Robust Digital Redesign of Continuous PID Controller for Power system Using Plant-Input-Mapping

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Abstract: - the digital redesign technique is one of the most popular approaches to the design of digital controllers in industries. Which converting a good-designed continuous time controller to a digital controller suitable for digital implementation. In this paper, the Plant-Input-Mapping algorithm (PIM) is used for converting the S-domain model of the PID controller to a Z-domain model counterpart. The proposed digital PID controller is used to enhance the damping of a single machine power system. The proposed method is based on a transfer function from the reference input to the plant input, which called continuous time plant input transfer function CT-PITF. All the poles of the transfer function that need to be controlled must appear in the CT-PITF. The results obtained from the proposed digital PID controller more convergence to the CT-PID controller especially for longer sampling period where Tustin's method is violated. The proposed algorithm is stable for any sampling rate, as well as it takes the closed loop characteristic into consideration. The computation algorithm is simple and can be implemented easily. The proposed digital PID controller is successfully applied to the linearized model of a single machine infinite bus system and the performances of the analog PID controller, Tustin's controller and the proposed digital PID controller are compared and their results are presented.

Key-Words:- Dynamic stability, Digital redesign, Discretization, Plant-Input-Mapping, Discrete systems.

1 Introduction

There are many different approaches to designing discrete-time controllers for a continuous-time system in a feedback configuration. There are two digital design approaches for digital control systems [1]. The first approach, called the direct digital design approach, is to discretize the analog plant and then determine a digital controller for the discretized plant. The second approach, called the digital redesigns approach [2, 3], is to design a good analog controller for the analog plant and then carry out the digital redesign for the good designed analog controller. Many digital devices have been put into practical use in power system such as digital PID, digital PSS and digital AVR. The analog PID controller is widely used in power system to generate supplementary control signal for the excitation system in order to damp the low frequency oscillations.

In the digital redesign technique, a good-designed continuous time controller is converted to a digital controller counterpart. It is based on an optimal matching of continuous-time closed loop step responses of both continuous-time and discretized systems. Different techniques are used to convert continuous systems into discrete systems. However, it is to be noted that continuous system can only be approximated and the discrete system can never be exactly equivalent. One of most popular digital redesign method is the bilinear transformation (Tustin’s) method [2]. This method is considered as local discretization and it produces satisfactory results when the sampling period is sufficiently low.

In recent years, applications of discrete time controllers to power systems were reported in a number of publications [4, 5, 6, 7, 8, and 9]. It solved the transient stability problem addressed by analog controller, except that discrete time controller is just a matter of reprogramming a software program. [10].

In [5] the design of a discrete power system stabilizer PSS, which has been presented by linear approximation for single-machine infinite-bus system, was represented by nonlinear differential equations, the transfer function of the PSS was discretized using Tustin’s discretization method. The method in [6] analyzed the asymptotical stability of the digital controls of power systems with a special emphasis on the digital PSS. It treated the power systems as nonlinear hybrid dynamical systems so the power systems can be analyzed in a more exact way. In [4] a technique based on sampled-data control was proposed for optimal discretization of analog controllers while taking into account both closed-loop and inters ample behavior. In [7] a discrete fuzzy PID excitation controller utilizing the bilinear transforms (Tustin's) was implemented. This controller was developed by first designing discrete time linear PID control law and then progressively driving the steps necessary to incorporate a fuzzy logic control mechanism into the modification of the PID structure. The method in [9] presented a digital redesign method for discretizing a continuous-time power system stabilizer PSS for a single machine power system using Plant-Input-Mapping PIM method. This technique guaranteed the stability for any sampling rate as well as it took closed-loop characteristics into consideration

In this paper the Plant Input-Mapping (PIM) is applied to redesign an analog PID controller. This analog PID must have good performances controller. Our goal in this paper is to develop a high performance digital PID controller for single machine power system that takes into consideration the closed loop performance, which cannot be attained when using the traditional digital redesign method. The PIM method is a discretization scheme that can guarantee the stability for virtually any sampling rates (non-pathological sampling rates) and that has good performances even for large sampling intervals [1, 3, 11, 12].

Overall, the PIM method paved the way to the digital re-design of a general analog controller with guaranteed stability and continuous-time performance recovery. Such generality and stability are not available by any other methods. On the other hand the major disadvantage of this algorithm is depending on plant model. This design technique provides the designer a useful alternative to existing digital re-design methods as well as to possibly a wide class of direct digital design methods including the model reference control as explained in this research.

This article is organized as follows. In section (2), describes the system configuration that consists of two subsections, which are driving a power system model and explains the continuous time proportional, integral and derivative controller (PID Controller) model. Section (3), describes the standard PIM digital redesign method is considered. The discretization of PID controller by using Tustin's method describes in section (4). The application of the PIM method to a single-machine power system is considered in section (5). Section (6) analysis the simulation results. Finally the conclusions are given in section (7).

2 The System Configuration

2.1 Power system model

Fig. 1 shows Schematic of the studied system, which a single machine infinite bus (SMIB) power system is considered. The SMIB system, called the plant which consists of a synchronous generator connected through transmission line to a very large power network approximated by an infinite bus. The synchronous generator is driven by a turbine with a governor and excited by an external excitation system. The excitation system is controlled by an automatic voltage regulator (AVR) and a PID controller. The power system considered in this study is the fourth order linearized one-machine and infinite bus system [13].

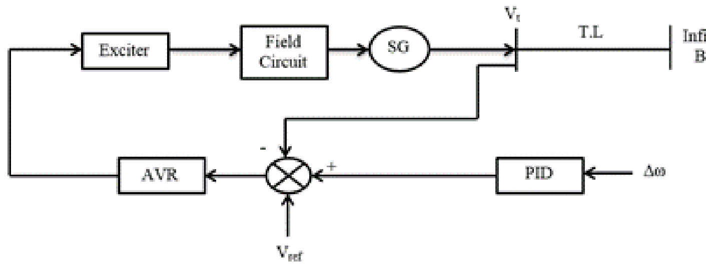

Fig. 1 Schematic of a single-machine infinite-bus (SMIB) power system.

Fig. 2 shows a block diagram of transfer functions describing the different subsystems of the one machine infinite bus power system. The different subsystems blocks are given as [9];

A. Excitation system

$$\frac{K_E}{1 + sT_E} \tag{1}$$

where K_E is the gain of exciter and T_E is time constant of exciter.

B. Field flux decay

$$\frac{K_3}{1 + sK_3T'_{d0}} \tag{2}$$

where T'_{d0} is the d-axis transient open circuit time constant.

C. Machine mechanical dynamics loop:

$$\frac{1}{2Hs + K_d} \tag{3}$$

where H is the inertia constant and K_d is damping coefficient.

Parameters K1……….K6 are the constant of linearized model of synchronous machine. From the block diagram shown in Fig. 2, and using Eqs. (1,2 and3) the following fourth order linearized one machine infinite bus system can be derived as described in [9,13]. The equations which describe the system are (Eqs: 4:7):-

$$\Delta\dot{\omega} = -\frac{K_d}{2H}\Delta\omega - \frac{K_1}{2H}\Delta\delta - \frac{K_2}{2H}\Delta\psi_{fd} + (0)\Delta E_{fd} + \frac{1}{2H}\Delta P_m + (0)V_{ref}$$

$$\Delta\dot{\delta} = \omega_B\Delta\omega + (0)\Delta\delta + (0)\Delta\psi_{fd} + (0)\Delta E_{fd} + (0)\Delta P_m + (0)V_{ref}$$

$$\Delta\dot{\psi}_{fd} = (0)\Delta\omega - \frac{K_4}{T'_{do}}\Delta\delta - \frac{1}{K_3T'_{do}}\Delta\psi_{fd} + \frac{1}{T'_{do}}\Delta E_{fd} + (0)\Delta P_m + (0)V_{ref}$$

$$\Delta\dot{E}_{fd} = (0)\Delta\omega - \frac{K_EK_5}{T_E}\Delta\delta - \frac{K_6K_E}{T_E}\Delta\psi_{fd} - \frac{1}{T_E}\Delta E_{fd} + (0)\Delta P_m + \frac{K_E}{T_E}V_{ref}$$

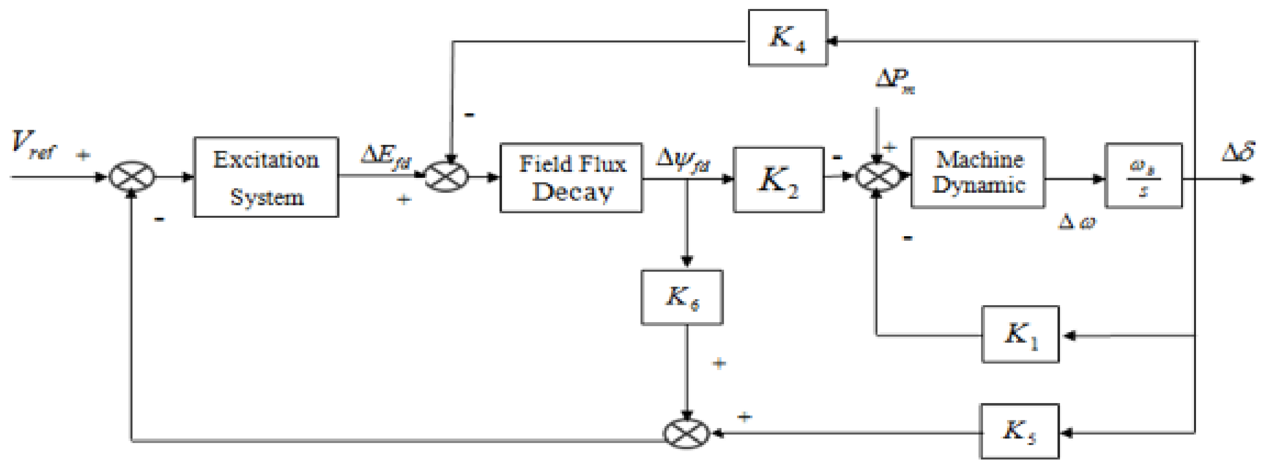

Fig.2 Block diagram of one machine infinite bus system

The following fourth order linearized model of a one machine with infinite bus system can be given in state variable form as follows

$$\dot{X} = AX + BU$$
$$Y = CX + DU \quad (8)$$

Where

$$A = \begin{bmatrix} -\frac{K_d}{2H} & -\frac{K_1}{2H} & -\frac{K_2}{2H} & 0 \\ \omega_B & 0 & 0 & 0 \\ 0 & -\frac{K_4}{T'_{do}} & -\frac{1}{K_3 T'_{do}} & \frac{1}{T'_{do}} \\ 0 & -\frac{K_5 K_E}{T_E} & -\frac{K_6 K_E}{T_E} & -\frac{1}{T_E} \end{bmatrix}$$

$$B^T = [\frac{1}{2H} \quad 0 \quad 0 \quad 0 \quad]$$
$$C = [1 \quad 0 \quad 0 \quad 0 \quad]$$
$$D = [0]$$

The state variables comprise the generator are speed deviation $\Delta\omega$, rotor angle deviation $\Delta\delta$, transient internal voltage deviation $\Delta E'_q$ and field voltage deviation ΔE_{fd} . The deviation of the angular velocity $\Delta\omega$ is assumed to be measured as the output of the system. The constants of the generation system and connected power system used for study are given in appendix I [9, 13]. The damping coefficient K_d is included in the swing equation. The eigenvalues of the matrix A should lie in LHP in the S-plane for the system to be stable. It is to be noted that the elements of matrix A are depended on the operating condition. The values of $K_1 : K_6$ in the matrix A are to be calculated according to the operating conditions of the generation system and connected power System [13]. Details of these constants are given in appendix II.

Using the data given above, the transfer function of the power system $\overline{G}(s)$ given by Fig. 2 and the state space equations given by Eq. 8 can be calculated using the MATLAB function SS2TF in the signal processing toolbox and are given by:

$$\overline{G}(s) = \frac{Y(s)}{U(s)} = \frac{0.108s^3 + 2.181s^2 + 12.33s}{s^4 + 20.67s^3 + 147.2s^2 + 525s + 2876} \quad (9)$$

The power system transfer function $\overline{G}(s)$ poles and zero is given in Table (1)

Table (1) power system transfer function poles and zeros

poles	zeros
$-10.2216 \pm j3.6926$	$-10.0990 \pm j3.4842$
$-0.1150 \pm j4.9334$	0.0

2.2 Continuous time PID controller model

The PID controller is simple and easy to implement. It is widely applied in industry to solve various control problems. PID controllers have been used for decades. During this time, many modifications have been presented in the literature [14, 15]. Then the transfer function of the modified continuous time PID controller

[16] is given by

$$PID\ (s) = K_P + K_I \frac{1}{s} + K_D \frac{s}{\tau s + 1} \quad (10)$$

where K_P is the proportional gain, K_I is the integral gain, K_D is the derivative gain and the term $\left[\frac{1}{\tau s + 1}\right]$ acts as an effective low-pass filter on the D regulator to attenuate noise in the derivative block. The individual effects of these three terms on the closed-loop performance are summarized in [16]. PID controller parameters are determined from the Matlab tuning given by

$K_p = 15.5, K_I = 5.0, K_D = 0.0115,\ \tau = 0.01$

where the speed deviation Δω is the input to the PID controller, and the filter is used to remove the controller effect at steady state conditions. Utilizing the parameter of the PID controller, the transfer function of the PID controller given can be calculated as;

$$G_{PID}(s) = \frac{a_2 s^2 + a_1 s + a_0}{b_2 s^2 + b_1 s + b_0} = \frac{0.1665 s^2 + 15.55 s + 5}{0.01 s^2 + s} \quad (11)$$

3 PIM Digital Redesign Method

Fig. 3 shows a SISO system which consists of plant with a transfer function $\overline{G}(s)$ and three analog controllers with rational, proper transfer functions $\overline{A}(s)$, $\overline{B}(s)$ and $\overline{C}(s)$ [16, 17].

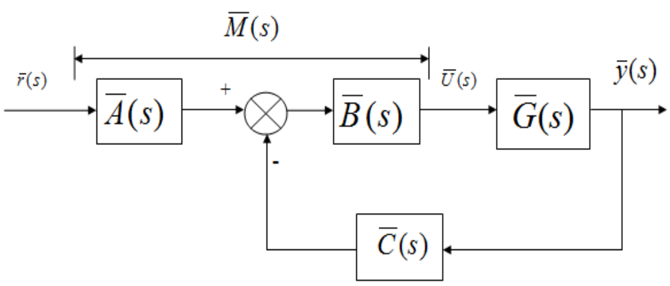

Fig. 3 Continuous-time control system.

The continuous-time plant is linear, time-invariant, and strictly proper, and is denoted as

$$\overline{G}(s) = \frac{\overline{n}_G(s)}{\overline{d}_G(s)} \quad (12)$$

The plant transfer function $\overline{G}(s)$ is now discretized using the step invariant-model (SIM), which is a combination of the zero-order-hold (ZOH), the plant and the sampler as shown in Fig. 4

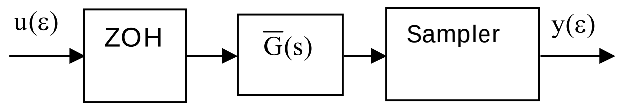

Fig. 4 Step invariant model (SIM) of the plant

Let the step-invariant model of this plant be expressed as

$$G(\varepsilon) = \frac{n_G(\varepsilon)}{d_G(\varepsilon)} = \frac{r_m \varepsilon^m + r_{m-1}\varepsilon^{m-1} + \dots\dots\dots + r_0}{p_n \varepsilon^n + p_{n-1}\varepsilon^{n-1} + \dots\dots\dots + p_0} \quad (13)$$

where $\partial[n_G(\varepsilon)] = m$, $\partial[d_G(\varepsilon)] = n$, and ∂ denotes the degree of its argument. The plant is expressed in Euler operator [18], which is defined as

$$\varepsilon = \frac{z-1}{T} \quad (14)$$

Where z is the usual zee operator and T is the sampling interval. The Euler operator is used here for better numerical properties in digital control implementation and ease of relating discrete-time results to continuous –time counterparts [12].

Assume that the analog control system is internally stable, satisfies all the design specifications, and is realized with proper transfer functions, which given as;

$$\overline{A}(s) = \frac{\overline{n}_A(s)}{\overline{d}_A(s)},\ \overline{B}(s) = \frac{\overline{n}_B(s)}{\overline{d}_B(s)},\ \overline{C}(s) = \frac{\overline{n}_C(s)}{\overline{d}_C(s)} \quad (15)$$

In the PIM method, both the closed-loop characteristics and plant information are used in the discretization process in the name of the Plant-Input-Transfer Function (PITF). The PITF is the transfer function from the reference input to the plant input and is given by

$$\overline{M}(s) = \frac{\overline{u}(s)}{\overline{r}(s)} = \frac{\overline{A}(s)\overline{C}(s)}{1 + \overline{B}(s)\overline{C}(s)\overline{G}(s)} \quad (16)$$

The PITF is discretized in the standard PIM method. This is carried out using the Matched-pole-zero (MPZ) method [19] and the resulting discrete time model becomes the target PITF.

The target discrete-time PITF can be expressed as

$$M(\varepsilon) = MPZ\ (\overline{M}(s)) = \frac{n_M(\varepsilon) d_G(\varepsilon)}{d_M(\varepsilon)} \quad (17)$$

It is found that the denominator of the SIM of the plant appears in the numerator of DT-PITF. Choosing the discrete-time controller blocks [12] as;

$$A(\varepsilon) = \frac{m(\varepsilon)}{\lambda(\varepsilon)}, B(\varepsilon) = \frac{\beta(\varepsilon)}{\lambda(\varepsilon)}, C(\varepsilon) = \frac{\lambda(\varepsilon)}{\alpha(\varepsilon)} \quad (18)$$

Once this discrete-time PITF is obtained, this must be realized in closed-loop configuration, such as one shown in Fig. 5.

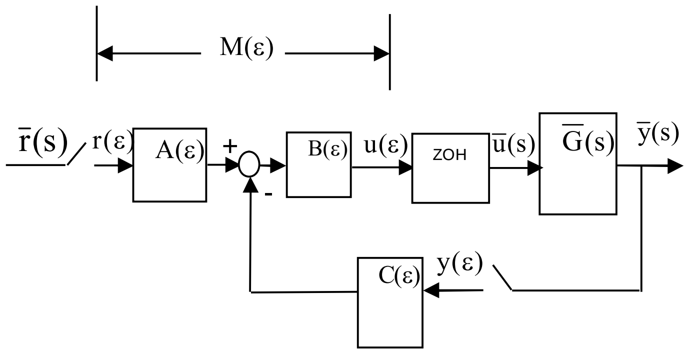

Fig. 5 Discrete-time control system redesigned using the PIM method.

And $\lambda(\varepsilon)$ is an arbitrary stable polynomial of appropriate degree [1, 16]. The actual PITF of this control system is given by

$$M(\varepsilon) = \frac{m(\varepsilon) d_G(\varepsilon)}{\beta(\varepsilon) n_G(\varepsilon) + \alpha(\varepsilon) d_G(\varepsilon)} \tag{19}$$

The polynomial $n_G(\varepsilon)$ and $d_G(\varepsilon)$ are known from of the plant (see Eq. 13). By equating the target and the actual PITF, it can be seen that the polynomial $m(\varepsilon)$ must be in the numerator of polynomial $n_M(\varepsilon)$, whereas $\alpha(\varepsilon)$ and $\beta(\varepsilon)$ must be determined by solving the following Diophantine equation:

$$\alpha(\varepsilon) d_G(\varepsilon) + \beta(\varepsilon) n_G(\varepsilon) = d_M(\varepsilon) \tag{20}$$

If the order of denominator of $M(\varepsilon)$ is P, where P≥2n -1, and n is the order of denominator of the plant $\overline{G}(s)$, which is not satisfied. The uniqueness of the solution of Eq. 20 is not assured. As in the case, a stable polynomial $\lambda(\varepsilon)$ of order q must be multiplied in the numerator and denominator of the target PITF $M(\varepsilon)$ to guarantee the solution of Eq. 21 [20].

$$\lambda(\varepsilon) = (\varepsilon + \frac{1}{2T})^q \quad \text{, where } q \geq 2n-1-p \tag{21}$$

The Diophantine equation after modification becomes;

$$\alpha(\varepsilon) d_G(\varepsilon) + \beta(\varepsilon) n_G(\varepsilon) = \lambda(\varepsilon) d_M(\varepsilon) \tag{22}$$

Equation (21) can be solved to find the unknown terms $\alpha(\varepsilon)$ and $\beta(\varepsilon)$ using for instance Eliminant matrix and a state space formulation [1]. Fig. 6 shows the three controller block of the PIM model.

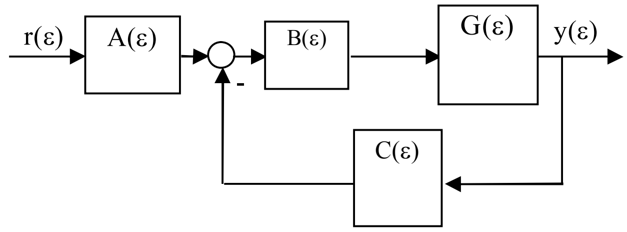

Fig. 6 PIM design method for a plant.

The PIM design guarantees the internal stability for any nonpathological sampling interval and that the performance of the resulting control system approaches that of the analog control system as T→0.

4 Discretization of PID controller by using (Tustin's Method)

Discretization of PID controllers by using bilinear method (Tustin's method) is investigated [21]. By replacing each S-domain in analog controllers to Z-domain, according to this relation.

$$\left(s = \frac{z-1}{\frac{T}{2}(z+1)} \right) \text{, Where T is sampling time} \tag{23}$$

Then, the transfer function of a digital PID controller (Tustin's method) is

$$PID(z) = \frac{\left(a_2 + \frac{T}{2}a_1 + a_0\frac{T^2}{4}\right)z^2 + \left(-2a_2 + a_0\frac{T^2}{2}\right)z + \left(a_2 - \frac{T}{2}a_1 + a_0\frac{T^2}{4}\right)}{\left(b_2 + \frac{T}{2}b_1\right)z^2 - 2b_2 z + \left(b_2 - \frac{T}{2}b_1\right)} \tag{24}$$

From Eq. 11, then; $a_2 = 0.1665$, $a_1 = 15.55$, $a_0 = 5$, $b_2 = 0.01$, $b_1 = 1$, $b_0 = 0$ .and the sampling interval selected according to the sampling theorem which defined as the sampling frequency should be at least twice the highest frequency contained in the signal [22]. Then the sampling time for digital control is 0.2sec selecting by sampling theory [22], then the transfer function of a digital PID controller is

$$PID(z) = \frac{1.772 z^2 - 0.233 z - 1.339}{0.11 z^2 - 0.02 z - 0.09} \tag{25}$$

After design of discrete-time PID controllers for discrete-time control systems by using traditional method (Tustin's method) compare it with design of discrete-time control system by using the proposed method (PIM) which presented in the next section.

5 Application of PIM Digital Redesign Method to Power System Model

To apply the design technique presented in section 3, the transfer function $\overline{G}(s)$ for the power system given by Eq. 9 and the transfer function for the PID controller given by Eq. 11 are used in the design Procedure with the blocks

$\overline{A}(s)$ and $\overline{C}(s)$ equal to one as shown in Fig. 3 [16]. Simulations responses of the power system based on the linear model given by state space representation are presented. The power system is subject to a step change in the mechanical torque denoted by ΔP_m . The signal to be controlled is the rotor speed denoted by Δω. The analog PID is placed on the block $\overline{B}(s)$ of Fig. 3 of the three block controllers PIM digital redesign method [9]. For comparison, results of the analog PID and the digital PID obtained by the bilinear transformation (Tustin's method) are investigated [21]. The CT-PITF is found to be

$$\overline{M}(s) = \frac{(s+100)(s^2+20.435s+118)(s^2+0.2305s+24.36)}{(s+99.53)(s^2+20.625s+119.6)(s^2+1.851s+24.68)} \quad (26)$$

It is clear that all power system poles and PID controller poles are appearing in the numerator of the CT-PITF. The CT-PITF poles and zero are given in Table (3).

Table (3) CT-PITF poles and zero

poles	zeros
$-10.3104\pm j3.6492$	$-10.22\pm j3.70$
$-0.9253\pm j4.8807$	$-0.120\pm j4.93$
-99.5287	-100.0

The SIM model of the power system is given by

$$G(\varepsilon) = \frac{0.087558\,\varepsilon(\varepsilon^2+8.963\varepsilon+20.25)}{(\varepsilon^2+9.0435\varepsilon+20.63)(\varepsilon^2+4.611\varepsilon+21.93)} \quad (27)$$

The SIM of the power system contains poles and zeros are given in Table (4)

Table (4) SIM poles and zeros

poles	zeros
$-2.3054\pm j4.0762$	$-4.4817\pm j0.4101$
$-4.5213\pm j0.4358$	-0.0000

The MPZ model of the ZOH type with its DC gain adjusted is used for discretizing the CT-PITF and is given as

$$M(\varepsilon) = \frac{0.84888(\varepsilon+5)(\varepsilon^2+9.0425\varepsilon+20.63)(\varepsilon^2+4.612\varepsilon+21.93)}{(\varepsilon+5)(\varepsilon^2+9.052\varepsilon+20.66)(\varepsilon^2+5.344\varepsilon+18.99)} \quad (28)$$

The DT-PITF (Case of PIM PID) contains poles and zeros are given in Table (5)

Table (5) DT-PITF poles and zeros

poles	zeros
$-2.6721\pm j3.4420$	$-4.5212\pm j0.4369$
$-4.5260\pm j0.04240$	$-2.3060\pm j4.0763$
-5.0000	-5.0000

The sampling interval selected for digital control is 0.2 sec, (any sampling interval T>0 is nonpathological), which is reasonable compared with the dynamic of the system. The condition P≥2n -1, where P is the order of denominator of $M(\varepsilon)$ and n is the order of denominator of the plant $\overline{G}(s)$ is required to assure uniqueness for solving the Diophantine equation Eq. 20, but in this study of PID controller the condition (P≥2n -1) is not satisfied, then the uniqueness of the solution of Eq. 20 is not assured. To account for this, C. A. Rabbath [20] proposed a modification of Diophantine equation to solve for this problem, a stable polynomial $\lambda(\varepsilon)$ of order q must be multiplied in the numerator and denominator of the target PITF $M(\varepsilon)$ to guarantee the solution of Eq. 20. According to Eq. 21 the polynomial $\lambda(\varepsilon)$ is selected as

$$\lambda(\varepsilon) = (\varepsilon + \frac{1}{2T})^3 \quad (29)$$

The polynomial $m(\varepsilon)$ is obtained from the numerator of $M(\varepsilon)$ which is defined

$$m(\varepsilon) = 0.84898\,\varepsilon + 4.2452 \quad (30)$$

It is clear that the numerator of $M(\varepsilon)$ includes the poles of the SIM of the power system and the polynomial $m(\varepsilon)$. The modified Diophantine equation in Eq. 22 can be solving by the Eliminant matrix method. Using the numerator and denominator of the SIM of power system the Eliminate matrix E can be constructed as follows:

$$E=\begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13.6534 & 1.0000 & 0 & 0 & 0.0876 & 0 & 0 & 0 \\ 84.2565 & 13.6534 & 1.0000 & 0 & 0.7848 & 0.0876 & 0 & 0 \\ 293.4392 & 84.2565 & 13.6534 & 1.0000 & 1.7734 & 0.7848 & 0.0876 & 0 \\ 452.4707 & 293.4392 & 84.2565 & 13.6534 & -0.0000 & 1.7734 & 0.7848 & 0.0876 \\ 0 & 452.4707 & 293.4392 & 84.2565 & 0 & -0.0000 & 1.7734 & 0.7848 \\ 0 & 0 & 452.4707 & 293.4392 & 0 & 0 & -0.0000 & 1.7734 \\ 0 & 0 & 0 & 452.4707 & 0 & 0 & 0 & -0.0000 \end{bmatrix} \quad (31)$$

Solving the modified Diophantine equation (Eq. 22) with the aid of the Eliminate matrix given by Eq. (31), the polynomial $\alpha(\varepsilon)$ and $\beta(\varepsilon)$ are obtained as follows;

$$\alpha(\varepsilon) = 1.000\varepsilon^3 + 9.7309\varepsilon^2 + 29.3496\varepsilon + 27.0993 \quad (32)$$

$$\beta(\varepsilon) = 18.1742\varepsilon^3 + 242.6006\varepsilon^2 + 625.9518\varepsilon - 967.3896 \quad (33)$$

To relate a discrete-time system to continuous time counterpart, the following operator is used

$$\varepsilon = \frac{z-1}{T}$$

(34)

Where T is the sampling interval and z is the usual shift operator. The three controller blocks, $A(z)$, $C(z)$ and $B(z)$ are calculated using the results obtained above by taking A(z) as a unity then C (z) and B (z) are;

$$B(z) = \frac{z^3 - 1.5z^2 + 0.75z - 0.125}{z^3 - 1.054z^2 + 0.2816z - 0.01101} \quad (35)$$

$$C(z) = \frac{11.56z^3 - 17.07z^2 + 5.425z - 0.5761}{z^3 - 1.5z^2 + 0.75z - 0.125} \quad (36)$$

6 Simulation Results

The test system has been modeled through Matlab programming. Fig. 7 to fig. 10 show simulations results of the proposed digital redesign technique PIM method by using the control sampling rates of 5Hz, 4Hz, 2.5Hz, and 2Hz, respectively. It is noticed that the PIM controllers is stable for any sampling rates and closely match those of the continuous-time PID controller. On other hand, it is found that Tustin's method is violated when sampling interval becomes large.

As shown in Fig. 7 the responses of Tustin's and PIM of PID controller produces a smaller overshoot than analog controller while the performance of Tustin's and PIM of PID controller converge to the analog case at the control sampling rate of 5Hz. At the 4Hz control sampling rate, the overshoot of the Tustin's and PIM of PID controller become larger than the corresponding case of 5Hz, though they are acceptable. Their plant inputs are still close to that of the analog controller as shown in Fig. 8. When the sampling rate becomes slow as shown in Fig. 9 the Tustin's response oscillates violently and is not satisfactory while the PIM of PID controller produces a different transient response from analog one and it has a small overshoot, it settles in 5sec at the same time as the analogue one with no steady state error and almost no oscillation at the 2.5Hz sampling rate. At the 2Hz control sampling rate, the Tustin's case oscillates to such an extent that it is not acceptable and doesn't settle even after 10 sec. Although the PIM of PID yields transient responses that are different from analog case, their performance is very good as shown in Fig.10.

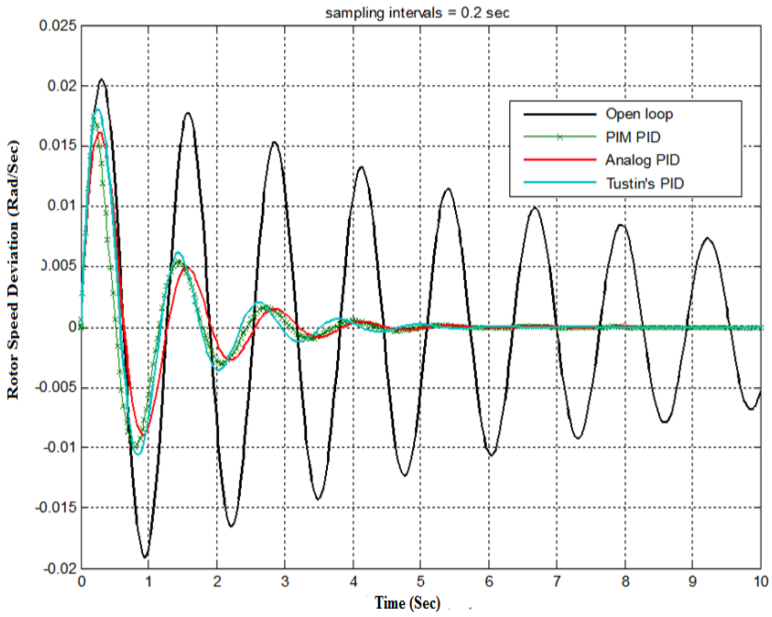

Fig. 7 Dynamic responses to step change in the mechanical torque (sampling interval 0.2 s) of PID controller.

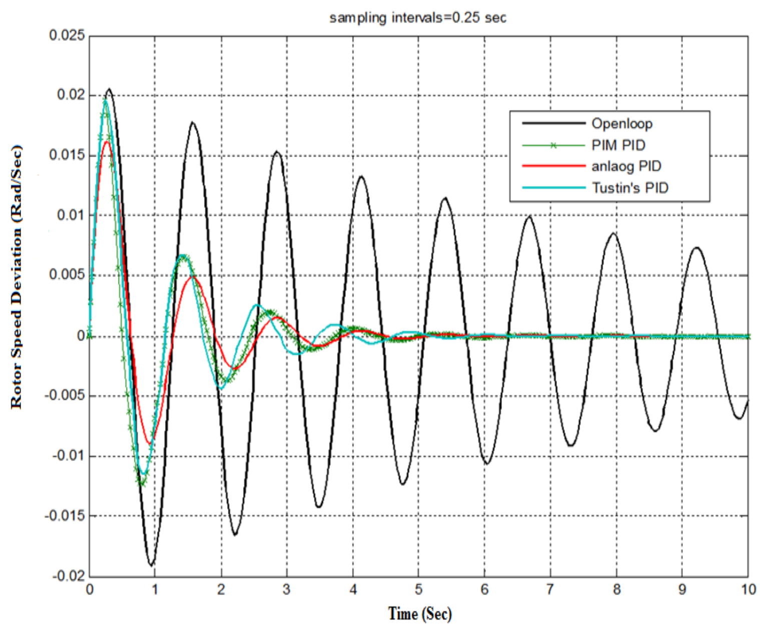

Fig. 8 Dynamic responses to step change in the mechanical torque (sampling interval 0.25 s) of PID controller.

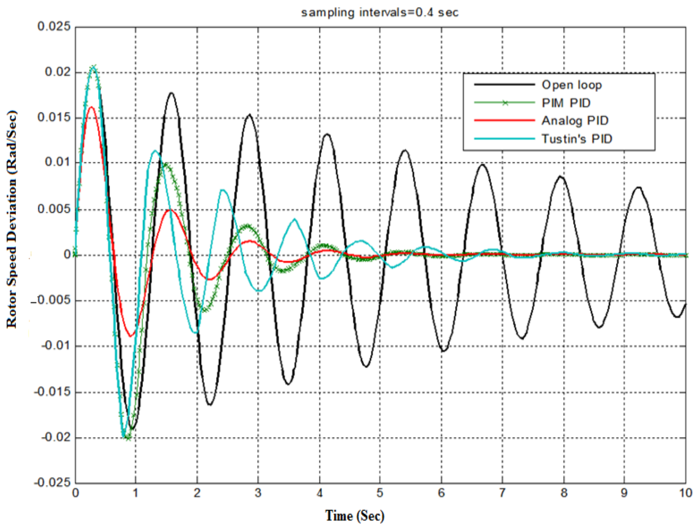

Fig. 9 Dynamic responses to step change in the mechanical torque (sampling interval 0.4 s) of PID controller.

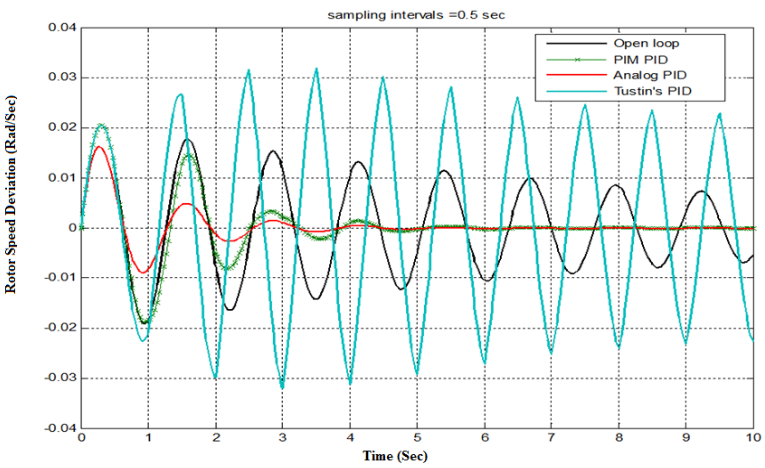

Fig. 10 Dynamic responses to step change in the mechanical torque (sampling interval 0.5 s) of PID controller.

7 Conclusion

The presented technique in this study guarantees the stability for any sampling rate as well as it takes closed-loop characteristics into consideration and that has good performances even for large sampling intervals, unlike the popular conventional such as Tustin's method of discretization. On the other hand the major disadvantage of the presented technique is depending on plant model. The proposed digital PID controller is applied to a single machine infinite power system for stability enhancement. Design PIM-PID controller require to design of the three discrete-time controllers $A(Z)$, $B(Z)$, $C(Z)$ which depend on solution of the Diophantine equation, the condition P≥2n -1, where P is the order of denominator of $M(\varepsilon)$ and n is the order of denominator of the plant $\overline{G}(s)$ must be satisfied assure uniqueness of the solution of this equation, but in this study of PID controller the condition (P≥2n -1) is not satisfied, then the uniqueness of the solution of this equation is not assured. To account for this, C. A. Rabbath [20] proposed a modification of Diophantine equation to solve for this problem. It enables us to solve the problem and design the three discrete-time controller. A comparison study of the proposed digital PID controller is carried out with conventional continuous-time PID controller and Tustin ' s PID controller. The results observed by simulations showed that the proposed digital PID controller converge to the CT-PID controller especially for longer sampling period where Tustin's method is violated.

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APPENDIX I

I.1 Generator parameters:

H=4.63, K_d=4.4, T'_{d0}=7.67, ω_B=377.0, X_d=0.973 pu, x'_d=0.19 pu, X_q=0.55 pu

I.2 Exciter parameters:

K_e=50.0, T_e=0.05.

I.3 The K's:

K_1=0.5758, K_2=0.9738, K_3=0.6584, K_4=0.5266, K_5=-0.0494, K_6=0.8450.

I.4 Transmission line:

R_e=0.0, X_e=0.997 pu.

I.5 Operating point:

Q_{e0}=0.015 pu, V_{t0}=1.05 pu, P_{e0}=0.75 pu.

APPENDIX II

The constants K_1 : K_6 are evaluated with transmission line resistance re=0 and are given as follows:

$$K_1 = \frac{X_q - X_d}{(X_e + X'_d)} I_{q0} V_0 \sin\delta_0 + \frac{E_{qo} V_0 \cos\delta_0}{(X_e + X_q)}$$

$$K_2 = \frac{V_0 \sin\delta_0}{(X_e + X'_d)}$$

$$K_3 = \frac{X'_d + X_e}{(X_d + X_e)}$$

$$K_4 = \frac{X_d - X'_d}{(X_e + X'_d)} V_0 \sin\delta_0$$

$$k_5 = \frac{X_q}{(X_e + X_q)} \frac{V_{d0}}{V_{t0}} V_0 \cos\delta_0 + \frac{X'_d}{(X_e + X'_d)} \frac{V_{q0}}{V_{t0}} V_0 \sin\delta_0$$

$$K_6 = \frac{X_e}{(X_e + X'_d)} \frac{V_{q0}}{V_{t0}}$$

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