

# A Mathematical Model of an Automated Control System for Heat Regulation in a Building

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**Abstract:** - In this study, a mathematical model of an automated control system for heat regulation in a building was developed. A method of mathematical modeling of the centralized heating control system based on mathematical models of distributed power systems and experimental studies has been developed, which allows for determining the parameters of the coolant when the outdoor temperature changes, qualitative regulation of heat in autonomous sources, quantitative regulation in automated individual heating devices, etc. The method makes it possible to study the interaction of an automated individual heating point to increase the efficiency of the management of distributed power systems of buildings. As a result of studying the controller created by the R2 control unit, for the PI controller, the calculated heat consumption of the building imperceptibly increases from 1.08502 kJ to 1.085888 kJ when an oscillatory transient occurs, and for the I controller, the calculated heat consumption of the building remains at the same level. The level, as for the PI controller, increases slightly during the oscillatory transition from 1.08456 GJ to 1.08535 GJ.

**Key-Words:** - individual heat point, automatic regulation, mathematical modeling

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## 1 Introduction

In an intelligent heating system, the heating system element requires intellectualization. Accurate calculations are necessary to ensure heat supply and energy saving. The development of mathematical models directly affects the user's convenience and economical operation of the heating system.

In, [1], the operation of heating systems was investigated, in addition to monitoring and adjusting the operating parameters, it is necessary to coordinate the heat supply in accordance with the season, outdoor air temperature, and the user's heat needs to adapt the purpose of heat from the heating system. In, [2], a heating system was investigated, which is designed to supply electricity at various temperature loads, protecting users from incredibly high or low temperatures in the house, guaranteeing the satisfaction of users' needs for heat, and avoiding unnecessary heat costs to ensure economical maintenance of heating systems. Several studies have been conducted on optimizing the

operation of heating systems, in which the main focus was on creating mathematical models of water supply temperature, fuel consumption, and outdoor air temperature, as well as regulating water supply temperature and fuel consumption.

In, [3], they developed a mathematical model of the heat point and found that to adapt to changes in heat load, frequent regulation of the water supply temperature can reduce operating costs, but they certainly did not focus on the frequency with which the water supply temperature was regulated.

In, [4], an optimization method was used for controls in the heating mains network region, and it was found that the optimization effect was different when the water supply temperature was regulated at different frequencies. In addition, the choice of the control cycle was an important part of the forecasting problem, [5].

In the article, [6], we checked the operation of the thermal system and found that, compared with traditional control, a significant temperature

discrepancy between the main supply and return water is possible to reduce pump consumption and increase overall fuel efficiency.

In, [7], a mathematical and dynamic model of the heating network was developed; based on this modification, the peak valley method and the ratio analysis method were introduced, respectively, and it also became possible to calculate two important parameters related to the dynamic data of the heating system - the delay time and the degree of comparative attenuation. In, [8], a hydraulic pump was compared with the classical main circulating pump with an adjustable rotation speed in the heating system, which can save at least 20% of energy. In particular, when using a pump with a distributed unstable speed with a low flow rate, more energy could be saved. In, [9], a method for controlling the radiator system was created based on mathematical research and computer modeling. By finding a rational combination of water supply temperature and flow rate in the heating system, a low temperature of the main recurrent water was obtained, which allowed to reduce operating costs. In, [10], a mathematical model was developed for an integrated direct heat supply system that combines wind energy, solar energy, natural gas, and electricity. By approving a purposeful function of rational management behavior under time-consuming operational constraints, it was allowed to minimize fuel consumption and increase the efficiency of the system. In, [11], [12], [13], the heat storage capacity of the district heating system was adapted to the large size of the renewable energy conversion in the system, which improved the flexibility and efficiency of the system. Basing on the monitoring of outdoor air temperature and the history of scientific and technical data. In, [14], a mathematical distribution model was created and an error-free optimizer was developed to minimize pumping costs and heat costs; by optimizing the water supply temperature and fuel consumption, the heating design could do efficiently and smoothly. In, [15], presented all possible approaches to simple forecasting of district heating networks to optimize using a mathematical model. In, [16], the analytical solution of Fourier and non-Fourier models of heat transfer in a longitudinal rib in the presence of internal heat release under a periodic boundary condition is studied. The entire review is given in the dimensionless form. These two mathematical models were solved analytically using the Laplace transform method. The temperature distribution in the longitudinal edge is measured using the residuals in a single plane theorem for the inverse

Laplace transform method. The nature of the temperature wave is revealed at a small value.

The temperature of the longitudinal edge is evaluated for various parameter values relative to the spatial coordinate. The effect of the variability of various parameters on the temperature distribution in the rib has been thoroughly studied. It has been observed that the cooling process proceeds quickly in a Fourier-free model compared to the Fourier model. Asymptotic methods are widely used in mathematical modeling of a thermal object. When describing the processes of heat propagation in a solid, three stages are distinguished. One of them is called the regular mode stage, which exists with a sufficiently large change in the process over time. Asymptotic methods are used, for example, in studies of regular thermal regimes corresponding to a developed stage of the process. Asymptotic methods have found application in the propagation of heat transfer processes not only in weakly curved rods, and cylinders of variable cross-sections but also in studies of composite materials, [17]. The paper presents an abbreviated analysis of various numerical methods applicable to the Casson fluid, based on the study of various kinds of experimental work over the past 10 years. Previous studies, outlined in various versions by various researchers, are used as a key source of information about numerical methods used to solve control equations.

The study is generally useful when searching for literature that studies the effect of the necessary control parameters on Casson flow profiles.

In addition, comparisons with classical methods are provided, and the results are carefully checked. It may be noted that some long-standing methods among all the various varieties of numerical methods are stable and well-known among researchers, such as the shooting technique, the Runge-Kutta method, the Keller Box method, etc. The results are tested in each of these cases, [18].

The purpose of this study is to develop a mathematical model of an automated control system for heat regulation in a building, differs in that the control unit is based on the calculated transient characteristics of the serial connection of the control object and the temperature sensor, it is also possible to determine the parameters of the coolant with possible changes both in the structure of the elements of an automated individual heating point and in the systems heating of a building or structure.

## 2 Research Methodology

The advantage of this analysis lies in the fact that when developing automated individual heating

points of houses, the use of heat by the building in accordance with standard standards is located perfectly at the same level, and, therefore, there is a possibility of utilitarian use of the I-regulator to regulate the flow of heating of premises and buildings, because it is easier to introduce and configure. It is also possible to say that the present study of the transient motion, taking into account the temperature of the coolant in the return pipeline of the structure, shows that an ultra-low-frequency filter arrives under control concerning full-fledged oscillations at its inlet (in the output pipeline). In contrast to the known methods of finding the controller parameters, the control source is based on the calculated transient characteristics of the alternating combination of the control object and the temperature sensor.

Using the studied mathematical model, it is possible to establish not only suitable options for the control unit but, above all, the characteristics of the coolant with probable changes both in the structure of components of an automated personal heating point and in the heating systems of a building or structure.

In this paper, the features of the control of an automated IHP with known standard regulators are investigated using mathematical modeling. The heat supply system is a complex of distributed heat exchange devices integrated into the whole system of generation, transportation, and consumption of thermal energy. The elements of the water heat supply system are divided by types of heat transfer (convection, thermal conductivity, radiation) and by design (direct current, counterflow, combined current). If we neglect a small change in the mass of the coolant, then the rate of change in its temperature will be proportional to the amount of heat:

$$\left\{ \begin{array}{l} cmdT = Q_k + Q_\lambda + Q_r + Q_f + Q_m \\ Q_k = \pm \sum_{i=1}^n \alpha_i \cdot F_i \cdot (T_i - T_{i-1}) \\ Q_\lambda = \sum_i^n \lambda_i \cdot F_i \cdot \frac{\partial^2 T_i}{\partial x_i^2} \\ Q_r = \sum_{i=1}^n F_i \cdot q_i(t) \\ Q_f = \sum_{i=1}^n k_{fi} \cdot F_i \cdot (T_i - T_{i-1}) \end{array} \right. \quad (1)$$

where  $Q_k, Q_\lambda, Q_r, Q_f, Q_m$  the total costs by convection, thermal conductivity, radiation, and

filtration,  $Q_m$  - heat source,  $c$  - heat capacity,  $m$  - mass of the medium,  $\alpha_i$  - heat transfer coefficient,  $F$  - heat exchange surface,  $\lambda_i$  - thermal conductivity coefficient of the medium,  $x$  - spatial coordinate,  $q_i(t)$  - specific heat flux,  $k_{fi}$  - filtration heat transfer coefficient.

The expression  $dT$  in the first equation (1) is a differential:

$$dT = \frac{\partial T_i}{\partial t} + w_i \frac{\partial T_i}{\partial x_i} \quad (2)$$

For each of the elements of the heat supply system, it is possible to obtain a system of partial differential equations by substituting the values in the expression (1).

$$\frac{\partial T}{\partial t} + w_x \frac{\partial T}{\partial x} + w_y \frac{\partial T}{\partial y} + w_z \frac{\partial T}{\partial z} = \frac{\lambda}{cp} \Delta T + \frac{Q_v}{cp} \quad (3)$$

$$\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \quad (4)$$

where  $T$  is the temperature of the coolant,  $t$  - time,  $\Delta T$  - coefficient of thermal conductivity,  $c$  - heat capacity of the coolant,  $p$  - density of the coolant,  $x, y, z$  - coordinates,  $w_x, w_y, w_z$  - projection of the velocity vector of the coolant,  $\lambda$  - Laplace operator in a rectangular coordinate system. In the case of solids, a differential equation of the thermal conductivity of the form is applied:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{cp} \Delta T + \frac{Q_v}{cp} \quad (5)$$

where  $\Delta T, c, p$  - coefficient of thermal conductivity, heat capacity, and density of the body. For the uniqueness of the solution of equations (3)-(5), geometric, physical, boundary, and time conditions should be supplemented. Geometric - determine the size and shape of the body, physical - includes numerical values and the nature of changes in the thermophysical parameters of the body and the environment, the intensity of internal heat sources. Boundary conditions determine the conditions of heat exchange at the body's boundary, temporarily - set the nature of changes in temperature or heat flow at the initial and final time. Suitable differential equations follow from conservation laws, while the material is considered a continuous continuous medium, the characteristics of the transfer processes are represented by constant functions of coordinates and time.

To study the non-stationary problems of forced convective heat transfer, a differential equation describing heat transfer in a moving medium with a constant velocity is used. The structure of a typical automated IHP of a heating system to a heat source,

shown in Figure 1, contains a process controller TC1, a block of circulation pumps C1 and C2 with electric drives ED1 and ED2, a control valve RV1 with an actuator A1, a check valve BV1, a direct-acting differential pressure regulator PR1 with an RV2 valve, temperature sensors of the coolant DT1 and TG2 accordingly, in the supply and return pipelines, pressure sensors PT1 and PT2, an outdoor temperature sensor TG3, as well as a thermal energy metering unit, for example, a heat meter with a set of sensors.

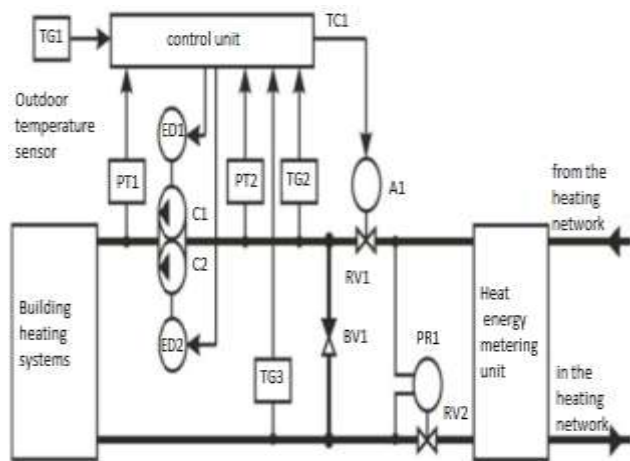


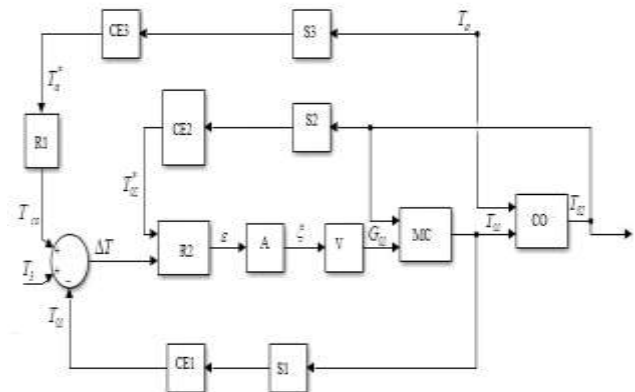
Fig. 1: Block diagram of an automated IHP building

A generalized functional diagram of the heating system of an automated building IHP is shown in Figure 2. The circuit contains the following elements: WC – weather compensation unit (interference control); CU – control unit (deviation control, Figure 2.); PU – protection unit (control of the permissible temperature range of the coolant in the return pipeline);

BS - logic control unit switching input signals and depending on temperature; Elements CE1 – CE3 converters of output values (resistances) of temperature sensors S1 – S3 to the physical quantities (temperatures) measured by them; the actuator in the form of an electric motor with a constant speed of rotation of the shaft; the regulating body V - in the form of a seat valve; the heating element of the mixing unit carriers – MK (mixing unit) (Figure 2) from the connected heating networks and the return pipeline of the building heating system through a jumper with a check valve; the control object is CO (control object), which is the building heating system (HS).

Designations of the main values of the functional scheme: – outdoor air temperature; – converted outdoor air temperature; – calculated temperature of the coolant in the supply pipeline of the building heating system; – control deviation of

the coolant temperature in the supply pipeline of the building heating system (set by the user to the dispatcher for correction);  $\varepsilon$  – temperature deviation from the set value;  $x_2$  – control signal of the control unit;  $v$  – output signal of the switching unit;  $x_3$  – reduced value of the movement of the regulatory body;  $G_l$  – calculated flow rate  $T_l$  – the temperature of the coolant at the entrance to the IHP, formed by the boiler power plant, depending on; and is the temperature of the coolant, respectively, in the supply and return pipelines of the heating system (SS) city. buildings;  $T_{01}^*$  and  $T_{02}^*$  converted temperatures and, respectively; R1-R3 – output resistances of temperature sensors S1-S3;  $u_1$  and  $u_2$  control signals of the PU unit, which set the movement of the actuator A in the direction of opening or closing the regulatory body V, respectively;  $u$  – output signal of the PU unit.



the movement of the regulatory body (RB);  $G_{0l}$  - the flow rate of the coolant after the RB, i.e. before the jumper with the check valve;  $T_{0l}$  - the temperature of the coolant in the supply pipeline of the internal circuit RB of the building;  $T_{0l}^*$  - the measured temperature of the coolant in the building heating system;  $T_{02}$  - the temperature of the coolant in the return pipeline heating systems of the building;  $T_{02}^*$  - the measured temperature of the coolant at the inlet to R2.

A mathematical model of a building HS based on an automated IHP in accordance with the functional scheme and taking into account the structures of regulators R1 and R2 (to simplify the scheme in Figure 2, their structures are not disclosed) is presented in the form of a system of equation (6). System (1) includes the following equations: Equations of motion of temperature sensors S3 and S1

$$\tau_{da} \frac{dT_a^*(t)}{dt} + T_a^*(t) = k_{da} T(t), \quad (6)$$

$$\tau_{dl} \frac{dT_{0l}^*(t)}{dt} + T_{0l}^*(t) = k_{dl} T_{0l}(t), \quad (7)$$

Equations for regulators R1 and R2 (equation of the heating schedule for calculating the design temperature of the coolant in the supply pipeline of the building HS  $T_{co}$

$$T_{co}(t) = f_l(T_a^*) \quad (8)$$

Coupling equations for determining  $\Delta T$

$$\Delta T(t) = T_{co}(t) + T_3(t) - T_{0l}^*(t) \quad (9)$$

Equations of the R2 controller for control in heat supply systems

$$x_l(\Delta T) = \begin{cases} 0, & -X_d \leq \Delta T(t) \leq X_d \\ k_l \Delta T(t), & |\Delta T(t)| > X_d \end{cases} \quad (10)$$

Nonlinear equation of the restriction zone

$$\varepsilon(t) = \frac{k_m}{X_p} \left[ x_l(\Delta T) T_u + \frac{dx_l(\Delta T)}{dt} \right] \quad (11)$$

The equation of the actuator

$$x_2(t) = \begin{cases} k_2 \varepsilon(t), & -k_m \leq \varepsilon(t) \leq k_m \\ k_m, & |\varepsilon(t)| > k_m \end{cases} \quad (12)$$

The equation of the regulator of the organ concerning the output value  $G_{0l}$

$$\frac{d\xi(t)}{dt} = \frac{k_u}{\tau_u} x_2(t) \quad (13)$$

Coupling equations for the mixi

$$G_{0l}(t) = G_l k_k e^{k_2 \xi(t)} \quad (14)$$

Equation of motion of the CO through the control channel

$$T_l G_{0l}(t) + T_{02}(t)(G_{co} - G_{0l}(t)) = G_{co} T_{0l}(t) \quad (15)$$

Equations of motion of the temperature sensor S2

$$\tau_{d2} \frac{dT_{02}^*(t)}{dt} + T_{02}^*(t) = k_{d2} T_{02}(t) \quad (16)$$

$$\tau_1 \tau_2 \frac{d^2 T_{02}(t)}{dt^2} + (\tau_1 + \tau_2) \frac{dT_{02}(t)}{dt} + T_{02}(t) = k T_{0l}(t) \quad (17)$$

The equation for determining the value of thermal power

$$W(t) = G_{0l}(t) T_l(t) - G_{02}(t) T_{02}(t) \quad (18)$$

Additional designations in the system of equations

(6) are as follows:  $\tau_{di}$  and  $k_{di}$  - accordingly, the time constant and the transmission coefficient of the  $i$ -th temperature sensor;  $G_{co}$  is the flow rate of the coolant in the internal circuit of the building HS, determined by the circulation pump (Figure 1);  $G_l$  - nominal flow rate of the coolant at the inlet of the RB;  $T_l$  - the temperature of the coolant in the supply pipeline at the input to the IHP;  $x_l(\Delta T)$  - the output value of the nonlinear dead zone of the regulator R2;  $x_2(t)$  - the output value of the nonlinear restriction zone (saturation) in the R2 regulator;  $k_1$  and  $k_2$  the proportionality coefficients, respectively, of the nonlinear dead zones and the limitations of the regulator R2;  $X_d$  - the dead zone of the regulator R2;  $X_p$  - the proportionality zone of the regulator R2;  $T_u$  - the constant of the regulator R2.

### 3 Results

Based on the mathematical model of building HS, the equation for determining the value of the

thermal power  $W(t)$  in a building, taking into account the use of an automated IHP, is as follows:

$$W(t) = G_{01}(t)T_1(t) - G_{02}(t)T_{02}(t) \quad (19)$$

Simulation parameters. We believe that at the initial moment of time  $t=0$ , the automated Individual heating point of the building is switched to the reduced heat consumption mode by reducing  $T_{co}$  by  $5^{\circ}C$ .

The initial parameters for modeling are presented in Table 1.

Table 1. Initial parameters for modeling

Name of the parameter, its designation	Dimension value
conversion coefficient of the controlled object, $k$	0.807
the time constant of the control object, $\tau_1$	1337 s
the time constant of the control object, $\tau_2$	759 s
the initial temperature of the coolant in the system, $T_2$	$48^{\circ}$
the maximum flow rate at the commissioning of an IHP, $G_l$	$14.7 \text{ m}^3/\text{h}$
maximum consumption in building HS, $G_{co}$	$16 \text{ m}^3/\text{h}$
outdoor air temperature, $T_a$	$-1^{\circ}$
the initial position of the actuator valve stem	74.6%
controller parameter R2, $k_m$	100%
controller parameter R2, $X_d$	0

The main characteristics of time sensors are presented in Table 2.

Table 2. Time constants of temperature sensors

Sensor type	Purpose and symbol.	Magnitude, dimension
ESMU-100	Submersible copper coolant temperature sensor in the sleeve, $\tau_{dl}$	32 s
ESMT	Outdoor air temperature sensor	900 s

The main characteristics of the control valve of the VB2 CB are presented in Table 3.

Table 3. Characteristics of the VB2 control valve

Name of the parameter, its designation	Magnitude, dimension
diameter, $D_v$	40 mm
ratio, $k_{kvs}$	$25 \text{ m}^3/\text{h}$
conditional pressure, $P_v$	2.5 MPa
temperature, $T_{min}$	$5^{\circ}C$
temperature, $T_{max}$	$150^{\circ}C$
rod stroke, $h$	10 mm

The technical characteristics of the AME 20 type actuator for operation with the VB2 control valve are presented in Table 4.

Table 4. Characteristics of the AME 20 actuator

The name of the parameter, designation	Magnitude, dimension
voltage	24 V
frequency	50/60 Hz
power consumption	4 Wt
type of control signal	analog
developed force	450 N
stroke of the rod	10 mm
time of movement of the rod by 1 mm	15 s/mm
input signal 1	0-10 V, $R_1=24$ kOm
input signal 2	0-20 mA; $R_i=500$ kOm
input signal	0(2)-10 V
the presence of a return spring	no
minimum ambient temperature	0
maximum ambient temperature	55 °C

The proportional-integral law of regulation (PI - regulator) formed by the control unit R2 of the controller with the use of an executive mechanism of the AME 20 type is investigated.

The equations of motion of the control unit R2 have the form (see the system of equations (6)):

$$\varepsilon(t) = \frac{k_m}{X_p} \left[ x_l(\Delta T)T_u + \frac{dx_l(\Delta T)}{dt} \right] \quad (20)$$

where  $X_p = 80^\circ \text{C}$ ,  $T_u = 10$  s. The initial equation (20) in the PI-controller is then integrated by the executive mechanism. This is an important feature of the controller in question.

The result of simulation modeling taking into account the  $X_p$  and  $T_u$  data for this research variant is shown in Figure 3.

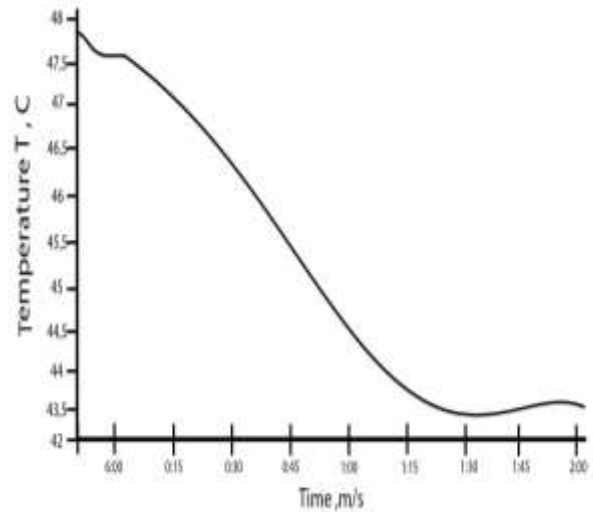


Fig. 3a: Dynamic characteristics in the form of changes in the temperature of the coolant  $T_{01}$  at the input with the building HS on an increased time scale

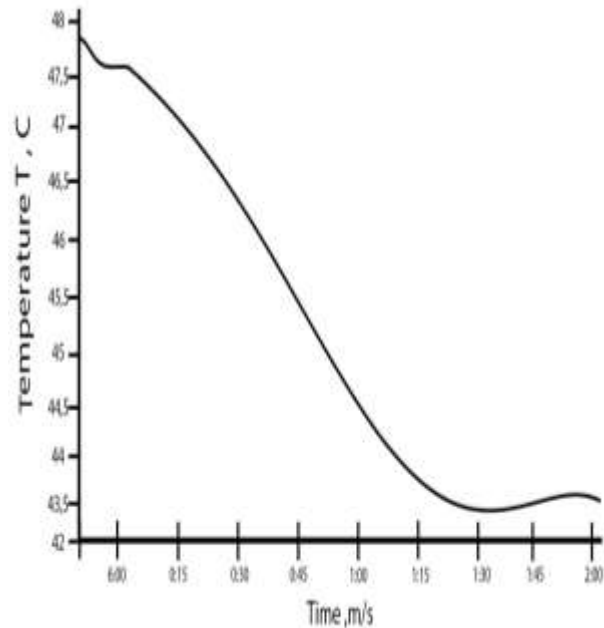


Fig. 3b: Temperature of the coolant  $T_{02}$  in the return pipeline at the outlet of the building heating systems

As can be seen from Figure 3a and Figure 3b analysis of the studied dynamic characteristics in the form of changes in the temperature of the coolant  $T_{01}$  at the input from the building HS on an increased time scale (Figure 3a) and the temperature of the coolant  $T_{02}$  in the return pipeline at the outlet

of the building heating systems (Figure 3b) shows that they have the form of aperiodic transients.

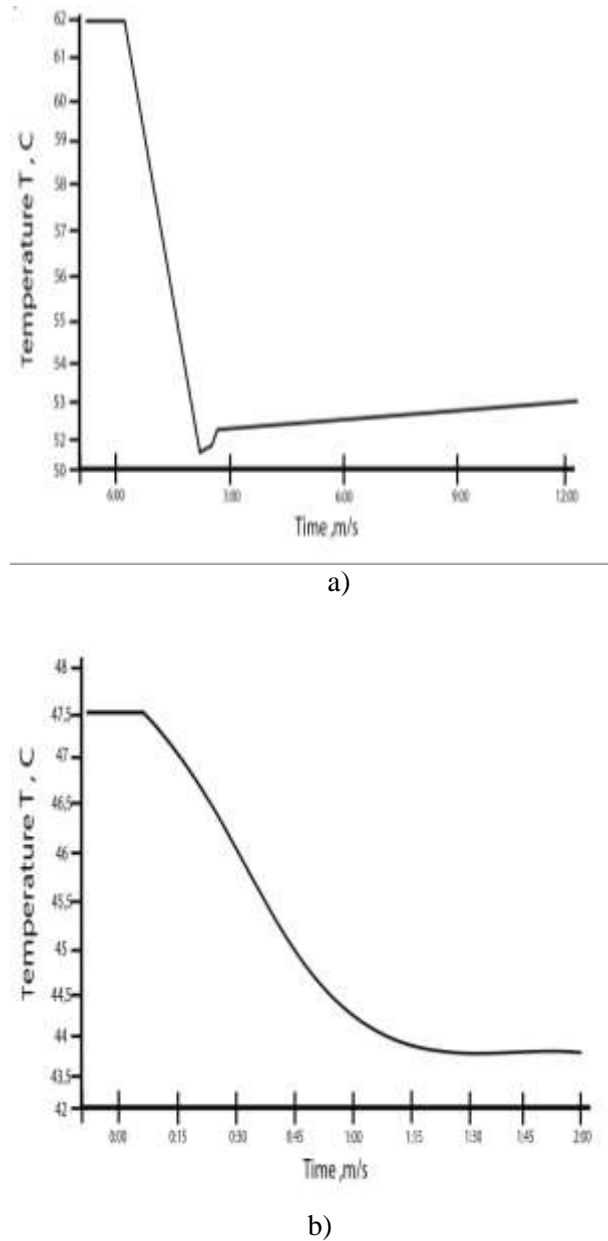


Fig. 4: Dependence of changes in  $T_{01}$  (a) and  $T_{02}$  (b) of building HS

As can be seen from Figure 4a and Figure 4b, the estimated heat consumption of the building is about 1.08502 KJ. We increase the time in equation (7), i.e. we take it equal to 60 s.

Analysis of the studied dynamic characteristics in the form of changes in  $T_{01}$  at the input to the heating systems of buildings on an increasing time scale (Figure 4a) and the temperature of the  $T_{02}$  coolant in the return pipeline at the outlet of the

building HS (Figure 4b) shows that a transient oscillatory process is observed for  $T_{01}$ . The estimated heat consumption is about 1.0858 GJ. The integral law of regulation (I-regulator) formed by the control unit R2 of the controller using a similar actuator is investigated. In this regard, the equation of motion of the control unit R2 is replaced in the system of equations (6) by an equation of the form:

$$\varepsilon(t) = \frac{k_m}{X_p} [x_l(\Delta T)T_u] \quad (21)$$

where  $X_p = 80^\circ\text{C}$ ,  $T_u = 20\text{ s}$ . The initial equation (21) in the I-regulator is then integrated by the actuator. The simulation results for this variant are shown in Figure 5.

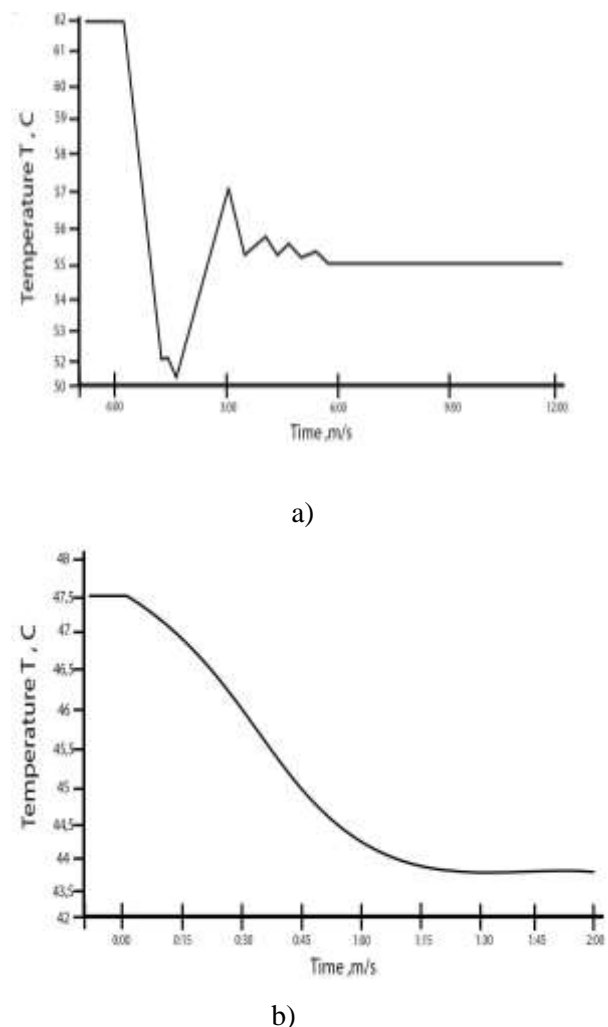


Fig. 5: Dependences of changes  $T_{01}$  (a) and  $T_{02}$  (b) of the building HS c.

Figure 5 shows an analysis of dynamic characteristics in the form of changes in the



temperature of the  $T_{01}$  coolant at the input to the building heating systems on an increased time scale (Figure 5a) and the temperature of the  $T_{02}$  coolant in the return pipeline at the outlet of the building heating systems (Figure 5b) shows that an oscillatory transition process is observed for. The estimated heat consumption of the building in the studied case is about 1.08535 GJ. In equation (3) we take the following parameter values:  $X_p = 100^\circ\text{C}$ ,  $T_u = 1\text{ s}$ . During all studies, the values of  $T_u = \text{const}$  were selected, taking into account that  $T_u$  is greater or less than the time of movement of the valve stem using the actuator (Table 4). The simulation results taking into account the selected values of  $X_p$  and  $T_u$  are shown in Figure 6.

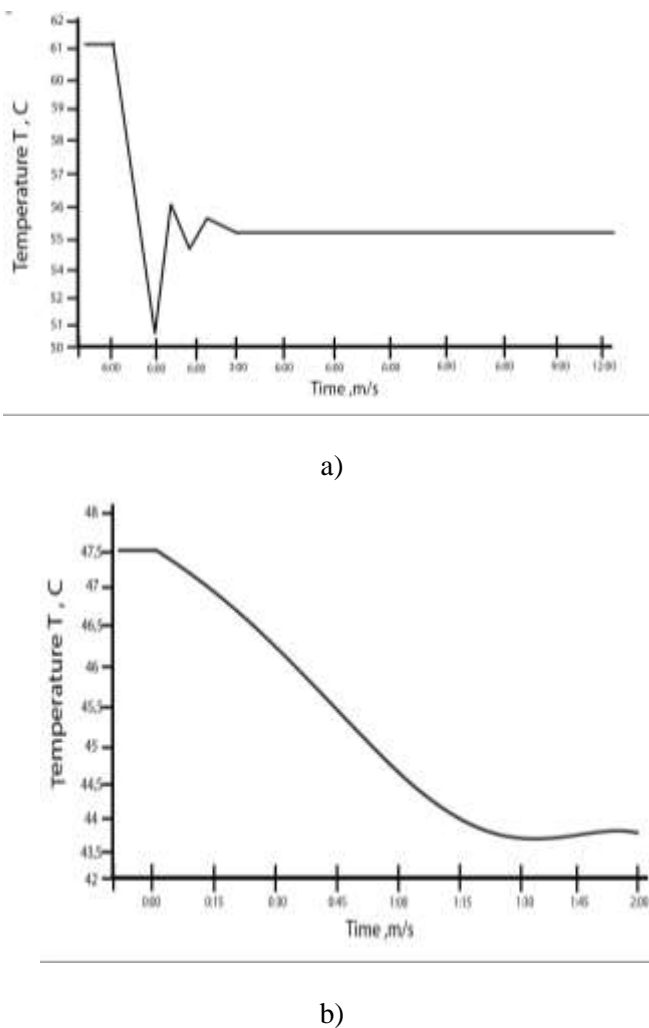


Fig. 6: Dependences of changes  $T_{01}$  (a) and  $T_{02}$  (b) of the building HS

Figure 6 shows an analysis of the studied dynamic characteristics in the form of changes in  $T_{01}$  at the input to the heating system of the building on an enlarged time scale (Figure 6a) and the temperature of the coolant  $T_{02}$  in the return pipeline at the outlet of the building HS (Figure 6b) shows that they have the form of aperiodic transients. As expected, the estimated heat consumption of the building decreased to 1.08456 GJ.

A comparative analysis of the results obtained for the studied 2 control laws formed by the control unit R2 of the controller using an AME 20 type actuator with different parameters of the controller unit showed the following:

1) for the PI controller, the calculated heat consumption of the building increases slightly from 1.08502 GJ to 1.08588 GJ when an oscillatory transient occurs;

2) for the I-regulator, the calculated heat consumption of the building is at the same level as for the PI-regulator and increases slightly with an oscillatory transient from 1.08456 GJ to 1.08535 GJ. Transient processes of an oscillatory type for the actuator should be excluded since they lead to premature failure of the electric motor of the actuator. To eliminate the oscillatory processes that have appeared in the automatic control system of the automated individual heat pump during the processes under study, it is necessary to change the tuning parameters of the regulator taking into account the specified time of movement of the rod using the actuator.

At the initial moment of time  $t = 0$ , the automated ITP is switched to the reduced heat consumption mode by reducing the temperature by  $5^\circ\text{C}$  (due to the deviation). The duration for the studied cases is 1 h. 45 min. Since the coolant mixing unit is characterized by significantly less inertia compared to the building heating system, the actual temperature  $T$  and the value of its deviation  $e$  from the calculated temperature in the heating system. Analysis of the temperature change  $T_{01}$  in the supply pipeline of the heating system shows that the duration of the transition process through the control channel does not exceed 5 minutes. The duration of the transition to a new steady state of the water temperature in the return pipeline exceeds 1.5 hours and is determined by transients in the heating system of the building. Consequently, transients in the coolant mixing unit can be neglected. In the case under study, the temperature overregulation is

0.93%, and the total calculated thermal energy consumption is 1.16256 GJ. Analysis of the characteristic G shows that in the mode of reduced heat consumption, when working out the commands of the BR unit using the AME 20 actuator through the VB 2 control valve, the flow rate G initially almost instantly decreases to 3.7 m<sup>3</sup>/h (53.3%), and then, according to the exponential law, partially increases to 5.54 m<sup>3</sup>/h (79.8%). This is due to the operation of the regulator and a decrease in the temperature of the coolant G02 in the return pipeline of the building heating system. Analysis of temperature changes in the supply pipeline of the building heating system shows that the process has become oscillatory, and the nature of the transition process for the temperature in the return pipeline has not changed. At the same time, the over-regulation for the temperature of G01 is 6.25%, and the total estimated thermal energy consumption is 1.08502 GW. A comparative analysis of dynamic processes for the automated ITP under study with various actuators shows that an increase in the speed of the actuator based on the AME 30 leads to the operation of its electric motor in the mode of frequent operation, however, transient processes of the oscillatory type in the system should be excluded, because they contribute to the premature failure of the IM electric motor. Therefore, when changing the heat consumption mode of a building, it is necessary to change the tuning coefficients of the BR control unit.

## 4 Discussion

The developed mathematical model of the building heating control system in the form of a block diagram takes into account the features of the mathematical models of the elements and their nonlinear characteristics in the structure of the regulator with an integrated actuator, switching units, and a regulatory body, as well as the nonlinearity in the area of mixing of heat carriers in the heating point. The model allows us to study the processes occurring in the heating system of a building with an automated when controlling and disturbing influences change. The developed method of mathematical modeling of the control system for decentralized heating of a complex of buildings, based on mathematical models of distributed power systems and experimental studies,

allows determining the parameters of the coolant when the outdoor temperature changes, qualitative regulation of heat in autonomous sources, quantitative regulation in automated, etc. The introduction of automated ITPS for buildings with the highest thermal load leads to noticeable savings in thermal energy under various operating modes of heat consumption systems. Thus, during dynamic processes in the thermal points of the building complex, significant fluctuations in the values of thermal power are observed, determined by changes in the flow rate of the coolant and the temperature difference in heating systems. This mathematical model allows you to determine the values of the coolant when the outdoor temperature changes, better heat regulation in autonomous sources, numerical control in automated individual heating devices, etc. The method allows you to study the coordination of an automated separate heating point to increase the performance of managing distributed power systems of buildings.

## 5 Conclusion

The possibilities of mathematical modeling of the control of an automated individual thermal point of a building with well-known standard regulators are presented. When creating automated individual heating points of buildings, it is necessary to take into account the results obtained, which showed that the heat consumption of a building under standard regulatory laws is approximately at the same level and therefore there is a possibility of the practical application of an I-regulator to regulate the heating process of a building since it is easier to implement and configure.

The analysis of the transient process taking into account the temperature of the coolant in the return pipeline of the building shows that the control object is a low-frequency filter with respect to significant fluctuations at its input (in the supply pipeline). In contrast to the known methods for determining the parameters of the controller control unit based on the calculated transient characteristics of the serial connection of the control object and the temperature sensor using the developed mathematical model, it is possible to determine not only the optimal settings of the control unit but first of all, the parameters of the coolant with possible changes both in the structure of the elements of an automated individual heating point and in the heating systems of a building or structure.

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### **Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)**

-Farida Telgozhayeva and Murat Kunelbayev carried out the simulation and the optimization.  
-Zhanara Spabekova and Gulnur Tyulepberdinova have implemented the concept.  
-Ainagul Berdygulova was responsible for the Statistics.

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### **Conflict of Interest**

The authors have no conflicts of interest to declare.

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