Propagation of Non-stationary Skew-Symmetric Waves from a Spherical Cavity in a Porous-elastic Half-space

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Abstract: - Problems of propagation and diffraction of non-stationary waves in porous-elastic mediums are of great theoretical and practical importance in such fields of science and technology as geophysics, seismic exploration of minerals, seismic resistance of structures, and many others. The work considers the problem of propagation of non-stationary skew-symmetric waves from a spherical cavity in a porous-elastic half-space saturated with liquid. To solve the problem, the integral Laplace transform in dimensionless time and the method of incomplete separation of variables were used. In the space of Laplace images in time, known and unknown functions are expanded into Gegenbauer polynomials. The problem is reduced to solving an infinite system of linear algebraic equations, the solution of which is sought in the form of an infinite exponential series. Recurrence relations for the coefficients of the series and initial conditions for them are obtained, which makes it possible to obtain a solution to the infinite system without using the reduction method. Recurrence relations make it possible to determine the coefficients of a series in the form of rational functions, which makes it possible to calculate their originals using the theory of residues. In image space, formulas are obtained for the coefficients of the series of components of the displacement vector and stress tensor. Numerical experiments were carried out, the results of which are presented in the form of graphs. The results obtained can be used in geophysics, seismology, and design organizations during the construction of structures, as well as in the design of underground reservoirs.

Key-Words: - propagation of shear wave, spherical cavity, Gegenbauer polynomials, Laplace transform, residues, unsteady wave, porous-elastic medium, stress, displacement.

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1 Introduction

The study of non-stationary wave processes in continuous media is a complex and, at the same time, relevant direction in the wave dynamics of continuous media. The relevance of the problems of continuum dynamics is due to the development of various fields of technology, the creation of new structures operating under dynamic loads, as well as problems of geophysics, seismology, gas exploration, oil exploration, mining industry, and construction of civil and industrial structures. Currently, there is a large number of scientific works devoted to the study of wave propagation and diffraction in continuous media. The existence of a contact layer of soil during shear interaction of a solid body with soil is demonstrated based on the analysis of the numerical solution of a one-dimensional non-stationary problem of the interaction of a rigid strip with a nonlinearly deformable soil medium, [1]. The appearance of a peak value of shear stress and subsequent structural destruction on the contact layer of soil is shown, which is consistent with the experimental results. Solutions to the problem of constant-velocity expansion of a spherical cavity in a soil medium are analyzed: the cavity expands from a point in the half-space occupied by an elastic-plastic soil medium, [2]. The previously obtained linearized analytical solution of this problem is presented, obtained under the assumption that the medium behind the shock wave front is incompressible. As a result of comparison with the results of the numerical solution of the problem in the full formulation, it is shown that the approximate solution is a good approximation of the dependence of the pressure at the boundary of the cavity on the rate of its expansion. The propagation of non-stationary transverse waves from a spherical inclusion in an elastic half-space was studied in the study [3]. Formulas for the components of the displacement vector and stress tensor are obtained. The stress-strain state of the medium in the vicinity of a spherical inclusion has been studied. An exact solution to the problem of joint (coupled) seismic vibrations of an underground pipeline and an infinite elastic medium is given in the article [4]. Based on the established theorem on the separation of boundary conditions for wave potentials on the surface of a cylinder, a method is proposed that significantly simplifies the solution of the external problem for the medium. In [5], an exact analytical solution to the problem of the dynamic expansion of a spherical cavity in an elastic medium (soil) with an arbitrary constant speed was obtained. The solution found makes it possible to judge the impact (or control the impact) of underground explosions on objects in the "far" zone, at distances significantly exceeding the size of the cavity. Article [6] presents a unified mathematical approach to describing the dynamic stress-strain state of mechanical structures made of heterogeneous materials with a double connected system of pore channels filled with fluid. New dynamic equations have been obtained that describe the vibrations of poroelastic systems based on the developed model of a continuous medium with additional degrees of freedom in the form of different pressures of the components that make up the liquid phase of the material. In [7], the propagation of non-stationary longitudinal waves from a spherical cavity supported by a thin spherical shell in an elastic-porous space saturated with liquid was studied. An analytical solution to the problem in the image space of the Laplace transform is obtained. Numerical results are presented in the form of graphs. The article [8] considered the problem of non-stationary transverse oscillations of an elastic half-space with a rigid ball. To solve the problem, the Laplace integral transform and the method of incomplete separation of variables were used. Formulas for the components of the displacement vector and stress tensor are obtained. The paper [9] studies the interaction of spherical elastic *SH* waves of harmonic type with a spherical layer when the source is placed outside the layer. The exact solution to this scattering problem is studied in detail after establishing the generalized Debye expansion. The problem of the diffraction of plane waves by a system of two concentric spherical shells surrounded by acoustic media was studied in the article [10]. In this case, the solution is represented in the form of a superposition of elementary waves. The study [11] considered the problem of the propagation of shear disturbances from a spherical cavity in an infinite elastic medium. In this case, the Fourier transform in time was used. Expressions are obtained for the displacement and stress over time caused by an axially symmetric shear stress applied to the inner surface of a spherical cavity in an infinite isotropic medium. In [12], the problems of scattering and diffraction of a cylindrical transverse shear wave in a viscoelastic isotropic medium by a spherical inhomogeneity are solved analytically. The waves are generated by harmonic longitudinal vibrations of the cylinder walls. The spherical inclusion is located in the radial center of the cylinder and differs from the cylindrical material only in its complex shear modulus. Numerical examples are given to show the effect of changes in inclusion rigidity on displacement fields. The paper [13] addresses the problem of transient elastodynamics analysis of a thick-walled, fluid-filled spherical shell embedded in an elastic medium with an analytical approach. Various constitutive relations for the elastic medium, shell material, and fill fluid are considered, as well as various excitation sources (including S/P wave or plane/spherical wave incident at different locations). The statements and solutions of parabolic problems modeling the physical phenomena in soils in the case of discontinuous velocity on the boundaries at the initial time are given in [14]. The notion of generalized vorticity diffusion is introduced and the cases of self-similarity existence are classified. In the case of a physically linear medium, new self-similar solutions are obtained which describe the process of unsteady axially symmetric shear in spherical coordinates. A semi-infinite circular cylindrical cavity filled with an ideal compressible liquid that contains a spherical body located near its face is considered in [15]. A plain acoustic wave propagates along the axis of the cavity. The problem of determining of hydrodynamic characteristics of the system depending on the frequency of the plane wave and geometrical parameters is solved. The developed method can detect the anomalous features of the diffraction of a plane wave due to the influence of the face wall. This study [16] considers the propagation of harmonic plane waves in a double-porosity solid saturated by a viscous fluid. Two different porosities are supported with different permeabilities to facilitate the wave-induced fluid flow in this composite material. Relevant equations of motion are solved to explain the propagation of three longitudinal waves and one transverse wave in the double-porosity dual-permeability medium. A numerical example is considered to illustrate dispersion in velocity and attenuation of the four waves. The effect of wave-induced fluid flow is analysed with changes in wave-inhomogeneity, pore-fluid viscosity, and double-porosity structure. In paper [17] slow motion of a porous spherical shell with radially varying permeability in a spherical container at the instant it passes through the center of the spherical container is discussed. The exact solution of the problem is obtained. The influence of the permeability parameter on the flow has been discussed and exhibited graphically. A statement of the problem of determination of the acoustic radiation force acting on the rigid spherical particle is formulated in the study [18]. The particle is located in the fluid-filled thin flexible tube The problem is solved by the use of the method of separation of variables. The characteristics of the acoustic radiation force are studied depending on the primary wave frequency, the radius of the rigid spherical particle, the radius of the compliant cylindrical elastic tube, the properties of the filling liquid, the thickness of the tube wall, and elastic properties of the tube material.

This work is devoted to the study of problems on the propagation of non-stationary skew-symmetric waves from a spherical cavity in a porous-elastic half-space saturated with liquid. The purpose of the work is to develop an algorithm for solving the problem and study non-stationary wave processes during the propagation of non-stationary skew-symmetric waves from a spherical cavity in a porous-elastic saturated half-space.

2 Problem Formulation

Let the centre O of a spherical cavity of radius R ($R < h$) be located in a saturated porous-elastic half-space $z \geq 0$ at a depth h from the plane $z = 0$ on the axis $O_2 z$ (the point O_2 lies on the boundary of the half-space) (Figure 1). We will consider two coordinate systems: spherical r, θ, ϑ with the centre at the point O and cylindrical ρ, ϑ, z with the origin at the point O_2.

At a moment $\tau = 0$ in time, an axisymmetric specified tangential surface load $q(\tau, \theta)$ is applied to the surface of the cavity, which forms a rotational motion of the medium around an axis passing through the center of the cavity (Figure 1).

$$\sigma_{r\vartheta}\big|_{r=R} = q(\tau, \theta). \tag{1}$$

Taking into account the axial symmetry of the problem, the motion of the medium relative to the non-zero component ψ of the vector displacement potential is described by the wave equation

$$\gamma^2 \ddot{\psi} = \Delta\psi - \frac{\psi}{r^2 \sin^2 \theta}. \tag{2}$$

Here Δ is the Laplace operator in a spherical coordinate system r, θ, ϑ.

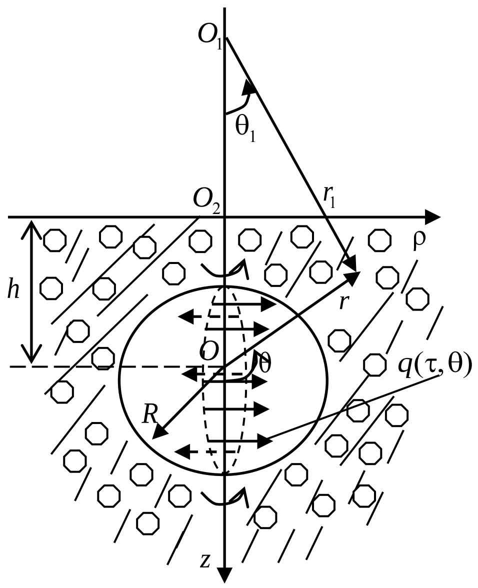

Fig. 1: Geometry of the problem

The flat boundary of a half-space is either a rigid wall:

$$u_\vartheta\big|_{z=0} = 0, \tag{3}$$

or free surface

$$\sigma_{z\vartheta}\big|_{z=0} = 0. \tag{4}$$

Initial conditions are homogeneous

$$\psi\big|_{\tau=0} = \dot{\psi}\big|_{\tau=0} = 0 \tag{5}$$

and the problem closes with the requirement that the solution is bounded at infinity

$$\lim_{r\to\infty} \psi = 0. \tag{6}$$

In this case, the functions u_ϑ, U_ϑ, $\sigma_{r\vartheta}$ and $\sigma_{\theta\vartheta}$ are connected by the following relations:

$$u_\vartheta = \frac{\partial\psi}{\partial r} + \frac{\psi}{r}, \qquad U_\vartheta = \beta u_\vartheta \tag{7}$$

$$\sigma_{r\vartheta} = \frac{\eta - \chi}{2}\left(\frac{\partial u_\vartheta}{\partial r} - \frac{u_\vartheta}{r}\right),$$

$$\sigma_{\theta\vartheta} = \frac{\eta - \chi}{2r}\left(\frac{\partial u_\vartheta}{\partial \theta} - u_\vartheta ctg\theta\right). \tag{8}$$

The problem statement is given in the following dimensionless quantities (the prime indicates a dimensionless quantity)

$$r' = \frac{r}{R},\ \tau = \frac{c_2 t}{R},\ \psi' = \frac{\psi}{R^2},\ \gamma = \frac{c_2}{c_2^*},\ u'_\vartheta = \frac{u_\vartheta}{R},$$

$$\sigma'_{\alpha\beta} = \frac{\sigma_{\alpha\beta}}{H},\ \beta = -\rho_{12}/\rho_{22},\ \chi = \frac{A}{H},\ \eta = \frac{P}{H},$$

$$H = P + 2Q + R,\ P = A + 2N,$$

where u_ϑ and U_ϑ are components of the displacement vector of the skeleton and fluid, respectively; $\sigma_{\alpha\beta}$ $(\alpha,\beta=r,\theta,\vartheta)$ are stress tensor components; A and N are the elastic constants of the medium's skeleton; R is pressure applied to the liquid; Q is the amount of adhesion between solid and liquid components; ρ_{12} is coefficient of dynamic coupling between solid and liquid components; ρ_{22} is the effective mass of the fluid during its relative motion; c_2 is the speed of propagation of transverse waves in the medium (below, the prime in the designations of dimensionless quantities is omitted).

3 Problem Solution

The initial-boundary value problem (1)-(6) is solved using the integral Laplace transform in time τ and the method of incomplete separation of variables. In the image space, we represent the potential ψ^L, components u_ϑ^L displacement vector and $\sigma_{r\vartheta}^L$ stress tensor in the form of infinite series of Gegenbauer polynomials $C_{n-1}^{3/2}(\cos\theta)$, [19], and the representation of the infinite series for the stress tensor component $\sigma_{\theta\vartheta}^{(l)L}$ has the following form (L denotes the transformant of the Laplace transform, s is transformation parameter), [20]

$$\sigma_{\theta\vartheta}^L=\sum_{n=1}^{\infty}\sigma_{\theta\vartheta n}^{(1)L}(r,s)P_n(\cos\theta)+$$
$$+\cos\theta\sum_{n=1}^{\infty}\sigma_{\theta\vartheta n}^{(2)L}(r,s)C_{n-1}^{3/2}(\cos\theta), \quad (9)$$
$$\sigma_{\theta\vartheta n}^{(1)L}=-\frac{n(n+1)(\eta-\chi)}{2r}u_{\vartheta n}^L, \qquad \sigma_{\theta\vartheta n}^{(2)L}=\frac{\eta-\chi}{r}u_{\vartheta n}^L,$$

where $P_n(\cos\theta)$ are the Legendre polynomials [19].

In image space, taking into account the boundedness condition (6) the solution of equation (2) is written in the form:

$$\psi^L=\psi_0^L+\psi_1^L, \quad (10)$$
$$\psi_0^L=-\frac{\sin\theta}{\sqrt{r}}\sum_{n=1}^{\infty}A_n^L(s)K_{n+1/2}(r\gamma s)C_{n-1}^{3/2}(\cos\theta),$$
$$\psi_1^L=-\frac{\sin\theta_1}{\sqrt{r_1}}\sum_{n=1}^{\infty}B_n^L(s)K_{n+1/2}(r_1\gamma s)C_{n-1}^{3/2}(\cos\theta_1).$$

Here $A_n^L(s)$ and $B_n^L(s)$ are arbitrary functions; r_1, θ_1, ϑ_1 is an additional spherical coordinate system obtained by transferring along the axis O_2z of the centre O of the original spherical system to a point O_1 symmetrical to the point O relative to the plane $z=0$.

Taking into account the connection between coordinates r, θ and r_1, θ_1 on flat boundary of the half-space

$$r\big|_{z=0}=r_1\big|_{z=0}, \qquad \theta\big|_{z=0}+\theta_1\big|_{z=0}=\pi, \quad (11)$$

as well as properties of Gegenbauer polynomials [19]:

$$C_{n-1}^{3/2}(-x)=(-1)^nC_{n-1}^{3/2}(x), \quad (12)$$

from the boundary conditions (3) and (4), we obtain the connection between the functions $A_n^L(s)$ and $B_n^L(s)$

$$B_n^L(s)=\pm(-1)^nA_n^L(s). \quad (13)$$

Here and below, the upper sign corresponds to a rigid wall, and the lower sign corresponds to the free boundary of the half-space.

Now, substituting (13) into (10) and using the addition theorem [21] for Bessel functions of the second kind $K_{n+1/2}(x)$, as well as expressing these functions in terms of elementary ones [19], we present the image of the potential in the form of the following series:

$$\psi^L=-\sin\theta\sum_{n=1}^{\infty}\psi_n^L(r,s)C_{n-1}^{3/2}(\cos\theta), \quad (14)$$

where

$$\psi_n^L(r,s)=\frac{1}{r^{n+1}(\gamma s)^n}\Big[R_{n0}(r\gamma s)A_n(s)e^{-r\gamma s}+$$
$$+G_{n0}(r\gamma s)\sum_{p=1}^{\infty}S_{np}(s)A_p(s)e^{-2h\gamma s}\Big], \quad (15)$$
$$S_{np}(s)=\frac{\pm(-1)^p(2n+1)}{2n(n+1)}\sum_{\sigma=|p-n|}^{p+n}b_\sigma^{(n1p1)}\frac{R_{\sigma0}(2h\gamma s)}{(2h\gamma s)^{\sigma+1}},$$
$$G_{n0}(s)=R_{n0}(-s)e^s-R_{n0}(s)e^{-s},$$
$$R_{n0}(s)=\sum_{k=0}^{n}D_{nk}s^{n-k}, \qquad D_{nk}=\frac{(n+k)!}{(n-k)!2^k},$$
$$D_{nk}=0, \qquad k<0,\ k>n.$$

Here $b_\sigma^{(n1p1)}$ are the Clebsch-Gordon coefficients, [21].

Similarly to (14), we expand the images of displacement u_ϑ^L, stress $\sigma_{r\vartheta}^L$ and a given function q^L into series using Gegenbauer polynomials, and arrive at the following expressions and the boundary condition regarding the coefficients of the series

$$u_{\vartheta n}^L=\frac{\partial\psi_n^L}{\partial r}+\frac{\psi_n^L}{r}, \quad \sigma_{r\vartheta n}^L=\frac{\eta-\chi}{2}\left(\frac{\partial u_{\vartheta n}^L}{\partial r}-\frac{u_{\vartheta n}^L}{r}\right), \quad (16)$$

$$\sigma^L_{r\vartheta n}\Big|_{r=1} = q^L_n(s). \quad (17)$$

Next, taking into account (15) from formulas (16) we arrive at the following expressions for the coefficients $u^L_{\vartheta n}$, $\sigma^L_{r\vartheta n}$

$$u^L_{\vartheta n}(r,s) = -\frac{1}{r^{n+2}(\gamma s)^n}\Big[R_{n3}(r\gamma s)A_n(s)e^{-r\gamma s} + \\ + J_{n3}(r\gamma s)S_n(s)\Big], \quad (18)$$

$$\sigma^L_{r\vartheta n}(r,s) = \frac{\eta-\chi}{2\gamma^{n+2}s^n r^{n+3}}\Big[R_{n4}(r\gamma s)A_n(s)e^{-r\gamma s} + \\ + J_{n4}(r\gamma s)S_n(s)\Big], \quad (19)$$

$$J_{nm}(s) = R_{nm}(-s)e^s - R_{nm}(s)e^{-s}, \ m = \overline{3,4},$$
$$R_{n3}(s) = R_{n1}(s) - 2R_{n0}(s), \ R_{n4}(s) = R_{n2}(s) - R_{n0}(s),$$
$$R_{n1}(s) = \sum_{k=0}^{n+1} B_{nk}s^{n+1-k}, \quad R_{n2}(s) = \sum_{k=0}^{n+2} C_{nk}s^{n+2-k},$$
$$B_{nk} = D_{nk} + kD_{n,k-1}, \quad C_{nk} = B_{nk} + kB_{n,k-1},$$
$$S_n(s) = \sum_{p=1}^{\infty} S_{np}(s)A_p(s)e^{-2h\gamma s}.$$

Substituting (19) into the boundary condition (17), we obtain an infinite system of linear algebraic equations for the functions $A^L_n(s)$, which we write in the form of a matrix equation:

$$\mathbf{M}(s)\mathbf{A}(s)y^2 + \mathbf{F}^{(1)}(s)\mathbf{A}(s)x + \\ + \mathbf{F}^{(2)}(s)\mathbf{A}(s)xy^2 = \mathbf{p}(s)y, \quad (20)$$
$$x = e^{-2h\gamma s}, \qquad y = e^{-\gamma s}.$$

Here $\mathbf{M}(s)$ is an infinite diagonal matrix with elements $M_n(s)$; $\mathbf{F}^{(l)}(s)$ are infinite matrices of elements $F^{(l)}_{np}(s)$ ($l = 1,2$); $\mathbf{p}(s)$ is an infinite column vector with elements $p^L_n(s)$; $\mathbf{A}(s)$ is an infinite unknown column vector with components $A^L_n(s)$, and functions $M_n(s)$, $F^{(l)}_{np}(s)$ and $p^L_n(s)$ have the form:

$$M_n(s) = R_{n4}(R\gamma s), \ p^L_n(s) = 2\gamma^{n+2}s^n q^L_n(s)/(\eta-\chi),$$
$$F^{(1)}_{np}(s) = M_n(-s)S_{np}(s), \ F^{(2)}_{np}(s) = M_n(s)S_{np}(s).$$

We look for the solution to matrix equation (20) in the form of an infinite exponential series:

$$\mathbf{A}(s) = \sum_{i,j=0}^{\infty} \mathbf{a}_{ij}(s)x^i y^{-j-1}, \quad (21)$$

Here $\mathbf{a}_{ij}(s)$ are infinite column vectors with elements $a^{(n)}_{ij}(s)$, $n = 1,2,3,\ldots$.

Substituting series (21) into equation (20) and equating the coefficients of the left and right sides for the same degrees of variables x and y (the series on the right side contains only one non-zero term), we obtain a recurrent system of equations for functions $a^{(n)}_{ij}(s)$ and the corresponding initial conditions for them:

$$\mathbf{a}_{ij}(s) = \mathbf{E}_1(s)\mathbf{a}_{i-1,j+1}(s) - \mathbf{E}_2(s)\mathbf{a}_{i-1,j-1}(s), \ i \ge 1, j \ge 1,$$
$$\mathbf{E}_1(s) = \left\| S_{np}(s) \right\|, \qquad \mathbf{E}_2(s) = \left\| \frac{M_n(-s)}{M_n(s)} S_{np}(s) \right\|,$$
$$\mathbf{a}_{i0}(s) = \mathbf{E}_1(s)\mathbf{a}_{i-1,0}(s), \ i \ge 1,$$
$$\mathbf{a}_{i1}(s) = 0, \ i \ge 0, \qquad \mathbf{a}_{0j}(s) = 0, \ j \ge 1,$$
$$a^{(n)}_{00}(s) = \frac{p^L_n(s)}{M_n(s)}, \qquad n, p = 1,2,\ldots .$$

These relations make it possible to determine all the required images without using the reduction of an infinite system of equations. Analysis of recurrence relations shows that images are rational functions of the Laplace transform parameter, which makes it possible to calculate their originals, and therefore the originals of the displacement and stress coefficients in the medium using the theory of residues, [22].

The final formulas for the representations of the coefficients of the series in Gegenbauer polynomials of the sought functions follow from (18), (19) and (21)

$$u^L_{\vartheta n}(r,s) = -\frac{1}{r^{n+2}(\gamma s)^n}\sum_{i,j=0}^{\infty}\Big\{R_{n3}(r\gamma s)a^{(n)}_{ij}(s)y^r + \\ + J_{n3}(R\gamma s)\sum_{p=1}^{\infty} S_{np}(s)a^{(p)}_{ij}(s)x\Big\}x^i y^{-j-1}, \quad (22)$$

$$\sigma^L_{r\vartheta n}(r,s) = \frac{\eta-\chi}{2r^{n+3}\gamma^{n+2}s^n}\sum_{i,j=0}^{\infty}\Big\{R_{n4}(r\gamma s)a^{(n)}_{ij}(s)y^r + \\ + J_{n4}(r\gamma s)\sum_{p=1}^{\infty} S_{np}(s)a^{(p)}_{ij}(s)x\Big\}x^i y^{-j-1}.$$

3.1 Numerical Experiments

As an example, we consider the propagation of non-stationary skew-symmetric waves from spherical cavity in a half-space of sandstone saturated with kerosene with parameters $A = 0.4026 \cdot 10^9$ Pa, $N = 0.2493 \cdot 10^9$ Pa, $R = 0.0672 \cdot 10^9$ Pa, $Q = 0.0295 \cdot 10^9$ Pa, $\beta_0 = 0.26$, $\rho_s = 2600\ kg/m^3$,

$\rho_f = 820\ kg/m^3$, $\rho_{12} = -1.9\ kg/m^3$, which corresponds to the following dimensionless parameters $\beta = 0.0088331$, $\gamma = 1$, $\eta = 0.8772$, $\chi = 0.392$. The center of the cavity is located at a distance $h = 1.8$, the flat boundary of the half-space is the free surface (4). The results of numerical experiments are presented in the form of graphs of changes in components $\sigma_{r\vartheta}$, $\sigma_{\theta\vartheta}$ of the stress tensor and u_ϑ of displacement over dimensionless time t. As the law of change of the given load $q(\tau,\theta) = q_0 H(\tau)$, $q_0 = 1$, a constant function in time was chosen, where h is the Heaviside function. Numerical results were obtained taking into account seven terms of the series of Gegenbauer polynomials.

In Figure 1, Figure 2 and Figure 3 were obtained, respectively, for coordinate values $r = 1.2; 1.4; 1.6$. In Figure 2, Figure 4 and Figure 5 were plotted at the coordinate value $\theta = \pi/4$, and the graphs presented in Figure 3 are constructed at the coordinate value $\theta = 3\pi/4$.

Consequently, curve 1 in Figure 2 demonstrate the change in voltage coordinate $\sigma_{r\vartheta}$ at a point in the environment with coordinates $r = 1.2$, $\theta = \pi/4$.

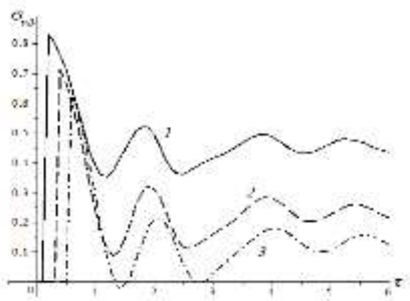

Fig. 2: Variation in time of the component $\sigma_{r\vartheta}$ when $q(\tau,\theta) = q_0 H(\tau)$, $\theta = \pi/4$

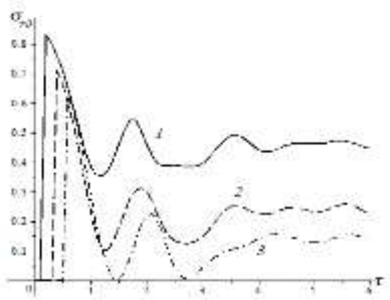

Fig. 3: Variation in time of the component $\sigma_{r\vartheta}$ when $q(\tau,\theta) = q_0 H(\tau)$, $\theta = 3\pi/4$

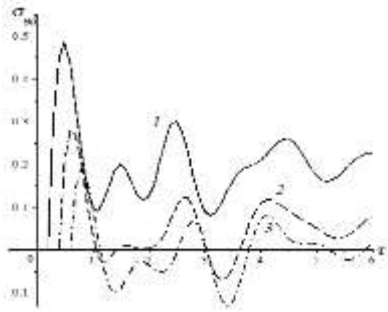

Fig. 4: Variation in time of the component $\sigma_{\theta\vartheta}$ when $q(\tau,\theta) = q_0 H(\tau)$, $\theta = \pi/4$

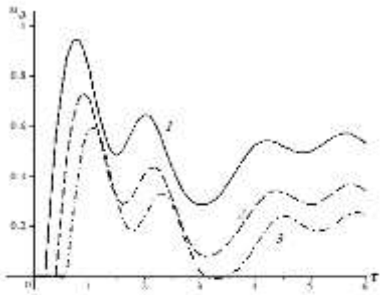

Fig. 5: Variation in time of the component u_ϑ when $q(\tau,\theta) = q_0 H(\tau)$, $\theta = \pi/4$

In the following example, in the form of a given load on the surface of the cavity, function $q(\tau,\theta) = e^{-\tau} H(\tau)$, exponentially decreases in time. For the initial values of the dimensionless parameters, numerical results were obtained for changes in the components $\sigma_{r\vartheta}$, $\sigma_{\theta\vartheta}$ of the stress tensor, and u_ϑ of the displacement over dimensionless time τ. In this case, the components of $\sigma_{r\vartheta}$, $\sigma_{\theta\vartheta}$ the stress tensors and u_ϑ displacement vector are demonstrated in Figure 6, Figure 7 and Figure 8. In all of them, Figure 1, Figure 2 and Figure 3 are plotted, respectively, at the above coordinate values r. Graphs in Figure 6 and Figure 8 were obtained at $\theta = \pi/4$, and the graphs presented in Figure 7 built at $\theta = 3\pi/4$.

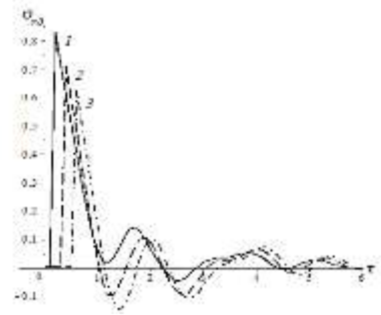

Fig. 6: Variation in time of the component $\sigma_{r\vartheta}$ when $q(\tau,\theta) = e^{-\tau} H(\tau)$, $\theta = \pi/4$

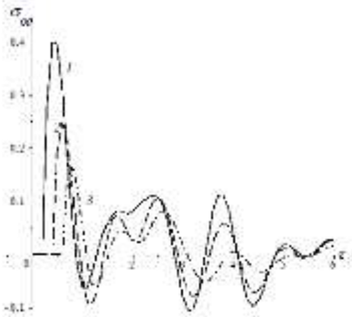

Fig. 7: Variation in time of the component $\sigma_{\theta\vartheta}$ when $q(\tau,\theta)=e^{-\tau}H(\tau)$, $\theta=\pi/4$

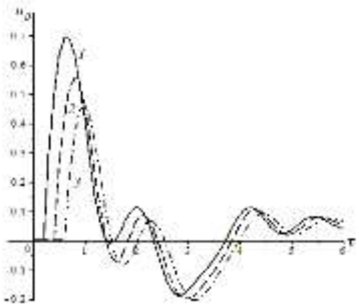

Fig. 8: Variation in time of the component u_ϑ when $q(\tau,\theta)=e^{-\tau}H(\tau)$, $\theta=\pi/4$

The graphs show that with the arrival of a wave, a jump appears. As the distance increases, the jumps $r=1.2, 1.4, 1.6$ decrease.

Waves reflected from the flat boundary of the half-space influence the stress-strain state of the medium. For the moments time $\tau>5$, the stress-strain state of the medium practically passes into a stationary state.

4 Conclusion

An algorithm has been developed for solving the problem of the propagation of non-stationary transverse shear waves from a spherical cavity in a porous-elastic half-space. The propagation of non-stationary skew-symmetric waves in the vicinity of a cavity under various given loads has been studied in the form of a constant and exponentially decreasing function.

Numerical results were obtained, which are presented in the form of graphs, which show the influence of the flat boundary of the half-space on the stress-strain state of the medium. The results obtained can be used in geophysics, seismology, and design organizations during the construction of structures, as well as in the design of underground reservoirs.

The proposed approach to solving the problem can be applied to similar problems without fundamental changes, such as the problem with a displacement specified at the boundary of a cavity, the problem of the diffraction of plane waves by a cavity, as well as the corresponding problems for a porous-elastic half-space.

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