# Bayesian modelling of summer daily maximum temperature data

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*Abstract:* - The extremes of summer daily maximum temperature was analyzed using the generalized Pareto distribution (GPD) to the Bisho weather station data, Eastern Cape Province, South Africa. Since the extreme events are naturally scarce it is expected that the use of a Bayesian inference may improve the efficiency of the parameters estimates of the distribution compared to the maximum likelihood method. Therefore, the Bayesian approach was also applied in the paper using the Markov Chain Monte Carlo for the generalized Pareto distribution. The expected improvement in efficiency is not fully achieved in this study using the non-informative and informative priors. However, the effects of informative prior constructed from historical data depends on the distance.

*Key-Words:* - Bayesian approach, Declustering, generalized extreme value distribution, generalized Pareto distribution, maximum temperature, peak-over threshold

## 1 Introduction

The extreme temperature events can cause significant effects on agriculture, such as crop drought and damage, health effects and power outages. All of these effects would lead to the economic loss [7]. The aim of this paper is to study the extreme of summer daily maximum temperature by applying the generalized Pareto Distribution (GPD) to the Bisho weather station data. Bisho is the capital city of the Eastern Cape Province, South Africa. The GPD is the distribution of the sample of excesses above a high threshold and it is commonly referred to as the peak-over-threshold (POT) method ([15], [5]). The POT method has been extensively used in finance and insurance (e.g., [11]), hydrology (e.g., [8]), precipitation (e.g., [9]) and extreme waves and wave parameters (e.g., [13]) just to mention a few. There are two statistical approaches for an extreme value analysis that might be used namely, frequentist and Bayesian ([3], [4], [6]).

The rest of the paper is organized as follows. The data used and methodology for the analyses are introduced in Section 2. The results are discussed in Section 3 and some concluding remarks and recommendations for future studies are given in Section 4.

## 2 Methodology

### 2.1 Extreme value distributions

The generalized extreme value (GEV) distribution and the generalized Pareto Distribution (GPD) are two commonly used distributions for modeling extremal events. To model the extreme values using the GEV a series of $N$ independent observations $y_1, y_2, \ldots, y_N$, first blocked into $m$ blocks of size $n$ with $n$ reasonably large and hence $N = mn$. For weather data the block size is usually one year, i.e. $n \approx 365$ days. Then from each block the maxima or extreme value, $M_i, i = 1, 2, \ldots, m$, is selected and this form a series of $m$ annual maxima data to which the GEV distribution family can be fitted. The extreme value analysis using GEV by block-maxima method is often wasteful of data, in particular when more data on the extremes are available, leading to large uncertainties on return level estimates. Unlike the block-maxima method, the POT method provides a more efficient use of data. In the POT method, first a threshold is chosen and all the data above the threshold are being considered and thus more than one event per year could be included in the analysis. From a statistical point, since the method includes more data points for extreme event into the model it would result in more precise estimate of the parameters [5].

Suppose $y_1, y_2, \ldots$ is a sequence of IID with a continuous distribution $F(.)$. Suppose that $M_n = \{y_1, y_2, \ldots, y_n\}$ and $y$ denote an arbitrary term of the sequence and that $F(.)$ satisfies the condition

$$\Pr\left\{\frac{(M_n - b_n)}{a_n} \le y\right\} \to F^n(a_n y + b_n) \to G(y) \qquad (1)$$

for some sequences of normalizing constants $\{a_n > 0\}$ and $b_n \in R$, as $n \to \infty$. The function $G$ is a non-degenerate distribution function. If the results in (1) hold, the distribution $F$ is said to be in the domain of attraction of the extreme value distribution $G$. Then $G$ belongs to family of distributions that can be summarized by the GEV distribution and has the distribution function

$$G(y,\mu,\sigma,\xi) = \exp\left[-\left\{1+\xi\left(\frac{y-\mu}{\sigma}\right)\right\}^{-1/\xi}\right] \qquad (2)$$

where $\{y: 1+\xi(y-\mu)/\sigma > 0\}$ and $\mu$, $\sigma > 0$ and $\xi$ are location, scale and shape parameters, respectively. Then, for suitably large $u$, the distribution function of $(y-u)$ condition on $y > u$, i.e. $P(y-u \mid y > u)$, can be approximated by the GPD, which has a distribution function of the form

$$H(y) = 1 - \left\{1+\xi\frac{y}{\sigma^*}\right\}^{-1/\xi}, \quad y > 0 \qquad (3)$$

where $\xi y/\sigma^* > 0$ and $\sigma^* = \sigma + \xi(u-\mu)$ [15]. The number of observations that exceeds the threshold $y-u$ is referred to as the exceedances. Note that $\mu$, $\sigma > 0$ and $\xi$ are location, scale and shape parameters, respectively as defined in expression (2). That is, if $G(y)$ is the approximating distribution of block maxima, then there is a corresponding approximate distribution for threshold exceedances from within the generalized Pareto family with shape parameter $\xi$ equal to that of the GEV distribution but the scale parameter $\sigma^* = \sigma + \xi(u-\mu)$ for any given threshold $u$. The distribution function in expression (3) for $\xi = 0$ is interpreted by taking the limit $\xi$ approaching zero, that is

$$\lim H(y) = 1 - \exp\left(-\frac{y}{\sigma^*}\right), \quad y > 0$$

an exponential distribution with parameter $1/\sigma^*$. The GPD usually expressed as a two parameter distribution in the form

$$H(y,\sigma^*,\xi) = \begin{cases} 1 - \left\{1+\xi\dfrac{y}{\sigma^*}\right\}^{-1/\xi} & , \ \xi \ne 0 \\ 1 - \exp\left(\dfrac{y}{\sigma^*}\right), & \xi = 0 \end{cases} \qquad (4)$$

where $y \in [0,\infty)$ for $\xi \le 0$ and $y \in [0, \sigma^*/\xi)$ for $\xi > 0$ [12]. By differentiating the GPD in expression (4) with respect to $y$ the density distribution is given by

$$h(y,\sigma^*,\xi) = \begin{cases} \dfrac{1}{\sigma^*}\left\{1+\xi\dfrac{y}{\sigma^*}\right\}^{-1/\xi-1} & , \ \xi \ne 0 \\ \dfrac{1}{\sigma^*} - \exp\left(\dfrac{y}{\sigma^*}\right), & \xi = 0 \end{cases}$$

where $y \in [0,\infty)$ for $\xi \le 0$ and $y \in [0, \sigma^*/\xi)$ for $\xi > 0$. If $y_1, y_2, \ldots, y_m$ are the $m$ exceedances of a threshold $u$, then the log of joint likelihood function associated with $y_1, y_2, \ldots, y_m$ is given by

$$\log L(y_1, y_2, \ldots, y_m, \sigma^*, \xi) = \begin{cases} -m\log\sigma^* - \left(\dfrac{1}{\xi}+1\right)\displaystyle\sum_{i=1}^{m}\log\left(1+\xi\frac{y}{\sigma^*}\right), & \xi \ne 0 \\ -m\log\sigma^* - \dfrac{1}{\sigma^*}\displaystyle\sum_{i=1}^{m} y_i, & \xi = 0, \end{cases} \qquad (5)$$

The maximum likelihood estimators (MLEs) of the parameters $\xi$ and $\sigma^*$, say $\hat{\xi}$ and $\hat{\sigma}^*$, for example when $\xi \ne 0$, are obtained by maximizing the log of joint likelihood function in (5) with respect to $\xi$ and $\sigma^*$. In practice the maximization is done by numerically iteration, e.g. using a quasi-Newton method. The standard errors of the MLEs can be approximated asymptotically using the inverse of the information matrix [1]. The goodness-of-fit of the GPD model can be examined by the probability and quantile plots.

### 2.1.1 Return level estimation for GPD

The focus of extreme weather events analysis usually lies not on estimates of the GPD parameters rather on application of the fitted model to estimate other quantities. Suppose $y_p$ be the $p$ year return level, i.e. it is the value occurring on average once in every $p$ years. The formula for $y_p$ can be derived from GEV theory for large $n$ and applying a Taylor series expansion, and is given by

$$y_p = \begin{cases} u + \left\{ \frac{\sigma^*}{\xi}\left[p \times \Pr\{Y > u\}\right]^{\xi} - 1 \right\}, & \xi \neq 0 \\ \mathrm{u} + \sigma^* \times \log\left[p \times \Pr\{Y > u\}\right], & \xi = 0, \end{cases} \quad (6)$$

The maximum likelihood estimate of the return level $y_p$ can be obtained using the MLEs of $\hat{\xi}$ and $\hat{\sigma}^*$, whereas $\Pr(Y > u)$ is estimated by the sample proportion of observations exceeding a threshold $u$. The standard errors for the return level estimate are obtained by means of the delta method [18].

## 2.2 Bayesian analysis of extreme values for GEV distribution

Since extreme data by their nature are scarce, the addition of other sources of information through a prior distribution may improve the statistical inference on extremes. Furthermore, unlike the maximum likelihood method a Bayesian analysis of extreme values is not dependent on the regularity assumptions required by the asymptotic theory of maximum likelihood [2]. The basic theory of Bayesian analysis of extreme values is well established and is presented in a number of excellent articles and texts such as those by [4] and [2]. Here we will focus on its application using the generalized Pareto distribution.

As in the likelihood approach, suppose the daily temperature maxima $M_n = \{y_1, y_2, \ldots, y_n\}$ and $y$ denote an arbitrary term of the sequence and that $F(.)$ satisfies the condition in expression (1) and their distribution fall within a GPD family given in expression (3). However, the parameters $\sigma$ and $\xi$ are now treated as random variables for which we specify prior distributions. The specification of priors enables us to supplement the information provided by the data. For this study, the joint prior density was chosen to be

$$f(\theta, \xi) = f_\theta(\theta) f_\xi(\xi)$$

where $\theta = \log(\sigma^*)$, $f_\theta(.)$ and $f_\xi(.)$ are marginal priors of $\theta$ and $\xi$, respectively. Then the posterior density has the form

$$\begin{aligned} f(\sigma^*, \xi \mid y) &\propto f(\theta, \xi) L(\sigma^*, \xi \mid y) \\ &= f_\theta(\theta) f_\xi(\xi) L(\sigma^*, \xi \mid y) \end{aligned}$$

where $L(\sigma^*, \xi \mid y)$ is the likelihood using the GPD given earlier with $\sigma^*$ replaced by $e^\theta$. Generally, in a Bayesian analysis, vectors of simulated values from the marginal posterior distributions of the GPD parameters are obtained. Then, for example, a realization from the posterior distribution of any specified $(1/\alpha)$-year return level $y_\alpha$ is obtained by substituting the simulated samples of $\sigma^*$ and $\xi$ into equation (6) [3], from which summary statistics can then be obtained.

Because of lack of expert information on rainfall extremes for the Bisho weather station, we have formulated the informative prior information for the Bayesian analysis from the maximum temperature characteristics of two weather stations, namely East London and Queenstown in the same province [3]. However, these prior information from external sources might not be adequately elicited directly in terms of the GPD parameters, for example if the marginal prior distributions for each parameter were available, it may not be clear how to build their joint prior distribution. Therefore, to avoid the independent priors approach [16] we have used [15] approach where the prior information on the parameters is elicited in terms of extreme quantiles rather than the extreme value model parameters themselves.

The *evd* [19], *evdbayes* [20], *extRemes* [10] and *ismev* [21] packages of R [17] were used for the data analyses and the results are presented in the following section.

# 3 Results and Discussion

## 3.1 Modelling extreme temperature using POT method and MLE

To analyse extreme maximum temperature using the POT method, first a threshold value $u_0$ is determined and then the GPD is fitted to the temperature values above $u_0$. The threshold value of 27°C has been chosen using the mean excess plot approach. The adequacy of the chosen threshold of $u_0$ can be checked using the plots of the ML estimates for the shape and modified scale parameters against a number of different thresholds.

Table 1 ML estimates and associated 95\% confidence intervals (CI) of scale and shape parameters of GPD model fitted to summer maximum daily temperature.

| Parameter | All exceedances | | Declustered | |
|---|---|---|---|---|
| | Estimate (se) | 95% CI | Estimate (se) | 95% CI |
| $\sigma^*$ | 4.255 (0.134) | (3.992, 4.518) | 5.467 (0.214) | (5.048, 5.886) |
| $\xi$ | -0.254 (0.018) | (-0.289, -0.219) | -0.346 (0.019) | (-0.384, -0.309) |

If the GPD is a reasonable model for the exceedances of a threshold $u_0$, then the estimates of the shape and modified scale parameters should be approximately constant to all threshold greater than $u_0$ (Coles, 2001). The plots of ML estimates of the shape and modified scale parameters versus threshold values for the summer daily maximum temperatures show that the selected threshold 27°C is adequate, as the estimates of the shape and modified scale parameters are approximately constant for thresholds greater than 27°C (plots are not given here).

On daily climate data such as maximum temperature it is possible that high values occur on clusters of consecutive days. This clustering induces temporal dependence in the observations and invalidates the independence assumption used to formulate the log-likelihood in equation (5). Furthermore, ignoring the dependence and applying the POT approach as if the data are independent will still leads to unbiased estimators but the estimators have standard errors that are small (Kearns and Pagan, 1997). The most commonly adopted approach to overcome this problem is declustering of the exceedances of the threshold to produce approximately independent data (see Coles, 2001, p 98-100). The results in Table 1 are the ML estimates for the scale and the shape parameters along with the associated 95% confidence intervals, from fitting the GPD to all threshold exceedances, i.e. ignoring temporal dependence, and using declustering with run length 2 and threshold 27°C.

The diagnostic plots, particularly probability and quantile plots, are approximately linear showing that the GPD models with threshold 27°C is adequate for the summer daily maximum temperatures at the Bisho weather station (see Fig. 1). The results in Table 1 show that when the analysis is done to declustered data, the GPD shape parameter $\xi$ underestimated and the scale parameter $\sigma^*$ is overestimated, relative to the analysis which uses all threshold exceedances. The width of confidence intervals for the parameters using declustered data are slightly wider than that of all exceedances data results. Therefore, as expected, fitting GPD model to all exceedances when there is temporal dependence will result in underestimation of the standard error of the parameter estimates.

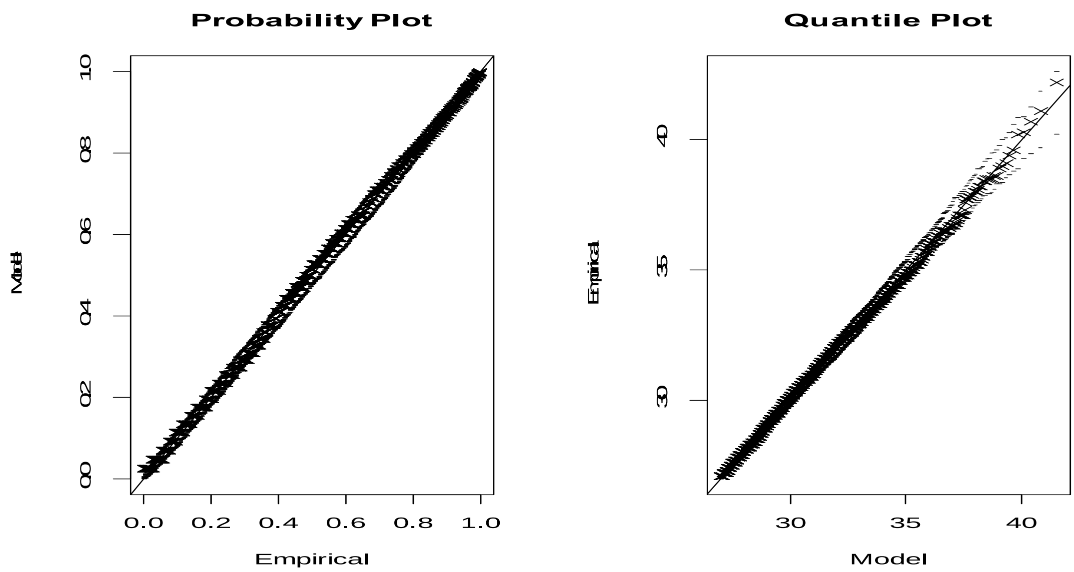


Fig. 1 Diagnostic plots of GPD for the summer daily maximum temperatures.

### 3.1.1 Return level estimation for GPD

The estimated return levels, using the ML method, for different return periods with 95% profile likelihood confidence intervals (CIs) are given in Table 2. It can be seen from Table 2 that the return levels for summer maximum daily temperatures increase slowly for higher return periods and further the intervals are increasingly wider as the return period is increasing. Note also that the return level estimates in the declustered data analysis are consistently greater for all three return periods compared to all exceedances data analysis estimates.

Table 2 Return levels and 95\% CIs (in °C) for maximum daily temperatures using GPD.

| Temperature | Return Period (years) | Analysis based on | |
|---|---|---|---|
| | | All exceedances | Declustered |
| Maximum | 10 | 26.231 (25.888, 26.574) | 30.528 (30.044, 31.012) |
| | 50 | 32.633 (32.196, 33.070) | 35.965 (35.461,36.469) |
| | 100 | 34.601 (34.129, 35.073) | 37.487 (36.968,38.006) |

## 3.2 Bayesian modelling of extreme maximum temperature data using non-informative and informative priors

The Markov Chain Monte Carlo (MCMC) method was applied to the summer daily maximum temperature data. The GPD scale parameter was re-parameterised as $\log(\hat{\sigma}^*)$ to retain the positivity of this parameter. Different starting points were used to check that the chains had converged to the correct place and all the chains are converged well. The following two independent non-informative priors were used

$$f(\log(\sigma^*)) \sim N(0,10000) \text{ and } f(\xi) \sim N(0,100)$$

for the two parameters of the generalized Pareto distribution, where, for example, $N(0,10000)$ denotes a Gaussian distribution with mean 0 and variance 10000. The large variances of the distributions impose flat-priors. The posterior means and standard deviations of these parameters are given in Table 3. The posterior means and standard deviations are close to the MLEs of the GPD parameters except for the declustered data where there a significant improvement in the standard deviation of the scale parameter. It is expected for flat-priors that posterior means would be close to the MLEs because they add little information to the likelihood. For the informative priors, the expert priors were formulated using the historical summer daily maximum temperature data of the East London and Queenstown weather stations from the same province. The posterior means for the scale parameter of GPD from the informative priors are greater than that of the posterior means of non-informative priors except for the declustered data of East London weather station whereas the shape parameters are underestimated in both East London and Queenstown cases (Table 3). Furthermore, only the precision of the scale and shape parameters for all exceedances data was improved by informative prior constructed using the East London data.

Table 3 Posterior means (standard errors) for the GPD parameters.

| Prior | All exceedances | | Declustered | |
|---|---|---|---|---|
| | $\hat{\sigma}^*$ | $\hat{\xi}$ | $\hat{\sigma}^*$ | $\hat{\xi}$ |
| Non-informative | 4.243 (0.133) | -0.249 (0.019) | 4.255 (0.134) | -0.254 (0.018) |
| Informative | | | | |
| East London | 4.731 (0.103) | -0.283 (0.017) | 2.568 (0.142) | -0.412 (0.034) |
| Queenstown | 4.743 (0.189) | -0.282 (0.020) | 5.808 (0.271) | -0.363 (0.022) |

To investigate the effects of the non-informative and informative priors on the return levels we have produced the posterior densities plots. The plots were done by substituting the vectors of observations from the marginal posterior distributions of $\hat{\sigma}^*$ and $\hat{\xi}$ in equation (6), for $0 < \alpha < 1$. This procedure was carried out for $\alpha = 0.1, 0.5$ and $\alpha = 0.01$ to obtain the posterior distributions of the 10-, 50- and 100-year return levels. Fig. 2 shows plots of the posterior densities of the 10-, 50- and 100-year return levels for the informative priors and non-informative priors of all exceedances data. It can be seen from the plots that the informative priors based on the East London has had effect on the densities of the return levels but those of Queenstown and non-informative priors have had little effect. A similar effect was found for the declustered data. This might suggest that the effect of informative priors constructed using historical data of other weather stations depend of the distance between the weather stations. The distance between Bisho weather station and East London weather station is about 55 kms whereas for the Queenstown weather station the distance is about 149 kms. The posterior densities of the 10- and 50-year return levels are symmetric whereas for the 100-year return levels densities are slightly skewed to the right. These skewness might reflect the uncertainty within the model for establishing upper limits of the return levels relative to lower limits for longer return periods (Coles and Tawn, 2005). Hence, the posterior medians would be more suitable estimates for the return levels than the posterior means, particularly for the 100-year return level. The posterior medians and 95% credibility intervals of the maximum temperature are given in Table 4.

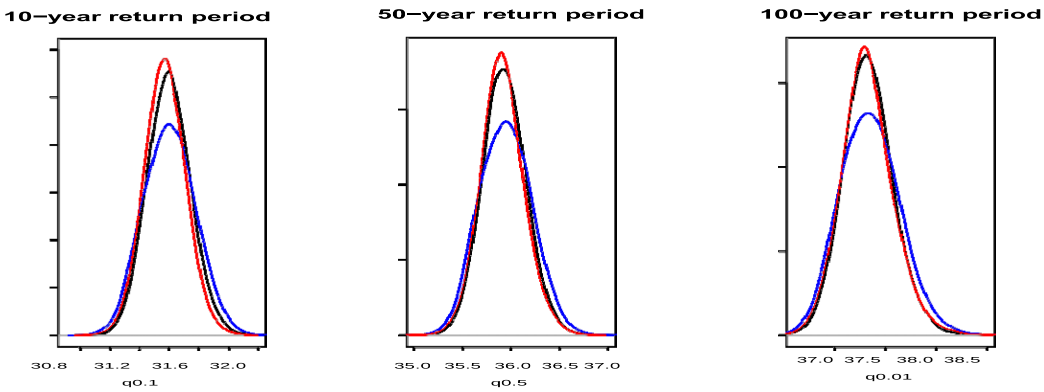


Fig. 2 Posterior densities for the 10-, 50- and 100-year return levels using all exceedences data, an informative prior (East London in blue and Queenstown in black) and a non-informative prior (in red).

Table 4 Posterior medians (95% credibility intervals) for the 10-, 50- and 100-year return levels (in mm) of the annual maximum rainfall using non-informative and informative priors.

| Priors | Return period (years) | | |
|---|---|---|---|
| | 10 | 50 | 100 |
| Non-informative | 31.372 (31.150, 31.599) | 35.912(35.560, 36.289) | 37.322 (36.928, 37.756) |
| Informative | | | |
| East London | 31.375 (31.205, 31.542) | 35.891 (35.639, 36.161) | 37.291 (37.010, 37.587) |
| Queenstown | 31.398(31.167, 31.639) | 35.932(35.573, 36.304) | 37.331 (36.943, 37.747) |

It can be seen from Table 4 that the median return levels for summer daily maximum temperature increase for higher return periods and also the credibility intervals are increasingly wider as the return periods are increasing. Furthermore, using the East London prior one would expected, for example, that the summer daily maximum temperature at Bisho will exceed 31.375°C once every 10 years, 35.891°C once every 50 years and 37.291°C every 100 years.

## 4 Conclusion

In this paper, we have conducted extreme value analysis using the generalized Pareto (GP) distribution for modelling summer daily maximum temperature data at the Bisho weather station, Eastern Cape province, South Africa by the ML and the Bayesian approaches. The MLEs of the model parameters have then been used to obtain the MLEs of the 10-, 50- and 100-year return levels. The Bayesian approach was used by simulating data from the posterior distributions of the GPD parameters by the MCMC method. Both the non-informative and the informative priors are imposed. The results obtained with the two priors were compared with the maximum likelihood results and also each other. The posterior means of the GPD parameters and their standard deviations obtained using the non-informative priors are close to the MLEs of the parameters for all exceedences case. The effect of the informative priors used in the analysis on the posterior means and standard deviations depend on the distance between the weather station used for the construction of informative prior and the weather station where the GPD model is fitted. That is, the shorter the distance the more closer the values of the posterior means of the GPD parameters to ML estimates and furthermore the standard deviations of the GPD parameters estimates get smaller. This reduction in the standard error reflects the decrease in uncertainty due to the informative priors. However, for the declustered case different observations were made, for example unlike the all exceedences case, the standard deviation of the posterior mean of shape parameter for the informative prior constructed using the East London data is greater than that of the informative prior constructed using the Queenstown data. The return level results show that the median return levels for summer daily maximum temperature will increase as the length of return period increases.

In general, the expected benefit of the Bayesian analysis is the improvement in precision of the parameter estimates over the MLEs, however in this study this is not fully achieved for the declustered data using the informative priors. The results of this study indicate that the effect of informative prior on the precision of parameter estimates depend on distance between stations. Therefore, the findings of the study could be improved using a spatial modeling and work is currently in progress to address this and will be reported elsewhere.

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