ideally. But there is transmission error in helical gear system in reality. The transmission error will lead the time-varying length of contact line, frictional force and frictional torque unsteady, which would following arouse the helical gear system vibration. So the transmission error was taken into account and the time-varying length of contact line, frictional force and frictional torque were calculated with transmission error despite the parameters were calculated using the method introduced in literature [2-3]. From the literature [1] [25], the transmission error could be expressed as Eq. (1):

$$e_n(t) = e_{n0} + e_{na}\sin[2\pi\omega_h(t)t + \phi] \tag{1}$$

Where: e_{n0} ——the mean value of the normal transmission error; e_{na} ——the amplitude of the normal transmission error; ϕ ——phase angle of meshing gear teeth; $\omega_h(t)$ ——meshing frequency of helical gear pair, which could be calculated as $\omega_h(t) = z_p \omega_p(t)$ in which z_p is the gear teeth number of pinion and $\omega_p(t)$ is the rotational frequency of pinion. Then the formula of $\omega_p(t)$ is $\omega_p(t) = \omega_{p0} + \omega_{pa} \sin(2\pi\omega_{pp} t)$, which stands on the fluctuation of rotational frequency of pinion, ω_{p0} is the mean value of $\omega_p(t)$, ω_{pa} is the fluctuation of $\omega_p(t)$, and ω_{pp} is the fluctuation frequency of $\omega_p(t)$.

Then the time-varying transmission error at x, y, z direction respectively could be expressed as following Eq. (2a) to Eq. (2c) shows.

$$e_x(t) = \sin[180 \frac{e_y(t)}{\pi r_a}] \tag{2a}$$

$$e_y(t) = e_n(t) \cos\beta \tag{2b}$$

$$e_z(t) = e_n(t) \sin\beta \tag{2c}$$

2.2 Time-Varying Meshing Stiffness

Refer to the literature [19] [25] and [28], the time-varying meshing stiffness was expressed as Fourier series show in Eq. (3a). This formula has sufficient precision in practice with the experiences in Refs. [13-25].

$$K_{mn}(t) = K_m + \sum_{\xi=1}^{\infty} \{a_{n1} \cos[\xi\omega_h(t)t] + b_{n1} \sin[\xi\omega_h(t)t]\} \tag{3a}$$

Where: K_m ——the mean value of helical gear pair meshing stiffness; $\omega_h(t)$ ——meshing frequency of helical gear pair; a_n, b_n —— the coefficient of Fourier series in Eq.(3).

Then the time-varying meshing stiffness at y, z direction respectively could be expressed as following Eq. (3b) to Eq. (3c) shows.

$$K_{my}(t) = K_{mn}(t) \cos\beta \tag{3b}$$

$$K_{mz}(t) = K_{mn}(t) \sin\beta \tag{3c}$$

2.3 Time-Varying Meshing Damping Coefficient

Because of the time-varying meshing damping coefficient have the smaller influence on the dynamic characteristic of helical gear system. The meshing damping coefficient $C_{mn}(t)$ was considered constant as C, just as Eq. (4a) shows.

$$C_{mn}(t) = C \tag{4a}$$

And the time-varying meshing damping coefficient at y, z direction respectively could be expressed as following Eq. (4b) to Eq. (4c) shows.

$$C_{my}(t) = C_{mn}(t) \cos\beta \tag{4b}$$

$$C_{mz}(t) = C_{mn}(t) \sin\beta \tag{4c}$$

2.4 Time-Varying Input and Output Torque

The input torque and output torque of the helical gear system will change due to the external reasons or internal reasons such as time-varying meshing stiffness, time-varying frictional damping coefficient and so on. To express the input torque and output torque in a clear way, the Fourier series also have been taken into consideration in this article to deliver the formula of input torque and output torque. Assume the formula of input torque is as shown in Eq. (5a) and the output torque is shown in Eq. (5b).

$$T_p(t) = T_{p0} + \sum_{\xi=1}^{\infty}[a_{n1}\cos(\xi\omega_{Tp}t) + b_{n1}\sin(\xi\omega_{Tp}t)] \quad (5a)$$

$$T_g(t) = T_{g0} + \sum_{\xi=1}^{\infty}[a_{n2}\cos(\xi\omega_{Tg}t) + b_{n2}\sin(\xi\omega_{Tg}t)] \quad (5b)$$

In order to calculate conveniently, just the first degree was left as shown in Eq. (5c) and Eq. (5d).

$$T_p(t) = T_{p0} + T_{pa}\sin(\omega_{Tp}t) \quad (5c)$$

$$T_g(t) = T_{g0} + T_{ga}\sin(\omega_{Tg}t) \quad (5d)$$

Where: ω_{Tp} ——fundamental frequency of input torque error; a_{n1}, b_{n1}——the coefficient of Fourier series in Eq.(5a); ω_{Tg} ——fundamental frequency of output torque error; a_{n2}, b_{n2} ——the coefficient of Fourier series in Eq.(5b).

2.5 Dynamic Meshing Force

The meshing process of the gear is a dynamic process, so there will be a delay from input torque to output torque. The reason of dynamic delay is due to time-varying meshing force, time-varying friction force and time-varying frictional torque et al. So the calculation of time-varying meshing force is very important in a dynamic helical gear system. From the Refs.[1] and Refs.[13-25], if the displacement of the meshing point, meshing stiffness and meshing damping coefficient could be found, the dynamic meshing force could be calculated. Refer to the Refs. [1], the generalized displacement matrix was established as $\{\delta\} = \{x_p, y_p, z_p, \theta_p, x_g, y_g, z_g, \theta_g\}^T$ shown in Fig.4 at section 3.1, in which the subscript p represents the meshing point on pinion and the subscript g represents the meshing point on gear. Then the vibration displacement of point p was derived as Eq. (6a) and Eq. (6b) shows, and the vibration displacement of point g was derived as Eq. (6c) and Eq. (6d) shows from Refs. [1].

$$\bar{y}_p = y_p + \theta_p r \quad (6a)$$

$$\bar{z}_p = z_p - \bar{y}_p \tan\beta = z_p - (y_p + \theta_p r)\tan\beta \quad (6b)$$

$$\bar{y}_g = y_g - \theta_g R \quad (6c)$$

$$\bar{z}_g = z_g - \bar{y}_g \tan\beta = z_g - (y_g - \theta_g R)\tan\beta \quad (6d)$$

The meshing stiffness, meshing damping coefficient and transmission error was calculated in section 2.3, 2.4 and 2.1 respectively. So the meshing force of the helical gear system was expressed in Eq. (7a) and Eq. (7b).

$$F_y(t) = K_{my} f(\bar{y}_p - \bar{y}_g - e_y) + C_{my}(\dot{\bar{y}}_p - \dot{\bar{y}}_g - \dot{e}_y) \quad (7a)$$

$$F_z(t) = K_{mz} f(\bar{z}_p - \bar{z}_g - e_z) + C_{mz}(\dot{\bar{z}}_p - \dot{\bar{z}}_g - \dot{e}_z) \quad (7b)$$

Where: $f(i_j)(i = x, y, z;\ j = 1, 2)$ is the nonlinear function which shown in Eq. (8a) and Eq. (8b) of backlash in circular tooth, and the backlash in circular tooth at normal direction was defined as $2b_n$. Then the backlash in circular tooth at transverse and axis direction was $2b_t = 2b_n\cos\beta$ and $2b_a = 2b_n\sin\beta$ respectively.

$$f(y) = \begin{cases} y - b_t & (y > b_t) \\ 0 & (|y| \le b_t) \\ y + b_t & (y < -b_t) \end{cases} \quad (8a)$$

$$f(z)=\begin{cases} z-b_a & (z>b_a) \\ 0 & (|z|\le b_a) \\ z+b_a & (z<-b_a) \end{cases} \quad (8b)$$

The dynamic meshing force at transverse direction $F_T(t)$ equals to meshing force at y direction $F_y(t)$.

$$F_T(t)=F_y(t) \quad (9)$$

2.6 Time-Varying Frictional Force and Frictional Torque

From the meshing force at transverse direction $F_T(t)$ and the helix angle β, the meshing force at normal direction could be formulation as Eq. (13) shows.

$$F_N(t)=\frac{F_T(t)}{\cos\beta}=\frac{F_y(t)}{\cos\beta} \quad (10)$$

Where β is helix angle.

Through the Ref. [2-3], the formula of friction force of pinion could be expressed as:

$$F_{pf}(t)=\mu L_{pF}(t)F_N(t) \quad (11)$$

Where $L_{pF}(t)$ was concerned with the length of contact line on the pinion.

Then the friction force of gear was shown in Eq. (12).

$$F_{gf}(t)=-F_{pf}(t) \quad (12)$$

The time-varying torque on the pinion could be expressed as Eq. (13) shown.

$$T_{pf}(t)=\mu L_{pT}(t)F_N(t) \quad (13)$$

Where $L_{pT}(t)$ was a parameter concerned with the length of contact line on the pinion.

Taking the same method, the time-varying torque on the gear could be expressed as Eq. [8] shows following.

$$T_{gf}(t)=\mu L_{gT}(t)F_N(t) \quad (14)$$

Where $L_{gT}(t)$ was another parameter concerned with the length of contact line on the gear.

3 Nonlinear Dynamic Vibration Model of Helical Gear System

3.1 The Nonlinear Dynamic Model

Considering the time-varying friction force, time-varying meshing force and frictional torque, the dynamic model of helical gear system was established as shown in Fig. 1. Let the generalized coordinates as:

$$\{\delta\}=\{x_p,y_p,z_p,\theta_p,x_g,y_g,z_g,\theta_g\}^T$$

And there were 8 degrees in which 6 degrees are moving degree and 2 degrees are rotation degree in the helical gear system. The dynamic model of helical gear system consisted of 8 degrees and shown in Fig.1 was established in Eq.(15a) and Eq.(15b) by newton's second law. The meshing impact force was neglected here.

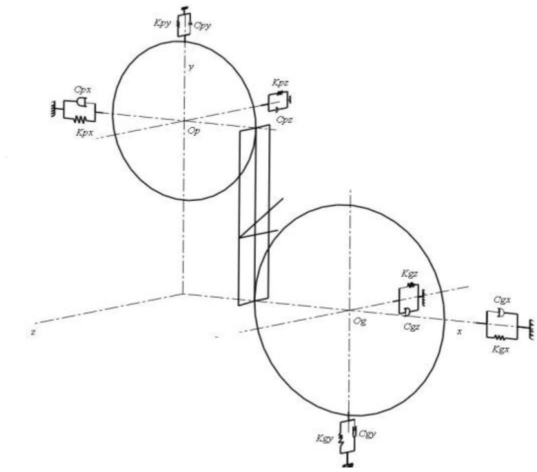


Fig.1. Dynamic model of helical gear system

$$\begin{cases} M_p\ddot{x}_p+C_{px}\dot{x}_p+K_{px}x_p=F_{pf}(t) \\ M_p\ddot{y}_p+C_{py}\dot{y}_p+K_{py}y_p=-F_y(t) \\ M_p\ddot{z}_p+C_{pz}\dot{z}_p+K_{pz}z_p=-F_z(t) \\ J_p\ddot{\theta}_p=T_p(t)+T_{pf}(t)-F_y(t)r \end{cases} \quad (15a)$$

And,

$$\begin{cases} M_g \ddot{x}_g + C_{gx} \dot{x}_g + K_{gx} x_g = F_{gf}(t) \\ M_g \ddot{y}_g + C_{gy} \dot{y}_g + K_{gy} y_g = F_y(t) \\ M_g \ddot{z}_g + C_{gz} \dot{z}_g + K_{gz} z_g = F_z(t) \\ J_g \ddot{\theta}_g = -T_g(t) + T_{gf}(t) + F_y(t) R \end{cases} \quad (15b)$$

Where: $M_i (i = p, g)$——the mass of pinion and gear respectively; $x_i, y_i, z_i (i = p, g)$ ——the displacement at x, y, z direction of pinion at point p and gear at point g respectively; $J_i (i = p, g)$ ——the mass moment of inertia of pinion and gear respectively; $C_{ij} (i = p, g; j = x, y, z)$ ——damping coefficient of support of pinion and gear in x, y, z direction respectively; $K_{ij} (i = p, g; j = x, y, z)$ ——stiffness of support of pinion and gear in x, y, z direction respectively; $F_i(t)(i = y, z)$ ——dynamic meshing force at y, z direction respectively; $F_{if}(t)(i = p, g)$——dynamic friction force of pinion and gear respectively; $T_i(t)(i = p, g)$ ——input and load torque of pinion and gear respectively; $T_{if}(t)(i = p, g)$—— dynamic friction torque of pinion and gear respectively.

3.2 Dimensionless of Kinetic Equation of the Helical Gear System with Transmission Error

In this study, the numerical computation method was employed to solve the differential equation. If we calculated directly, the computational error will be too big. So the Eq. (15a) and Eq. (15b) was managed by dimensionless. Take the steps as shown in Refs. [25].

Define dimensionless time as:

$$\tau = t \cdot \omega_n \quad (16a)$$

And the dimensionless excitation frequency of the system is:

$$\omega = \omega_h / \omega_n \quad (16b)$$

Where: ω_n——the inherent vibration frequency of the helical gear system, and $\omega_n = \sqrt{K_m / m_e}$; K_m ——the mean value of time-varying meshing stiffness; m_e——the equivalent mass of the helical gear system, and $m_e = \dfrac{J_p J_g}{J_g R_b^2 + J_p r_b^2}$.

Take the b_n as the nominal dimension to take the Eq. (15) dimensionless. And the dimensionless displacement of helical gear system could be expressed as:

$$\begin{aligned} & p_1 = x_p / b_n, p_2 = y_p / b_n, p_3 = z_p / b_n, p_4 = r\theta_p / b_n \\ & p_5 = x_g / b_n, p_6 = y_g / b_n, p_7 = z_g / b_n, p_8 = R\theta_g / b_n \\ & p_{11} = y / b_n, p_{12} = z / b_n \end{aligned} \quad (17)$$

Then the Eq. (15a) and Eq. (15b) could be transformed to:

$$\begin{cases} \ddot{p}_1 + 2\xi_{px} \dot{p}_1 + \eta_{px} p_1 + \tilde{L}_1(\tau)[\eta_{pmy} f(p_{11}) \\ \quad + 2\xi_{pmy} \dot{p}_{11}] = 0 \\ \ddot{p}_2 + 2\xi_{py} \dot{p}_2 + \eta_{py} p_1 + \eta_{pmy} f(p_{11}) + 2\xi_{pmy} \dot{p}_{11} = 0 \\ \ddot{p}_3 + 2\xi_{pz} \dot{p}_3 + \eta_{pz} p_1 + \eta_{pmz} f(p_{12}) + 2\xi_{pmz} \dot{p}_{12} = 0 \\ \ddot{p}_4 + 2\eta_{pmy} f(p_{11}) + 2\xi_{pmy} \dot{p}_{11} - 2\tilde{L}_2(\tau)[\eta_{pmy} f(p_{11}) \\ \quad + 2\xi_{pmy} \dot{p}_{11}] = g_p(\tau) \end{cases} \quad (18a)$$

And,

$$\begin{cases} \ddot{p}_5 + 2\xi_{gx}\dot{p}_5 + \eta_{gx}p_5 - \tilde{L}_1(\tau)[\eta_{gmy}f(p_{11}) \\ \quad + 2\xi_{gmy}\dot{p}_{11}] = 0 \\ \ddot{p}_6 + 2\xi_{gy}\dot{p}_6 + \eta_{gy}p_6 - \eta_{gmy}f(p_{11}) - 2\xi_{gmy}\dot{p}_{11} = 0 \\ \ddot{p}_7 + 2\xi_{gz}\dot{p}_7 + \eta_{gz}p_7 - \eta_{gmz}f(p_{12}) - 2\xi_{gmz}\dot{p}_{12} = 0 \\ \ddot{p}_8 - 2\eta_{gmy}f(p_{11}) - 2\xi_{gmy}\dot{p}_{11} + 2\tilde{L}_3(\tau)[\eta_{gmy}f(p_{11}) \\ \quad + 2\xi_{gmy}\dot{p}_{11}] = -g_g(\tau) \end{cases} \tag{18b}$$

Where, in Eq. (18a) and Eq. (18b), the symbol ξ_{ij} and $\eta_{ij}(i = p, g; j = x, y, z)$ was the dimensionless support damping and support stiffness of pinion and gear at the x, y, z direction respectively, the symbol ξ_{imj} and $\eta_{imj}(i = p, g; j = y, z)$ was the dimensionless meshing damping and meshing stiffness at the y, z direction respectively, the symbol $\tilde{L}_i(t)(i = 1,2,3)$ was the dimensionless coefficients related to the time-varying length of contact line, and the $g_i(i = p, g)$ was the driving torque and load torque respectively. Then the expressions of these coefficients could be expressed as following shows.

$\xi_{px} = \dfrac{C_{px}}{2M_p\omega_n}$; $\xi_{py} = \dfrac{C_{py}}{2M_p\omega_n}$; $\xi_{pz} = \dfrac{C_{pz}}{2M_p\omega_n}$;

$\xi_{gx} = \dfrac{C_{gx}}{2M_g\omega_n}$; $\xi_{gy} = \dfrac{C_{gy}}{2M_g\omega_n}$; $\xi_{gz} = \dfrac{C_{gz}}{2M_g\omega_n}$;

$\xi_{pmy} = \dfrac{C_{my}}{2M_p\omega_n}$; $\xi_{pmz} = \dfrac{C_{mz}}{2M_p\omega_n}$; $\xi_{gmy} = \dfrac{C_{my}}{2M_g\omega_n}$;

$\xi_{gmz} = \dfrac{C_{mz}}{2M_g\omega_n}$; $\eta_{px} = \dfrac{K_{px}}{M_p\omega_n^2}$; $\eta_{py} = \dfrac{K_{py}}{M_p\omega_n^2}$;

$\eta_{pz} = \dfrac{K_{pz}}{M_p\omega_n^2}$; $\eta_{gx} = \dfrac{K_{gx}}{M_g\omega_n^2}$; $\eta_{gy} = \dfrac{K_{gy}}{M_g\omega_n^2}$;

$\eta_{gz} = \dfrac{K_{gz}}{M_g\omega_n^2}$; $\eta_{pmy} = \dfrac{K_{my}}{M_p\omega_n^2}$; $\eta_{pmz} = \dfrac{K_{mz}}{M_p\omega_n^2}$;

$\eta_{gmy} = \dfrac{K_{my}}{M_g\omega_n^2}$; $\eta_{gmz} = \dfrac{K_{mz}}{M_g\omega_n^2}$; $g_p = \dfrac{2T_p}{b_n M_p\omega_n^2}$;

$g_g = \dfrac{2T_g}{b_n M_g\omega_n^2}$.

$\tilde{L}_1(\tau)$ and $\tilde{L}_2(\tau)$ are just the dimensionless length of contact line, and could be calculated by using $\tau = t \cdot \omega_n$ in the formulas respectively. And the backlash-nonlinear function could be expressed as:

$$f(p_{11}) = \begin{cases} p_{11} - \dfrac{b_t}{b_n} & (p_{11} > \dfrac{b_t}{b_n}) \\ 0 & (|p_{11}| \le \dfrac{b_t}{b_n}) \\ p_{11} + \dfrac{b_t}{b_n} & (p_{11} < -\dfrac{b_t}{b_n}) \end{cases} \tag{19a}$$

$$f(p_{12}) = \begin{cases} p_{12} - \dfrac{b_a}{b_n} & (p_{12} > \dfrac{b_a}{b_n}) \\ 0 & (|p_{12}| \le \dfrac{b_a}{b_n}) \\ p_{12} + b_a & (p_{12} < -\dfrac{b_a}{b_n}) \end{cases} \tag{19b}$$

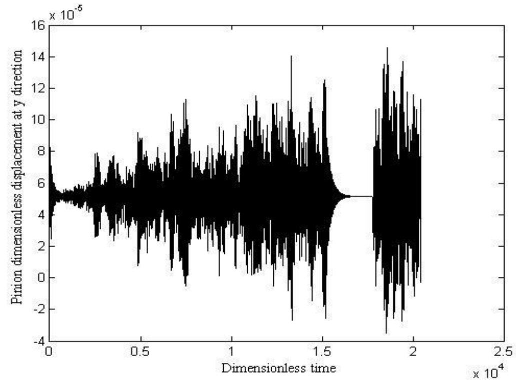
4 Numerical Result

To simplify the numerical calculation, the transmission error $e_{ij}(\tau)$ is ignored. For subsequent numerical study, the basic data used are:

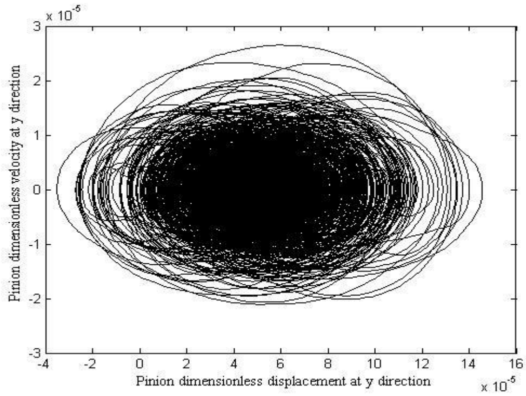
$z_p = 30$, $z_g = 90$, $m = 3$, $\alpha = 20°$, $\beta = 10°$, $g = 9.8N/kg$, $b_n = 0.1mm$, $B = 60mm$, T_p=$300N \cdot m$, $T_g = 900N \cdot m$, $f = 1000r/\min$, $m_p = 5kg$, $m_g = 45kg$,

$K_{mna} = 0.2\times10^9 N/m$, $K_{mn} = 5\times10^9 N/m$.

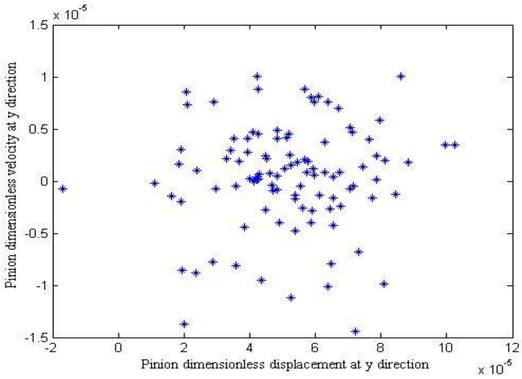
Then we could obtain the time history, speed-displacement phase diagram and Poincare section of pinion at y direction in Fig.2. Also the time history, speed-displacement phase diagram and Poincare section of gear at y and z direction was shown in Fig.3 and Fig.4 respectively.



(a)Time history

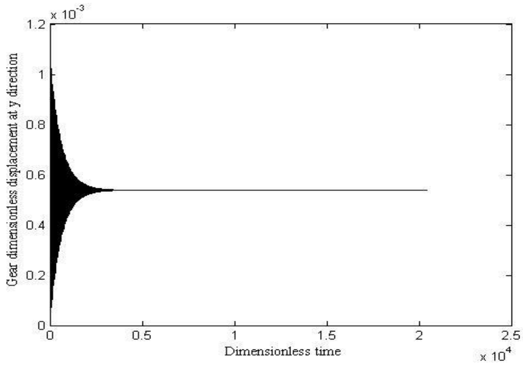


(b)Speed-displacement phase diagram

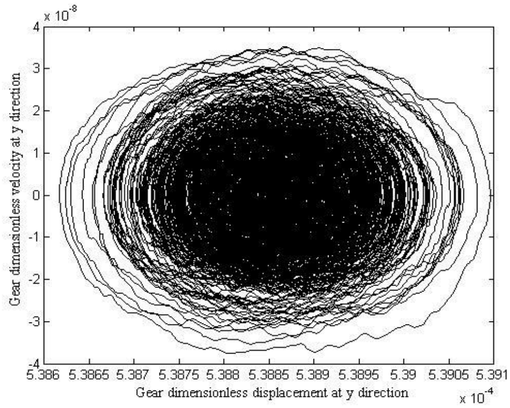


(c) Poincare section

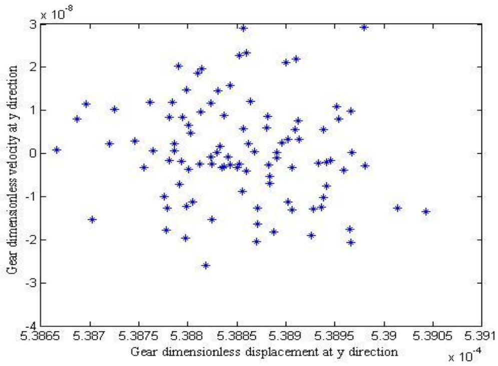
Figure.2. The numerical result of pinion at y direction



(a)Time history

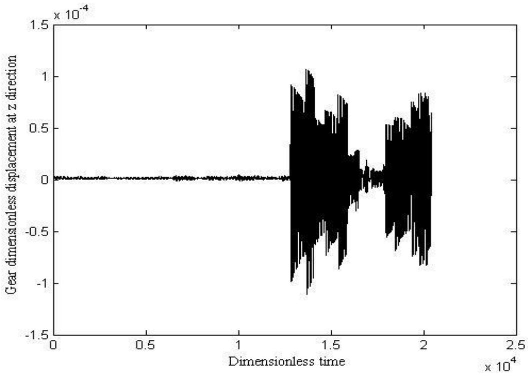


(b) Speed-displacement phase diagram

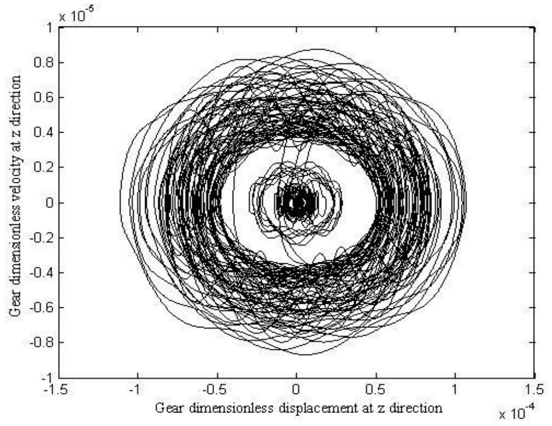


(c) Poincare section

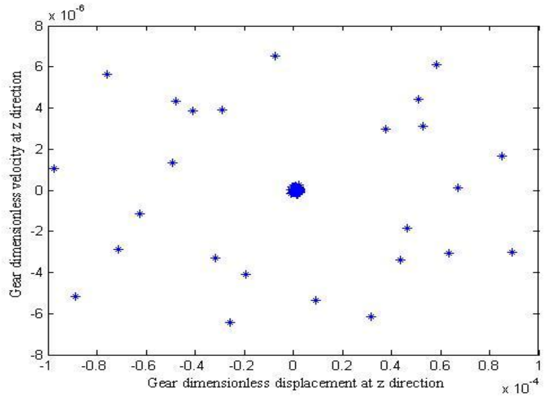
Figure.3. The numerical result of gear at y direction



(a)Time history



(b) Speed-displacement phase diagram



(c) Poincare section

Figure.4. The numerical result of gear at *z* direction

From the Fig.2 to Fig.4, we could find that there are abundant nonlinear characteristics in the helical gear system. Due to that the nonlinear characteristics is very important in transmission system, the further study is very necessary. To find the bifurcation and chaos characteristics and the influence of parameters, bifurcation diagram will be taken into consideration.

5 Conclusions

(1) The method to calculate the various time-varying parameters was presented in the section 2. In the section 2, the time-varying transmission error, time-varying length of contact line, time-varying meshing stiffness, time-varying meshing damping coefficient time-varying input and output torque, dynamic meshing force, dynamic friction force and time-varying fictional torque were considered in this dynamical model. Especially, when calculated the dynamic meshing force, friction force and frictional torque, the nonlinear of space also was considered a main nonlinear excitation source.

(2) The nonlinear dynamic model of helical gear pair with friction force and frictional torque has been established through centralized parameter method. 8 degrees was taking into consideration, especially the two degrees due to the frictional force.

(3) The non-linear equation of the helical gear system has been simplified and the dimensionless was adopted. Through the dimensionless equation, the dynamic characteristic could be revealed and studied by taking numerical method.

(4)Through the numerical method, we find that there are abundant nonlinear characteristics in the helical gear system when taking the frictional force and frictional torque into consideration. So the parameters select is very important in the helical gear system design.

Acknowledgments

We would like to thank to the reviewers for their encouraging comments and constructive suggestions to improve the manuscript.

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