Singular Perturbation Method Applied to Power Factor Correction Converter Application

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Abstract: - A linear discrete stable control system is considered. The Power Factor Correction (PFC) converter to allow independent control of current and voltage. It converter are fast and slow states to inheres sty present small parameters inductor and capacitor its computes stiffness and to include switching ripple effects. As an alternative a Singular Perturbation Method (SPM) is presented Initial Value Problem (IVP) and Boundary Value Problem (BVP). It is applied to two state switching power converters to provide rigorous justification of the time scale separation. It is modeled as a one parameter singularly perturbed system. SPM consists of an outer series solution and one boundary layer correction (BLC) solution. A boundary layer correction is required to recover the initial conditions lost in the process of degeneration and to improve the solution. SPM is carried out up to second-order approximate solution for the PFC converter model for IVP and BVP. The results are compared with the exact solution (between with and without parameters). The results substantiate the application.

Key-Words: - Power Factor Correction; Singular Perturbation Method; Boundary Layer Correction; Initial Value Problem; Boundary Value Problem.

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1 Introduction

The singularly perturbed systems are ill-conditioned systems with computational stiffness. Hence exact solution of these systems requires special numerical methods to overcome this stiffness. The SPM removes the stiffness of the system, reduces the order of the system and satisfies all the specified boundary conditions thereby giving a solution very close to the exact solution. The singularly perturbed systems are time-scale systems. A two-time scale system results in one parameter singularly perturbed system. A three-time scale system results in a two parameter singularly perturbed system. Similarly a Multi Time Scale (MTS) system results in n parameter singularly perturbed system. MTS systems are highly stiff exhibiting chaotic behavior with butterfly phenomenon. Obviously these systems need an alternative. The alternative is SPM. The singular perturbation theory is well developed for continuous-time control systems [1], [2], [3], [4], [5] compared to discrete-time control systems [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17]. Chaos associated with multiple time-scales exhibiting butterfly phenomenon [18]. [19], [20], [21] creates hurdles for finding the solution.

Conventional wisdom in power electronics is that in DC–DC converters and many other applications, inductor currents are “fast” state variables, while capacitor voltages are “slow” state variables. Often, this is used as justification for a particular design methodology or control scheme as singular perturbation method. As power conversion densities increase, switching frequencies increase, control bandwidths increase, and components are miniaturized, a designer should wonder whether the conventional wisdom is still valid. Frequently, controllers for DC–DC converters use two loops: an inner current loop and an outer voltage loop. The current loop can take many forms. If there is a separation in timescales between the current dynamics and voltage dynamics, the two loops can be designed independently.

While timescale separation, removes the stiffness of the system and reduces the order of the system are important for many DC–DC converters, power factor correction converters require singular perturbation method for proper operation. The objective of a PFC converter is to force an inductor current to follow the input voltage wave shape (normally a rectified sinusoid), while the output capacitor voltage is as close to dc as possible [22]. The typical solution is to use a large output capacitor to smooth out the power fluctuations from the input. Hence, reduce error between with separation and without separation by SPM.

Here a singular perturbation theory for DC–DC Converters and application to PFC Converters with two time scales is considered as a case study. It is modeled as a one parameter SPS then IVP and BVP are studied using the SPM extended up to second-order approximation

Singular perturbation theory [2] is a tool for formally partitioning a dynamic system into slow and fast variables. The two timescales differ in scale by a small parameterε. The fast variables, denoted here as x1, are related to the slow variables, denoted as x0, by an integral manifold (an algebraic relation) plus a small dynamic error parameter ε.

2. Singular Perturbation Method

I. Discrete Multi-Parameter Problem

The multi-parameter discrete control systems are being studied extensively. From control view here we present the Discrete Multi-Parameter Singular Perturbation Method (DMPSPM) in state space form. Consider the linear, singularly perturbed multi-parameter discrete control system. This can be represented as

$$x_j(k+1) = A\,[\mu_0 \dots \mu_j\ x_j(k)] + B\,u(k) \qquad (1a)$$

$$x_j(k+1) = A\,[\varepsilon_0 \dots \varepsilon_j\ x_j(k)] + B\,u(k) \qquad (1b)$$

$$x_j(k=0) = x_j(0),\ j = 0, 1, \dots, n.$$

where $A=\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1} & A_{m2} & \dots & A_{mm} \end{bmatrix}$, $B=\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_m \end{bmatrix}$

and state vector $x_{j-1}(k) \in R^{n_j}$, j=1, 2, …, m; m = n+1. Aij and Bi are matrices of suitable dimensionality. The control vector free of the small parameters is $u(k) \in R^r$. Redefined the parameters as $\varepsilon_1 = \mu_1$ and $\varepsilon_j = \frac{\mu_j}{\mu_{j-1}}, j = 2, \dots, n.$ The parameter $\varepsilon_0 = \mu_0,$ ε_1 is not a small parameter and is introduced to facilitate the presentation of the multi-parameter problem. The initial conditions of the system (1b) are

$$x_j(k=0) = x_j(0),\ j = 0, 1, \dots, n. \qquad (1c)$$

The $(n_1+n_2+\dots+n_m)$ order discrete TPBVP represented by (1) is said to be in singularly perturbed form based on the degenerate TPBVP

$$\begin{bmatrix} x_0^{0\dots0}(k+1) \\ x_j^{0\dots0}(k+1) \end{bmatrix} = A \begin{bmatrix} x_0^{0\dots0}(k) \\ 0 \end{bmatrix} + B\,u(k),$$

$$j = 1, 2, \dots, n. \qquad (2a)$$

obtained by suppressing the small parameters $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ in (2a) is of order n1 and can satisfy the boundary conditions of slow modes only, resulting in

$$x_0^{00\dots0}(k=0) = x_0(0) \text{ and } x_j^{00\dots0}(k=0) \neq x_j(0),$$

$$j = 1, 2, \dots, n. \qquad (2b)$$

The $(n_2+\dots+n_m)$ initial condition missing in the process of degeneration are restored by the following singular perturbation method.

II. Initial and Boundary Value Problems for Singular Perturbation Method

(a).Outer solution

Asymptotic expansions for the outer solution are expressed in terms of the small parameters as

$$\{x_{v,o}(k)\} = \sum_{i,j,\dots,r\geq 0}^{q} [x_v^{ij\dots r}(k)]\,\varepsilon_1^i \varepsilon_2^j \dots \varepsilon_n^r,$$

$$v = 0, 1, \dots, n. \qquad (3)$$

for qth order of approximation. By substituting (3) in (1b) and equating like powers of the small parameter a set of equations may be obtained. (2a) is the resulting zero-order equation. We can get equations may be obtained for n≥3.

(b). Boundary layer correction (BLC) solutions

In order to get back the boundary conditions lost due to degeneration, to supply the required boundary conditions to solve the outer and BLC equations and to get a distinctive solution, the following transformations need to be applied for the n boundary layer corrections.

Transformations for BLC:

$x_{0ci}(k) = x_0(k)/(\varepsilon_1 \dots \varepsilon_i)^{k+1}$, i=1, 2…, n;

$x_{1c1}(k) = x_1(k)/(\varepsilon_1)^k$;

$x_{1ci}(k) = x_1(k)/(\varepsilon_1^k(\varepsilon_2 \dots \varepsilon_i)^{k+1})$, i=2, 3, …, n.

$x_{2ci}(k) = x_2(k)/(\varepsilon_1 \dots \varepsilon_i)^k$, i=1, 2;

$x_{2ci}(k) = x_2(k)/((\varepsilon_1\varepsilon_2)^k(\varepsilon_3 \dots \varepsilon_i)^{k+1})$,i=3,..., n;

… …

$x_{nci}(k) = x_n(k)/(\varepsilon_1 \dots \varepsilon_i)^k$,i=1,2,…,n. (4)

Here suffix c refers to initial boundary layer correction.

BLC Equations:

BLC equations may be obtained by seeking asymptotic expansions for n initial BLC as

$$\{x_{vcs}(k)\} = \sum_{ij\ldots r\geq 0}^{q} \left\{x_{vcs}^{ij\ldots r}(k)\right\} \varepsilon_1^i \varepsilon_2^j \ldots \varepsilon_n^r;$$

v = 0, 1, … n. s = 1, 2, …, n. (5)

for qth order of approximation. By substituting (5) in (4) and collecting the coefficients of like powers of the small parameters $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$; a set of subsystems for BLC may be obtained.

(c). Total series solution (TSS)

For a desired order of approximation q, the total series solution of states x(k) may be obtained from outer and BLC solutions as

$$x_0^q(k) = \sum_{i,j,\ldots,r\geq 0}^{q} \{x_0^{ij\ldots r}(k)\}\varepsilon_1^i \varepsilon_2^j \ldots \varepsilon_n^r + \sum_{s=1}^{n} (\varepsilon_1 \ldots \varepsilon_s)^{k+1} \sum_{i,j,\ldots,r\geq 0}^{q} \{x_{0cs}^{ij\ldots r}(k)\}\varepsilon_1^i \varepsilon_2^j \ldots \varepsilon_n^r$$

$$x_f^q(k) = \sum_{i,j,\ldots,r\geq 0}^{q} \{x_f^{ij\ldots r}(k)\} \varepsilon_1^i \varepsilon_2^j \ldots \varepsilon_n^r + \sum_{s=1}^{f} (\varepsilon_1 \ldots \varepsilon_s)^{k} \sum_{i,j,\ldots,r\geq 0}^{q} \{x_{fcs}^{ij\ldots r}(k)\}\varepsilon_1^i \varepsilon_2^j \ldots \varepsilon_n^r + \sum_{s=f+1}^{n} (\varepsilon_1 \ldots \varepsilon_f)^{k} (\varepsilon_{f+1} \ldots \varepsilon_s)^{k+1} * \sum_{i,j,\ldots,r\geq 0}^{q}\{x_{fcs}^{ij\ldots r}(k)\}\varepsilon_1^i \varepsilon_2^j \ldots \varepsilon_n^r \quad f=1,2,\ldots,n. \quad (6)$$

Here terms with negative power for singular perturbation parameters ε_i are defined to be zero, if any.

(d). Boundary conditions

The boundary conditions to solve outer equations (2a, 3) and BLC equation are to be provided in advance. These are determined uniquely from the fact that the total series solution (6) should satisfy the specified boundary conditions (1c). Consequently the following boundary conditions result [10], [11], [17].

(e). Algorithm

The algorithm is similar to other SPM [15], [17]. First start with zero-order solution to improve the degenerate solution. Here Outer and BLC solutions to be found using outer and BLC equations and conditions (5, 6).Then added according to TSS to get the zero-order solution. Similar procedure to be followed for first and higher order approximate solutions for further improvement.

3. Boost Converter Analysis

The PFC boost converter shown in Fig. 1 can be modeled as a switched linear system.

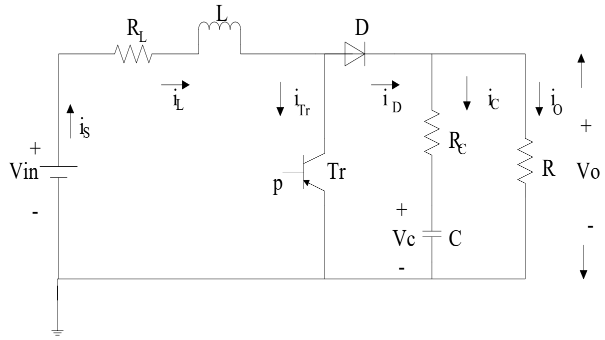


Fig. 1: Boost converter

The PFC application, implies large variation in input voltage and inductor current; otherwise, the dynamics are identical to any other boost converter. The switched linear model is

$$\frac{dv_C}{dt} = -\frac{1}{C(R+R_c)}v_C + S_d\frac{1}{C(R+R_c)}i_L$$

$$\frac{di_L}{dt} = -S_d\frac{R}{L(R+R_c)}v_C - \frac{R_L + S_d(RR_L/(R+R_L))}{L}i_L + \frac{V_{in}}{L} \quad (7)$$

Here, Sd is the switching function of the diode. All variables and coefficients must be normalized to put the system into standard form. The nominal output voltage is V_0, the nominal output current is $I_0 = V_0/R$, and the switching period is T. With these definitions, the other variables can be normalized on the basis

$$\varepsilon = \frac{L}{CR^2}$$

$$w = \frac{V_{in}}{V_o}$$

$$\delta = \frac{R_L}{R}\frac{(R+R_C)}{R}$$

$$S_d = 1 - d = u$$

$$\hat{v}_C = \frac{v_C}{V_O}$$

$$\hat{i}_C = \frac{i_L}{i_O}$$

$$p = \frac{T}{C(R_C+R)}$$

$$\hat{t} = \frac{T}{C(R_C+R)} \quad (8)$$

The first two variables are the normalized states. Normalized input voltage w is a disturbance input. The moving average of S_d, shown as u, is used as the input in the following analysis; often, the actual input is d, the duty cycle of the controlled switch. Then, the switching period T must be transformed into p on the $\hat{t}$ timescale. The last two variables accumulate the various parameters of the physical system. The normalized switched dynamical system is

$$\frac{d}{dt}\begin{bmatrix}\hat{v}_C\\ \hat{i}_L\end{bmatrix}=\begin{bmatrix}-1 & S_d\\ -\frac{S_d}{\varepsilon} & -\frac{\delta+S_d\frac{R_C}{R}}{\varepsilon}\end{bmatrix}\begin{bmatrix}\hat{v}_c\\ \hat{i}_L\end{bmatrix}+\begin{bmatrix}0\\ \frac{R_C+R}{\varepsilon R}\end{bmatrix}w \qquad (9)$$

Averaging can be applied to (9) to enable further analysis. Although singular perturbation theory may be applied to time-varying systems, the switching power converter results are more readily applied if the system is first converted to an equivalent time-invariant system. Since Sd is a switching function, the model (9) is linear in the states but time-varying. State-space averaging [25], which removes all knowledge of switching frequency, is typically used to form a nonlinear time-invariant converter model. Other averaging methods retain switching information in time-invariant models.

4. Sample Data Discrete Analysis

To study singular perturbation in digital controls, a sampled data model [23], [24] can be analyzed for timescale separation. Generic discrete-time systems have been analyzed with singular perturbation theory [17]. The boost converter of Fig. 1 demonstrates the basic problem of timescale separation with a digital control. First, the continuous-time model of (9) needs to be converted to discrete time. For notational convenience, rewrite the continuous-time model as

$$\dot{x}(t) = Fx(t) + Gu(t) \qquad (10)$$

Where F and G are respectively m x m and m x p real constant matrices with initial time is t_0 and sampling time is t.

$$x(t) = e^{F(t-t0)}\,x(t_0) + \int_{t_0}^{t} e^{F(t-\tau)}\,Gu(\tau)\,d\tau \qquad (11a)$$

In our case the input is sampled so we shall establish the solution going from one sampling instant t_0=kT to the next sampling instant t= (k+1) T.

$$x(t) = e^{F(t-kT)}\,x(kT) + \int_{kT}^{t} e^{F(t-\tau)}\,Gu(\tau)\,d\tau;$$
$$kT\leq t < (k+1)T \qquad (11b)$$

If we are interested in response at the sampling instants only, we set t= (k+1)T. In response to u(k), the state settles to the value x(k+1) prior to the application of input u(k+1).

$$x(k+1) = A\,[x(k)] + B\,u(k) \qquad (12)$$
$$\text{where } A= e^{FT}$$
$$B= \int_{kT}^{(k+1)T} e^{F((k+1)T-\tau)}\,G\,d\tau$$
$$B= \int_{kT}^{(k+1)T} e^{F((kT+T-\tau)}\,G\,d\tau \qquad (13a)$$

Letting $\mu = (\tau - kT)$ in (24a), we have

$$B= \int_{0}^{T} e^{F(T-\mu)}\,G\,d\mu \text{ with } \theta = T-\mu$$

We get

$$B= \int_{0}^{T} e^{F\theta}\,G\,d\theta \qquad (13b)$$

If we are interested in the value of x(k) between sampling instants, we first solve for x(kT) any k using state above equation and then use (13) x(t) to determine x(t) for $kT \leq t < (k+1)T$.

Algorithm for evaluation of matrix series:

We evaluate A by a series in the form

$$A= e^{FT} = I+FT\left(I+\frac{FT}{2}\left\{I+\frac{FT}{3}\left[I+\cdots+\frac{FT}{N-1}\left(I+\frac{FT}{N}\right)\right]\cdots\right\}\right) \qquad (14a)$$

which has better numerical properties than the direct series of powers. The empirical relation giving the number of terms N is

$$N=\min\{3\,!!\,FT\,!!+6, 100\} \qquad (14b)$$

This relation assures that no more than 100 terms are included. The B integral in (13b) can be evaluated term by term to give

$$B=\sum_{N=0}^{\infty}\frac{F^{i}T^{i+1}}{(i+1)!}G \qquad (15a)$$
$$B= (I+\frac{FT}{2!}+\frac{F^2T^2}{3!}+\ldots)TG$$
$$B= (e^{FT}-1)\,F^{-1}G \qquad (15b)$$

The transition (26) is possible only for a nonsingular matrix F. For a singular F, we may evaluate B from (15) by the approximation technique described above.

5. Application To A PFC Converter

PFC boost converters rely on timescale separation for effective operation. The boost converter parameter has 657uH of line inductance (L), 77uF of output capacitance (C), 584m ohm of line resistance (RL), 381m ohm of capacitor output resistance (RC), 100 ohm of output resistance (R) and switches at 25 kHz. Switching period (Sampling time) T=0.7msec [25]. The resulting system is given by

$$\begin{bmatrix}x_0(k+1)\\ x_1(k+1)\end{bmatrix}=\begin{bmatrix}0.9993 & 0.0002\\ -0.2471 & 0.0538\end{bmatrix}\begin{bmatrix}x_0(k)\\ x_1(k)\end{bmatrix}+\begin{bmatrix}0\\ 3.9612\end{bmatrix}u(k) \qquad (16a)$$

Here x_0 slow state variable, x_1 fast state variable and u(k) is unit step control function. The eigen spectrum of this system

(0.9992, 0.0539)

clearly indicates two-time-scale nature with one slow mode and one fast mode. Hence it is represented as a one-parameter system as shown below.

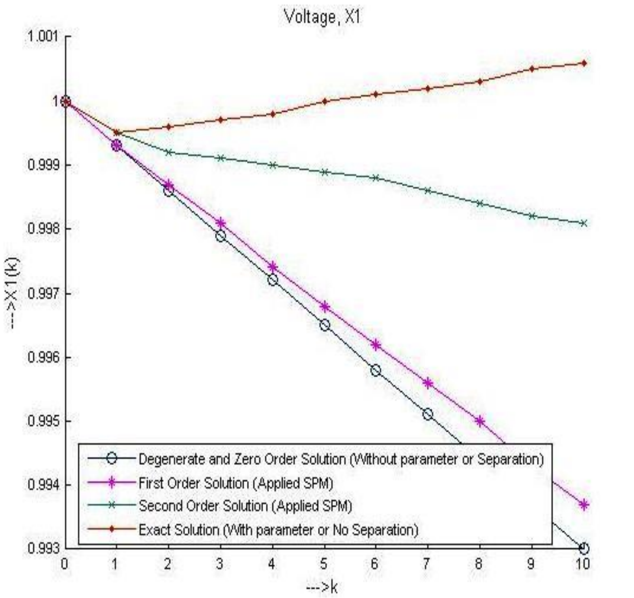
$$\begin{bmatrix}x_0(k+1)\\ x_1(k+1)\end{bmatrix}=\begin{bmatrix}0.9993 & 0.002\\ -0.2471 & 0.538\end{bmatrix}\begin{bmatrix}x_0(k)\\ \varepsilon x_1(k)\end{bmatrix}+\begin{bmatrix}0\\ 3.9612\end{bmatrix}u(k) \qquad (16b)$$

where $\varepsilon = 0.1$.

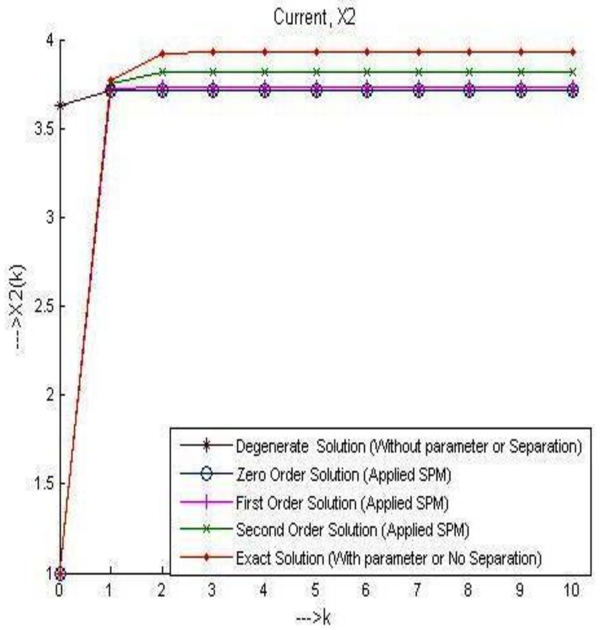
IVP:

$x_0(0)=1$, $x_1(0)=1$, This is an IVP where all the conditions are specified at initial point (k=0). The solutions for zero, first, second-order approximations are obtained and compared with the exact solution as shown in the Graph. 1 and 2 for IVP. From these Graph's we observe that

- The degenerate solution, obtained by making ε equal to zero in (16a), is unable to satisfy the initial conditions $x_1(0)$.
- The zero-order solution, obtained from (6), incorporates BLCs and hence it recovers the initial conditions $x_1(0)$. Thereafter, i.e., $k \geq 1$, it remains equal to the degenerate solution.
- The first-order solution improves the zero-order solution and is closer to the exact solution.
- The second-order solution improves the first-order solution and is much closer to the exact solution.
- Boundary layer (region of rapid transition) is formed at k = 0 for x_1 (the change from exact to degenerate solution is 1 to 3.6258 in IVP.



Graph .1: Comparison of various series solutions with the exact solution for voltage (X1 or x_0) for IVP



Graph .2: Comparison of various series solutions with the exact solution for current (X2 or x_1) for IVP

BVP:

x0(10)=2, x1(0)=1, This is TPBVP as x0 is specified at k =10 and x1 is specified at initial point (k=0).

The solutions for zero, first, second-order approximations are obtained and compared with the exact solution as shown in the Table I for BVP. From these tables we observe that

- The degenerate solution, obtained by making ε equal to zero in (16a), is unable to satisfy the initial conditions x1(0).
- The zero-order solution, obtained from (6), incorporates BLCs and hence it recovers the initial conditions x1(0). Thereafter, i.e., $k \geq 1$, it remains equal to the degenerate solution.
- The first-order solution improves the zero-order solution and is closer to the exact solution.
- The second-order solution improves the first-order solution and is much closer to the exact solution.
- Boundary layer (region of rapid transition) is formed at k = 0 for x1 (the change from exact to degenerate solution is 1 to -0.4980.

Table I: Comparison of various series solutions with the exact solution for BVP

x(k)	Degenerate Solution	Zero Order Solution	First Order Solution	Second Order Solution	Exact Solution
x1(0)	2.0141	2.0141	2.0134	2.0104	2.0073
x2(0)	-0.4980	1.0000	1.0000	1.0000	1.0000
x1(1)	2.0126	2.0126	2.0120	2.0097	2.0061
x2(1)	3.4635	3.4635	3.4691	3.4936	3.5190
x1(2)	2.0112	2.0112	2.0107	2.0082	2.0054
x2(2)	3.4639	3.4639	3.4827	3.5927	3.6548
x1(3)	2.0098	2.0098	2.0093	2.0075	2.0047
x2(3)	3.4642	3.4642	3.4830	3.5931	3.6623
x1(4)	2.0084	2.0084	2.0080	2.0063	2.0040
x2(4)	3.4646	3.4646	3.4833	3.5934	3.6629
x1(5)	2.0070	2.0070	2.0067	2.0051	2.0033
x2(5)	3.4649	3.4649	3.4837	3.5938	3.6631
x1(6)	2.0056	2.0056	2.0053	2.0047	2.0027
x2(6)	3.4653	3.4653	3.4840	3.5941	3.6632
x1(7)	2.0042	2.0042	2.0040	2.0034	2.0020
x2(7)	3.4656	3.4656	3.4843	3.5944	3.6634
x1(8)	2.0028	2.0028	2.0027	2.0021	2.0013
x2(8)	3.4660	3.4660	3.4847	3.5948	3.6636
x1(9)	2.0014	2.0014	2.0013	2.0009	2.0007
x2(9)	3.4663	3.4663	3.4850	3.5951	3.6638
x1(10)	2.0000	2.0000	2.0000	2.0000	2.0000
x2(10)	3.4667	3.4667	3.4853	3.5954	3.6639

6. Conclusion

Time-scale separation is important in many applications, from PFC to low-voltage dc–dc converters. Separation criteria were derived for buck converter in both continuous-time and discrete-time formulations. The relationship among inductance, capacitance, and the inductor's parasitic resistance dominated the small parameters. An experimental boost converter, with both resistive and constant-power loads, demonstrated the effects of various design choices. A simulated PFC converter showed that extremely simple controllers can produce good line current waveforms if there is timescale separation. Designers may use these results in several ways. If a particular converter is already designed, then the control designer may check the criteria before choosing a particular control methodology. Alternatively, if a particular control scheme is desired, the power designer can make component choices that ensure separation. The separation criteria can also be used as constraints to improve a converter optimization problem. Future work will explore similar concepts for other converter topologies and closed-loop systems. Depending on component selection, there may be two timescale. In a closed-loop system, the input u is no longer exogenous, but instead is a function of the states x0 and x1, and the disturbance input u. The feedback system may itself contain extra states, and may either enhance or detract from timescale separation. As shown in the example PFC controller, though, a controller built entirely on the slow timescale will usually be effective if there is timescale separation and further to applications in Electric Motors.

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