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Interval Observers for Switched Metzler-Takagi-Sugeno Fuzzy Systems

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Abstract: - This paper is concerned with the design of switching observers for nonlinear positive systems, where the basic parameters of the Metzler-Takagi-Sugeno fuzzy models are intervally defined. With consideration of the measurable set of premise variables the associated structure of the switching fuzzy interval observers is proposed for system state estimation to maintain stability and H_∞ performance level under the influence of norm-bounded additive disturbance. The design conditions take into account the lower and upper bounds of nonnegative system state. A numerical example is included to demonstrate the effectiveness of the developed theory for the condisered class of systems.

Key-Words: - Metzler matrices, Takagi-Sugeno models, switched systems, interval observers, state estimation, linear matrix inequalities.

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1 Introduction

Many complex systems are usually nonlinear and the Takagi-Sugeno (T-S) fuzzy models, [1], address many related practical purposes preferring the sector nonlinearity concept and the system state-space approach. Typically, the mathematical formalizations presented in [2], reflect many practical design problems, where synthesis is con-stituted on linear matrix inequalities (LMI), [3].

Focusing on the systems with positive states, [4], [5], their mathematical models should be able to reflect strict constraints implying from the Metzler matrix theory, [6]. Their main reflections mean that the design is conditioned by additional constraints, [7], to tolerate the system positiveness. A strictly LMI-based approach for design under Metzler constraints is given in [8], reflecting the diagonal stabilization principle (DSP). Developing methods to provide system state estimation of positive systems if only system matrix bounds are known, [9], some approaches for interval observer (IO) analysis are presented in [10], [11], interpreting the above design problems for T-S fuzzy IOs, [12]. The main limitations are measurable premise variables when implementing the such kind of observers.

The estimation performances of positive observers for switched fuzzy positive systems carry out the equivalent difficulties, [13], and so fuzzy approaches have to be formulated over the system positivity and the premise variables availability, [14]. The problem of positive properties of the response of IOs for switched T-S fuzzy positive systems remains relevant, where a new design goal is the satisfactory IO dynamics, [15], [16]. It should be pointed out that the measurable premise variables remain important issues.

The compatibility of an LMIs set in design of positive Metzler-Takagi-Sugeno (M-T-S) fuzzy switching IOs is solved in the paper, forming a new algorithmic platform with relationships to switching IO stability, positivity, upper and lower state bounds, H_∞ disturbance attenuation and the Metzler parametric constraints. Despite the specific limitations from the concept of DSP and the structural constraints that are imposed on the observer dynamics, the design is formulated as a parameter feasibility problem. The resulting design conditions are given in terms of LMIs, which by existing programming tools can be easily solved.

The paper is structured in the following way. In Section 2 the main characterization of M-T-S positive fuzzy switching models are presented. The properties of positive M-T-S fuzzy switching IOs are analyzed in Section 3 and the observer parameter design conditions for this class of observers are derived Section 4. The approach is illustrated by application to a M-T-S model with interval Metzler system matrices in Section 5 and the conclusions with relation properties are presented in Section 6.

Throughout the paper $\boldsymbol{x}^{\mathrm{T}}$, $\boldsymbol{X}^{\mathrm{T}}$ denotes the transpose of the vector $\boldsymbol{x}$, and the matrix $\boldsymbol{X}$, respectively, $\mathsf{diag}[\cdot]$ marks a (block) diagonal matrix, for a square symmetric matrix $\boldsymbol{X} \prec 0$ means that $\boldsymbol{X}$ is negative definite matrix, $\boldsymbol{I}_n$ labels the n-th order unit matrix, $\mathbb{R}$ ($\mathbb{R}_+$) marks the set of (nonnegative) real numbers, $\mathbb{R}^{n\times n}$ ($\mathbb{R}_+^{n\times n}$) refers to the set of (nonnegative) real matrices and $\mathbb{R}_{-+}^{n\times n}$ indicates the set of matrices with the strictly Metzler structure.

2 M-T-S Fuzzy Switching Models

This paper considers a class of switching dynamical structures with M-T-S fuzzy models

$$\dot{\boldsymbol{q}}(t) = \sum_{i=1}^{s} \mathrm{h}_i^{\sigma}(\boldsymbol{\vartheta}(t))\boldsymbol{A}_i^{\sigma}\boldsymbol{q}(t) + \boldsymbol{B}^{\sigma}\boldsymbol{u} + \boldsymbol{D}\boldsymbol{d}(t) \quad (1)$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \quad (2)$$

where $\boldsymbol{q}(t) \in \mathbb{R}_+^n$, $\boldsymbol{y}(t) \in \mathbb{R}_+^m$ are nonnegative, $\boldsymbol{d}(t) \in \mathbb{R}_+^d$ is positive and bounded, $\boldsymbol{C} \in \mathbb{R}_+^{m\times n}$, $\boldsymbol{D} \in \mathbb{R}_+^{n\times d}$ are nonnegative and $\boldsymbol{A}_i^{\sigma} \in \mathbb{M}_{-+}^{n\times n}$ are strictly Metzler and Hurwitz. Index $\sigma \in \Sigma$ marks an active switching mode from the list $\Sigma = \{1, \ldots n_s\}$ while n_s is an integer, $\mathrm{h}_i^{\sigma}(\boldsymbol{\theta}(t))$ is a normalized membership function satisfying for all items $i \in \{1, \ldots, s\}$, $\sigma \in \{1, \ldots, n_s\}$ that

$$0 \leq \mathrm{h}_i^{\sigma}(\boldsymbol{\vartheta}(t)) \leq 1, \quad \sum_{i=1}^{s} \mathrm{h}_i^{\sigma}(\boldsymbol{\vartheta}(t)) = 1 \quad (3)$$

to contain the number of fuzzy rules s and to overlay the o-dimensional premise variables vector $\boldsymbol{\vartheta}(t) = [\theta_1(t)\ \theta_2(t)\ \cdots\ \theta_o(t)]$. It is supposed that the premise variables are measurable.

A strictly Metzler matrix $\boldsymbol{A}_i^{\sigma}$ is characterized by its negative diagonal elements and by its strictly positive off diagonal elements. Consequently, a strictly Metzler $\boldsymbol{A}_i^{\sigma}$ is so limited by n^2 parametric constraints

$$a_{lh}^{\sigma} < 0,\ l = h,\ a_{lh}^{\sigma} > 0, l \neq h,\ \forall\, l, h \in \langle 1, n\rangle \quad (4)$$

To guaranty () in design task the DSPs have to be used, [17].

Definition 1 [8] If a strictly Metzler $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ is represented in the following rhombic form, where the diagonal exactness are constructed by the column index defined multiple circular shifts of elements of the columns of $\boldsymbol{A}$ as follows

$$\boldsymbol{A}_{\Theta} = \begin{bmatrix} a_{11} & & & & \\ a_{21} & a_{22} & & & \\ a_{31} & a_{32} & a_{33} & & \\ \vdots & \vdots & \vdots & \ddots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \\ & a_{12} & a_{13} & \cdots & a_{1n} \\ & & a_{23} & \cdots & a_{2n} \\ & & & \ddots & \vdots \\ & & & & a_{n-1,n} \end{bmatrix}, \quad (5)$$

then the following diagonal matrix structures

$$\begin{aligned} &\boldsymbol{A}(\nu + h, \nu) \\ &= diag\,[a_{1+h,1} \cdots a_{n,n-h}\, a_{1,n-h+1} \cdots a_{h,n}] \succ 0 \end{aligned} \quad (6)$$

$$\boldsymbol{A}(\nu, \nu) = diag\,[a_{11}\ a_{22}\ \cdots\ a_{nn}] \prec 0 \quad (7)$$

represent Metzler parametric constraints (2).

Remark 1 The DSP leads to parameterizations of a Metzler matrix $\boldsymbol{A}$ as, [18],

$$\boldsymbol{A} = \sum_{h=0}^{n-1} \boldsymbol{A}(\nu + h, \nu)\boldsymbol{L}^{h\mathrm{T}}, \quad \boldsymbol{L} = \begin{bmatrix} \boldsymbol{0}^{\mathrm{T}} & 1 \\ \boldsymbol{I}_{n-1} & \boldsymbol{0} \end{bmatrix} \quad (8)$$

where $\boldsymbol{L} \in \mathbb{R}^{n\times n}$ is the circulant permutation matrix, [19]. Applying (8) for any $\boldsymbol{A}_e = \boldsymbol{A} - \boldsymbol{J}\boldsymbol{C} \in \mathbb{R}_{-+}^{n\times n}$ address the following parametrization (see, for example, [18])

$$\boldsymbol{A}_e = \sum_{h=0}^{n-1} \big(\boldsymbol{A}_i(\nu + h, \nu) - \sum_{k=0}^{m} \boldsymbol{J}_{kh}\boldsymbol{C}_{dk}\big)\boldsymbol{L}^{h\mathrm{T}} \quad (9)$$

where, with relation to (7), (6), the representing diagonal matrices $\boldsymbol{J}_{kh}, \boldsymbol{C}_{dk} \in \mathbb{R}_+^{n\times n}$ are defined as follows:

$$\boldsymbol{C}^{\mathrm{T}} = [\boldsymbol{c}_1 \cdots \boldsymbol{c}_m], \quad \boldsymbol{C}_{dk} = diag\,[\boldsymbol{c}_k^{\mathrm{T}}] \quad (10)$$

$$\boldsymbol{J} = [\boldsymbol{j}_1 \cdots \boldsymbol{j}_m],\ \boldsymbol{J}_k = diag\,[\boldsymbol{j}_{ik}] \quad (11)$$

where $\boldsymbol{J}_{kh} = \boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}_k\boldsymbol{L}^h$. Note the above mentioned matrix $\boldsymbol{A}_e = \boldsymbol{A} - \boldsymbol{J}\boldsymbol{C}$ has to be strictly Metzler and Hurwitz.

3 M-T-S Switching Fuzzy IO

In this case there are considered that $\boldsymbol{q}(0)$ as well as $\boldsymbol{A}_i^{\sigma}$ are unknown but bounded by known constant bounding vectors and known constant bounding matrices of appropriate dimensions in such a way that for all $i \in \langle 1, s\rangle$, $\sigma \in \{1, \ldots, n_s\}$ (these inequalities are understood elementwise)

$$0 \leq \underline{\boldsymbol{q}}(0) \leq \boldsymbol{q}(0) \leq \overline{\boldsymbol{q}}(0), \quad \underline{\boldsymbol{A}}_i^{\sigma} \leq \boldsymbol{A}_i^{\sigma} \leq \overline{\boldsymbol{A}}_i^{\sigma} \quad (12)$$

$$0 \leq \underline{\boldsymbol{\vartheta}}(t) \leq \boldsymbol{\vartheta}(t) \leq \overline{\boldsymbol{\vartheta}}(t),\ \boldsymbol{0} \leq \underline{\boldsymbol{q}}_e(t) \leq \boldsymbol{q}(t) \leq \overline{\boldsymbol{q}}_e(t) \quad (13)$$

Due to by interval defined system parameters and measurable premise variables, it can be used the IO structure for an M-T-S switched strictly positive system

$$\begin{aligned} \dot{\overline{\boldsymbol{q}}}_e(t) =& \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))(\overline{\boldsymbol{A}}_i^{\sigma} - \boldsymbol{J}_i^{\sigma}\boldsymbol{C})\overline{\boldsymbol{q}}_e(t) \\ &+ \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\big(\boldsymbol{J}_i^{\sigma}\boldsymbol{C}\boldsymbol{q}(t) + \boldsymbol{B}^{\sigma}\boldsymbol{u}(t)\big) \end{aligned} \quad (14)$$

$$\begin{aligned}\dot{\underline{\boldsymbol{q}}}_e(t) =& \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\big(\underline{\boldsymbol{A}}_i^{\sigma} - \boldsymbol{J}_i^{\sigma}\boldsymbol{C}\big)\underline{\boldsymbol{q}}_e(t) \\ &+ \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\big(\boldsymbol{J}_i^{\sigma}\boldsymbol{C}\boldsymbol{q}(t) + \boldsymbol{B}^{\sigma}\boldsymbol{u}(t)\big)\end{aligned} \tag{15}$$

where (2) yields together with

$$\overline{\boldsymbol{y}}_e(t) = \boldsymbol{C}\overline{\boldsymbol{q}}_e(t), \quad \underline{\boldsymbol{y}}_e(t) = \boldsymbol{C}\underline{\boldsymbol{q}}_e(t) \tag{16}$$

and for $t \ge 0$ if $\overline{\boldsymbol{q}}_e(0) = \overline{\boldsymbol{q}}(0)$, $\underline{\boldsymbol{q}}_e(0) = \underline{\boldsymbol{q}}(0)$ it is

$$\overline{\boldsymbol{A}}_{ei}^{\sigma} = \overline{\boldsymbol{A}}_i^{\sigma} - \boldsymbol{J}_i^{\sigma}\boldsymbol{C}, \quad \underline{\boldsymbol{A}}_{ei}^{\sigma} = \underline{\boldsymbol{A}}_i^{\sigma} - \boldsymbol{J}_i^{\sigma}\boldsymbol{C} \tag{17}$$

Using the observation errors

$$\overline{\boldsymbol{e}}(t) = \boldsymbol{q}(t) - \overline{\boldsymbol{q}}_e(t), \quad \underline{\boldsymbol{e}}(t) = \boldsymbol{q}(t) - \underline{\boldsymbol{q}}_e(t) \tag{18}$$

it follows from (1), (14), (15), (18) that

$$\begin{aligned}\dot{\overline{\boldsymbol{e}}}(t) =& \boldsymbol{D}\boldsymbol{d}(t) + \\ &+ \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\overline{\boldsymbol{A}}_{ei}^{\sigma}\overline{\boldsymbol{e}}(t) - \overline{\boldsymbol{\Lambda}}^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\boldsymbol{q}(t)\end{aligned} \tag{19}$$

$$\begin{aligned}\dot{\underline{\boldsymbol{e}}}(t) =& \boldsymbol{D}\boldsymbol{d}(t) + \\ &+ \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{A}}_{ei}^{\sigma}\underline{\boldsymbol{e}}(t) + \underline{\boldsymbol{\Lambda}}^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\boldsymbol{q}(t)\end{aligned} \tag{20}$$

where

$$\begin{aligned}\overline{\boldsymbol{\Lambda}}^{\sigma}(\overline{\boldsymbol{\vartheta}}(t)) =& \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))(\overline{\boldsymbol{A}}_{ei}^{\sigma} - \boldsymbol{A}_{ei}^{\sigma}) \\ &> \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))(\overline{\boldsymbol{A}}_{ei}^{\sigma} - \underline{\boldsymbol{A}}_{ei}^{\sigma})\end{aligned} \tag{21}$$

$$\begin{aligned}\underline{\boldsymbol{\Lambda}}^{\sigma}(\underline{\boldsymbol{\vartheta}}(t)) =& \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))(\boldsymbol{A}_{ei}^{\sigma} - \underline{\boldsymbol{A}}_{ei}^{\sigma}) \\ &< \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))(\overline{\boldsymbol{A}}_{ei}^{\sigma} - \underline{\boldsymbol{A}}_{ei}^{\sigma})\end{aligned} \tag{22}$$

owing to that (12) implies

$$\underline{\boldsymbol{A}}_i^{\sigma} - \boldsymbol{J}_i^{\sigma}\boldsymbol{C} \le \boldsymbol{A}_i^{\sigma} - \boldsymbol{J}_i^{\sigma}\boldsymbol{C} \le \overline{\boldsymbol{A}}_i^{\sigma} - \boldsymbol{J}_i^{\sigma}\boldsymbol{C} \tag{23}$$

Remark 2 Since $\overline{\boldsymbol{A}}_{ei}^{\sigma}$, $\underline{\boldsymbol{A}}_{ei}^{\sigma}$ must be strictly Metzler and Hurwitz and if $\boldsymbol{q}(t)$ is at its upper limit $\overline{\boldsymbol{q}}(t)$, then

$$\dot{\overline{\boldsymbol{e}}}(t) = \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\overline{\boldsymbol{A}}_{ei}^{\sigma}\overline{\boldsymbol{e}}(t) + \boldsymbol{D}\boldsymbol{d}(t) \tag{24}$$

$$\begin{aligned}\dot{\underline{\boldsymbol{e}}}(t) =& \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{A}}_{ei}^{\sigma}\underline{\boldsymbol{e}}(t) + \boldsymbol{D}\boldsymbol{d}(t) + \\ &+ \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\big(\overline{\boldsymbol{A}}_i^{\sigma} - \underline{\boldsymbol{A}}_i^{\sigma}\big)\boldsymbol{q}(t)\end{aligned} \tag{25}$$

Otherwise, if the current state of the system $\boldsymbol{q}(t)$ is at its lower limit $\underline{\boldsymbol{q}}(t)$, then the IO error dynamics $\overline{\boldsymbol{e}}(t)$ is approximated by the equation

$$\begin{aligned}\dot{\overline{\boldsymbol{e}}}(t) =& \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\overline{\boldsymbol{A}}_{ei}^{\sigma}\overline{\boldsymbol{e}}(t) + \boldsymbol{D}\boldsymbol{d}(t) - \\ &- \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\big(\overline{\boldsymbol{A}}_i^{\sigma} - \underline{\boldsymbol{A}}_i^{\sigma}\big)\boldsymbol{q}(t)\end{aligned} \tag{26}$$

while the dynamics of $\underline{\boldsymbol{e}}(t)$ at the lower $\underline{\boldsymbol{q}}(t)$ is given as

$$\dot{\underline{\boldsymbol{e}}}(t) = \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{A}}_{ei}^{\sigma}\underline{\boldsymbol{e}}(t) + \boldsymbol{D}\boldsymbol{d}(t) \tag{27}$$

The second elements of (24)-(27) express the effect of the disturbances (generalized disturbances) on the dynamics of the estimation error. It can be seen that all these components are non-negative and bounded, so they cannot cause IO instability.

Assumption 1 Denoting the lower limit of $\boldsymbol{q}(t)$ as $\underline{\boldsymbol{q}}(t)$ and performing (27) as

$$\begin{aligned}&\dot{\underline{\boldsymbol{q}}}(t) - \dot{\underline{\boldsymbol{q}}}_e(t) \\ &= \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{A}}_{ei}^{\sigma}(\underline{\boldsymbol{q}}(t) - \underline{\boldsymbol{q}}_e(t)) + \boldsymbol{D}\boldsymbol{d}(t)\end{aligned} \tag{28}$$

it follows from (28)

$$\begin{aligned}\dot{\underline{\boldsymbol{q}}}_e(t) =& \dot{\underline{\boldsymbol{q}}}(t) - \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{A}}_{ei}^{\sigma}\underline{\boldsymbol{q}}(t) + \\ &+ \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{A}}_{ei}^{\sigma}\underline{\boldsymbol{q}}_e(t) - \boldsymbol{D}\boldsymbol{d}(t)\end{aligned} \tag{29}$$

Rewriting (1) for this limit case in the form

$$\begin{aligned}&\dot{\underline{\boldsymbol{q}}}(t) \\ &= \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))(\boldsymbol{A}_i^{\sigma}\underline{\boldsymbol{q}}(t) + \boldsymbol{B}^{\sigma}\boldsymbol{u}(t) + \boldsymbol{D}\boldsymbol{d}(t))\end{aligned} \tag{30}$$

then, substituting (30) in (29) it yields

$$\begin{aligned}&\dot{\underline{\boldsymbol{q}}}_e(t) \\ &= \sum_{i=1}^{p} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))(\boldsymbol{A}_i^{\sigma} - \underline{\boldsymbol{A}}_i^{\sigma} + \boldsymbol{J}^{\sigma}\boldsymbol{C}))\underline{\boldsymbol{q}}(t) + \\ &+ \sum_{i=1}^{s} h_i^{\sigma}(\underline{\boldsymbol{\vartheta}}(t))\underline{\boldsymbol{A}}_{ei}^{\sigma}\underline{\boldsymbol{q}}_e(t) + \boldsymbol{B}^{\sigma}\boldsymbol{u}(t)\end{aligned} \tag{31}$$

Thus, for a positive M-T-S model, where $\boldsymbol{C} \in \mathbb{R}_+^{m \times n}$ is nonnegative and $\boldsymbol{q}(t) \in \mathbb{R}_+^n$ is positive, the lower system state estimate by M-T-S fuzzy positive IO is nonnegative if $\boldsymbol{J}^{\sigma} \in \mathbb{R}_+^{n \times m}$ is nonnegative, $\underline{\boldsymbol{A}}_i^{\sigma} \le \boldsymbol{A}_i^{\sigma}$ and $\underline{\boldsymbol{A}}_{ei}^{\sigma}$ are Metzler and Hurwitz.

Consequence 1 Adaptation of Remark 1 and (17) entail the following matrix parameterizations

$$\overline{\boldsymbol{A}}_i^{\sigma}(\nu,\nu) = diag\,[\overline{a}_{i11}^{\sigma}\ \overline{a}_{i22}^{\sigma}\ \cdots\ \overline{a}_{inn}^{\sigma}] \prec 0 \tag{32}$$

$$\underline{\boldsymbol{A}}_i^{\sigma}(\nu,\nu) = diag\,[\underline{a}_{i11}^{\sigma}\ \underline{a}_{i22}^{\sigma}\ \cdots\ \underline{a}_{inn}^{\sigma}] \prec 0 \tag{33}$$

$$\begin{aligned}&\overline{\boldsymbol{A}}_i^{\sigma}(\nu+h,\nu)\\ &= diag\left[\overline{a}_{i,1+h,1}^{\sigma}\cdots\overline{a}_{i,n,n-h}^{\sigma}\,\overline{a}_{i,1,n-h+1}^{\sigma}\cdots\overline{a}_{ihn}^{\sigma}\right]\\ &\succ 0\end{aligned} \tag{34}$$

$$\begin{aligned}&\underline{\boldsymbol{A}}_i^{\sigma}(\nu+h,\nu)\\ &= diag\left[\underline{a}_{i,1+h,1}^{\sigma}\cdots\underline{a}_{i,n,n-h}^{\sigma}\,\underline{a}_{i,1,n-h+1}^{\sigma}\cdots\underline{a}_{ihn}^{\sigma}\right]\\ &\succ 0\end{aligned} \tag{35}$$

$$\boldsymbol{C}^{\mathrm{T}} = [\boldsymbol{c}_1\ \cdots\ \boldsymbol{c}_m], \quad \boldsymbol{C}_{dk} = diag\left[\boldsymbol{c}_k^{\mathrm{T}}\right] \tag{36}$$

$$\boldsymbol{J}_i^{\sigma} = [\boldsymbol{j}_{i1}^{\sigma}\cdots\boldsymbol{j}_{im}^{\sigma}], \ \boldsymbol{J}_{ik}^{\sigma} = diag\,[\boldsymbol{j}_{ik}^{\sigma}] \tag{37}$$

$$\boldsymbol{J}_{ikh}^{\sigma} = \boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}_{ik}^{\sigma}\boldsymbol{L}^{h} \tag{38}$$

$$\overline{\boldsymbol{A}}_{ei} = \sum_{h=0}^{n-1}\left(\overline{\boldsymbol{A}}_i^{\sigma}(\nu+h,\nu) - \sum_{k=0}^{m}\boldsymbol{J}_{kh}^{\sigma}\boldsymbol{C}_{dk}\right)\boldsymbol{L}^{h\mathrm{T}} \tag{39}$$

$$\underline{\boldsymbol{A}}_{ei} = \sum_{h=0}^{n-1}\left(\underline{\boldsymbol{A}}_i^{\sigma}(\nu+h,\nu) - \sum_{k=0}^{m}\boldsymbol{J}_{kh}^{\sigma}\boldsymbol{C}_{dk}\right)\boldsymbol{L}^{h\mathrm{T}} \tag{40}$$

To perform an M-T-S fuzzy switching positive IO, the observer synthesis must be able to offer strictly positive IO parameters as it is given by the following affirmation.

4 Design of M-T-S Switched IO

A statement of positive M-T-S fuzzy switching IOs is provided by the following theorem.

Theorem 1 Suppose that $\overline{\boldsymbol{A}}_i^{\sigma}, \underline{\boldsymbol{A}}_i^{\sigma} \in \mathbb{R}_{-+}^{n\times n}$ are strictly Metzler and $\boldsymbol{C} \in \mathbb{R}_{+}^{m\times n}$ is non-negative. If there exist positive definite diagonal matrices $\boldsymbol{P}, \boldsymbol{V}_{ik}^{\sigma} \in \mathbb{R}_{+}^{n\times n}$ and positive scalar $\eta \in \mathbb{R}_+$ such that for $i = 1,2,\ldots,s,\ h = 1,2,\ldots,n-1$, $\sigma \in \{1,\ldots,n_s\}$

$$\begin{bmatrix}\overline{\boldsymbol{\Omega}}_i^{\sigma} & * & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\eta\boldsymbol{I}_d & * \\ \boldsymbol{C}^{\sigma} & \boldsymbol{0} & -\eta\boldsymbol{I}_m\end{bmatrix} \prec 0, \quad \boldsymbol{P} \succ 0 \tag{41}$$

$$\begin{bmatrix}\underline{\boldsymbol{\Omega}}_i^{\sigma} & * & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\eta\boldsymbol{I}_d & * \\ \boldsymbol{C}^{\sigma} & \boldsymbol{0} & -\eta\boldsymbol{I}_m\end{bmatrix} \prec 0, \quad \boldsymbol{V}_{ik} \succ 0 \tag{42}$$

$$\boldsymbol{P}\overline{\boldsymbol{A}}_i^{\sigma}(\nu,\nu) - \sum_{k=1}^{m}\boldsymbol{V}_{ik}^{\sigma}\boldsymbol{C}_{dk} \prec 0 \tag{43}$$

$$\boldsymbol{P}\underline{\boldsymbol{A}}_i^{\sigma}(\nu,\nu) - \sum_{k=1}^{m}\boldsymbol{V}_{ik}^{\sigma}\boldsymbol{C}_{dk} \prec 0 \tag{44}$$

$$\boldsymbol{P}\boldsymbol{L}^{h}\overline{\boldsymbol{A}}_i^{\sigma}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=1}^{m}\boldsymbol{V}_{ik}^{\sigma}\boldsymbol{L}^{h}\underline{\boldsymbol{C}}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{45}$$

$$\boldsymbol{P}\boldsymbol{L}^{h}\underline{\boldsymbol{A}}_i^{\sigma}(\nu+h,p)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=1}^{m}\boldsymbol{V}_{ik}^{\sigma}\boldsymbol{L}^{h}\overline{\boldsymbol{C}}_{dk}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{46}$$

where

$$\begin{aligned}\overline{\boldsymbol{\Omega}}_i^{\sigma} &= \boldsymbol{P}\overline{\boldsymbol{A}}_i^{\sigma} + \overline{\boldsymbol{A}}_i^{\sigma\mathrm{T}}\boldsymbol{P} - \\ &- \sum_{k=1}^{m}\boldsymbol{V}_{ik}^{\sigma}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_{dk} - \sum_{k=1}^{m}\boldsymbol{C}_{dk}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{V}_{ik}^{\sigma}\end{aligned} \tag{47}$$

$$\begin{aligned}\underline{\boldsymbol{\Omega}}_i^{\sigma} &= \boldsymbol{P}\underline{\boldsymbol{A}}_i^{\sigma} + \underline{\boldsymbol{A}}_i^{\sigma\mathrm{T}}\boldsymbol{P} - \\ &- \sum_{k=1}^{m}\boldsymbol{V}_{ik}^{\sigma}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_{dk} - \sum_{k=1}^{m}\boldsymbol{C}_{dk}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{V}_{ik}^{\sigma}\end{aligned} \tag{48}$$

and, if the task is feasible, the positive gains for all $i \in \{1,\ldots,s\}$, $\sigma \in \{1,\ldots,n_s\}$ are given as

$$\boldsymbol{J}_{ik}^{\sigma} = \boldsymbol{P}^{-1}\boldsymbol{V}_{ik}^{\sigma}, \quad \boldsymbol{j}_{ik}^{\sigma} = \boldsymbol{J}_{ik}^{\sigma}\boldsymbol{l} \tag{49}$$

$$\boldsymbol{J}_i^{\sigma} = [\boldsymbol{j}_{i1}^{\sigma}\cdots\boldsymbol{j}_{im}^{\sigma}], \quad \boldsymbol{l}^{\mathrm{T}} = [1\cdots 1] \tag{50}$$

and $\overline{\boldsymbol{A}}_{ei}^{\sigma}, \underline{\boldsymbol{A}}_{ei}^{\sigma} \in \mathbb{R}_{-+}^{n\times n}$ are strictly Metzler and Hurwitz.

Hereafter, $*$ denotes the symmetric item in a symmetric matrix.

Proof: Choosing Lyapunov function in the following form

$$\begin{aligned}v(\overline{\boldsymbol{e}}(t)) &= \overline{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}\overline{\boldsymbol{e}}(t) - \eta\int_0^t \boldsymbol{d}^{\mathrm{T}}(\tau)\boldsymbol{d}(\tau)\mathrm{d}\tau + \\ &+ \eta^{-1}\int_0^t \overline{\boldsymbol{e}}_y^{\mathrm{T}}(\tau)\overline{\boldsymbol{e}}_y(\tau)\mathrm{d}\tau \\ &> 0\end{aligned} \tag{51}$$

where $\boldsymbol{P} \in \mathbb{R}_{+}^{n\times m}$ is a diagonal positive definite matrix (PDDM) and $\eta \in \mathbb{R}_+$ is a positive scalar, then the time derivative of (51) along all the observer error trajectories is computed as follows

$$\begin{aligned}\dot{v}(\overline{\boldsymbol{e}}(t)) &= \dot{\overline{\boldsymbol{e}}}^{\mathrm{T}}(t)\boldsymbol{P}\overline{\boldsymbol{e}}(t) + \overline{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\overline{\boldsymbol{e}}}(t) + \\ &+ \eta^{-1}\overline{\boldsymbol{e}}_y^{\mathrm{T}}(t)\overline{\boldsymbol{e}}_y(t) - \eta\boldsymbol{d}^{\mathrm{T}}(t)\boldsymbol{d}(t) \\ &< 0\end{aligned} \tag{52}$$

When substituting the lower limit of $\boldsymbol{e}(t)$ given in (24), it can be fixed that

$$\begin{aligned}
&\dot{v}(\overline{\boldsymbol{e}}(t)) \\
&= \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\overline{\boldsymbol{e}}^{\mathrm{T}}(t)(\overline{\boldsymbol{A}}_{ei}^{\sigma\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\overline{\boldsymbol{A}}_{ei}^{\sigma})\overline{\boldsymbol{e}}(t)+ \\
&+ \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))(\overline{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{D}\boldsymbol{d}(t)+ \\
&+ \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\overline{\boldsymbol{e}}^{\mathrm{T}}(t)\eta^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}\overline{\boldsymbol{e}}(t)- \\
&- \eta\boldsymbol{d}^{\mathrm{T}}(t)\boldsymbol{d}(t) \\
&< 0
\end{aligned} \tag{53}$$

Denoting as following

$$\overline{\boldsymbol{e}}_d^{\mathrm{T}}(t) = \begin{bmatrix}\overline{\boldsymbol{e}}^{\mathrm{T}}(t) & \boldsymbol{d}^{\mathrm{T}}(t)\end{bmatrix} \tag{54}$$

then in terms of (54) one can easily have

$$\dot{v}(\overline{\boldsymbol{e}}_d(t)) = \sum_{i=1}^{s} h_i^{\sigma}(\overline{\boldsymbol{\vartheta}}(t))\overline{\boldsymbol{e}}_d^{\mathrm{T}}(t)\overline{\boldsymbol{\Omega}}_i^{\sigma}\overline{\boldsymbol{e}}_d(t) < 0 \tag{55}$$

where, by the nomenclature,

$$\overline{\boldsymbol{\Omega}}_i^{\sigma} = \begin{bmatrix}\overline{\boldsymbol{A}}_{ei}^{\sigma\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\overline{\boldsymbol{A}}_{ei}^{\sigma} + \eta^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C} & * \\ \boldsymbol{D}^{\mathrm{T}}\boldsymbol{P} & -\eta\boldsymbol{I}_d\end{bmatrix} \tag{56}$$

is negative definite.

Reflecting (17) for all $i \in \{1,\ldots,s\}$ and $\sigma \in \{1,\ldots,n_s\}$, then

$$\begin{aligned}
&\boldsymbol{P}(\overline{\boldsymbol{A}}_i^{\sigma} - \boldsymbol{J}_i^{\sigma}\boldsymbol{C}) + (\overline{\boldsymbol{A}}_i^{\sigma} - \boldsymbol{J}_i^{\sigma}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{P} \\
&= \boldsymbol{P}\big(\overline{\boldsymbol{A}}_i^{\sigma} - \sum_{k=1}^{m}\boldsymbol{j}_{ik}^{\sigma}\boldsymbol{c}_k^{\sigma\mathrm{T}}\big)+ \\
&+\big(\overline{\boldsymbol{A}}_i^{\sigma} - \sum_{k=1}^{m}\boldsymbol{j}_{ik}^{\sigma}\boldsymbol{c}_k^{\sigma\mathrm{T}}\big)^{\mathrm{T}}\boldsymbol{P} \\
&= \boldsymbol{P}\Big(\overline{\boldsymbol{A}}_i^{\sigma} - \sum_{k=1}^{m}\boldsymbol{J}_{ik}^{\sigma}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_{dk}\Big)+ \\
&+\big(\boldsymbol{A}_i^{\sigma} - \sum_{k=1}^{m}\boldsymbol{J}_{ik}^{\sigma}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_{dk}\big)^{\mathrm{T}}\boldsymbol{P}
\end{aligned} \tag{57}$$

and setting

$$\boldsymbol{V}_i^{\sigma} = \boldsymbol{P}\boldsymbol{J}_i^{\sigma} \tag{58}$$

then (57) defines (47) and (41) can be constructed from (56) by the Schur complement property.

According to the parametrization of $\overline{\boldsymbol{A}}_{ei}^{\sigma}$, (32), (39) it has to yield

$$\overline{\boldsymbol{A}}_i^{\sigma}(\nu,\nu) - \sum_{k=0}^{m}\boldsymbol{J}_{ik}^{\sigma}\boldsymbol{C}_{dk} \prec 0 \tag{59}$$

$$\overline{\boldsymbol{A}}_i^{\sigma}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=0}^{m}\boldsymbol{J}_{ikh}^{\sigma}\boldsymbol{C}_{dk}^{\sigma}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{60}$$

Multiplying by PDDM $\boldsymbol{P}$ the left side of (59) in turn this implies that

$$\boldsymbol{P}\boldsymbol{A}_i^{\sigma}(\nu,\nu) - \sum_{k=0}^{m}\boldsymbol{P}\boldsymbol{J}_{ik}^{\sigma}\boldsymbol{C}_{dk}^{\sigma} \prec 0 \tag{61}$$

and using (58) then (61) implies (43).

Multiplying by $\boldsymbol{P}\boldsymbol{L}^h$ the left side of (60) it yields

$$\boldsymbol{P}\boldsymbol{L}^h\boldsymbol{A}_i^{\sigma}(\nu+h,\nu)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=0}^{m}\boldsymbol{P}\boldsymbol{J}_{ik}^{\sigma}\boldsymbol{L}^h\boldsymbol{C}_{dk}^{\sigma}\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{62}$$

and with (58) then (62) implies (45), since $\boldsymbol{L}^h\boldsymbol{L}^{h\mathrm{T}} = \boldsymbol{I}_n$.

Since analogously can be set the LMIs working on $\underline{\boldsymbol{A}}_{ei}^{\sigma}$, this closes the proof. ■

Note, in the given sense (43)-(46) enforce Metzler parametric constraints in the observer gains design problem.

5 Illustrative Example

The considered M-T-S system (1), (2) is built on the parameters

$$\overline{\boldsymbol{A}}_1^1 = \begin{bmatrix}-0.2580 & 2.0160 & 1.5570\\ 0.1420 & -3.6480 & 0.0720\\ 0.2060 & 0.0730 & -2.5540\end{bmatrix}$$

$$\overline{\boldsymbol{A}}_2^1 = \begin{bmatrix}-0.2580 & 2.0660 & 1.5530\\ 0.1420 & -3.6450 & 0.2010\\ 0.2120 & 0.0510 & -2.5560\end{bmatrix}$$

$$\overline{\boldsymbol{A}}_1^2 = \begin{bmatrix}-0.2410 & 2.1600 & 1.4450\\ 0.1450 & -3.6420 & 0.1170\\ 0.1830 & 0.0970 & -2.5950\end{bmatrix}$$

$$\overline{\boldsymbol{A}}_2^2 = \begin{bmatrix}-0.2680 & 2.1640 & 1.5560\\ 0.1570 & -3.6390 & 0.1720\\ 0.2020 & 0.0810 & -2.5750\end{bmatrix}$$

$$\underline{\boldsymbol{A}}_1^1 = \begin{bmatrix}-0.2720 & 1.9380 & 1.4540\\ 0.0580 & -3.9610 & 0.0650\\ 0.1100 & 0.0580 & -2.9080\end{bmatrix}, \ \underline{\boldsymbol{C}} = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\underline{\boldsymbol{A}}_2^1 = \begin{bmatrix}-0.2730 & 1.9440 & 1.4510\\ 0.0590 & -3.9610 & 0.1070\\ 0.1090 & 0.0510 & -2.9180\end{bmatrix}, \ \boldsymbol{D} = \begin{bmatrix}0.045\\ 0.080\\ 0.053\end{bmatrix}$$

$$\underline{\boldsymbol{A}}_1^2 = \begin{bmatrix}-0.2760 & 2.0940 & 1.4450\\ 0.0520 & -3.9510 & 0.0920\\ 0.1250 & 0.0840 & -2.9380\end{bmatrix}, \ \boldsymbol{L} = \begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

$$\underline{\boldsymbol{A}}_2^2 = \begin{bmatrix}-0.2720 & 2.1020 & 1.4150\\ 0.0570 & -3.9510 & 0.1200\\ 0.1000 & 0.0770 & -2.9420\end{bmatrix}, \ \boldsymbol{l} = \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$$

Tt is not hard to attest that $\overline{\boldsymbol{A}}_i^{\sigma}$, $\underline{\boldsymbol{A}}_i^{\sigma}$ are strictly Metzler and Hurwitz for all i, σ and $\underline{\boldsymbol{A}}_i^{\sigma} \le \overline{\boldsymbol{A}}_i^{\sigma}$.

To reflect DSP, it is necessary the diagonal representations of the system parameters such that

$$\boldsymbol{C}_{d1} = \text{diag}\,[1\ 0\ 0]\,, \quad \boldsymbol{C}_{d2} = \text{diag}\,[0\ 0\ 1]$$

$$\overline{\boldsymbol{A}}_2^1(\nu,\nu) = \text{diag}\,[-0.2680\ -3.6450\ -2.5560]$$

$$\overline{\boldsymbol{A}}_2^2(\nu+1,\nu) = \text{diag}\,[0.1420\ 0.0510\ 1.5530]$$

$$\overline{\boldsymbol{A}}_2^2(\nu+2,\nu) = \text{diag}\,[0.2120\ 2.0660\ 0.2010]$$

where only the set of desired diagonal representations of the matrix $\overline{\boldsymbol{A}}_2^1$ is presented.

It can be found that for the number of matrix inequalities $N = 43$ the feasible variables which provide by using SeDuMi, [20], a solution for the problem are

$$\boldsymbol{P} = \text{diag}\,[3.1300\ 2.7628\ 3.0234]\,, \quad \eta = 5.8844$$

$$\boldsymbol{V}_{11}^1 = \text{diag}\,[4.1263\ \ 0.0530\ \ 0.1279]$$

$$\boldsymbol{V}_{12}^1 = \text{diag}\,[2.0099\ \ 0.0630\ \ 1.3823]$$

$$\boldsymbol{V}_{21}^1 = \text{diag}\,[4.1493\ \ 0.0533\ \ 0.1280]$$

$$\boldsymbol{V}_{22}^1 = \text{diag}\,[2.0078\ \ 0.1239\ \ 1.3821]$$

$$\boldsymbol{V}_{11}^2 = \text{diag}\,[4.2250\ \ 0.0472\ \ 0.1350]$$

$$\boldsymbol{V}_{12}^2 = \text{diag}\,[1.9133\ \ 0.0936\ \ 1.3743]$$

$$\boldsymbol{V}_{21}^2 = \text{diag}\,[4.2174\ \ 0.0515\ \ 0.1185]$$

$$\boldsymbol{V}_{22}^2 = \text{diag}\,[1.9764\ 0.1274\ 1.3777]$$

These results fulfil the diagonal positiveness criterion on the LMI variables and it can be observed that such parameters produce strictly positive observer matrix gains

$$\boldsymbol{J}_1^1 = \begin{bmatrix} 1.3183 & 0.6421 \\ 0.0192 & 0.0228 \\ 0.0423 & 0.4572 \end{bmatrix}, \ \boldsymbol{J}_2^1 = \begin{bmatrix} 1.3257 & 0.6415 \\ 0.0193 & 0.0448 \\ 0.0424 & 0.4571 \end{bmatrix}$$

$$\boldsymbol{J}_1^2 = \begin{bmatrix} 1.3499 & 0.6113 \\ 0.0171 & 0.0339 \\ 0.0447 & 0.4546 \end{bmatrix}, \ \boldsymbol{J}_2^1 = \begin{bmatrix} 1.3474 & 0.6314 \\ 0.0186 & 0.0461 \\ 0.0392 & 0.4557 \end{bmatrix}$$

It is worth to mention that the positive observer gains have to be directly used in the IO fuzzy switched IOs. This limitation is due to the M-T-S fuzzy switched system positiveness.

From those, it can be presented for comparison that

$$\underline{\boldsymbol{A}}_{e2}^1 = \begin{bmatrix} -1.6259 & 2.0940 & 0.8337 \\ 0.0349 & -3.9510 & 0.0581 \\ 0.0803 & 0.0840 & -3.3926 \end{bmatrix}$$

$$\rho(\underline{\boldsymbol{A}}_{e2}^1) = \{-1.5560\ -3.4304\ -3.9831\}$$

$$\overline{\boldsymbol{A}}_{e2}^1 = \begin{bmatrix} -1.5909 & 2.1600 & 0.8337 \\ 0.1279 & -3.6420 & 0.0831 \\ 0.1383 & 0.0970 & -3.0496 \end{bmatrix}$$

$$\rho(\overline{\boldsymbol{A}}_{e2}^1) = \{-1.3889\ -3.1266\ -3.7669\}$$

where, evidently, $\underline{\boldsymbol{A}}_{e2}^1 < \overline{\boldsymbol{A}}_{e2}^1$ and $\underline{\boldsymbol{A}}_{ei}^\sigma \leq \overline{\boldsymbol{A}}_{ei}^\sigma$ for all $i = 1, 2$ and $\sigma = 1, 2$.

The proposed LMI conditions with diagonal matrix variables and measurable premise variable vector in design of M-T-S fuzzy positive switched IOs are illustrated in this example.

However, if the switched M-T-S fuzzy IO observer needs to be designed for purely Metzler systems with interval-specified parameters, it requires the use of structured non-negative diagonal matrix variables $\boldsymbol{V}_{ik}^\sigma$. This manifests itself in the fact that the stable matrices $\underline{\boldsymbol{A}}_{ei}^\sigma, \overline{\boldsymbol{A}}_{ei}^\sigma$ of the switched interval observer will be purely Metzler and Hurwitz, since the matrix gains $\boldsymbol{J}_i^\sigma$ will be non-negative. Unfortunately, the design of the structure of non-negative diagonal matrix variables $\boldsymbol{V}_{ik}^\sigma$ may not be unambiguous, [21].

6 Concluding Remarks

The main objectives in this paper are the synthesis conditions of IOs design for positive M-T-S fuzzy switched systems with interval-specified dynamics of the form of strictly Metzler matrices and bounded system disturbances. The DSP, and the proposed LMI structures reflect the key idea to obtain the Metzler and Hurwitz matrix structures, while Lyapunov function and the related LMIs form the base of the observer stability. The results are presented to the case for the underlying system under arbitrary switching and under influence of the bounded unknown disturbance, the used Lyapunov function guaranties quadratic stability of the observer in all switched modes. Despite its design conditions complexity, estimation using positive switched M-T-S fuzzy IOs is robust to the changes covered in plant dynamics by given interval bounds, taking into account that the positivity of the lower state estimation need to be keep.

In future works, the proposed method will be extended to the issue of M-T-S fuzzy switched systems with ostensible Metzler interval-specified parameters, also parameterizing the M-T-S model performance requirements for partly unmeasurable premise variables.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

Anna Filasová elaborated the principles of matrix constraints representation in the positive fuzzy IO gain synthesis and implemented their linear matrix structures, Dušan Krokavec addressed the incidence of diagonal stabilisation principle into set of LMIs for stability of positive fuzzy IOs and converted the lower bound system state limit to an LMI problem. Both authors have read and agreed to the proposed version of the manuscript.

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