Passive Attitude Control for Spacecraft

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Abstract: - The stability of spacecraft attitude is studied with considering sinusoidal disturbance. A passive attitude controller without angular velocity measurement for spacecraft described by quaternion is designed. The passive controller has no information related to system parameters. So it has robustness to model error and uncertainty of model parameters. The stability of spacecraft attitude control with considering external sinusoidal disturbance can be proved by applying Lyapunov approach and LaSalle Invariance Principle. Simulation results demonstrate the effectiveness of the designed attitude controller.

Key-Words: - Spacecraft attitude, Passive control, Quaternion, Sinusoidal disturbance

1 Introduction

The attitude of spacecraft can be expressed by matrix, Euler angle, or quaternion. The method of matrix representation is complicated in calculation; Euler angle also exist some limitations. For example, the rotation matrix is not interchangeable, Euler angle rotation must be in a particular order, and equivalent to Euler angle change may not cause equal rotation, which leads to a rotating unevenness. When Euler angle is equal to $\pm\pi/2$, there will be a singular point, leading to the loss of degrees of freedom, which is called as the phenomenon of gimbal lock. But expressing 3D rotation with quaternion can avoid these limitations, and also has clear geometric meaning and simple calculation. In the past few decades, scholars have been used different control methods based on quaternion representation to solve the attitude stabilization of spacecraft, such as robust control approach [1,2], Lyapunov-based approach [3-5], variable structure control approach [6-10], adaptive control approach [11,12].

Often, angular velocity and quaternion, are used to deal with the stabilization of feedback control. However, the angular velocity measurement is not necessary in some of the previous works. For example, in [7], a design criterion for a class of PD controllers was firstly proposed by using the Lyapunov-based approach, and then a design criterion of controller without angular velocity measurement was presented based on the passivity approach. The approach proposed in [7] was further extended to the system described by the Rodrigues and modified Rodrigues parameters [8].

The external disturbances, which inevitably affect the motion of spacecraft, are ignored in the above-mentioned literatures. This paper focuses on the issue of spacecraft attitude stabilization in the case of the bounded external disturbances. We use quaternion to represent the spacecraft attitude and design a passive attitude controller without angular velocity measurement. In addition, the designed controller does not contain information related to the system parameters, which makes it robust to the model error and the model parameter uncertainty.

2 Mathematical Model of Spacecraft System

2.1 Spacecraft attitude kinematics

We use the unit quaternion to represent spacecraft attitude in order to avoid singularity. Define the unit quaternion as

$$\bar{q} = \begin{pmatrix} q \\ q_0 \end{pmatrix} = \begin{pmatrix} \sin(\theta/2)\hat{n} \\ \cos(\theta/2) \end{pmatrix} \quad (1)$$

where, $\hat{n} \in R^3$ is the rotation axis represented by unit vector, θ is the rotation angular, $q \in R^3$ and $q_0 \in R$ are the components of the unit quaternion, which subject to the following constraint:

$$q^{\mathrm{T}} q + q_0^2 = 1 \quad (2)$$

The kinematic equation represented by the unit quaternion is given by

$$\dot{q} = \frac{1}{2}E(q)\omega = \frac{1}{2}(q_0 I + q^{\times})\omega$$
$$\dot{q}_0 = -\frac{1}{2}q^{\mathrm{T}}\omega \tag{3}$$

where, $\omega = [\omega_1\ \omega_2\ \omega_3]^{\mathrm{T}}$ is the spacecraft angular velocity vector with respect to the inertial reference frame, expressed in the spacecraft body-fixed reference frame, I is the 3×3 unit matrix, $q^{\times}$ is the skew symmetric matrix which is defined by Eq.(4).

$$q^{\times} = \begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix} \tag{4}$$

2.2 Spacecraft attitude dynamics

The dynamic model of the spacecraft attitude control system is described by the differential equation in Eq.(5).

$$J\dot{\omega} = -\omega^{\times}J\omega + u + d \tag{5}$$

where, $J = J^{\mathrm{T}} \in R^{3\times3}$ is the inertia matrix which is a symmetric and positive define matrix, $u \in R^3$ is the vector of control torque, $d = [d_1\ d_2\ d_3]^{\mathrm{T}}$ is the vector of external disturbance.

Assuming that d is sinusoidal disturbance and satisfies the following condition.

$$\ddot{d} + \gamma^2 d = 0 \tag{6}$$

where, $\gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \gamma_3\}$ is the disturbance frequency matrix.

3 Passive Attitude Control without Angular Velocity Measurement

3.1 Attitude control without considering external disturbance

In [7], a passive controller without angular velocity measurement was proposed. Along the line of [7], we construct a controller as follows.

$$\begin{cases} \dot{x} = Ax + BCq \\ y = B^{\mathrm{T}}P(Ax + BCq) = B^{\mathrm{T}}P\dot{x} \\ u = -k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}}(q)Cy - E^{\mathrm{T}}Fq \end{cases} \tag{7}$$

where, k is a positive constant, B is a full rank matrix, $C = \mathrm{diag}\{c_1, c_2, c_3\}$ and $F = \mathrm{diag}\{f_1, f_2, f_3\}$ are the symmetric positive definite gain matrices, $E(q)$ is the Jacobian, which satisfies the following property.

$$E(q) = q_0 I + q^{\times} \tag{8}$$

There exist positive definite matrices P and Q such that

$$A^{\mathrm{T}}P + PA = -Q \tag{9}$$

The symbolic function $\mathrm{sgn}(q_0)$ is defined by

$$\mathrm{sgn}(q_0) = \begin{cases} 1 & q_0 > 0 \\ -1 & q_0 < 0 \end{cases} \tag{10}$$

For attitude stabilization control, $\bar{q} = [\pm1, 0, 0, 0]^{\mathrm{T}}$ is the stable equilibrium of system when ω is zero, since q and $-q$ represent the same attitude. In fact, the equilibrium tended to is related to the initial symbol of q_0. The different the initial symbol of q_0, the different the control law. Therefore, consider the following Lyapunov function candidate

$$V = \frac{1}{2}\omega^{\mathrm{T}}J\omega + q^{\mathrm{T}}Fq + k(\mathrm{sgn}(q_0) - q_0)^2 + (Ax + BCq)^{\mathrm{T}}P(Ax + BCq) \tag{11}$$

Using Eqs.(3), (5), (7) and (9), the time derivative of V can be computed to be

$$\begin{aligned} \dot{V} &= \omega^{\mathrm{T}}J\dot{\omega} + 2q^{\mathrm{T}}F\dot{q} - 2k[(\mathrm{sgn}(q_0) - q_0)\dot{q}_0] + \\ &\quad (A\dot{x} + BC\dot{q})^{\mathrm{T}}P\dot{x} + \dot{x}^{\mathrm{T}}P(A\dot{x} + BC\dot{q}) \\ &= -\omega^{\mathrm{T}}E^{\mathrm{T}}(q)Cy - \dot{x}^{\mathrm{T}}Q\dot{x} + 2\dot{q}^{\mathrm{T}}Cy \\ &= -2\dot{q}^{\mathrm{T}}Cy - \dot{x}^{\mathrm{T}}Q\dot{x} + 2\dot{q}^{\mathrm{T}}Cy \\ &= -\dot{x}^{\mathrm{T}}Q\dot{x} \le 0 \end{aligned} \tag{12}$$

The Lyapunov function candidate V is positive definite and radially unbounded from Eq.(12). And $\dot{V}$ is less than or equal to zero in the whole state space. By the LaSalle Invariance Principle [13], all trajectories converge to the largest invariant set $\psi = \{(\omega, q, x):\ \dot{V} = 0\} = \{(\omega, q, x):\ \dot{x} = 0\}$. On the invariant set we have that $y = B^{\mathrm{T}}P\dot{x} = 0$, $Ax + BCq = \dot{x} = 0$ from Eq.(5). Since $\dot{x} = 0$, then $A\dot{x} + BC\dot{q} = \ddot{x} = 0$, namely $BC\dot{q} = 0$. The matrices B and C are reversible and Eq.(2), therefore $\dot{q} + \dot{q}_0 = 0$. It is clear that $\dot{q}_0 = 0$. From Eq.(3), $\omega = 0$ in the invariant set implies $\dot{\omega} = 0$. And it can be obtained that $J\dot{\omega} = -\omega^{\times}J\omega + u$ (d=0) or $u = J\dot{\omega} + \omega^{\times}J\omega = 0$ from Eq.(5). From Eq.(7), we have that $u = -k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}}(q)Cy - E^{\mathrm{T}}Fq$. That is

$$-k[\mathrm{sgn}(q_0) - q_0]q - E^{\mathrm{T}}Fq = 0 \tag{13}$$

Both sides of Eq.(13) is multiplied by q^{T}, we have that $-k[\mathrm{sgn}(q_0) - q_0]q^{\mathrm{T}}q - q^{\mathrm{T}}(q_0 I - q^{\times})Fq = 0$. Then the above equation can be simplified as $-k[\mathrm{sgn}(q_0) - q_0]q^{\mathrm{T}}q - q_0 q^{\mathrm{T}}Fq = 0$. That is

$$q^{\mathrm{T}}\{k[\mathrm{sgn}(q_0) - q_0]I + q_0 F\}q = 0 \tag{14}$$

Therefore, $k[\mathrm{sgn}(q_0) - q_0] + q_0 f_i = 0$ (i=1,2,3), q=0 and x=0. The largest invariant set is

$\psi = \{(\omega, q, x):\ \omega = 0, q = 0, x = 0\}$, which corresponds to the desired equilibrium. The designed control law can guarantee that the closed-loop system tends to the globally asymptotically stable equilibrium.

3.2 Attitude control with considering external disturbance

The stability of spacecraft is analyzed in the small area of equilibrium ($|q_0|$=1, q=0) when there exists the external disturbance. The controller is constructed as follows.

$$\begin{cases} \dot{x} = Ax + BCq \\ y = B^{\mathrm{T}}P(Ax + BCq) = B^{\mathrm{T}}P\dot{x} \\ u = -k[\operatorname{sgn}(q_0) - q_0]q - E^{\mathrm{T}}(q)Cy - E^{\mathrm{T}}Fq + E^{\mathrm{T}}G\alpha \\ \ddot{\alpha} + \gamma^2\alpha = q \\ \beta = \alpha + \operatorname{sgn}(q_0)G^{-1}d \end{cases} \quad (15)$$

where, $G = \operatorname{diag}\{g_1, g_2, g_3\}$ is the symmetric and positive define matrix, β is a new incoming variable.

And the following condition is satisfied.

$$\ddot{\beta} + \gamma^2\beta = q \quad (16)$$

The control law u includes two parts. $-k[\operatorname{sgn}(q_0) - q_0]q - E^{\mathrm{T}}(q)Cy - E^{\mathrm{T}}Fq$ is the passive control part, which can guarantee that the system achieves global asymptotic stable to the equilibrium point. $E^{\mathrm{T}}G\alpha$ is the suppress vector, which can suppress the sinusoidal disturbance.

Consider the following Lyapunov function candidate

$$\begin{aligned} V = {} & \frac{1}{2}\omega^{\mathrm{T}}J\omega + q^{\mathrm{T}}Fq + k(\operatorname{sgn}(q_0) - q_0)^2 + \\ & (Ax + BCq)^{\mathrm{T}}P(Ax + BCq) + \\ & \dot{\beta}^{\mathrm{T}}G\dot{\beta} + \beta^{\mathrm{T}}G\gamma^2\beta - 2\beta^{\mathrm{T}}Gq \end{aligned} \quad (17)$$

Using Eqs.(3), (5) and (17), the time derivative of V can be computed to be

$$\begin{aligned} \dot{V} = {} & \omega^{\mathrm{T}}J\dot{\omega} + 2q^{\mathrm{T}}F\dot{q} - 2k[(\operatorname{sgn}(q_0) - q_0)\dot{q}_0] + \\ & (A\dot{x} + BC\dot{q})^{\mathrm{T}}P\dot{x} + \dot{x}^{\mathrm{T}}P(A\dot{x} + BC\dot{q}) + \\ & 2\dot{\beta}^{\mathrm{T}}G\ddot{\beta} + 2\beta^{\mathrm{T}}G\gamma^2\dot{\beta} - 2\beta^{\mathrm{T}}G\dot{q} - 2\dot{\beta}^{\mathrm{T}}Gq \\ = {} & \omega^{\mathrm{T}}(-\omega^{\times}J\omega + u + d) + q^{\mathrm{T}}FE\omega + \\ & k(\operatorname{sgn}(q_0) - q_0)q^{\mathrm{T}}\omega + \dot{x}^{\mathrm{T}}(A^{\mathrm{T}}P + PA)\dot{x} + \\ & 2\dot{q}^{\mathrm{T}}Cy + 2\dot{\beta}^{\mathrm{T}}G(\ddot{\beta} + \gamma^2\beta - q) - \dot{\beta}^{\mathrm{T}}GE^{\mathrm{T}}\omega \\ = {} & \omega^{\mathrm{T}}\{-\omega^{\times}J\omega + u + E^{\mathrm{T}}Fq + k(\operatorname{sgn}(q_0) - q_0)q - \\ & E^{\mathrm{T}}G\alpha - d\} - \dot{x}^{\mathrm{T}}Q\dot{x} + 2\dot{q}^{\mathrm{T}}Cy \\ = {} & -\omega^{\mathrm{T}}E^{\mathrm{T}}(q)Cy - \dot{x}^{\mathrm{T}}Q\dot{x} + 2\dot{q}^{\mathrm{T}}Cy \\ = {} & -2\dot{q}^{\mathrm{T}}Cy - \dot{x}^{\mathrm{T}}Q\dot{x} + 2\dot{q}^{\mathrm{T}}Cy \\ = {} & -\dot{x}^{\mathrm{T}}Q\dot{x} \le 0 \end{aligned} \quad (18)$$

It can be known that the Lyapunov function candidate V is positive definite and radially unbounded. And $\dot{V}$ is less than or equal to zero in the whole state space. By the LaSalle Invariance Principle, all trajectories converge to the largest invariant set $\psi = \{(\omega, q, x, \beta):\ \dot{V} = 0\} = \{(\omega, q, x, \beta):\ \dot{x} = 0\}$. On the invariant set we have that $y = B^{\mathrm{T}}P\dot{x} = 0$, $Ax + BCq = \dot{x} = 0$ from Eq.(15). So $A\dot{x} + BC\dot{q} = \ddot{x} = 0$, namely $BC\dot{q} = 0$. The matrices B and C are reversible and Eq.(2), therefore $\dot{q} + \dot{q}_0 = 0$. It is clear that $\dot{q}_0 = 0$. From Eq.(3), $\omega = 0$ in the invariant set implies $\dot{\omega} = 0$. And it can be obtained that $J\dot{\omega} = -\omega^{\times}J\omega + u + d$ or $u + d = J\dot{\omega} + \omega^{\times}J\omega = 0$ from Eq.(5). From Eq.(15), we have that $-k[\operatorname{sgn}(q_0) - q_0]q - E^{\mathrm{T}}(q)Cy - E^{\mathrm{T}}Fq - E^{\mathrm{T}}G\alpha + d = 0$. That is

$$-k[\operatorname{sgn}(q_0) - q_0]q - E^{\mathrm{T}}Fq - E^{\mathrm{T}}G[\alpha + \operatorname{sgn}(q_0)G^{-1}d] = 0$$

Then the above equation can be simplified as

$$-k[\operatorname{sgn}(q_0) - q_0]q - E^{\mathrm{T}}Fq - E^{\mathrm{T}}G\beta = 0 \quad (19)$$

Differentiate Eq.(19), then $\dot{\beta} = 0$, which is in the invariant set, so $\ddot{\beta} = 0$. According to Eq.(16), we know

$$\gamma^2\beta = q \quad (20)$$

Substituting Eq.(20) into Eq.(19), we have

$$-k[\operatorname{sgn}(q_0) - q_0]q^{\mathrm{T}}q - q^{\mathrm{T}}(q_0I - q^{\times})Fq + q^{\mathrm{T}}(q_0I - q^{\times})G\gamma^{-2}q = 0$$

Therefore, $k[\operatorname{sgn}(q_0) - q_0] + q_0\left(f_i - \frac{g_i}{\gamma_i^2}\right) \neq 0$ (i=1, 2, 3), q=0 and x=0. The largest invariant set is $\psi = \{(\omega, q, x, \beta):\ \omega = 0, q = 0, x = 0, \beta = 0\}$, which corresponds to the desired equilibrium. The designed control law can guarantee that the closed-loop system tends to the globally asymptotically stable equilibrium and suppress the effect of external disturbance. Therefore the spacecraft attitude achieves stabilization.

4 Simulations and Results

In order to demonstrate the effectiveness of the designed attitude controller, several numerical simulations are presented in this section.

A spacecraft with the following inertia matrix is considered: $J = \begin{pmatrix} 15 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{pmatrix} (\mathrm{kg \cdot m^2})$. The other main parameters are k=8, -A=P=B=I, $C = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 8 \end{pmatrix}$,

$F=\begin{pmatrix}3&0&0\\0&4&0\\0&0&5\end{pmatrix}$. Assuming that the sinusoidal disturbance torque is $d=\begin{pmatrix}0.1\sin(0.2t)\\-0.06\sin(0.3t)\\0.14\sin(0.4t)\end{pmatrix}(\mathrm{N\cdot m})$. Then $\gamma=\mathrm{diag}(0.2,0.3,0.4)$.

The initial states are $\hat{n}=\begin{pmatrix}0.5345\\0.2673\\0.8018\end{pmatrix}$, $\theta=\dfrac{11\pi}{6}$, $\omega(0)=0$. Then the corresponding quaternion representations are $q(0)=\begin{pmatrix}0.1383\\0.0692\\0.2075\end{pmatrix}$ and $q_0(0)=-0.9659$. At this time, yaw angle, roll angle and pitch angle are 23.56°, 17.21° and 4.98°, respectively.

Simulations are carried out in the following cases.

(1) Consider $G=\begin{pmatrix}0&0&0\\0&0.&0\\0&0&0\end{pmatrix}$ and $G=\begin{pmatrix}0.1&0&0\\0&0.35&0\\0&0&0.7\end{pmatrix}$. The simulation results are shown from Figs. 1-4.

(2) Consider $G=\begin{pmatrix}0.1&0&0\\0&0.35&0\\0&0&0.7\end{pmatrix}$ and $\theta=\dfrac{\pi}{6}$. The value of $q_0(0)$ is changed to 0.9695 and other parameters remain unchanged. And the simulation result is shown in Fig. 5.

(3) Consider $G=\begin{pmatrix}0.1&0&0\\0&0.35&0\\0&0&0.7\end{pmatrix}$. And assume that there exist the model error and model parameter uncertainty, that is $J=\begin{pmatrix}15+0.5&0&0\\0&20+0.4&0\\0&0&10+0.6\end{pmatrix}(\mathrm{kg\cdot m^2})$. The simulation result is shown in Fig. 6.

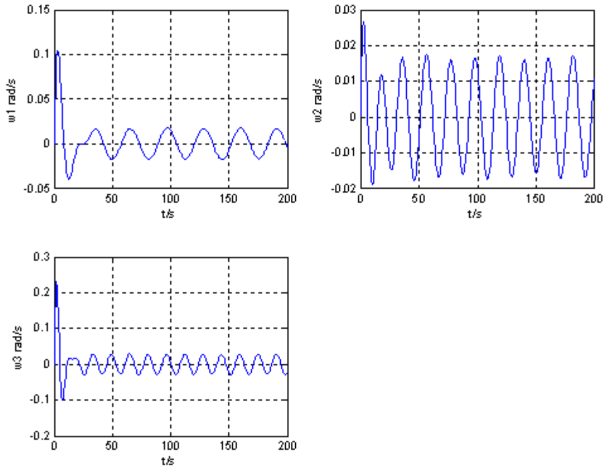

Fig.1 The angular velocity curve without suppress of external disturbance

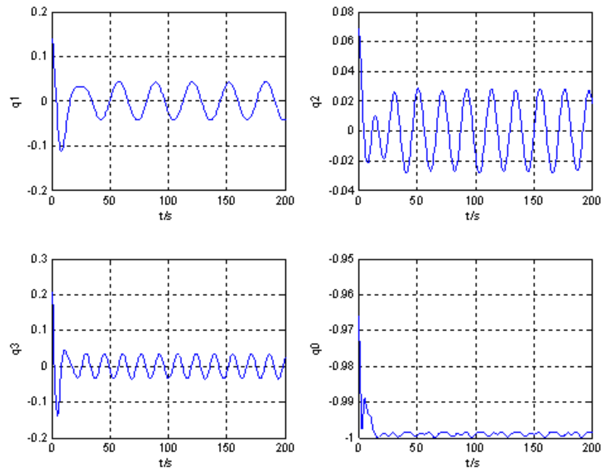

Fig.2 The quaternion curve without suppress of external disturbance

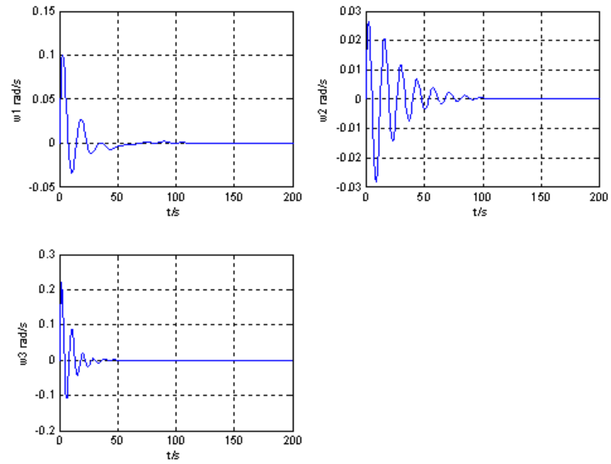

Fig.3 The angular velocity curve with suppress of external disturbance

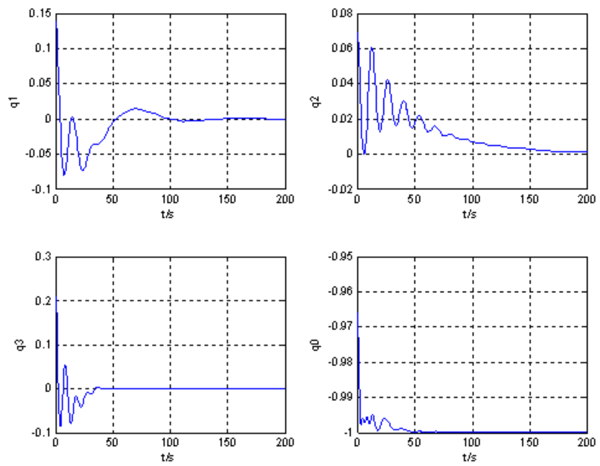

Fig.4 The quaternion curve with suppress of external disturbance

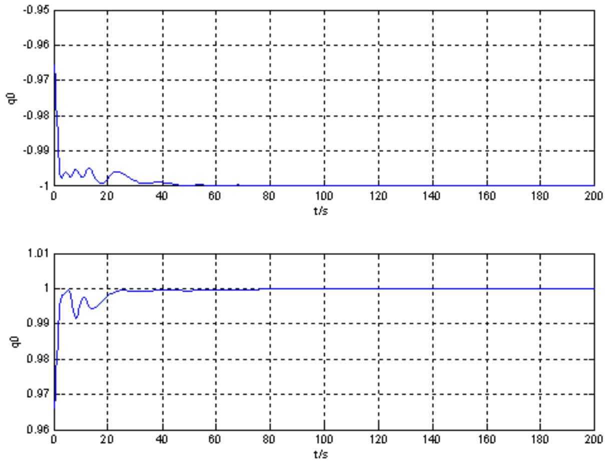

Fig.5 The q_0 curve with different initial value of q_0

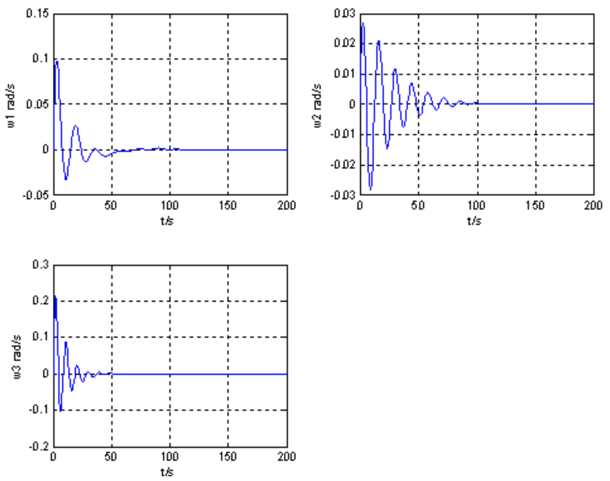

Fig.6 The angular velocity curve with the existence of model error and model parameter uncertainty

The observation from Figs.1-2 is that the designed passive controller without angular velocity measurement can make the closed-loop system described by Eqs.(3), (5) and (15) stable near the equilibrium point and suppress the effect of external disturbance. By comparing Fig.1 with Fig.3 and comparing Fig.2 with Fig.4, it can be seen that the closed-loop system without the suppression vector of external disturbance is greatly affected by the external disturbance and can not converge to the equilibrium point and be not any more stable. However, the closed-loop system with the suppression vector of external disturbance can converge fastly to the equilibrium point. Fig.5 shows that the equilibrium tended to is related to the initial symbol of q_0. When there exist model error and model parameter uncertainty, the performance of the closed-loop system under the control torque is given in Fig.6. Obviously, the system can be still stable at equilibrium point. It shows that the designed controller is robust to model error and model parameter uncertainty.

5 Conclusion

We considered the problem of spacecraft attitude stabilization with the existence of external disturbances and uncertain inertia in this paper. A passive attitude controller is designed, which introduces the suppression vector of external disturbance into the control law. The designed controller does not need the angular velocity measurement and can suppress the effect of external disturbance to a certain extent. In addition, the control law doesn't contain information related to the system parameters, which makes the attitude control system robust to model error and model parameter uncertainty.

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