Method of recognition the radar emitting sources based on the naive Bayesian classifier

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Abstract: The method of recognition the radar emitting sources based on the naive Bayesian classifier is considered in the article. A feature of the method is the comparison of the received radar parameters of the object with the parameters that are contained in the a priori database. The probability of object recognition is revealed based on the results of the comparison. The article shows an algorithm for applying the method in the case of the normal distribution of each radar parameters. The applied conditions of the method are determined depended on the number of non-zero features and the size of the database. Simulation modeling was performed, which showed the possibility of using the recognition algorithm in software and hardware radar systems.

Key-words: radar emitting sources, naive Bayesian classifier, method of radar's recognition, radio-technical signal

1 Introduction

At present, the problem of pattern recognition is one of the main problems in many branches of science and technology. One of the solutions to this problem is the conceptual approach. The object under study, which has a number of properties, is compared with objects from the a priori database. The task is to identify the object that exists in the database, which with the greatest probability corresponds to the object under study. Based on this, a decision is made to recognize (classify) the object.

In many applied radio technical tasks, there is the problem of classification (recognition) of radar emitting sources (RES) [1-4]. It consists in a detailed analysis of the signal structure, which makes it possible to predict the class of RES. The RES class is a set of characteristics of the radar emitting source, which allows it to be attributed to a radio electronic device that solves problems unique to this class [5]. The functional characteristics of the RES characterize one or another radar class. Appendix 1 shows the sequence of the radar classification using the example of the long-range search radar AN / FPS-35.

It is known the processing of radar information can be divided into several stages [6]. At first stage, the microwave signal processing is carried out. Also there is a selection of spatial and frequency characteristics. At second stage, the structure of radio-technical signal is analyzed. Such characteristics of a emitting radio signal as the carrier frequency, signal receiving power, the duration of radio pulses, the pulse repetition period, the signal spectrum are formed [7]. These functional features determine one or another class of modern RES [8]. A set of knowledge what functional features characterize a particular class of radar is contained in the database (classifier).

Commonly the analysis of the functional signal features give an exhaustive answer which class of objects the RES belongs to. For example, if the following parameters are specified as the original data:

- frequency range $\Delta f = 9050 \ldots 10000 MHz$
- pulse duration $\tau = 1 \mu c$
- pulse repetition frequency $F = 750 Hz$

It can be argued with a high probability that this station is also a surface search radar AN/SPS-55 [5].

At present, there are several theoretical methods for solving the problems of classification [1]:

1 - deterministic methods based on handling of logical operators;

2 - stochastical methods using finite stochastical automats and hidden Markov models;

3 - neural methods using artificial neural networks.

There are a number of approaches that make it possible to achieve the required result for these methods: the Dempster-Schafer rule, Bayesian networks, etc.

Actually the application of statistical methods has a number of advantages for modern radar systems. Firstly, the computational capability allows high-speed processing of radar data streams. Secondly, the recognition of the object can be carried out in real time. Therefore Bayes's theorem can be used as one of the special methods of recognition, which allows us to update the probability of a statistical event depending on the incoming information.

This article analyzes the method of classification based on the naive Bayesian classifier. This method assumes the independence of the functional features of the received radio signal. The conditions for its application require solving a number of problems:

1) Evaluation the scope of the method, including the number of functional features to be examined and their relative impact on obtaining a reliable result.

2) Calculation of the recognized probability under the conditions the normal distribution of functional characteristics. Algorithm Development allows this method to be used in software and hardware systems.

3) Evaluation of method's effectiveness using simulation modeling.

It should be said that such a problem formulation places high demands on the database, where the explored objects are contained. The presence of incorrect or inaccurate data in the classifier can lead to significant errors in the evaluation of the final results.

2 Theoretical analysis of the method

A naive Bayesian approach is one of the basic in the theory of classification and underlies many methods [9]. It relies on the assumption that the distribution densities of each evaluated classes are known. The class distribution density expresses the distribution of any radar source parameter (for example, the pulse duration), which is statistically estimated. Therefore it is possible to write out in an explicit form a classification algorithm having a minimum probability of errors.

Suppose we have a set of classes $X = X\{x_1 \quad x_2 \quad \ldots \quad x_K\}$ and a finite set of name classes $Y = Y\{y_1 \quad y_2 \quad \ldots \quad y_S\}$. The conditional (a posteriori) probability $p(y|x)$ can be calculated by the Bayes formula if the values $p(x|y)$ and $P(y)$ are known:

$$p(y|x) = \frac{p(x,y)}{p(x)} = \frac{P(y)\,p(x|y)}{\sum_{S\in Y} p_S(x) P_S} \quad (1)$$

Where $P(y)$ a priori probability of the object recognition;

$p(x|y)$ - the probability of the recognized object $x \in X$ if the object $y \in Y$ is truth;

$p(x) = \sum_{S\in Y} p_S(x) P_S$ - the full probability of class definition X.

The problem of classifying an object can be solved on based of an probabilistic estimate $p(y|x)$ if a priori information is known $P(y)$ and as a result of observation additional data $p(x|y)$ are formed.

The denominator of formula (1) is the sum of all probabilities in the variable $y \in Y$. Expression (1) defines the case when an object is on the probability boundary and can be assigned to several classes. Actually when an object x belongs to one of the classes, you can use a simplified expression

$$p(x,y) = P(y)\,p(x|y) = P(x)\,p(y|x) \quad (2)$$

Where $P(y) \equiv P_y$ - a priori probability of the class y;

$p(x|y) \equiv p_y(x)$ - likelihood function of the class y;

$p(y|x)$ - a posteriori probability of a class y.

Expression (2) defines Bayes' theorem with respect to classification methods: the probability of identifying an object is equal to the product of existing information multiplied by information obtained in the process of studying a given object. It is necessary to find such a transformation algorithm a, for which $X \to Y$ with a minimum probability of error. Therefore it is fair to write down the principle of maximum a posteriori probability

$$a(x) = \arg\max_{y \in Y} p(y|x) = \arg\max_{y \in Y} P_y p_y(x) \quad (3)$$

The expression is called the Bayesian decisive rule.

A particular case of this expression can be written as follows $a(x) = \arg\max_{y \in Y} p_y(x)$, when $P_y = const$. Fig. 1 shows a graphical interpretation of this rule, where the line divides the class boundaries $Y = Y\{y_1 \quad y_2 \quad \dots \quad y_S\}$ in the case of their mutual intersection $\bigcap_S y_S$.

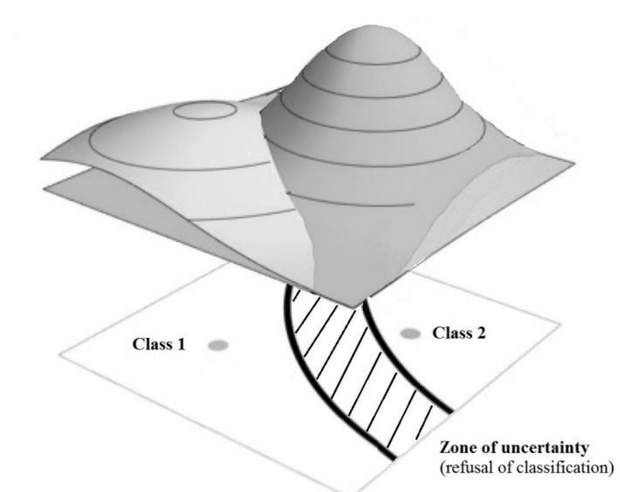


Fig. 1 The class intersection boundary of probability densities

It is also worth noting it is customary to single out a zone of uncertainty in practice that allows for a refusal to classify. In essence, this is zone of false alarm (false positive) or the zone of the event's omission (false negatives). If the features of the probability density are independent $p_{y1}(x_1) \quad p_{y2}(x_2) \quad \dots \quad p_{yN}(x_N)$, then the zones of false alarm or pass of the event are zero, otherwise, when there are zones of false positive or false negative. In the following we shall consider the zone of uncertainty is zero.

In many applied problems, the likelihood function $p(x|y) \equiv p_y(x)$ is determined by several attributes of the class $X = X(x_1 \quad x_2 \quad \dots \quad x_N)$. If the probability densities of these features $p_1(x_1) \quad p_2(x_2) \quad \dots \quad p_N(x_N)$ are independent random variables, the likelihood functions of the classes can be represented [9]

$$p_y(x) = p_{y1}(x_1) \times \dots \times p_{yN}(x_N) = \prod_{i=1}^{N} p_{yi}(x_i|y_i) \quad (5)$$

where $p_{yi}(x_i|y_i)$ - the distribution density of the values of the i attribute for the class y.

Equation (5) expresses the condition for the independence of classes. It is the way to apply and use the naive Bayesian classifier. If the probability density features are independent $p_{y1}(x_1) \quad p_{y2}(x_2) \quad \dots \quad p_{yN}(x_N)$, then the false alarm zones are zero, otherwise, when $p_{y1}(x_1) \cap p_{y2}(x_2) \cap \dots \cap p_{yN}(x_N) \neq \varnothing$ there are zones of false positive or false negative.

3 Algorithm for recognizing by the naive Bayesian classifier

Consider the problem of classifying RES by its radio-technical features. The class distribution densities are independent and there is no zone of uncertainty.

Let $M = \{m_1 \quad m_2 \quad \dots \quad m_k\}$ - the available features (parameter) classes obtained as a result of radio-technical analysis of the electronic environment;

$N = \{n_1 \quad n_2 \quad \dots \quad n_L\}$ - name classes obtained as result of a priori research of all radio electronic means.

It is required to set such a unique possible ratio between the class of features $m_i \in M$ and the class of names $n_j \in N$ according to the classifier of radio-electronic and radio-technical means, which set the maximum a posteriori probability $p(n_j | m_i)$ for each class of names.

Any class of features of a radio-technical means is limited to a number of parameters. For RES, it is customary to evaluate a number of parameters that most clearly give an idea of the test signal [10]: (Appendix 2):

- Carrier signal frequency $m_1 = f$;

- Pulse duration $m_2 = \tau$;
- Pulse repetition period $m_3 = T$;
- Radar survey period $m_4 = \upsilon$.

This sequence is analyzed by the receiver of the radar station and provides the measured signal parameters - $f_{CARRIER}$, $\tau_{DURATION}$, $T_{REPETITION}$, υ_{SURVEY}. These results in the target form about the object write in main memory of radar processing computer.

Obviously there is a potential accuracy in the estimation of parameters in the process of measuring. If it is dealing with a radio signal, the parameters of which do not change significantly within the classification stage, in this case it is expedient to use estimated values. Each of the parameters of the radio signal is measured with a specified accuracy. The probability of parameter detection can be estimated according to the normal distribution law with known values of the root-mean-square deviation and the mathematical expectation:

$$P_1(m_i) = \frac{1}{\sigma_{n1}[m_i]\sqrt{2\pi}} \exp\left(-\frac{\left(\mathrm{E}[m_i] - \overline{\mathrm{E}}[m_i]\right)^2}{2\sigma_{n1}^2[m_i]} \right) \quad (6)$$

Where $P_1(m_i)$ - the probability of identifying the signal parameter by its statistical characteristics;

$\overline{\mathrm{E}}[m_i]$ - mathematical expectation of the signal parameter, set according to the classifier $a(M)$;

$\mathrm{E}[m_i]$ - the averaged value of the signal parameter obtained as a result of observing the object;

σ_{m_i} - root-mean-square deviation of the signal parameter, set according to the measuring unit.

The received target form contains the average value $\mathrm{E}[m_i]$, which must be compared with known value from the classifier $\overline{\mathrm{E}}[m_i]$. Hence the parameter probability must be belonged to a given class of objects. When four radio-technical parameters are observed, the probability is determined according to the normal distribution law (Fig. 2-5)

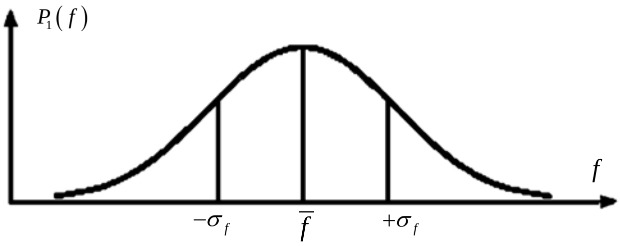


Fig. 2 Normal distribution of frequency

$$P_1(f) = \frac{1}{\sigma_f\sqrt{2\pi}} \exp\left(-\frac{\left(f - \overline{f}\right)^2}{2\sigma_f^2} \right)$$

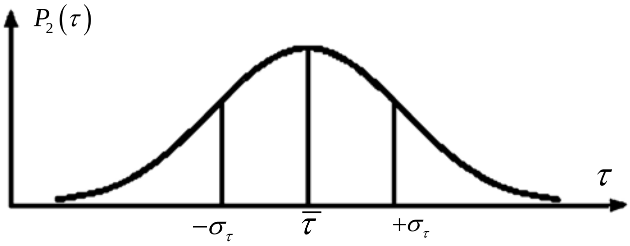


Fig. 3 Normal distribution of frequency pulse duration $P_2(\tau) = \frac{1}{\sigma_\tau\sqrt{2\pi}} \exp\left(-\frac{\left(\tau - \overline{\tau}\right)^2}{2\sigma_\tau^2} \right)$

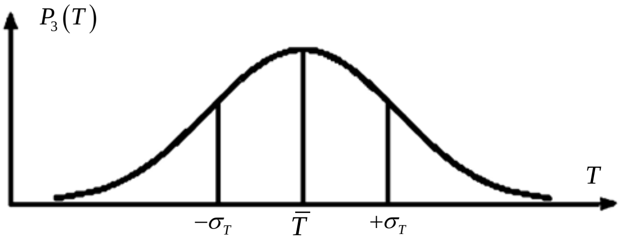


Fig. 4 Normal distribution of frequency pulse repetition period $P_3(T) = \frac{1}{\sigma_T\sqrt{2\pi}} \exp\left(-\frac{\left(T - \overline{T}\right)^2}{2\sigma_T^2} \right)$

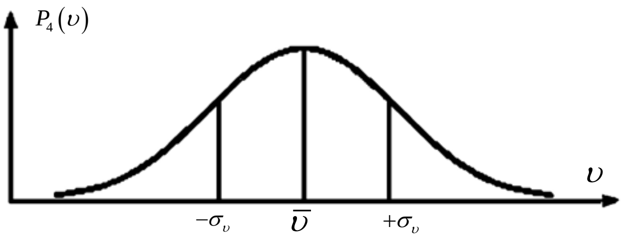


Fig. 5 Normal distribution of frequency radar survey period $P_4(\upsilon) = \frac{1}{\sigma_\upsilon\sqrt{2\pi}} \exp\left(-\frac{\left(\upsilon - \overline{\upsilon}\right)^2}{2\sigma_\upsilon^2} \right)$

Consider the case where a passive radar station detects RES with the following radio-technical parameters (features) $M_0 = \{m_{10} \quad m_{20} \quad \dots \quad m_{k0}\}$. So the detection probability with RES-signal will be:

Table 1 Probability of detecting RES

Source	Carrier signal frequency, f	Pulse duration, τ	Pulse repetition period, T	Radar survey period, T
RES1	$P\left(f_{RES1} \middle| f_0\right)$	$P\left(\tau_{RES1} \middle| \tau_0\right)$	$P\left(T_{RES1} \middle| T_0\right)$	$P\left(\upsilon_{RES1} \middle| \upsilon_0\right)$
RES2	$P\left(f_{RES2} \middle| f_0\right)$	$P\left(\tau_{RES2} \middle| \tau_0\right)$	$P\left(T_{RES2} \middle| T_0\right)$	$P\left(\upsilon_{RES2} \middle| \upsilon_0\right)$
RES3	$P\left(f_{RES3} \middle| f_0\right)$	$P\left(\tau_{RES3} \middle| \tau_0\right)$	$P\left(T_{RES3} \middle| T_0\right)$	$P\left(\upsilon_{RES3} \middle| \upsilon_0\right)$
…	…	…	…	…
RESL	$P\left(f_{RESL} \middle| f_0\right)$	$P\left(\tau_{RESL} \middle| \tau_0\right)$	$P\left(T_{RESL} \middle| T_0\right)$	$P\left(\upsilon_{RESL} \middle| \upsilon_0\right)$

The basic hypothesis of the naive Bayesian classifier is the assumption the independence of the conditional probabilities obtained for each parameters of the detected signal. Expression (5) allows us to find the probability value for each object from the class of names that characterize the overall probability of identifying the object. Analysis of the signal shows that the receiver can detect the carrier frequency of the signal, while the pulse duration (or pulse repetition period, or frequency of the radar survey) doesn't depend on this frequency.

Thus the following expressions can be written for conditional probabilities of detecting a signal parameter:

$$p_1(M) = p_1(m_1) \times p_1(m_2) \times \ldots \times p_1(m_k) = \prod_{i=1}^{k} p_1\left(m_k \middle| m_0\right) \quad (7)$$

$$p_2(M) = p_2(m_1) \times p_2(m_2) \times \ldots \times p_2(m_k) = \prod_{i=1}^{k} p_2\left(m_k \middle| m_0\right) \quad (8)$$

……

$$p_L(M) = p_L(m_1) \times p_L(m_2) \times \ldots \times p_L(m_k) = \prod_{i=1}^{k} p_L\left(m_k \middle| m_{i0}\right) \quad (9)$$

In this expression $p_1(M) \quad p_2(M) \quad \ldots \quad p_L(M)$ - the probability of classifying a name classes n_L for all parameters $M = \{m_1 \quad m_2 \quad \ldots \quad m_k\}$ of the detected signal, which has features $M_0 = \{m_{10} \quad m_{20} \quad \ldots \quad m_{k0}\}$.

If we substitute the values of (6) into equations (7) - (9) and simplify them, then we obtain the following system of equations:

$$p_1(M) = \frac{1}{\prod_{i=1}^{k}\left(\sqrt{2\pi}\sigma_i[m_i]\right)} \exp\left[-\sum_{i=1}^{k} \frac{\left(\mathrm{E}[m_i] - \bar{\mathrm{E}}_1[m_i]\right)^2}{2\sigma_i^2[m_i]}\right] \quad (10)$$

$$p_2(M) = \frac{1}{\prod_{i=1}^{k}\left(\sqrt{2\pi}\sigma_i[m_i]\right)} \exp\left[-\sum_{i=1}^{k} \frac{\left(\mathrm{E}[m_i] - \bar{\mathrm{E}}_2[m_i]\right)^2}{2\sigma_i^2[m_i]}\right] \quad (11)$$

……

$$p_L(M) = \frac{1}{\prod_{i=1}^{k}\left(\sqrt{2\pi}\sigma_i[m_i]\right)} \exp\left[-\sum_{i=1}^{k} \frac{\left(\mathrm{E}[m_i] - \bar{\mathrm{E}}_L[m_i]\right)^2}{2\sigma_i^2[m_i]}\right] \quad (12)$$

The obtained expressions determine the probability that the class of objects $M_0 = \{m_{10} \quad m_{20} \quad \ldots \quad m_{k0}\}$ is consistent with the parameters of an arbitrary class $M_j = \{m_{j1} \quad m_{j2} \quad \ldots \quad m_{jk}\}$ that are selected according to the classifier $a(M)$. It means the received signal with radio technical parameters $M_0 = [f_0 \quad \tau_0 \quad T_0 \quad \upsilon_0]^T$ is subsequently processed with a set of parameters $M = [f_j \quad \tau_j \quad T_j \quad \upsilon_j]^T$ belonging to each of the stations $1 \ldots L$ that are present in the classifier $a(M)$.

In addition to the obtained probabilities $p_1(M) \quad p_2(M) \quad \ldots \quad p_L(M)$, the classification evaluation takes into account available information on the name classes, which is determined by the values of a priori probability P_y. According to expression (4) in this case, we can write the equation of maximum a posteriori probability, which allows us to choose from the classifier that name class that most closely matches the calculated data:

$$a_1(M) = \frac{P_1}{\prod_{i=1}^{k}\left(\sqrt{2\pi}\sigma_i[m_i]\right)} \exp\left[-\sum_{i=1}^{k} \frac{\left(\mathrm{E}[m_i] - \bar{\mathrm{E}}_1[m_i]\right)^2}{2\sigma_i^2[m_i]}\right] \quad (13)$$

$$a_2(M) = \frac{P_{i2}}{\prod_{i=1}^{k}\left(\sqrt{2\pi}\sigma_i[m_i]\right)} \exp\left[-\sum_{i=1}^{k} \frac{\left(\mathrm{E}[m_i] - \bar{\mathrm{E}}_2[m_i]\right)^2}{2\sigma_i^2[m_i]}\right] \quad (14)$$

……

$$a_L(M) = \frac{P_L}{\prod_{i=1}^{k}\left(\sqrt{2\pi}\sigma_i[m_i]\right)} \exp\left[-\sum_{i=1}^{k} \frac{\left(\mathrm{E}[m_i] - \bar{\mathrm{E}}_L[m_i]\right)^2}{2\sigma_i^2[m_i]}\right] \quad (15)$$

Where $P_1 \quad P_2 \quad \ldots \quad P_L$ the value of the a priori probability for each of the name classes $1 \quad 2 \quad \ldots \quad L$.

The resulting values of the a posteriori probability $a_1(M) \quad a_2(M) \quad \ldots \quad a_L(M)$ make it possible to find out which object from the name classes $1 \quad 2 \quad \ldots \quad L$ most plausibly corresponds to the features of the objects classes with the obtained parameters $M_0 = \{m_{10} \quad m_{20} \quad \ldots \quad m_{k0}\}$. Essentially, the algorithm of the naive "Bayesian" classification consists in selection the largest value $a_j(M) = \max$

from the set of obtained values of the name classes $\{a_1(M) \quad a_2(M) \quad \dots \quad a_N(M)\} \in [0,1]$.

It is important to note that the obtained probabilities are absolute values. In real conditions, they must be normalized with respect to a known value $\prod_{i=1}^{k}\left(\sigma_j[m_i]\times\sqrt{2\pi}\right)$.

Formulas (13) – (15) reflect the probability of "comparing" the tested RES with each of the objects listed in the classifier. The final decision on the comparison is taken based on the maximum probability obtained for a particular type of station j:

$$a_j(M) = \max\arg\left\{\frac{P_j \exp\left[-\sum_{i=1}^{k}\frac{\left(\mathrm{E}[m_i]-\bar{\mathrm{E}}_j[m_i]\right)^2}{2\sigma_i^2[m_i]}\right]}{\prod_{i=1}^{k}\left(\sqrt{2\pi}\sigma_i[m_i]\right)}\right\} \quad (16)$$

The choice of a classified object is carried out in two alternative ways:

1) the maximum value from the whole name classes $a_j(M) = \max\{a_j : j \in L\}$;

2) in case of exceeding the threshold level $a_j(M) > P_{TH}$

It should be noted that often has to deal with incomplete information of features in practical problems of analyzing detected radio emission. The value of a priori probability for a name classes can then have different values $\{P_1 \quad P_2 \quad \dots \quad P_L\} \in [0,1]$. For example, a radio source generates a signal with a random value of the pulse duration. In this case, estimating the magnitude τ is extremely problematic. For this reason, the feature of RES will not be informationally complete $M = \{f_0 \quad \varnothing \quad T_0 \quad \upsilon_0\}$. Another example RES generates a chirp signal for analyzing the Doppler shift, where a variable carrier frequency is present $(f \neq const)$ and there is a change in the pulse duration $(T \sim inv)$. In this case, the features will have the following information set $M = \{\varnothing \quad \tau_0 \quad \varnothing \quad \upsilon_0\}$.

4 Evaluation of the effectiveness

The estimated efficiency of the method was carried out by modeling the algorithm on the automated operator workplace which the radio signals imitation sensors were connected to (Fig. 6, 7). The signal sensors generated arbitrary values of the parameters f, τ, T, υ in a pseudo-random manner.

Fig. 6 Automated operator workplace

	RADAR TYPE	STATION	PROBABILITY
1	(surface search)	APS-124	0.0388
2		radar-1	0.0281
3	(air search)	APS-115	0.0172
4	(landed control)	SPN-35	0.0141
5	(countermeasure)	ALQ-165	7.3741e-16
6	(countermeasure)	ALQ-214	7.3741e-16
7		SPQ-9B	4.4085e-73
8		radar-2	3.0656e-142

Fig. 7 Software classification of RES

A test database was used from the source [11] where the radio technical features were introduced for a set of on-board, offshore and ground-based radars. Software was written in MatLAB 2012b to assess the classification probability of a particular RES.

Modeling showed that the estimation of a priori probability P_L depends on the size of the database L and the number of non-zero features k. Typically, the database is used when $L > 1000$. Therefore the value of the joint probability increases $\alpha = \bigcap_{j=1}^{L} p_j(M)$. It shows a type 1 error for a name classes and does the ability incorrectly to classify several different stations. The value α depends on the accuracy of the detected signal parameters σ_i.

A more significant contribution to the accuracy of the a priori probability P_L is made by the number of parameters k. In fact the quantity of evaluated features for RES usually does not exceed $4\dots7$. The mathematical expression for the value can be set heuristically:

$$P_L = \bigcup_{i=1}^{k}\frac{\Delta m_i}{\sigma_i \times L} \quad (11)$$

Where Δm_i - the range of the feature;

σ_i - root-mean-square deviation in the evaluation of the feature;

L - the number of name classes in the database.

Table 2 shows the experimental values P_L obtained as a result of modeling with $L > 1000$ and $k = 4$.

Table 2 Experimental values P_L

The case of detecting 1 parameter out of 4 possible parameters	$P_{(1\mid 4)} \approx 0,003$
The case of detecting 2 parameter out of 4 possible parameters	$P_{(2\mid 4)} \approx 0,04$
The case of detecting 3 parameter out of 4 possible parameters	$P_{(3\mid 4)} \approx 0,15$
The case of detecting 4 parameter out of 4 possible parameters	$P_{(4\mid 4)} \approx 0,52$

It is often necessary to choose a priori probability P_L in practical problems based on experimental data. Actually each estimated signal value according to the classifier can belong to several types of RES. For example, the pulse duration can have a signal $\tau_{ИМП} = 0,5\mu s$ transmitted by both the navigation station Marconi LN66 and the marine radar SPS-64 [5]. Then it is necessary to choose a probability such that the appearance of a type 1 error is avoided.

It should be noted that the method of naive "Bayesian" classification can be widely applied in software and hardware systems. For example, control systems of airborne, surface and sea complexes use for air and surface analysis about detected emitted targets. At the same time, structural data of objects classes (signal parameters, types of radar stations) can be modified depending on the conditions of use.

5 Conclusions

The article describes a method that allows to identify received radar emitting sources with features of radio signal, which is available in the database. On the basis of a naive Bayesian classifier, it is possible to determine which radar classes one or another radio source can be assigned with a given probability level. In the case of a normal "Gaussian" distribution of parameters, RES is classified according to the maximum a posteriori probability:

$$a_j(M) = \max \arg \left\{ \frac{P_j \exp\left[-\sum_{i=1}^{k} \frac{\left(\mathrm{E}[m_i] - \bar{\mathrm{E}}_j[m_i]\right)^2}{2\sigma_i^2[m_i]} \right]}{\prod_{i=1}^{k} \left(\sigma_i[m_i] \times \sqrt{2\pi}\right)} \right\} \quad (12)$$

Where P_j the conditional probabi~~lity~~ for given number of radio-technical features;

$\bar{\mathrm{E}}[m_i]$ - mathematical expectation of the signal parameter according to the classifier;

$\mathrm{E}[m_i]$ - the averaged value of the signal parameter obtained as a result of observation;

σ_{m_i} - root-mean-square deviation of the signal parameter.

There are limitations in the application of the method:

- Probability density of radio technical features should be non-overlapping $\bigcap_{i=1}^{k} p_i(M) = \varnothing$. This provides an independent evaluation of each parameter, which may include the carrier frequency, pulse duration, pulse repetition period etc.;

- Fullness of the database determines the classification probability of the RES. On the other hand, the probability of type 1 error increases, which leads to an incorrect classification of RES;

- Increase of radar features makes it possible to more accurately recognize RES. The number of parameters is associated with the conditional probability of the classification error. The absence of a parameter reduces this probability. It was experimentally established that for a database with more than 1000 objects and the 4 features, the distribution will be:

Table 3 Probability values for 4 features

1 feature of 4	$P_{(1\mid 4)} \approx 0,003$
2 feature of 4	$P_{(2\mid 4)} \approx 0,04$
3 feature of 4	$P_{(3\mid 4)} \approx 0,15$
4 feature of 4	$P_{(4\mid 4)} \approx 0,52$

The method of the naive Bayesian classifier can be used to create algorithms. It is possible to evaluate the probability of RES recognition. The task of determining the class of a radio object simplified in software and hardware.

References:

[1] Kouemou G. (ed.) *Radar Technology*. InTech, 2010. – 430 p. – ISBN 978-953-307-029-2.

[2] Anderson S. J. Target classicfiation, recognition and identification with HF radar / S. J. Anderson // *Proceedings of the NATO Research and Technology Agency Sensors and Electronics Technology Panel Symposium* SET–080/RSY17/RFT: «Target identification and recognition using RF systems», 11–13 October, 2004, Oslo, Norway. RTO–MP–SET–080. — Oslo, 2004. — P. 1–20.

[3] Greco M. Radar detection and classification of jamming signals / M. Greco, F. Gini, A. Farina, V. Ravenni // *14th European Signal Processing Conference* (EUSIPCO 2006), Florence, Italy, September 4-8, 2006. – P. 21-25.

[4] Lu D. A survey of image classification methods and techniques for improving classification performance / D. Lu, Q. Weng // *International Journal of Remote Sensing* Vol. 28, No. 5, 10 March 2007, P. 823–870.

[5] Friedman N. Naval Radar. Conway Maritime Press, 1981. – 240 p.

[6] *Electronic Warfare and Radar Systems Engineering Handbook*. Naval Air Warfare Center Weapons Division. – Point Mugu. – California, 2013. – 455 p.

[7] Kvasnov A. Methods the integration of information obtained from radioelectronic and optoelectronic sensors of the coverage and detection / Anton Kvasnov // *Proceedings of the International Symposium. Automated systems and technologies AST'2017*. Peter the Great St. Petersburg Poltechnic University. – Saint Petersburg, 14-17 June 2017. – P. 67-74.

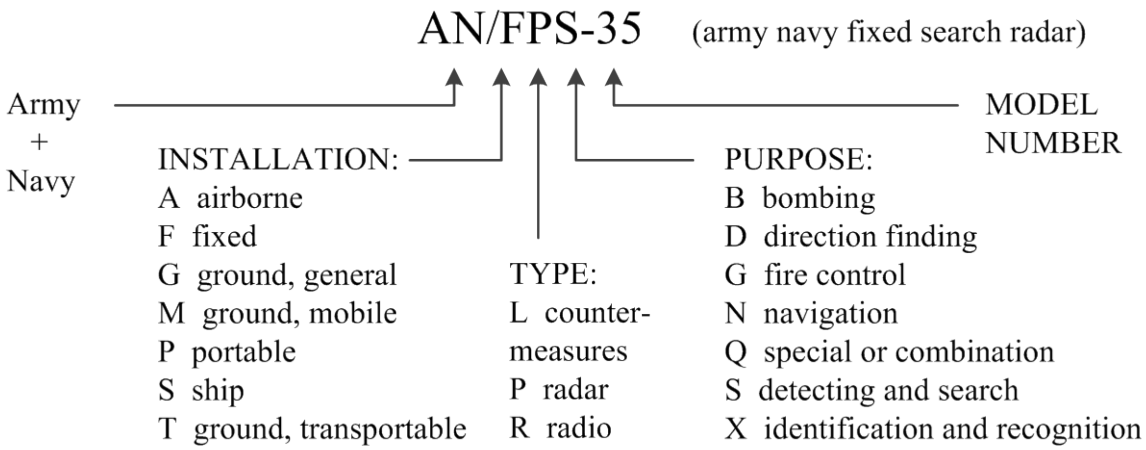
[8] Jochumsen L. W. *Radar Target Classification using Recursive Knowledge-Based Methods*. PhD Dissertation. Aalborg University Press, 2016. – 133 p.

[9] Rosner B. *Fundamentals of Biostatistics*. 7th edition. Brooks/Cole Cengage Learning, 2010. – 859 p.

[10] Skolnik M. I. *Radar handbook 3rd edition* / Merill I. Skolnik. – New-York: The McGraw-Hill Companies, 2008. – P. 1327.

[11] AN Equipment Forecast Overview. Accessed 06 January 2017, <https://www.forecastinternational.com>.

Appendix 1. Joint service classification system



Appendix 2. Estimated parameters of RES