Stability Analysis for Complex Rotational Flow

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Abstract: - Based on the earlier developed mathematical model of the complex flow due to the double rotations in two perpendicular directions, the stability analysis is performed in the paper. The Navier-Stokes equations are derived in the coordinate system rotating around the two perpendicular different axes, the vertical one of them is arranged on some distance from the other axis of rotation, the horizontal axis is directed along the tangential line to the circle of the vertical rotation. The two centrifugal and Coriolis forces create the unique features in high oscillating flow, with localities of the stretched liquid, due to their action varying by the circumferential cylindrical coordinate in the channel flow. Stability analysis for the complex rotational flow under double rotations creating strongly varying mass forces and stretching of the liquid is considered at first.

Key-Words: - Double Rotation; Stretched Liquid; Stability; Varying by Angle; Centrifugal Forces
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1 Introduction and Task Statement

1.1 The Double Rotating Coordinate System

The earlier developed mathematical model for the flow in double rotating coordinate system [1], [2] is implemented for the stability analysis. The first rotation is going around z-axis as shown in Fig. 1, with the rotation speed Ω. The other rotation is done with the rotation speed ω regarding the tangential axis to the main rotation circle above-mentioned, at the distance R_0 from the central axis z (x=y=0) of the device.

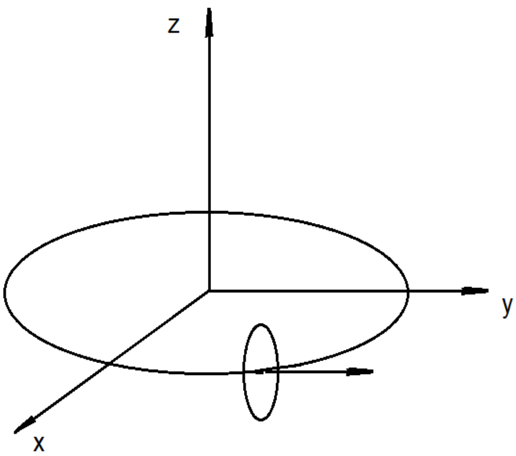

Fig. 1 The coordinate systems in a flow of the double rotating coordinate system

There are 3 cylinders rotating with the rotation speed   installed around the axis z on the distance R_0 from the centre, equally distributed by the circle of the radius R_0. Their radiuses are r_0 [1],[2]. The flow situation and the varying centrifugal forces are shown in Fig. 2:

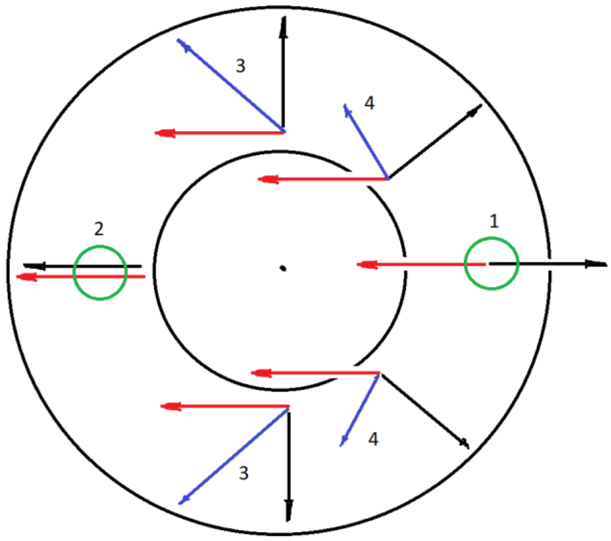

Fig. 2 The schematic directions of centrifugal forces in flow of the double rotating coordinate system

The above centrifugal forces are acting as shown in Fig. 2, where the centrifugal forces due to the main rotation (red colour) are directed in all points of the domain to the left in channel (edge of the main rotation circle), while the centrifugal forces due to rotation of the cylindrical channel (black) act by the radius of the channel. Therefore, in the position 1, the forces act counter currently causing a high strength of liquid (condition for cavitation!).

In the left side of the cylindrical channel (the situation 2) both forces act in the same direction causing the dramatic increase of pressure and its oscillation (condition for bursting of bubbles born due to cavitation). In all the other points of flow region (the situations 3 and 4 for example) a liquid is forced from the point 1 to the point 2 counter currently from the top and bottom of the channel.

The channel is rotating around its axis and moving in the main rotation around the vertical axis at the same time. Flow in the above described system is due to the liquid pumped into the rotating device from below creating depressurization at the entrance to the central vertical channel around the vertical axis. Water is supplied to the curvilinear helicoidally channel, which uniformly spreads it to the three channels (cylindrical cavitators rotating around their horizontal axes). More in details it was described in [1],[2].

The rotating coordinate system has the vertical axis z ($x=y=0$) on distance R_0 as shown in Fig. 1. Also rotation is going on around the axis tangential to the circle of the radius R_0. Intensive rotational movement and mixing flow are fascinating phenomena and may be highly effective in a number of applications [3],[4],[5].

Many theoretical aspects have been studied for the diverse exciting rotational flows including accounting the cavitations effects and stretching of the liquid [6],[7],[8],[9],[10],[11],[12],[13],[14]. But it requires deeper understanding of the phenomena for complex rotational flow with stretched liquid and alternating fast changing mass forces and flow conditions. It is interesting both from the theoretical point of view as the new class of flow, as well as from the practical applications, e.g. for the new technologies and devices [2].

In the cylindrical system (r,φ,z), the coordinate surfaces are cylinders r = constant,semi-planes $\varphi = const$ and planes $z = const$. Differential equation array for fluid flow is as follows [1]:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial r}+\frac{\rho u}{r}+\frac{\partial(\rho v)}{r\partial\varphi}+\frac{\partial(\rho w)}{\partial z}=0,$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v\partial u}{r\partial\varphi}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}+2\left(v\omega-w\Omega\cos\varphi\right)+$$

$$+(r_0\cos\varphi-R_0)\Omega^2\cos\varphi+r_0\omega^2=-\frac{1}{\rho}\frac{\partial p}{\partial r}+$$

$$+\frac{\partial}{r^2\partial\varphi}(\nu\frac{\partial u}{\partial\varphi})+\frac{\partial}{\partial r}(\nu\frac{\partial u}{\partial r})+\frac{\partial}{\partial z}(\nu\frac{\partial u}{\partial z})+$$

$$+\frac{\partial(\nu u)}{r\partial r}-\frac{\nu}{r^2}\left(2\frac{\partial v}{\partial\varphi}+u\right), \qquad (1)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v\partial v}{r\partial\varphi}+\frac{w\partial v}{\partial z}+\frac{uv}{r}+2\left(w\Omega\sin\varphi-u\omega\right)+$$

$$+(R_0-r_0\cos\varphi)\Omega^2\sin\varphi=-\frac{1}{\rho r}\frac{\partial p}{\partial\varphi}+\frac{\partial}{\partial r}(\nu\frac{\partial v}{\partial r})+$$

$$+\frac{\partial}{r^2\partial\varphi}(\nu\frac{\partial v}{\partial\varphi})+\frac{\partial}{\partial z}(\nu\frac{\partial v}{\partial z})+\frac{\partial(\nu v)}{r\partial r}+\frac{\nu}{r^2}(2\frac{\partial u}{\partial\varphi}\text{-}v),$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+\frac{v\partial w}{r\partial\varphi}+w\frac{\partial w}{\partial z}+2\Omega\left(v\sin\varphi-u\cos\varphi\right)=$$

$$=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\partial}{\partial r}(\nu\frac{\partial w}{\partial r})+\frac{\partial}{r^2\partial\varphi}(\nu\frac{\partial w}{\partial\varphi})+$$

$$+\frac{\partial}{\partial z}(\nu\frac{\partial w}{\partial z})+\frac{\partial(\nu w)}{r\partial r}.$$

Here are: $\rho, \vec{v}=\{u,v,w\}, p$ - density, velocity vector and pressure, μ, $\nu=\mu/\rho$ - dynamic and kinematic viscosity coefficients, respectively, The water is incompressible liquid, therefore, density is constant by comparably small variation of the pressure and temperature.

The careful study of the literature did not reveal any papers about the fluid flows under double rotations like in was presented in the invented and tested by Kujtim Hyseni device [2]. In this case, as shown in Fig. 2, the centrifugal forces are varying by the angle of rotation of the flow in a turbine located on the horizontal disk rotating around the vertical axis. Volumetrically distributed can be only mass forces, which are rarely met, e.g. electromagnetic or centrifugal ones. In the unique considered case, as shown in Fig. 2, there are positions in a flow with the stretching liquid, which may cause negative pressure and strong cavitations effect nearly unknown for the moment. The first attempts in this direction were made in [1], [2]. Further study is presented here by stability analysis of rotational flow.

1.2 Statement of the Problem

The equation array (1) is considered for isothermal incompressible flow. Parameters of the flow are stated as a stable average part plus small deviation in a wavy form $e^{i(kz+m\varphi-\alpha t)}$: $u=\overline{u}+u'$, $v=\overline{v}+v'$, $w=\overline{w}+w'$, $p=\overline{p}+p'$, with the amplitudes of perturbations, respectively: $v'-V$, $w'-W$, $p'-P$.

After substitution of the introduced parameters in (1), the linearized approximation for perturbations of the parameters is obtained:

$$U+imV+ikr_0W=0,\ -i\alpha U+\overline{v}U\frac{im}{r_0}-2V\frac{\overline{v}}{r_0}+$$

$$+\overline{w}ikU=2\left(W\Omega\cos\varphi-V\omega\right)-\frac{\nu}{r_0^2}\left(2Vim+U\right),$$

$$\frac{im}{r_0}\overline{v}V + \overline{w}ikV + W\frac{d\overline{v}}{dz} + \frac{im}{\rho r_0}P - i\alpha V =$$

$$= 2\left(U\omega - W\Omega\sin\varphi\right) - \nu\left(\frac{m^2+1}{r_0^2}V + k^2V\right), \quad (2)$$

$$\frac{im}{r_0}\left(\overline{v}W + V\overline{w}\right) + W\frac{d\overline{w}}{dz} + ikW\overline{w} - i\alpha W =$$

$$= 2\Omega\left(V\sin\varphi - U\cos\varphi\right) - \frac{ik}{\rho}P - \nu\left(\frac{m^2}{r_0^2}W + k^2W\right).$$

It is seen from (2) that such wavy oscillations do not satisfy the equations due to dependence on angle φ.

1.3 Substantiation of the Problem Statement

The corresponding above peculiarities must be substantiated. Consider the supplied water flow rate q (l/s or kg/s) due to the main rotation with the speed Ω (Hz, 1/s).

The power introduced by this rotation is approximately N=$q\left(R_0\Omega\right)^2$ (W). It is distributed by three turbines, the power of each of is ideally about N_t=N/3. Without the looses in the system, the power of the turbines by their rotation can be estimated as 1/3 of the total flow rate, N_t=$q\left(r_0\omega\right)^2/3$. This yields the following approximate correlation:

$$\omega = \Omega R_0 / r_0. \quad (3)$$

We have R_0=187 mm, r_0=63 mm - for the outer channel of the turbine and 57 for the inner channel of the turbine of the width 3 mm. In turbine it is distributed flow by the rotation and movement along the axes. Rotational flow is the most because it is arranged for cavitation inside the turbines. Let us take for the estimation r_0=60 mm, then it results from (3): $\omega = 3.1\Omega$. Accounting the losses it may be taken approximately $\omega \approx 3\Omega$. Then from (2):

$$U + imV + ikr_0W = 0, \quad -i\alpha U + \overline{v}U\frac{im}{r_0} - 2V\frac{\overline{v}}{r_0} +$$

$$+\overline{w}ikU = 2\Omega\left(W\cos\varphi - 3V\right) - \frac{\nu}{r_0^2}\left(2Vim + U\right), \quad (4)$$

$$\frac{im}{r_0}\left(\overline{v}W + V\overline{w}\right) + W\frac{d\overline{w}}{dz} + ikW\overline{w} - i\alpha W =$$

$$= 2\Omega\left(V\sin\varphi - U\cos\varphi\right) - \frac{ik}{\rho}P - \nu\left(\frac{m^2}{r_0^2}W + k^2W\right),$$

$$\frac{im}{r_0}\overline{v}V + \overline{w}ikV + W\frac{d\overline{v}}{dz} + \frac{im}{\rho r_0}P - i\alpha V =$$

$$= 2\Omega\left(3U - W\sin\varphi\right) - \nu\left(\frac{m^2+1}{r_0^2}V + k^2V\right).$$

2 Stability Modelling and Analysis

The equation array (4) shows that dependence from the angle of rotation is principal and cannot be neglected. Therefore, it can be used for the estimation of the oscillations but not for the precise solution. If m, k are real values, then only harmonic oscillations are available varying in time. From (4), accounting that all equations must satisfy separately for their real and imaginary parts yields:

$$U = 0, \quad W = -\frac{m}{r_0k}V, \quad V\left(\frac{\overline{v}}{r_0} + 3\Omega\right) = -\Omega W\cos\varphi,$$

$$P = \alpha\frac{\rho}{k}W - \frac{\rho}{k}\frac{m}{r_0}\left(\overline{v}W + V\overline{w}\right) - \rho W\overline{w}, \quad (5)$$

$$W\left[\frac{d\overline{w}}{dz} + \nu\left(\frac{m^2}{r_0^2} + k^2\right)\right] = 2\Omega\left(V\sin\varphi - U\cos\varphi\right),$$

$$P = \frac{\rho r_0}{m}\left[\alpha - \left(\frac{m}{r_0}\overline{v} + \overline{w}k\right)\right]V, \quad \alpha = \frac{2\nu}{Ur_0^2}m,$$

$$W\left(\frac{d\overline{v}}{dz} + 2\Omega\sin\varphi\right) = 6\Omega U - \nu\left(\frac{m^2+1}{r_0^2}V + k^2V\right).$$

2.1 Simplification of the Equations

By $U = 0$ (estimation: $U \approx 0$), $\alpha = 2\nu m/(Ur_0^2) >> 1$ or $\alpha = \infty$, which means the very high frequency of oscillations without instability (strong shaking).

Further the equation array (5) is simplified to

$$m = \left(\frac{\overline{v}}{\Omega} + 3r_0\right)\frac{k}{\cos\varphi}, \quad P = \rho V\frac{m}{r_0k}\left(\frac{m\overline{v}}{r_0k} - \frac{\alpha}{k}\right),$$

$$W = -\frac{m}{r_0k}V, \quad k^2 + 2\sin\varphi\frac{r_0}{m\nu}\Omega k + \frac{1}{\nu}\frac{d\overline{w}}{dz} + \frac{m^2}{r_0^2} = 0,$$

$$\alpha\left(1 + \frac{m^2}{r_0^2k^2}\right) = \overline{v}\frac{m^3}{r_0^3k^2} + \frac{m}{r_0}\overline{v} + \overline{w}k, \quad (6)$$

$$\frac{m}{r_0k}\left(\frac{d\overline{v}}{dz} + 2\Omega\sin\varphi\right) = \nu\left(\frac{m^2+1}{r_0^2} + k^2\right).$$

The two contradictions to the assumptions made are seen from (6): dependence of the perturbations on the angle φ and impossibility to satisfy the equations 3 and 5 due to $\alpha = \infty$. Therefore, the perturbations $e^{i(kz+m\varphi-\alpha t)}$ are considered with the parameters: $k=k_r+ik_i$, $\alpha=\alpha_r+i\alpha_i$, where k_r,k_i are the real and imagine parts of the wave number k, similarly - α_r,α_i.

Then perturbations have the following form:

$$e^{i(kz+m\varphi-\alpha t)}=e^{i(k_r z+m\varphi-\alpha_r t)}e^{-k_i z+\alpha_i t},$$

may be wavy by z,φ and time, and exponentially growing or decreasing by the axis of the channel, while their varying in time is got from solution of the equations. Thus, from (4) yields:

$$\alpha_r=\frac{2\nu}{Ur_0^2}mV-m\frac{\bar{v}}{r_0}-\bar{w}k_r,$$

$$\alpha_i=\frac{2\Omega}{U}\left(W\cos\varphi-3V\right)-\frac{\nu}{r_0^2}-2V\frac{\bar{v}}{Ur_0}-\bar{w}k_i,$$

$$U=k_i r_0 W,\ W=-\frac{m}{k_r r_0}V, \qquad (7)$$

$$\frac{im}{r_0}\bar{v}V+\bar{w}\left(ik_r-k_i\right)V+W\frac{d\bar{v}}{dz}+\frac{im}{\rho r_0}P+$$

$$+\left(\alpha_i-i\alpha_r\right)V=2\Omega\left(3U-W\sin\varphi\right)+$$

$$-\nu\left[\frac{m^2+1}{r_0^2}V+\left(k_r^2-k_i^2+2ik_rk_i\right)V\right],$$

$$\frac{im}{r_0}\left(\bar{v}W+V\bar{w}\right)+W\frac{d\bar{w}}{dz}+\left(ik_r-k_i\right)W\bar{w}+$$

$$+\left(\alpha_i-i\alpha_r\right)W=2\Omega\left(V\sin\varphi-U\cos\varphi\right)+$$

$$-\frac{ik_r-k_i}{\rho}P-\nu\left[\frac{m^2}{r_0^2}W+\left(k_r^2-k_i^2+2ik_rk_i\right)W\right].$$

Here α_r determines the oscillation frequency by time, while α_i is responsible for growing of the oscillations ($\alpha_i>0$) or their decreasing ($\alpha_i<0$).

The first four correlations for α_r,α_i and U,W may be transformed from (7) as follows:

$$U=-k_i\frac{mV}{k_r},W=-\frac{mV}{k_r r_0},\alpha_r=-\frac{2\nu k_r}{k_i r_0^2}-m\frac{\bar{v}}{r_0}-\bar{w}k_r,$$

$$\alpha_i=\frac{2\Omega}{k_i}\left(\frac{\cos\varphi}{r_0}+3\frac{k_r}{m}\right)-\frac{\nu}{r_0^2}+2\frac{\bar{v}k_r}{k_i m r_0}-\bar{w}k_i. \qquad (8)$$

The correlations (8) shows that the parameters α_r,α_i expressed through perturbations' parameters, radius of the channel and average velocities of the flow. This is very good result! The relation between the velocity amplitudes is comparably simple. Only one term in the parameter α_i, which depends on the angle φ, breaks the assumptions made. Thus, equation for α_i is not accurate and may be used only for approximate estimations.

Inaccuracy caused by dependence α_i on φ in the last equation (8) is estimated as follows. The characteristic oscillations of the curvilinear channel are taken as the values similar to the waves $e^{i(k_r z+m\varphi-\alpha_r t)}$: $k_r=10^3$, $m=3$, $r_0=6\cdot10^{-2}$. It results $\cos\varphi/r_0+3k_r/m=0.17\cdot10^2\cos\varphi+10^3$. Thus, the first term in the last correlation is less than 1.7% comparing to the second one, so that can be neglected. The approximate correlation is the next:

$$\alpha_i=\frac{6\Omega k_r}{k_i m}-\frac{\nu}{r_0^2}+2\frac{\bar{v}k_r}{k_i m r_0}-\bar{w}k_i=$$

$$=\frac{2k_r}{k_i m}\left(3\Omega+\frac{\bar{v}}{r_0}\right)-\frac{\nu}{r_0^2}-\bar{w}k_i\approx\frac{12k_r}{k_i m}\Omega-\frac{\nu}{r_0^2}-\bar{w}k_i.$$

Thus, in the above estimations ($\nu=10^{-6}$ m^2/s - kinematic viscosity coefficient for the water):

$$\alpha_i=\frac{4}{k_i}10^3\Omega-2.8\cdot10^{-4}-\bar{w}k_i\approx\frac{4}{k_i}10^3\Omega$$

because $\Omega=100$ 1/s (6000 rpm) and $k_i=40$ leads to 10^4-40 $\bar{w}$. Then the last two equations (7) lead to:

$$P=\rho V\left[\frac{r_0}{m}\left(\alpha_r+2\nu k_rk_i-k_r\bar{w}\right)-\bar{v}\right],$$

$$\alpha_r=\frac{m}{r_0}\bar{v}+\frac{k_r}{m^2+k_r^2r_0^2}\left[k_i\left(m^2-k_r^2r_0^2\right)\nu+k_r^2r_0^2\bar{w}\right],$$

$$m^2+\frac{r_0}{\nu k_r}\left[2\Omega\left(3k_ir_0-\sin\varphi\right)-\frac{d\bar{v}}{dz}\right]m+1+$$

$$r_0^2\left(k_r^2-k_i^2\right)+\frac{r_0^2}{\nu}\left(\alpha_i-k_i\bar{w}\right)=0, \qquad (9)$$

$$\alpha_i=2r_0\Omega\left(\frac{k_r}{m}\sin\varphi+k_i\cos\varphi\right)+\nu\left(\frac{m^2}{r_0^2}+k_r^2-k_i^2\right)+$$

$$+\frac{d\bar{w}}{dz}-k_i\bar{w}+r_0k_i\frac{k_r}{m}\left[\frac{r_0}{m}\left(\alpha_r+2\nu k_rk_i-k_r\bar{w}\right)-\bar{v}\right].$$

2.1.2 Equations for Amplitudes of Perturbations

Now from (7), with account (9), the following is got

$$P=\frac{2}{m}\rho V\left[k_rr_0\left(\nu k_i-\bar{w}\right)-\frac{\nu k_r}{k_ir_0}-\bar{v}m\right],$$

$$k_i=\frac{\left(m^2+k_r^2r_0^2\right)\left(2m\bar{v}+\bar{w}k_rr_0\right)+k_r^3r_0^3\bar{w}\pm\sqrt{D_{ki}}}{2k_rr_0\nu\left(m^2-k_r^2r_0^2\right)},$$

$$D_{ki}=\left[\left(m^2+k_r^2r_0^2\right)\left(2m\bar{v}+\bar{w}k_rr_0\right)+k_r^3r_0^3\bar{w}\right]^2+$$

$$+8k_r^2\nu^2\left(m^2-k_r^4r_0^4\right),\quad W=-\frac{m}{k_rr_0}V\,,\qquad(10)$$

$$U=-\frac{m}{k_r}k_iV\,,\ \alpha_r=-\frac{2\nu k_r}{k_ir_0^2}-m\frac{\bar{v}}{r_0}-\bar{w}k_r\,,$$

$$\alpha_i=\frac{2\Omega}{mk_i}\left(\frac{m}{r_0}\cos\varphi+3k_r\right)-\frac{\nu}{r_0^2}+2\frac{\bar{v}k_r}{mk_ir_0}-\bar{w}k_i\,,$$

$$m^3+\frac{r_0}{\nu k_r}\left[2\Omega\left(3k_ir_0-\sin\varphi\right)-\frac{d\bar{v}}{dz}\right]m^2+$$

$$+\left[r_0^2\left(k_i^2-k_r^2\right)+2\frac{\Omega r_0}{\nu k_i}\cos\varphi-2\frac{\bar{w}r_0^2}{\nu}k_i\right]m+$$

$$+2\frac{k_rr_0}{\nu k_i}\left(3\Omega r_0+\bar{v}\right)=0\,,$$

$$\left[\nu\left(\frac{r_0^2k_i^2}{m^2}+1\right)+2\nu\frac{r_0^2k_i^2}{m^2+r_0^2k_r^2}-\frac{r_0^2k_i\bar{w}}{m^2+r_0^2k_r^2}\right]k_r^2+$$

$$2\left[\Omega\left(\frac{r_0}{m}\sin\varphi-\frac{3}{mk_i}\right)-\frac{\bar{v}}{mk_ir_0}\right]k_r+$$

$$+\frac{d\bar{w}}{dz}+2\Omega\left(r_0k_i-\frac{1}{r_0k_i}\right)\cos\varphi+\nu\left(\frac{m^2+1}{r_0^2}-k_i^2\right)=0\,.$$

All parameters in (10) were got, except m,k_r, calculating from the last two equations, which are in general the cubic and the fourth degree algebraic equations, correspondingly. With the estimations above: $k_r=10^3$, $m=3$, $r_0=6\cdot10^{-2}$, thus, $r_0^2k_r^2>>m^2$ (3600>>9!). Therefore, the second term in the brackets of k_r^2 can be neglected comparing to the first one, and m^2 - comparing to $r_0^2k_r^2$. Then yields

$$2\left[\Omega\left(\frac{r_0}{m}\sin\varphi-\frac{3}{mk_i}\right)-\frac{\bar{v}}{mk_ir_0}\right]k_r+$$

$$+\frac{d\bar{w}}{dz}-\bar{w}k_i+2\Omega\left(r_0k_i-\frac{1}{r_0k_i}\right)\cos\varphi+\qquad(11)$$

$$+\nu\left[\left(\frac{r_0^2k_i^2}{m^2}+1\right)k_r^2+\frac{m^2+1}{r_0^2}-k_i^2\right]=0\,.$$

2.2 Calculation of the Wave Parameters

Solution of the equation (11) is as follows

$$k_r=m\frac{\left[\bar{v}+\Omega r_0\left(3-k_ir_0\sin\varphi\right)\right]\pm\sqrt{D_{kr}}}{\nu k_ir_0\left(r_0^2k_i^2+m^2\right)}\,,\qquad(12)$$

$$D_{kr}=2\nu\left(r_0^2k_i^2+m^2\right)k_i\left[r_0^2k_i\left(k_i\bar{w}-\frac{d\bar{w}}{dz}\right)+\right.$$

$$\left.+2\Omega r_0\left(1-r_0^2k_i^2\right)\cos\varphi+\nu k_i\left(r_0^2k_i^2-m^2-1\right)\right]+$$

$$+\left[\bar{v}+\Omega r_0\left(3-k_ir_0\sin\varphi\right)\right]^2.$$

Then the cubic equation in (10) is solved for the parameter *m*. A *cubic formula* for the roots of cubic equation ax^3+bx^2+cx+d (with $a\neq0$) is

$$x_n=-\frac{1}{3a}\left(b+C\xi^n+\frac{\Delta_0}{C\xi^n}\right),\quad \xi=\frac{-1+i\sqrt{3}}{2}\,,$$

where $n\in\{0,1,2\}$, $i=\sqrt{-1}$, so that the real root is only one in our case (a=1), so that:

$$m=-\frac{1}{3}\left(b+C+\frac{\Delta_0}{C}\right),\quad d=2\frac{k_rr_0}{\nu k_i}\left(3\Omega r_0+\bar{v}\right),$$

$$b=\frac{r_0}{\nu k_r}\left[2\Omega\left(3k_ir_0-\sin\varphi\right)-\frac{d\bar{v}}{dz}\right],\qquad(13)$$

$$c=r_0^2\left(k_i^2-k_r^2\right)+2\frac{\Omega r_0}{\nu k_i}\cos\varphi-2\frac{\bar{w}r_0^2}{\nu}k_i\,,$$

$$C\approx0.7937\sqrt[3]{\Delta_1\pm\sqrt{\Delta_1^2-4\Delta_0^3}}\,,\quad \Delta_0=b^2-3c\,,$$

$$\Delta_1=2b^3-9bc+27d\,.$$

Here are the following expressions for the functions:

$$\Delta_0=\left\{\frac{r_0}{\nu k_r}\left[2\Omega\left(3k_ir_0-\sin\varphi\right)-\frac{d\bar{v}}{dz}\right]\right\}^2+$$

$$-3\left[r_0^2\left(k_i^2-k_r^2\right)+2\frac{\Omega r_0}{\nu k_i}\cos\varphi-2\frac{\bar{w}r_0^2}{\nu}k_i\right],$$

$$\Delta_1=2\left\{\frac{r_0}{\nu k_r}\left[2\Omega\left(3k_ir_0-\sin\varphi\right)-\frac{d\bar{v}}{dz}\right]\right\}^3+$$

$$-9\frac{r_0}{\nu k_r}\left[2\Omega\left(3k_ir_0-\sin\varphi\right)-\frac{d\bar{v}}{dz}\right]\left[r_0^2\left(k_i^2-k_r^2\right)+\right.$$

$$\left.+2\frac{\Omega r_0}{\nu k_i}\cos\varphi-2\frac{\bar{w}r_0^2}{\nu}k_i\right]+54\frac{k_rr_0}{\nu k_i}\left(3\Omega r_0+\bar{v}\right).$$

The value under the square root in (13) must be positive because m is supposed to be real value: $D_m=\Delta_1^2-4\Delta_0^3\geq0$, or $\Delta_1^2\geq4\Delta_0^3$. This results in $4\left(b^3d-b^2c^2+c^3\right)+3\left(3d-bc\right)^2\geq0$.

In general case from the last fourth degree equation of the system (10) yields

$$2m\left[\Omega r_0^2\left(r_0\sin\varphi-\frac{3}{k_i}\right)-\frac{\bar{v}r_0}{k_i}\right]k_r^3+\left[\frac{d\bar{w}}{dz}r_0^2+\right.$$

$$+2\Omega r_0\left(k_i r_0^2-\frac{1}{k_i}\right)\cos\varphi - r_0^2 k_i \bar{w}\Bigg] m^2 k_r^2 + \quad (14)$$
$$+2\nu\left(r_0^2 k_i^2+m^2+0.5\right)+\nu\left(r_0^2 k_i^2+m^2\right)r_0^2 k_r^4+$$
$$+2\left[\Omega\left(r_0\sin\varphi-\frac{3}{k_i}\right)-\frac{\bar{v}}{k_i r_0}\right]m^3 k_r+\left[\frac{d\bar{w}}{dz}+\right.$$
$$\left.+2\Omega\left(k_i r_0-\frac{1}{k_i r_0}\right)\cos\varphi+\nu\left(\frac{m^2+1}{r_0^2}-k_i^2\right)\right]m^4=0,$$

where from expression for the parameter k_r is got.

All parameters in (10), (12) are real values, therefore D_{ki}, D_m, D_{kr} cannot be negative values:

$$\left[\left(m^2+k_r^2 r_0^2\right)\left(2m\bar{v}+\bar{w}k_r r_0\right)+k_r^3 r_0^3\bar{w}\right]^2+$$
$$+8k_r^2\nu^2 m^4 \ge 8k_r^6 r_0^4\nu^2;\quad \Delta_1^2\ge 4\Delta_0^3,\quad (15)$$
$$\Delta_1=2\left\{\frac{r_0}{\nu k_r}\left[2\Omega\left(3k_i r_0-\sin\varphi\right)-\frac{d\bar{v}}{dz}\right]\right\}^3+$$
$$-9\frac{r_0}{\nu k_r}\left[2\Omega\left(3k_i r_0-\sin\varphi\right)-\frac{d\bar{v}}{dz}\right]\left[r_0^2\left(k_i^2-k_r^2\right)+\right.$$
$$\left.+2\frac{\Omega r_0}{\nu k_i}\cos\varphi-2\frac{\bar{w}r_0^2}{\nu}k_i\right]+54\frac{k_r r_0}{\nu k_i}\left(3\Omega r_0+\bar{v}\right),$$
$$\Delta_0=\left\{\frac{r_0}{\nu k_r}\left[2\Omega\left(3k_i r_0-\sin\varphi\right)-\frac{d\bar{v}}{dz}\right]\right\}^2+$$
$$-3\left[r_0^2\left(k_i^2-k_r^2\right)+2\frac{\Omega r_0}{\nu k_i}\cos\varphi-2\frac{\bar{w}r_0^2}{\nu}k_i\right];$$
$$2\nu\left(r_0^2k_i^2+m^2\right)k_i\left[r_0^2 k_i\left(k_i\bar{w}-\frac{d\bar{w}}{dz}\right)+\right.$$
$$\left.+2\Omega r_0\left(1-r_0^2k_i^2\right)\cos\varphi+\nu k_i\left(r_0^2k_i^2-m^2-1\right)\right]+$$
$$+\left[\bar{v}+\Omega r_0\left(3-k_i r_0\sin\varphi\right)\right]^2\ge 0.$$

The first condition (15) is always satisfied by the range of considering parameters ($k_r=10^3$, $k_i=\pm 5$, $m=3$, $r_0=6\cdot 10^{-2}$) because part of the positive value to the left in the inequality results $8k_r^6 r_0^4\left(r_0^2\bar{w}^2-\nu^2\right)+8k_r^2\nu^2m^4\ge 0$, where from follows $8k_r^6 r_0^4\left(36\cdot 10^{-4}-10^{-12}\right)+8k_r^2\nu^2m^4\ge 0$.

2.3 Analysis of the Flow Stability

Estimation for the velocity is as follows. The flow rate is totally 5.1 l/s, so that for each of the 3 turbines is $1.7*10^{-3}$ m^3/s. The cross section is estimated as a gap of the area S=$2\pi r_0*6*10^{-3}$ m^2=$2\pi\cdot 6\cdot 10^{-2}\cdot 6\cdot 10^{-3}$ m^2. Thus, $\bar{w}$=-0.752 m/s, $\bar{v}=\omega r_0$ =18 m/s. $\bar{w}$<0 because flow is going against the axis *z*, which is the axis of the turbine and tangential to the main rotation circular plate with turbine.

By these estimations,

$$\Delta_1\approx -10^{10}\left[2.33+3.46\left(0.9+\sin\varphi\right)^3\right],$$

thus, $\max\Delta_1\approx -10^{10}\left(2.33-0.346\right)=-1.98\cdot 10^{10}$ by $\sin\varphi$=-1; $\min\Delta_1\approx -2.6\cdot 10^{11}$ by $\sin\varphi$=1. And max $\Delta_0\approx 5.2\cdot 10^8$ by $\sin\varphi$=1; $\min\Delta_0\approx 3.2\cdot 10^6$ by $\sin\varphi$=-0.9. Therefore, $\Delta_1^2\ge 4\Delta_0^3$ is not satisfied by $\sin\varphi$=1 ($\Delta_1^2\approx 6.76\cdot 10^{22}$ while $4\Delta_0^3\approx 5.6\cdot 10^{26}$). By $\sin\varphi$=-1 it is approximately $\Delta_1^2\approx 3.92\cdot 10^{20}$ and $4\Delta_0^3\approx 1.31\cdot 10^{20}$, so that $\Delta_1^2\ge 4\Delta_0^3$ is satisfied. Thus, solution for the *m* is not correct for all values of φ and we have to find the other conditions for it.

The other transformation of (7) is done excluding dependence of the equations on φ in accordance with the assumptions made. Let assume correlations for the perturbations similar to the above velocities and neglect the first term in $W\cos\varphi-3V$ comparing to the second one. Then in $3U-W\sin\varphi$ neglect the second term comparing to the first by $|k_i|$>>1, e.g. k_i=-10 leads to $e^{-k_i z}\approx 2.7$ by z=0.1. By intensive perturbations, $|k_i|$ may be higher.

$-6*\Omega V/(k_i r_0)$ in the expression $\alpha_i W$ in the last equation is compared to $2\Omega V\left(\sin\varphi+mk_i\cos\varphi/k_r\right)$. It was shown $3/(k_i r_0)>>1$, $|mk_i/k_r\cos\varphi|<<1$ and $|\sin\varphi|\le 1$ ($|mk_i/k_r|=0.1, k_r=100, m=1, k_i=-10$). The first term exceeds the second one. Finally, (7):

$$\alpha_r=-\frac{2\nu k_r}{k_i r_0^2}-m\frac{\bar{v}}{r_0}-\bar{w}k_r,\ \ U=-k_i\frac{m}{k_r}V,$$
$$\alpha_i=2\frac{k_r}{k_i m}\left(\frac{\bar{v}}{r_0}+3\Omega\right)-\frac{\nu}{r_0^2}-\bar{w}k_i,\ \ W=-\frac{m}{k_r r_0}V,$$
$$2\frac{k_r}{k_i m}\left(\frac{\bar{v}}{r_0}+3\Omega\right)-\frac{m}{k_r r_0}\frac{d\bar{v}}{dz}-2\bar{w}k_i+$$
$$+2i\left(\frac{\nu k_r}{k_i r_0^2}+m\frac{\bar{v}}{r_0}+\bar{w}k_r\right)+\frac{imP}{\rho r_0 V}=\quad (16)$$
$$=-6k_i\frac{m}{k_r}\Omega-\nu\left[\frac{m^2}{r_0^2}+\left(k_r^2-k_i^2+2ik_rk_i\right)\right],$$

$$\frac{\nu}{k_r r_0^2}-\frac{2}{k_i m}\left(\frac{\overline{v}}{r_0}+3\Omega\right)-i\frac{2}{k_r r_0}\left(\frac{\nu}{r_0}+m\overline{v}\right)+$$
$$+2\frac{k_i}{k_r}\overline{w}-i\overline{w}-\frac{1}{k_r}\frac{d\overline{w}}{dz}=-\frac{ik_r-k_i}{\rho m}r_0\frac{P}{V}+$$
$$+\frac{\nu}{k_r}\left[\frac{m^2}{r_0^2}+\frac{m}{k_r r_0}\left(k_r^2-k_i^2+2ik_r k_i\right)\right].$$

Separating the imaginary and real parts in the last two equations of the system (16) yields

$$P=-2\rho r_0 V\left[\left(k_i+\frac{1}{mk_i r_0^2}\right)k_r\nu+\frac{\overline{v}}{r_0}+\frac{\overline{w}k_r}{m}\right],$$
$$2\frac{k_r}{k_i m}\left(\frac{\overline{v}}{r_0}+3\Omega\right)-\frac{m}{k_r r_0}\frac{d\overline{v}}{dz}-2\overline{w}k_i+$$
$$+6k_i\frac{m}{k_r}\Omega+\nu\left[\frac{m^2}{r_0^2}+\left(k_r^2-k_i^2\right)\right]=0, \qquad (17)$$
$$\frac{\nu}{k_r r_0^2}\left(1-m^2\right)-\frac{2}{k_i m}\left(\frac{\overline{v}}{r_0}+3\Omega\right)-\frac{1}{k_r}\frac{d\overline{w}}{dz}+2\frac{k_i}{k_r}\overline{w}=$$
$$=-2r_0\left[\left(k_r\frac{k_i r_0}{m}+\frac{1}{m^2 r_0}\right)k_r\nu+\frac{k_i\overline{v}}{m}+r_0\frac{\overline{w}k_r k_i}{m^2}\right]+$$
$$+\frac{m\nu}{k_r^2 r_0}\left(k_r^2-k_i^2\right), \quad k_i\nu+\frac{1}{k_r r_0}\left(\frac{\nu}{r_0}+m\overline{v}\right)+$$
$$+\frac{k_r}{m}r_0^2\left[\left(k_i+\frac{1}{mk_i r_0^2}\right)k_r\nu+\frac{\overline{v}}{r_0}+\frac{\overline{w}k_r}{m}\right]+\frac{\overline{w}}{2}=0.$$

Viscous terms are negligibly small, therefore

$$\left(6k_i r_0\Omega-\frac{d\overline{v}}{dz}\right)\frac{m^2}{2}+\left(\overline{v}+3\Omega r_0\right)\frac{k_r^2}{k_i}=r_0\overline{w}k_i k_r m,$$
$$r_0\left[\left(m^2+k_r^2 r_0^2\right)\overline{w}+mk_r r_0\overline{v}\right]k_i^2-m^2\frac{r_0}{2}\frac{d\overline{w}}{dz}k_i+$$
$$-k_r m\left(\overline{v}+3\Omega r_0\right)=0, \quad P=-2m\rho V\left(m\overline{v}+r_0 k_r\overline{w}\right),$$
$$k_r^2 r_0^2\left(m\overline{v}+r_0\overline{w}k_r\right)+m^3\overline{v}+m^2 k_r r_0\frac{\overline{w}}{2}=0. \qquad (18)$$

Solution of the equation array (18) is as follows

$$P=-2m\rho V\left(m\overline{v}+r_0 k_r\overline{w}\right), \; m=\frac{k_r\left(r_0 k_i\overline{w}\pm\sqrt{D_m}\right)}{3r_0 k_i\Omega-d\overline{v}/dz},$$
$$k_i=\frac{m^2 r_0 d\overline{w}/dz\pm\sqrt{D_{ki}}}{4r_0\left[\left(m^2+k_r^2 r_0^2\right)\overline{w}+mk_r r_0\overline{v}\right]}, \qquad (19)$$
$$D_m=\left(r_0\overline{w}k_i^2\right)^2-2k_i\left(6k_i r_0\Omega-\frac{d\overline{v}}{dz}\right)\left(\overline{v}+3\Omega r_0\right),$$
$$D_{ki}=16r_0\left[\left(m^2+k_r^2 r_0^2\right)\overline{w}+mk_r r_0\overline{v}\right]\cdot$$
$$\cdot k_r m\left(\overline{v}+3\Omega r_0\right)+\left(m^2 r_0\frac{d\overline{w}}{dz}\right)^2,$$
$$k_r^2 r_0^2\left(m\overline{v}+r_0\overline{w}k_r\right)+m^3\overline{v}+m^2 k_r r_0\frac{\overline{w}}{2}=0,$$

D_m, D_{ki} must be positive (k_i, m are real). Then

$$\left(r_0\overline{w}\right)^2 k_i^3\ge 2\left(6k_i r_0\Omega-\frac{d\overline{v}}{dz}\right)\left(\overline{v}+3\Omega r_0\right), \qquad (20)$$
$$\frac{r_0 m^3\left(d\overline{w}/dz\right)^2}{16\left(\overline{v}+3\Omega r_0\right)k_r}+\left(m^2+k_r^2 r_0^2\right)\overline{w}+mk_r r_0\overline{v}\ge 0.$$

Accounting $\overline{v}\approx 3\Omega r_0$, $d\overline{v}/dz\approx 0$ for constant by z rotation, which is the main reason for this velocity, and $\overline{w}<0$ (flow in turbine is opposite to *z*-axis), then the first inequality (20) results by $k_i>0$: $\overline{w}^2 k_i^2\ge 72\Omega^2$ or $\overline{w}^2 k_i^2\le 72\Omega^2$ by $k_i<0$.

By $k_i>0$, the perturbations decrease along the axis, and by the above estimations approximately yields $\Omega\le 0.09k_i$, where from for the above estimation $\Omega\le 0.9$. As far as $\Omega=100$, k_i must be too big, which is not real in our case.

By $k_i<0$, $\overline{w}^2 k_i^2\le 72\Omega^2$, or $\Omega\ge 0.09\left|k_i\right|$, which is satisfied in our situation [1],[2]. The second condition (20) yields by the above estimations: $\Omega\le 3.86\left(d\overline{w}/dz\right)^2 m^3/k_r^3$, $\left(d\overline{w}/dz\right)^2\ge 0.26\Omega k_r^3/m^3$. The last can satisfy only by nearly the same order of k_r and m because the frequency is high.

The solution (19) with the restrictions (20) may be used for calculation of the flow perturbation parameters. Also it is interesting to get solution of the problem for complex values *m*, which mean that 2 counter-current waves by the angle φ going from the position φ=0 to $\varphi=\pm\pi$ from the top and bottom are interacting and growing.

Suppose $m=m_r+im_i$ and consider perturbations: $e^{i(kz+m\varphi-\alpha t)}=e^{i(k_r z+m_r\varphi-\alpha_r t)}e^{-k_i z-m_i\varphi+\alpha_i t}$, where the wave numbers by *z,φ,t* are k_r, m_r, α_r, and k_i, m_i, α_i are the increments by *z,φ* and time *t*. Thus, we get

$$U=\left(m_i-\frac{m_r}{k_r}k_i\right)V, \; W=-\frac{m_r}{k_r r_0}V,$$
$$\left(m_i-\frac{m_r}{k_r}k_i\right)\left(\overline{v}\frac{m_r}{r_0}+k_r\overline{w}-\alpha_r\right)=-2\frac{\nu}{r_0^2}m_r,$$
$$\left(m_i-\frac{m_r}{k_r}k_i\right)\left(\alpha_i-\frac{m_i}{r_0}\overline{v}-k_i\overline{w}\right)-2\frac{\overline{v}}{r_0}=$$

$$= \frac{\nu}{r_0^2}\left(m_i + \frac{m_r}{k_r} k_i \right) - 6\Omega\,,\ \frac{m_r}{r_0}\left(\overline{w} - \frac{m_r}{k_r r_0}\overline{v} \right) - \frac{m_r}{r_0}\overline{w} +$$

$$+\alpha_r \frac{m_r}{k_r r_0} = 2\nu \frac{m_r}{k_r r_0}\left(\frac{m_r m_i}{r_0^2} + k_r k_i \right) - \frac{k_r}{\rho V} P\,,$$

$$\frac{m_i}{r_0}\left(\frac{m_r}{k_r r_0}\overline{v} - \overline{w} \right) + \frac{m_r}{k_r r_0}\left(k_i \overline{w} - \frac{d\overline{w}}{dz} - \alpha_i \right) =$$

$$= \frac{k_i}{\rho V} P + \nu \frac{m_r}{k_r r_0}\left(\frac{m_r^2 - m_i^2}{r_0^2} + k_r^2 - k_i^2 \right), \quad (21)$$

$$\frac{m_r}{r_0}\overline{v} + \overline{w}k_r + \frac{m_r}{\rho r_0 V} P - \alpha_r = -2\nu\left(\frac{m_r m_i}{r_0^2} + k_r k_i \right),$$

$$\alpha_i - \frac{m_i}{r_0}\overline{v} - \overline{w}k_i - \frac{m_r}{k_r r_0}\frac{d\overline{v}}{dz} - \frac{m_i}{\rho r_0 V} P =$$

$$6\Omega\left(m_i - \frac{m_r}{k_r} k_i \right) - \nu\left(\frac{m_r^2 - m_i^2 + 1}{r_0^2} + k_r^2 - k_i^2 \right).$$

2.3.1 Nonviscous Solution of Instability Problem

Neglecting the viscous terms as it was done above we simplify the equation array (21) as follows:

$$\left(\frac{m_r}{k_r} k_i - m_i \right)\left(\overline{v}\frac{m_r}{r_0} + k_r \overline{w} - \alpha_r \right) = 0\,,$$

$$\left(m_i - \frac{m_r}{k_r} k_i \right)\left(\alpha_i - \frac{m_i}{r_0}\overline{v} - k_i \overline{w} \right) = 0\,,$$

$$\left(\overline{w} - \frac{m_r}{k_r r_0}\overline{v} \right) - \overline{w} + \frac{\alpha_r}{k_r} = -\frac{k_r r_0}{\rho m_r V} P, \quad (22)$$

$$\alpha_i = -\frac{k_i k_r r_0}{m_r \rho V} P - \frac{d\overline{w}}{dz} + m_i\left(\frac{\overline{v}}{r_0} - \frac{k_r}{m_r}\overline{w} \right) + k_i \overline{w}\,,$$

$$\alpha_r = \frac{m_r}{r_0}\overline{v} + \overline{w}k_r + \frac{m_r}{\rho r_0 V} P, \quad 6\Omega\left(m_i - \frac{m_r}{k_r} k_i \right) =$$

$$= \alpha_i - \frac{m_i}{r_0}\overline{v} - \overline{w}k_i - \frac{m_r}{k_r r_0}\frac{d\overline{v}}{dz} - \frac{m_i}{\rho r_0 V} P\,.$$

Analysis of (22) shows that $m_r k_i = m_i k_r$ is the trivial solution, which leads to U =0 and identically satisfies the fourth equation reducing the number of equations creating the indefinite solution. Thus, it is excluded ($m_r k_i \neq m_i k_r$). Then follows:

$$P = -\frac{\overline{w}m_r}{k_r r_0}\rho V\,,\ k_i = \frac{1}{\overline{w}}\frac{d\overline{w}}{dz} + \frac{k_r}{m_r} m_i\,,\ \frac{m_r}{\rho r_0 V} P = 0\,,$$

$$\alpha_r = \overline{v}\frac{m_r}{r_0} + k_r \overline{w}\,,\quad \alpha_i = \overline{v}\frac{m_i}{r_0} + k_i \overline{w}\,, \qquad (23)$$

$$\frac{m_r}{k_r}\frac{d\overline{v}}{dz} + \frac{m_i}{\rho V} P = 6 r_0 \Omega\left(\frac{m_r}{k_r} k_i - m_i \right).$$

From equation $m_r P = 0$ follows $m_r = 0$ because $P = 0$ gives also the trivial solution. Then (23) is

$$m_r = 0\,,\ U = m_i V\,,\ W = 0\,,\ \alpha_r = k_r \overline{w}\,,$$
$$\alpha_i = \overline{v}m_i / r_0 + k_i \overline{w}\,,\ P = 0\,,\ k_r m_i = 0\,. \qquad (24)$$

If in (24) $k_r = 0$, then $W = P = 0$, $U = m_i V$, $\alpha_r = 0$, $\alpha_i = \overline{v}m_i / r_0 + k_i \overline{w}$, so that perturbations are only in two interconnected velocity components (by φ and z), which are not oscillating but just growing or decreasing by time and along axis of the turbine: $e^{-(\overline{v}m_i / r_0 + k_i \overline{w})t} e^{-k_i z}$. If $m_i = 0$, then $U = W = P = 0$, $\alpha_r = k_r \overline{w}$, $\alpha_i = k_i \overline{w}$, only rotation velocity has perturbation $e^{ik_r(z - \overline{w}t)} e^{k_i(\overline{w}t - z)}$. A simple sinusoidal kinematic wave $z = \overline{w}t + const$ of length $2\pi / k_r$ spreads along axis with a speed of flow $\overline{w}$.

2.3.2 Effect of Viscosity on the Flow Instability

Solution of (21) with viscous term in third equation to the right (product to the left is small nonzero) is:

$$P = -\frac{\rho k_r r_0 \overline{w} m_r}{k_r^2 r_0^2 + m_r^2} V\,,\ \alpha_r = \frac{m_r}{r_0}\overline{v} + k_r \overline{w} - \frac{m_r^2 k_r \overline{w}}{k_r^2 r_0^2 + m_r^2}\,,$$

$$\alpha_i = \left[k_i \frac{m_r}{k_r} - \frac{2\nu}{\overline{w}}\left(\frac{k_r}{m_r} + \frac{m_r}{k_r r_0^2} \right) \right]\frac{\overline{v}}{r_0} +$$

$$+ k_i \overline{w} + \frac{k_r r_0 \left(3\Omega r_0 - \overline{v} \right)\overline{w}m_r}{\nu\left(k_r^2 r_0^2 + m_r^2 \right)}\,,$$

$$m_i = k_i \frac{m_r}{k_r} - \frac{2\nu}{\overline{w}}\left(\frac{k_r}{m_r} + \frac{m_r}{k_r r_0^2} \right), \qquad (25)$$

$$\left(\frac{k_r^2}{m_r} + \frac{m_r}{r_0^2} \right)^2 - \frac{m_r}{\nu}\frac{d\overline{w}}{dz}\left(\frac{k_r^2}{m_r} + \frac{m_r}{r_0^2} \right) +$$

$$+ \frac{k_r r_0 \left(\overline{v} - 3\Omega r_0 \right) - \nu m_r k_i}{\nu^2 r_0^2} m_r \overline{w} = 0\,,$$

$$\left(\frac{k_r^2}{m_r} + \frac{m_r}{r_0^2} \right)\left[\frac{m_r}{\nu}\frac{d\overline{w}}{dz}\left(1 - \frac{\overline{w}}{12 r_0 \Omega} \right) - \frac{k_r \overline{w}}{6 r_0^2 \Omega} \right]\cdot$$

$$\cdot \frac{12\Omega \nu r_0^2}{m_r \overline{w}\left(12\Omega m_r + k_r \overline{w} \right)} + \frac{k_r r_0}{\nu m_r}\left(3\Omega r_0 - \overline{v} \right) + k_i = 0\,.$$

The equation array (25) thus obtained allows computing the parameters of the perturbations: $U, W, P, m_i, m_r, k_i, \alpha_i, \alpha_r$. Here k_r can be stated according to curvilinear channel, $k_r = 2\pi \cdot 100 = 628$, which means one wave by z on 1 sm distance along the axis. All amplitudes of perturbations are expressed through *V*. The last equations (25) yield

$$k_i = \frac{\nu}{\overline{w}} r_0 \left(\frac{r_0^2 k_r^2}{m_r^2} + 1 \right)^2 - \frac{1}{\overline{w}} \frac{d\overline{w}}{dz} \left(\frac{r_0^2 k_r^2}{m_r^2} + 1 \right) +$$

$$+ \frac{r_0 k_r}{m_r \nu} \left(\overline{v} - 3\Omega r_0 \right), \qquad (26)$$

$$k_i = \left(\frac{r_0^2 k_r^2}{m_r^2} + 1 \right) \left[\frac{k_r \overline{w}}{6 r_0^2 \Omega} - \frac{m_r}{\nu} \frac{d\overline{w}}{dz} \left(1 - \frac{\overline{w}}{12 r_0 \Omega} \right) \right] \cdot$$

$$\frac{12\Omega\nu}{\overline{w}\left(12\Omega m_r + k_r \overline{w}\right)} + \frac{k_r r_0}{\nu m_r} \left(\overline{v} - 3\Omega r_0 \right).$$

The last equation (26) has $1 - \overline{w} / \left(12 r_0 \Omega\right)$ estimated as 1+0.75/(12*6*10^{-2}*10^{2})$\approx$ 1+0.010$\approx$ 1. It yields:

$$k_i = \left(\frac{r_0^2 k_r^2}{m_r^2} + 1 \right) \left(\frac{k_r \overline{w}}{6 r_0^2 \Omega} - \frac{m_r}{\nu} \frac{d\overline{w}}{dz} \right) \cdot$$

$$\cdot \frac{12\Omega\nu}{\overline{w}\left(12\Omega m_r + k_r \overline{w}\right)} + \frac{k_r r_0}{\nu m_r} \left(\overline{v} - 3\Omega r_0 \right), \quad (27)$$

where from with account of the first equation (26):

$$\left(1 + \frac{1}{k_i \overline{w}} \frac{d\overline{w}}{dz} \right) m_r^3 + \left(\frac{\overline{w}}{2} - \frac{\nu}{k_i r_0^2} \right) \frac{k_r}{6\Omega} m_r^2 +$$

$$+ \frac{k_r r_0}{k_i} \left(\frac{k_r r_0}{\overline{w}} \frac{d\overline{w}}{dz} + \frac{k_r r_0 \overline{w}}{4\nu} - \frac{\overline{v}}{\nu} \right) m_r + \qquad (28)$$

$$+ \frac{k_r}{2k_i} \left(\frac{r_0^2 \Omega}{2\nu} - \frac{\nu k_r^2}{3\Omega} - \frac{k_r r_0 \overline{w}\overline{v}}{6\nu\Omega} \right) = 0 .$$

With the estimation above, (28) is simplified to:

$$\left(1 + \frac{1}{k_i \overline{w}} \frac{d\overline{w}}{dz} \right) m_r^3 + \frac{\overline{w} k_r}{12\Omega} m_r^2 + \qquad (29)$$

$$+ \frac{k_r r_0}{k_i \nu} \left(\frac{k_r r_0 \overline{w}}{4} - \overline{v} \right) m_r + \frac{k_r}{4\nu k_i} \left(r_0^2 \Omega - \frac{k_r r_0 \overline{w}\overline{v}}{3\Omega} \right) = 0 .$$

The estimation of the (29) is as follows:

$$\left(1 - \frac{4}{3k_i} \frac{d\overline{w}}{dz} \right) m_r^3 - \frac{5}{8} m_r^2 + \frac{10^6}{k_i} \left(765 - 1760 m_r \right) = 0 ,$$

therefore, the substantially big term is the last one because even $k_i = -100$ results huge growing of the perturbations by *z*. And m_r is supposed to be less than 10, because by high frequency of rotation it is not available high frequency of oscillations by *φ*. If so, then from the above $m_r \approx 0.435$. Thus, the cycles by *φ* are $\varphi_n \approx 4.6\pi n$, so that it is slightly more than 2 total cycles, and the first is 0.31 radian after $\pi / 2$, then after another 4π and $\pi + \pi / 5$, so that it moves after each 4π . From (27) follows

$$k_i = -133 \left(0.35 + 435 \frac{d\overline{w}}{dz} \right) + 138 \cdot 10^6 \left(\overline{v} - 3\Omega r_0 \right).$$

A specific peculiarity of the system is seen: the last term has huge multiplier in front of the small value $\overline{v} - 3\Omega r_0$, because $\overline{v} - 3\Omega r_0 \approx 0$. Flow has not the same velocity as the rotating turbine but close to it. If in a first approach we neglect it, we get

$$k_i = -133 \left(0.35 + 435 d\overline{w} / dz \right).$$

Estimation $d\overline{w} / dz \approx 0$ due to nearly constant axial velocity (on a short distance 0.1m) yields $k_i \approx -47$, which is close to the assumptions made. Due to small viscous forces comparing to the inertial by high rotation, if they are neglected, (21) yields:

$$U = \left(m_i - \frac{m_r}{k_r} k_i \right) V , \quad \frac{m_r}{r_0} \overline{v} + \overline{w} k_r + \frac{m_r}{\rho r_0 V} P = \alpha_r ,$$

$$\left(m_i - \frac{m_r}{k_r} k_i \right) \left(\alpha_i - \frac{m_i}{r_0} \overline{v} - k_i \overline{w} \right) - 2 \frac{\overline{v}}{r_0} = -6\Omega ,$$

$$\frac{m_r}{r_0} \left(\overline{w} - \frac{m_r}{k_r r_0} \overline{v} \right) - \frac{m_r}{r_0} \overline{w} + \alpha_r \frac{m_r}{k_r r_0} = -\frac{k_r}{\rho V} P ,$$

$$\left(m_i - \frac{m_r}{k_r} k_i \right) \left(\overline{v} \frac{m_r}{r_0} + k_r \overline{w} - \alpha_r \right) = 0 , \qquad (30)$$

$$\frac{m_i}{r_0} \left(\frac{m_r}{k_r r_0} \overline{v} - \overline{w} \right) + \frac{m_r}{k_r r_0} \left(k_i \overline{w} - \frac{d\overline{w}}{dz} - \alpha_i \right) = \frac{k_i}{\rho V} P ,$$

$$\alpha_i - \frac{m_i}{r_0} \overline{v} - \overline{w} k_i - \frac{m_r}{k_r r_0} \frac{d\overline{v}}{dz} - \frac{m_i}{\rho r_0 V} P =$$

$$= 6\Omega \left(m_i - \frac{m_r}{k_r} k_i \right), \quad W = -\frac{m_r}{k_r r_0} V .$$

Excluding $m_r k_i = m_i k_r$ as the trivial solution leads to U =0 and identically satisfies the fourth equation of the (30) reducing the number of the equations and thus doing the indefinite solution:

$$W = -\frac{m_i}{k_i r_0} V , \ \frac{d\overline{w}}{dz} = 0 , \ \overline{v} = 3\Omega r_0 , \ m_i = m_r \frac{k_i}{k_r} ,$$

$$U = 0 , \ \alpha_r = \frac{k_r}{k_i^2 r_0^2 + m_i^2} \left(m_i k_i r_0 \overline{v} + k_i^2 r_0^2 \overline{w} + \frac{m_i^3 \overline{v}}{k_i r_0} \right),$$

$$\alpha_i = \overline{w} k_i - m_i^2 \frac{m_i k_i r_0 \overline{v} + k_i^2 r_0^2 \overline{w} + m_i^3 \overline{v}}{\left(k_i^2 r_0^2 + m_i^2 \right) k_i r_0^2} + \frac{m_i}{k_i r_0} \frac{d\overline{v}}{dz} +$$

$$\frac{m_i}{r_0} \overline{v} \left(1 + \frac{m_i^2}{k_i^2 r_0^2} \right), \quad \frac{P}{\rho V} = \frac{m_i}{k_i r_0} \left[\overline{v} \frac{m_i}{k_i r_0} + \right.$$

$$-\frac{1}{k_i^2 r_0^2+m_i^2}\left(m_i k_i r_0 \overline{v}+k_i^2 r_0^2 \overline{w}+\frac{m_i^3 \overline{v}}{k_i r_0}\right)\Bigg].$$

In this case we have arbitrary wave numbers both by z and φ (arbitrary wave lengths). We have only m_i expressed through the other wave parameters and the explicit expressions for the wave increment α_i and their oscillation frequency in time α_r. This corresponds to $\overline{v}=3\Omega r_0$ when the average velocity by φ is coinciding with the same velocity of the rotation. And $d\overline{w}/dz=0$ - constant flow velocity along the axis. Quite specific conditions and substantially indefinite solution of the problem.

Thus, this case is excluded ($m_r k_i \neq m_i k_r$) as before. Then from (30) yields the next solution:

$$\alpha_i=\pm\frac{\sqrt{\overline{v}-3\Omega r_0}}{r_0}\left(2\sqrt{3\Omega r_0}+\frac{\overline{v}}{\sqrt{3\Omega r_0}}\right)+k_i\overline{w},$$

$$\alpha_r=k_r\overline{w},\ U=m_i V,\ W=0,\ P=0, \qquad (31)$$

$$m_r=0,\ \ m_i=\pm\sqrt{\frac{\overline{v}}{3\Omega r_0}-1},\ \ k_i=\frac{1}{\overline{w}}\frac{d\overline{w}}{dz}+$$

$$\pm\frac{1}{r_0}\left(\frac{\overline{v}+\overline{w}}{\overline{w}}\sqrt{\frac{\overline{v}}{3\Omega r_0}-1}+\frac{2}{\overline{w}}\sqrt{3\Omega r_0\left(\overline{v}-3\Omega r_0\right)}\right).$$

Only 2 velocity components (by φ and across the channel) are nonzero. The perturbations are:

$$\left\{\frac{u'}{V},\frac{v'}{V}\right\}=\left\{\pm\sqrt{\frac{\overline{v}-3\Omega r_0}{3\Omega r_0}},1\right\}e^{ik_r(z-\overline{w}t)}e^{k_i(\overline{w}t-z)}, \quad (32)$$

where k_r is the wave number, e.g. by the wavy form of channel 1 sm, k_r=628.

4 Conclusion

The mathematical modeling and analysis revealed a few interesting features of the stability for complex rotational flow due to rotations in two perpendicular directions. For the flow in rotational channel with wavy walls, the nonzero oscillations were got only for the cross sectional and rotation velocities. Both are kinematic waves spreading along the axis of the curvilinear channel with the average flow velocity. And both have the growing forefront $e^{k_i(\overline{w}t-z)}$. The amplitude of oscillations across the channel depends on rotation frequency, while the oscillation amplitude of rotation velocity does not. The revealed features of the flow stability are important for the testing and further application of the new device [2]. Also it is of interest in theoretical development of the fluid flow under double rotations absolutely unknown for the moment.

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