Geometrically Non-Linear Dynamic Behavior of Simply Supported Rectangular Plates Carrying a Concentrated Mass

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Abstract: - Simply supported plates carrying an added point mass are encountered in many engineering fields, like circuit boards or slabs carrying machines at different locations. Determination of the plate modified dynamic characteristics is a quite laborious task, especially in the non-linear regime, which is rarely treated in the literature. The added mass effect on the plate linear parameters was first examined using Hamilton's principle and spectral analysis. The modified plate's non-linear fundamental mode was then calculated and its non-linear response to high levels of harmonic excitation was determined. The non-linear formulation, involving a fourth order tensor due to the membrane forces induced in the plate mid-plane by large vibration amplitudes, led to a non-linear algebraic amplitude equation. The iterative solution gave the free vibration case a better qualitative understanding and a quantitative evaluation of the effect of the added mass. The non-linear forced response of the modified plate, examined for a wide frequency range, shows that the added eccentric mass induces changes in the area between the mass location and the simple supports and decreases the non-linear hardening effect. The numerical results, covering new situations, are expected to be useful in engineering applications necessitating for some reason the addition to the plate of a point mass or an adaptation of the plate frequencies in order to avoid the occurrence of undesirable resonances.

Key-Words: - Bifurcation, Simply supported rectangular plate, Added masse, Non-linear free vibrations, Non-linear forced vibrations, Mode shape.

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1 Introduction

In various industrial fields, a point mass may be added to a structure for given practical reasons. Sometimes, this is done to shift, generally to the left, the natural frequencies and avoid an undesirable resonance. It also happens that the dynamic characteristics of a structural component, like a slab carrying machines at different locations or a pinned circuit board with electronic components, have to be identified using theoretical models involving plates with added point masses.

An added point mass to a plate may significantly and unexpectedly affect its natural frequencies, mode shapes, free and forced linear and non-linear vibrations. As the location of the mass added modifies the mode configurations, it induces a displacement of their nodal lines and unexpected modal participation to the plate response to a given external loading. As will appear through the following review, the previous works on the subject are mainly concerned with linear frequencies and ignore most of the time the above-mentioned effects, especially the non-linear behavior. The purpose of the present investigations was to remedy, in a unified and systematic manner, the lack of information concerning these aspects of the subject. This paper is distinguished by:

1- The derivation of equations: The model is based on Hamilton's principle and a spectral analysis approach using the linear modes of the plate as basic functions in the series expansion. This permits a significant reduction in the number of functions used which appears to be very efficient in the treatment of the geometrically non-linear problem that would be otherwise very difficult to deal with. The latter explains why such papers investigating this aspect are rare.

2- The numerical procedure: The linear analysis made, requires a classical eigenvalue solution procedure. However, the non-linear analysis involves many numerical details mentioned partially in the manuscript, especially in the vicinity of bifurcation points when determining the non-linear frequency response functions.

3- Parametric analysis: In this paper, the purpose was not restricted to the effect of the added mass on the plate frequencies. More details are given concerning the mode shapes, very often omitted in the analyses made, in spite of their crucial importance with respect to the plate dynamic response. Also, the plate modified forced response, due to the mass addition, was qualitatively and quantitatively addressed in both the linear and the non-linear regimes. On the other hand, the method presented, and the associated and the developed software allow treating the problem systematically and in a unified manner.P. A. A. Laura, [1], investigated the vibrations of beams and plates elastically restrained against rotations at the supports with added masses, taking into account the rotational inertia. In a series of papers, K. H. Low and his co-author, [2], [3], and, [4], studied experimentally the vibration of rectangular plates carrying multiple masses at different locations and estimated the equivalent-center weight factor (ECWF). A good agreement was found between experimental and theoretical results in which the change in the strain energy was included. M. Amabili, [5], investigated theoretically Non-linear forced vibrations of rectangular plates carrying a point mass. The Von-Kármán non-linear plate theory was used and the results showed that the presence of the mass amplifies the response at its location and decreases the natural frequency. D. R. Avalos et al, [6], studied the exact solution of a simply supported plate carrying an elastically mounted concentrated mass, using the Dirac delta function. The paper is dealing only with linear vibration which corresponds to small amplitude oscillations. D. R. Avalos et al, [7], treated the same situation cited before with plates with rectangular cutouts only in linear vibration, they used the Rayleigh-Ritz method. H. A. Larrondo et al, [8], treated the transverse vibration of an anisotropic plate with elastically mounted concentrated mass using the Rayleigh-Ritz method focusing on the study of the convergence of the procedure and analyzing the behavior of the plate, a good convergence is achieved as the number of functions is increased from 100 to 900 terms. Z. Zhong et al, [9], investigated the transverse dynamic instability of a rectangular simply supported plate with arbitrary concentrated mass excited by an external distributed in-plane force along two opposite edges using the Von-Kármán large deflection theory, they found that the concentrated mass affects the out-of-plane dynamic instability of the plate. R. H. Gutiérrez and P. A. A. Laura, [10], presented a solution for simply supported and clamped rectangular plates based on an approximate solution of the non-linear partial derivative equation of motion obtained as an extension of an old work of HN. Chu and G. Herrmann, [11]. Y. Kubota et al, [12], discussed the high frequency response of a simply supported plate carrying a point mass under random forces, and an approximate expression has been developed using Asymptotic Modal Analysis. The response of the whole plate except near the added mass attachment point is the same as that of the plate without mass and the local response of the point mass is multiplied by a factor less than unity. Chai Gin Boay, [13], investigated the natural frequencies of plates with various combinations of clamped and simply supported edge conditions with and without added mass. The frequency of a plate with an added mass placed away from the center was not well predicted. T. Mizusawa, [14], dealt with simply supported and clamped skew plate carrying a concentrated mass in different locations in linear vibrations using the spline element method. He found that the natural frequency is affected by the mass ratio and it depends on the mass location and aspect ratio and skew angle, the natural frequency decreases with increasing the mass ratio. J. W. Nicholson and L. A. Bergman, [15], treated the vibration of a simply supported square rectangular thick plate with added mass at the plate center. They gave the natural linear frequencies of a thin and thick plate. In [16] X. Pang et al studied the vibrations of the elastically added masses to a plate by using the non-linear eigenvalue for a new model. In [17] P. Mahadevaswamy and B. S. Suresh treated by experiment the transverse vibrations of a clamped plate by vibratory flap excited harmonically and then compared their results with those based on a finite element analysis. In [18] M. Hamdani et al studied the non-linear free and forced vibration of an SCSC plate with added mass. P.A. Martin and A. J. Hull, [19], dealt with the dynamic response of a thin plate with concentrated masses in a linear regime. Then the results were compared to computations by using the finite element method. D. Wang and M.I. Friswell, [20], analyzed the minimum support stiffness to raise the plate's natural frequency and to get the optimal attachment point, therefore, the minimum related stiffness. In [21] H. Zhaoyang et al presented a new method of linear buckling of a thin rectangular plate side cracked by dividing it into sub-plates. In [22] Z. Xinran et al developed a new method that can be applicable to non- Lévy- type thick plates, the symplectic superposition method is applied to the free vibration of a thick rectangular plate. Hu, Zhaoyang et al, [23], worked with the symplectic space and Hamiltonian-system framework. B. Wang et al, [24], presented, for the first time, the solution of the rectangular thin plate by using the symplectic superposition method-based analytic buckling. In [25] A. Dongqi et al developed a new double finite integral transform method for non-Lévy-type cylindrical shell panels. In [26] M. Hamdani et al treated, in brief, the effect of added centric mass on the free non-linear vibration of a simply supported plate, also the bending stress distributions.

In the present work, a linear analysis modal is first made of simply supported rectangular plates carrying an added mass and the results are compared to the literature. Then, a numerical model is developed for non-linear free vibrations leading to the non-linear mode shapes and associated backbone curves of simply supported plates with no added mass and with an added mass placed at the plate center and at another location. The results were compared when it is possible with the literature. Finally, the non-linear forced vibration of plates subjected to a harmonic point force has been investigated and the results obtained were presented corresponding to plates with no added mass and plates with an added centric or an eccentric mass for a wide range frequency.

2 Theoretical Formulation

Consider the transverse vibrations of an isotropic rectangular plate carrying a point mass m at the point of coordinates (x, y) as shown in Fig. 1. The plate bending strain energy V_b has the following expression, [27]:

$$V_b = \frac{1}{2}\int D\left[\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}\right)^2 + 2(1-\nu)\left(\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 - \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2}\right)\right] dS \quad (1)$$

By neglecting in-plane displacements, the membrane strain energy V_a induced by large vibration amplitudes can be written as follows:

$$V_a = \frac{3D}{2H^2}\int\left[\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right]^2 dS \quad (2)$$

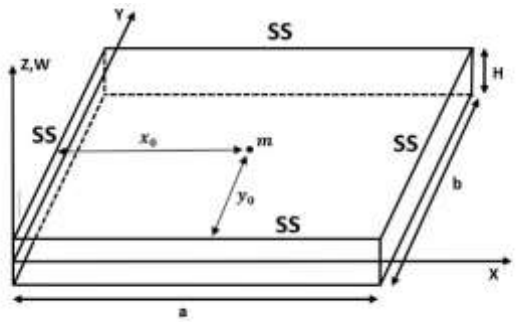

Fig. 1: A simply supported plate with an added mass at (x_0, y_0)

D is the bending stiffness $D = \frac{EH^3}{12(1-\nu^2)}$, $ds = dxdy$ is the elementary surface area, and H is the plate thickness. the total kinetic energy of the plate and the added point masses can be written as:

$$T = \frac{1}{2}\rho H \int\left(\frac{\partial W}{\partial t}\right)^2 dxdy + \frac{1}{2}m\left(\frac{\partial W(x_0,y_0)}{\partial t}\right)^2 \quad (3)$$

The plate transverse displacement W depends on time and space. If these are assumed to be separable and the motion is assumed to be harmonic, one can write:

$$W(x,y,t) = w(x,y)\sin(\omega t) \quad (4)$$

The spatial function $w(x,y)$ is presented as a finite series of N basic functions $w_{ij}(x,y)$:

$$w(x,y) = a_k w_k(x,y) = a_{ij} w_{ij}(\mathrm{x},\mathrm{y}) \quad (5)$$

With $\mathrm{k} = \mathrm{N}(\mathrm{i}-1)+\mathrm{j}$. The summation convention is used in which i and j are summed over $1,2,\ldots,\mathrm{N}$ with N representing the number of functions. The functions $w_{ij}(\mathrm{x},\mathrm{y})$ are obtained as a product of simply supported-simply supported beam functions $f_k(x)$ in the x and y directions:

$$w_{ij}(x,y) = f_i(x) f_j(y) \quad (6)$$

The bending strain, the membrane strain and the kinetic energy expressions become after discretization:

$$V_b = \frac{1}{2}a_i a_j k_{ij}\sin^2(\omega t) \quad (7)$$

$$V_a = \frac{1}{2}a_i a_j a_k a_l b_{ijkl}\sin^4(\omega t) \quad (8)$$

$$T = \frac{1}{2}\omega^2 a_i a_j m_{ij}\cos^2(\omega t) \quad (9)$$

k_{ij}, b_{ijkl} and m_{ij} are the rigidity, the geometrical non-linear rigidity and the mass tensors respectively. Their expressions are:

$$k_{ij} = \int D\left[\left(\frac{\partial^2 w_i}{\partial x^2}+\frac{\partial^2 w_i}{\partial y^2}\right)\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{\partial^2 w_j}{\partial y^2}\right) + 2(1-\nu)\left(\frac{\partial^2 W_i}{\partial x\partial y}\frac{\partial^2 W_j}{\partial x\partial y} - \frac{\partial^2 W_i}{\partial x^2}\frac{\partial^2 W_j}{\partial y^2}\right)\right] dxdy \quad (10)$$

$$b_{ijkl} = \frac{3D}{H^2}\int\left(\frac{\partial w_i}{\partial x}\frac{\partial w_j}{\partial x} + \frac{\partial w_i}{\partial y}\frac{\partial w_j}{\partial y}\right)\left(\frac{\partial w_k}{\partial x}\frac{\partial w_l}{\partial x} + \frac{\partial w_k}{\partial y}\frac{\partial w_l}{\partial y}\right) dxdy \quad (11)$$

$$m_{ij} = \rho H\int w_i w_j\, dxdy + m w_i(x_0,y_0) w_j(x_0,y_0) \quad (12)$$

In a non-dimensional form, we put:

$$w_i(x,y) = H{w^*}_i\left(\frac{x}{a},\frac{y}{b}\right) = H{w^*}_i(x^*,y^*) \quad (13)$$

a and b are the plate length and width along the x and y directions respectively. Non-dimensional tensors can be defined as follows:

$${b^*}_{ijkl} = 3\int\left(\alpha^2\frac{\partial {w^*}_i}{\partial x^*}\frac{\partial {w^*}_j}{\partial x^*} + \frac{\partial {w^*}_i}{\partial y^*}\frac{\partial {w^*}_j}{\partial y^*}\right)\left(\alpha^2\frac{\partial {w^*}_k}{\partial x^*}\frac{\partial {w^*}_l}{\partial x^*} + \frac{\partial {w^*}_k}{\partial y^*}\frac{\partial {w^*}_l}{\partial y^*}\right) dx^* dy^* \quad (14)$$

$${k^*}_{ij} = \int\left(\alpha^2\frac{\partial^2 {w^*}_i}{\partial {x^*}^2} + \frac{\partial^2 {w^*}_i}{\partial {y^*}^2}\right)\left(\alpha^2\frac{\partial^2 {w^*}_j}{\partial {x^*}^2} + \frac{\partial^2 {w^*}_j}{\partial {y^*}^2}\right) + 2(1-\nu)\alpha^2\left(\frac{\partial^2 {w^*}_i}{\partial x\partial y}\frac{\partial^2 {w^*}_j}{\partial x\partial y} - \frac{\partial^2 {w^*}_i}{\partial x^2}\frac{\partial^2 {w^*}_j}{\partial y^2}\right) dx^* dy^* \quad (15)$$

$$m^*_{ij} = \int w^*_i w^*_j \, dx^* dy^* + \eta w^*_i(x^*_0, y^*_0) w^*_j(x^*_0, y^*_0) \quad (16)$$

η presents the ratio of the mass added to the plate total mass $\eta = \frac{m}{\rho H a b}$ and α is the plate aspect ratio $\alpha = \frac{b}{a}$. The non-dimensional and dimensional tensors are related by:

$$b_{ijkl} = \frac{DaH^2}{b^3} b^*_{ijkl} \quad (a)$$

$$k_{ij} = \frac{DaH^2}{b^3} k^*_{ij} \quad (b)$$

$$m_{ij} = \rho H^3 a b m^*_{ij} \quad (c)$$

The plate motion is governed by Hamilton's principle symbolically written as:

$$\delta \int_0^{2\pi/\omega} (V - T) dt = 0 \quad (17)$$

Where δ indicates the variation of the integral. V and T are the plate's total strain and kinetic energies. This leads to the following set of n non-linear algebraic equations:

$$3a_i a_j a_k b^*_{ijkr} + 2a_i k^*_{ir} - 2\omega^{*2} a_i m^*_{ir} = 0 \quad r = 1, .., n \quad (18)$$

which can be written in a matrix form as:

$$3[B^*(A)]\{A\} + 2[K^*]\{A\} - 2\omega^{*2}[M^*]\{A\} = \{0\} \quad (19)$$

premultiplying the last equation by $\{A\}^T$, the expression of ω^{*2} is obtained:

$$\omega^{*2} = \frac{\{A\}^T[K^*]\{A\} + \frac{3}{2}\{A\}^T[B^*(A)]\{A\}}{\{A\}^T[M^*]\{A\}} \quad (20)$$

in a tensorial form:

$$\omega^{*2} = \frac{a_i a_j k^*_{ij} + \frac{3}{2} a_i a_j a_k a_l b^*_{ijkl}}{a_i a_j m^*_{ij}} \quad (21)$$

to the dimensional way:

$$\omega^2 = \frac{D}{\rho h b^4} \omega^{*2} \quad (22)$$

Substituting equation (21) into the non-linear algebraic systems (18) gives:

$$3a_i a_j a_k b^*_{ijkr} + 2a_i k^*_{ir} - 2 \frac{a_i a_j k_{ij}{}^* + \frac{3}{2} a_i a_j a_k a_l b_{ijkl}{}^*}{a_i a_j m_{ij}{}^*} a_i m^*_{ir} = 0 \quad r = 2, .., n \quad (23)$$

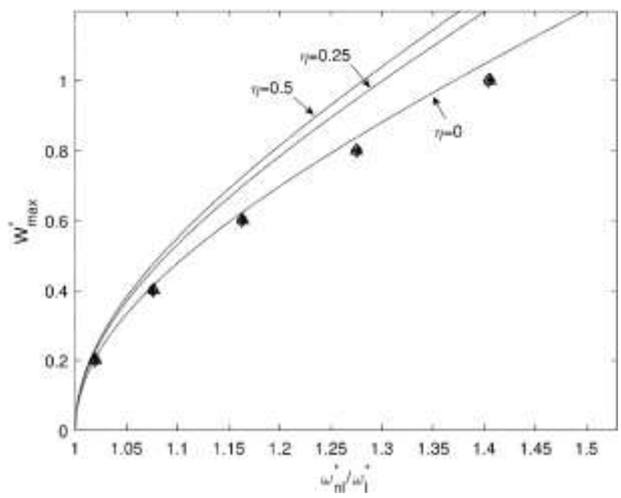

Fig. 2: Presentation of values of Table 2, Table 3, Table 4 in case of $\alpha = 1$, results of R. H. Gutiérrez and P. A. A. Laura [10] are presented by symbols: ($\diamond$) in case of without mass, (Δ) for $\eta = 0.25$ and ($*$) for $\eta = 0.5$. The results of the present work are presented by a line

The non-linear algebraic system (23) has been solved numerically using a Harwell Library routine called NS01A, [28], based on an iterative procedure, involving a combination of the steepest descent and Newton's method.

Considering the forced response, with a concentrated harmonic excitation force, the forcing term has to be added to the right-hand side of equation (18), which leads to:

$$\frac{3}{2} a_i a_j a_k b^*_{ijkr} + a_i k^*_{ir} - \omega^{*2} a_i m^*_{ir} = f^*_r \quad r = 1, .., n \quad (24)$$

f^*_r $(r = 1 \dots n)$, are the dimensionless generalized forces, whose expressions, for a concentrated force F applied at the point of coordinates (x^*_1, y^*_1):

$$f^*_r = \frac{b^3 \mathrm{F}}{aDH} w^*_r(x^*_1, y^*_1) \quad (25)$$

The n non-linear algebraic equations with n unknowns (24) have been solved by the NS01A routine, [28]. The method relies on fixing the non-dimensional excitation frequency ω^{*2} and then giving an initial estimate for the n contributions $(a_1, a_2, .., a_n)$. The solution obtained is taken as a new initial estimate for the following step corresponding to an excitation frequency $\omega^* + \Delta\omega^*$. This process is repeated until the desired frequency segment is covered. It is worth observing here that the routing may diverge when passing through the bifurcation point because the given estimate is too far from the nearest solution, [18].

3 Numerical Results and Discussion

The plate vibration is determined by the edge conditions and the mass distribution all along the plate. In this study, the plate is simply supported and a single mass is placed either at its center or out of the center. The effects of the added mass on the plate's linear and non-linear free and forced vibrations are examined.

The first part of this section is devoted to linear analysis. The linear frequencies and mode shapes of the plates examined, necessary to tackle the non-linear problem, are obtained by solving the eigenvalue equation (26) in which the effect of the geometrical non-linearity is neglected and the mass and the rigidity tensors are calculated before the equation is solved using Matlab Software.

$$a_i k^*_{ir} - \omega^{*2} a_i m^*_{ir} = 0 \quad r = 1,..,n \qquad (26)$$

Various values of the fundamental frequencies obtained here are compared with results found in the literature in Table 1. The plates concerned carrying a centric mass have aspect ratios $\alpha = 1,0.25,0.5,0.75,1$ and mass ratios $\eta = 0,0.25,0.5,1$. Comparison is made with the results of R. H. Gutiérrez and P. A. A. Laura, [10], T. Mizusawa, [14], J.W. Nicholson and L.A. Bergman, [15], and W. Soedel, [29]. A very good agreement is found in most cases.

The added mass at the plate center deforms the plate's fundamental mode shape, mainly at the region near the plate center, and this effect increases and becomes clearer with increasing the mass ratio η or decreasing the plate aspect ratio α. In the case of an eccentric added mass, it can be noticed that the maximum mode is displaced towards the added mass location and this phenomenon can be observed for small plate aspect ratios or big added mass ratios.

Considering non-linear free vibrations, we replace the right-hand side of the equation (23) by g_r, then the residual is calculated using the following formulation:

$$Res = \sqrt{\sum_{r=1}^{n} {g_r}^2} \qquad (27)$$

The routine needs a first estimation close enough to the real solution. The first estimation has been taken from the linear solution and the residual was fixed at $Res = 10^{-27}$ which is a strong convergence test.

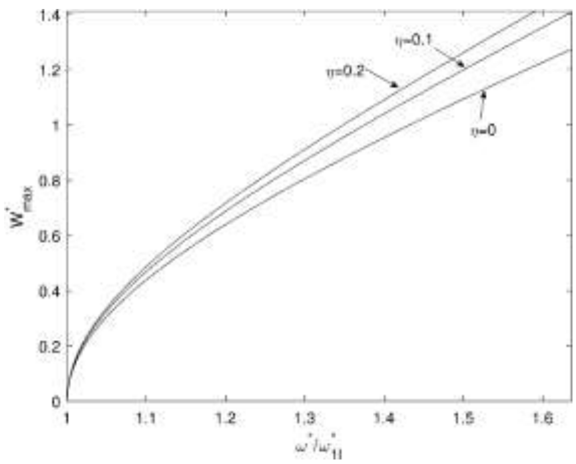

Fig. 3: the backbone curve of a plate $\alpha = 0.6$ carrying a centric mass

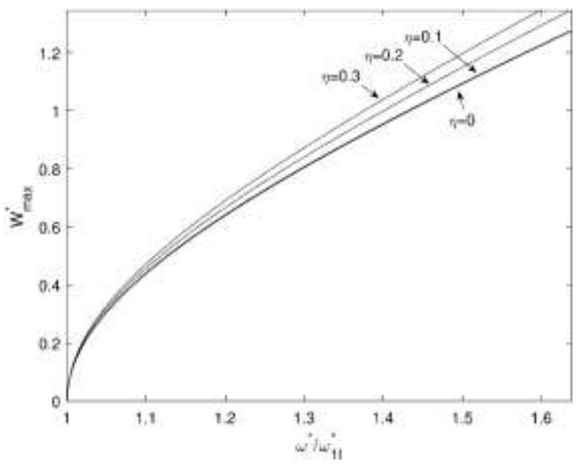

Fig. 4: the backbone curve of a plate $\alpha = 0.6$ carrying an added mass at (0.25,0.5)

The only data found in the literature, corresponding to the non-linear free vibration of simply supported plates with a centric added mass, are those of Gutiérrez and P. A. A. Laura, [10]. Their results are compared with the present ones in Table 2, Table 3, Table 4 giving the dependence of the non-linear to linear frequency ratio on the non-dimensional vibration maximum amplitude W_{max}, for different plate aspect ratios $\alpha = 1,0.75,0.5,0.25$ with no added mass and with an added mass $\eta = 0.25$ and $\eta = 0.5$ in Table 2, Table 3, Table 4 respectively.

Table 1. Comparison of the fundamental non-dimensional frequency of the first mode, (1) R. H. Gutiérrez and P. A. A. Laura, [10], (2) T. Mizusawa, [14], (3) J.W. Nicholson and L.A. Bergman, [15], (4) W. Soedel, [29].

η	α	Present work	(1)	(2)	(3)	(4)
0	1	19.74	19.74	19.74		
	0.75	15.42	15.42			
	0.5	12.34	12.33			
	0.25	10.49	10.48			
0.25	1	13.74	13.95	13.74	13.37	13.96
	0.75	10.71	10.90			
	0.5	8.44	8.72			
	0.25	6.56	7.41			
0.75	1	11.09	11.39	11.09		11.40
	0.75	8.63	8.90			
	0.5	6.74	7.12			
	0.25	5.05	6.05			
1	1	8.49		8.49		8.83

Table 2. Comparison of the non-dimensional frequency ratio $(\omega_{nl}{}^*/\omega_l{}^*)$ of a rectangular plate without added mass for different aspect ratios during multiple non-dimensional maximum amplitudes. (1): results of R. H. Gutiérrez and P. A. A. Laura, [10], (2) present work

	$\alpha = 1$		$\alpha = 0.75$		$\alpha = 0.5$		$\alpha = 0.25$	
$W_{max}{}^*$	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
0	1	1	1	1	1	1	1	1
0.2	1.019	1.019	1.020	1.019	1.024	1.024	1.030	1.030
0.4	1.076	1.072	1.080	1.076	1.093	1.092	1.113	1.119
0.6	1.163	1.155	1.171	1.163	1.198	1.196	1.237	1.264
0.8	1.275	1.255	1.287	1.274	1.330	1.329	1.391	1.439
1	1.404	1.372	1.422	1.395	1.482	1.479	1.567	1.623

Table 3. Comparison of the non-dimensional frequency ratio $(\omega_{nl}{}^*/\omega_l{}^*)$ of a rectangular plate with added mass $\eta = 0.25$ at the center for different aspect ratios during multiple non-dimensional maximum amplitudes. (1): results of R. H. Gutiérrez and P. A. A. Laura, [10], (2) present work

	$\alpha = 1$		$\alpha = 0.75$		$\alpha = 0.5$		$\alpha = 0.25$	
$W_{max}{}^*$	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
0	1	1	1	1	1	1	1	1
0.2	1.020	1.015	1.021	1.016	1.024	1.019	1.030	1.020
0.4	1.077	1.059	1.080	1.062	1.093	1.072	1.112	1.081
0.6	1.164	1.125	1.171	1.162	1.197	1.153	1.236	1.168
0.8	1.275	1.207	1.288	1.220	1.330	1.251	1.391	1.284
1	1.406	1.305	1.422	1.324	1.482	1.360	1.567	1.409

Table 4. Comparison of the non-dimensional frequency ratio $(\omega_{nl}{}^*/\omega_l{}^*)$ of a rectangular plate with added mass $\eta = 0.5$ at the center for different aspect ratios during multiple non-dimensional maximum amplitudes. (1): results of R. H. Gutiérrez and P. A. A. Laura, [10], (2) present work

	$\alpha = 1$		$\alpha = 0.75$		$\alpha = 0.5$		$\alpha = 0.25$	
$W_{max}{}^*$	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
0	1	1	1	1	1	1	1	1
0.2	1.020	1.015	1.020	1.015	1.024	1.017	1.030	1.019
0.4	1.076	1.056	1.080	1.058	1.093	1.068	1.112	1.075
0.6	1.164	1.119	1.171	1.124	1.198	1.142	1.236	1.159
0.8	1.276	1.196	1.288	1.205	1.330	1.231	1.392	1.264
1	1.406	1.285	1.422	1.296	1.482	1.335	1.567	1.389

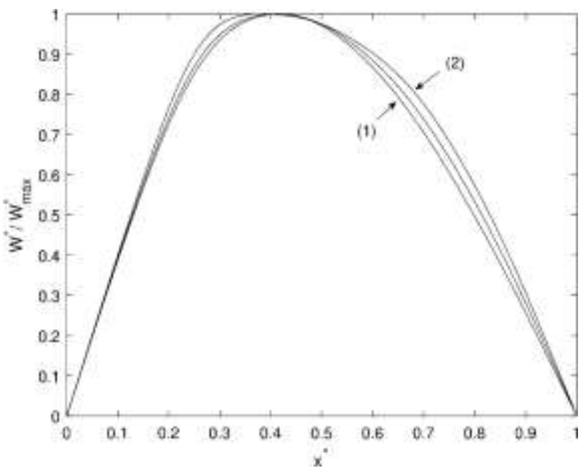

Fig. 5: Comparison of the first mode shape in large amplitude of a simply supported rectangular plate $\alpha = 0.6$ carrying an eccentric mass $\eta = 0.1$ at (0.25,0.5), (1) lowest amplitude, (2) highest amplitude

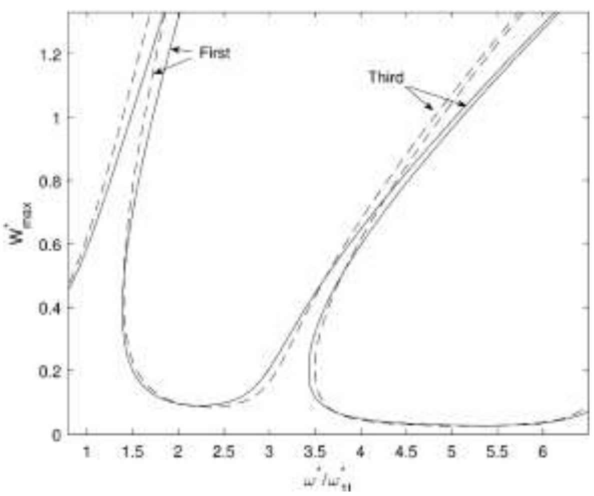

Fig. 6: The frequency response of a plate $\alpha = 0.6$ subjected to a harmonic concentrated force $F = 1N$ applied at the plate center. The continuous line presents the frequency response of a simply supported plate with no added mass. The discrete line presents the frequency response of a simply supported plate carrying an added centric mass with added mass ratio $\eta = 0.1$

According to the results of R. H. Gutiérrez and P. A. A. Laura, [10], the added centric mass does not affect on the non-linearity in simply supported plates, in contrast with the present work which showed that the added centric mass, as may be expected, decreases the non-linearity, as can be seen in Fig. 2 presenting the backbone curves of a square plate without added mass and with an added centric mass $\eta = 0.25$,0.5. Also, Fig.3 and Fig.4 give the backbone curve for a simply supported rectangular plate $\alpha = 0.6$ with an added centric and eccentric mass placed at (0.25,0.5) respectively.

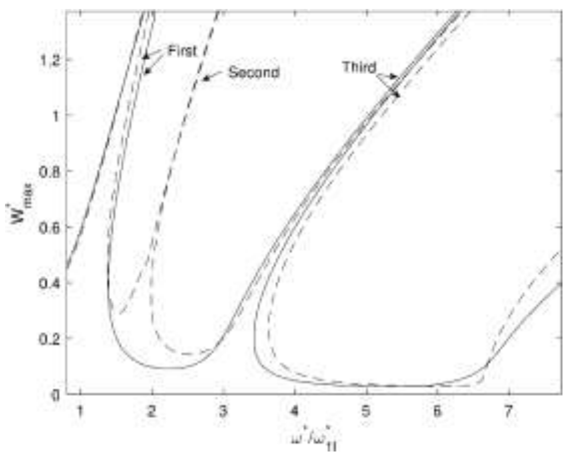

Fig. 7: The frequency response of a plate $\alpha = 0.6$ subjected to a harmonic concentrated force $F = 1N$ applied at the plate center. The continuous line presents the frequency response of a simply supported plate with no added mass. The discrete line presents the frequency response of a simply supported plate carrying an added eccentric mass at (0.25,0.5) with added mass ratio $\eta = 0.1$

It is clear that in Fig. 4 for a mass ratio $\eta = 0.1$, the non-linearity is slightly affected while for mass ratios $\eta = 0.2$,0.3, the effect on non-linearity is more significant. In general, a mass added to a simply supported rectangular plate at large amplitudes tends to reduce the hardening type of non-linearity by increasing the added mass ratio.

Fig. 5 presents the normalized mode shape of a rectangular plate $\alpha = 0.6$ carrying an added mass at (0.25,0.5) along the line $y^* = 0.5$ at large amplitudes, the mode shape is changed with a rise in the mode shape curvature y^* in the area between the mass location and the simple supports, due to the nature of the edge conditions.
The response of a simply supported plate to a harmonic excitation is discussed in what follows. Equation (24) is a set of n non-linear equations with n unknowns which are the contributions $(a_1, a_2, .., a_n)$. The mechanical characteristics are taken in all examples as follows: $a = 0.25m$, $b = 0.15m$, $h = 0.0005m$, $\rho = 7850kg/m^3$, $E = 198.10^9\,Pa$ and $\nu = 0.3$. By giving an initial estimation of the n contributions $(a_1, a_2, .., a_n)$ for ω^{*2}, then implemented into the non-linear algebraic equation (24), the subroutine NS01A, [28], is used to solve the equation numerically. The new contributions are used as an estimate for $\omega^* + \Delta\omega^*$. This process is repeated until the desired segment is achieved. Fig. 6 presents the frequency response of a simply supported plate with an aspect ratio $\alpha =$

0.6 with no mass and with an excited by a centric force, also decreases the hardening type of nonlinearity. Similarly, Fig. 7 presents the frequency response of a plate α = 0.6 subjected to a harmonic concentrated force F = 1 applied at the plate center. The continuous line presents the frequency response of a simply supported plate with no added mass. The discrete line presents the frequency response of a simply supported plate carrying an added eccentric mass at (0.25,0.5) with added mass ratio n = 0.1.

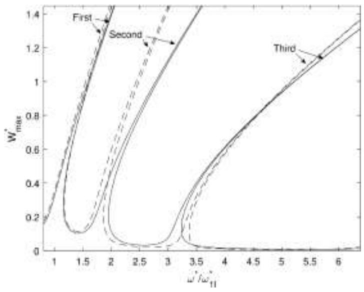

Fig. 8: The frequency response of a plate $\alpha = 0.6$ subjected to a harmonic concentrated force $F = 0.3N$ applied at $(0.25{,}0.5)$. The continuous line presents the frequency response of a simply supported plate with no added mass. The discrete line presents the frequency response of a simply supported plate carrying an added eccentric mass at $(0.25{,}0.5)$ with added mass ratio $\eta = 0.1$

Fig. 8 shows the frequency response of a simply supported rectangular plate with aspect ratio $\alpha = 0.6$ carrying a point mass at $(0.25{,}0.5)$ with $\eta = 0.1$ separately subjected to a point harmonic excitation $F = 0.3N$ at $(0.25{,}0.5)$. It shows that the added mass decreases also the hardening type of non-linearity in the neighborhood of the first, second and third modes. The new method used here showed up again a great accuracy in studying a different condition limit with an external load.

4 Conclusion

Linear and non-linear free and forced vibrations of a simply supported plate carrying a point mass have been analyzed and compared with the results available in the literature. Consequently, a good agreement has been remarked on for the linear frequencies. Firstly, the added centric mass concentrates the deformation at the plate center but in the case of an eccentric mass, it has been remarked that the maximum mode was displaced from the plate center towards the mass location. Secondly, in non-linear vibrations, an increase in the curvatures of a simply supported plate carrying an added centric mass appears in the smallest area between the mass location and the simply supported edges. Finally, the backbone curves show that the presence of the added mass decreases the hardening type of non-linearity and activates the response of certain modes to concentrated harmonic excitations.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

-Mustapha Hamdani developed the theory, performed the computations, analysed the data and wrote the manuscript.
-Mounia El Kadiri contributed to the design and implementation of the research.
-Rhali Benamar verified the numerical results, aided in interpreting and analysis of the results and worked on the manuscript.

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