Possibility of Quenching of Limit Cycles in Multi Variable Nonlinear Systems with Special Attention to 3X3 Systems

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Abstract: - The present work proposes novel methods of Quenching self-sustained oscillations in the event of the existence of limit cycles (LC) in 3x3 non-linear systems. It explores the possibility of Stabilising/Quenching the LC by way of signal stabilization using high frequency dither signals both deterministic and random when 3X3 systems exhibit such self-sustained nonlinear oscillations under autonomous state. The present work also explores the suppression limit cycles of 3X3 systems using state feedback by either arbitrary pole placement or optimal selection of pole placement. The complexity involved, in implicit non-memory type nonlinearity for memory type nonlinearities, it is extremely difficult to formulate the problem. Under this circumstance, the harmonic linearization/harmonic balance reduces the complexity considerably. Furthermore, the method is made simpler assuming the whole 3X3 system exhibits the LC predominantly at a single frequency. It is equally a formidable task to make an attempt to suppress the limit cycles for 3X3 systems with memory type nonlinearity in particular. Backlash is one of the nonlinearities commonly occurring in physical systems that limit the performance of speed and position control in robotics, the automation industry, and other occasions of modern applications. The proposed methods are well illustrated through examples and substantiated by digital simulation (a program developed using MATLAB CODES) and the use of the SIMULINK Toolbox of MATLAB software.

Key-Words: - Describing Function, Pole Placement, 3x3 nonlinear systems, limit cycles, harmonic linearization, signal stabilization, Random Input, Gaussian Signals, suppression limit cycles, Ricatti Equation.

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1 Introduction

In the present scenario, nonlinear self-sustained oscillations or LC are the basic features of instability. The importance and weight-age of this problem were felt among the researchers, [1], [2], [3], [4], [5] in the decades past, where they were mostly focussing on single input and single output (SISO) systems. However, for the last five to six decades, the analysis of 2X2 Multi Input and Multi Output (MIMO) Nonlinear Systems gained importance and quite a good amount of literature is available, [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], addressing this area of research. The analysis and prediction of limit cycles in both SISO and MIMO systems, a means of increasing the reliability of the describing function (DF) are well established, [4], [5], [10], [13], [16], [23], [49], [50] and others used harmonic linearization/harmonic balance, [29], [33], [51].

In several cases in physical 2X2 nonlinear systems limit cycles are observed such as a couple Core Reactor [12], Pressurised Water Reactor (PWR) nuclear Reactor System [20], Radar Antenna pointing system, [11], and Inter Connected power system [39], which can fit the structure, [1], [24] of a MIMO two-dimensional nonlinear system.

Backlash is a most remarkable nonlinearity, commonly existing in physical systems that limit the performance of speed and positions, this has been extensively discussed for 2X2 MIMO systems, [7], [8], [35], [39], [43], [44], [45], [46], [48]. The recent literature describes some facts of multidisciplinary applications where limit cycle oscillations have been discussed. The authors, [52], presented three possible scenarios, namely, stable limit cycles and chaos arise naturally in the flow and thermal dynamics of the device. The researchers, [53], formulated/initialized the cell model to the limit cycle, running one-dimensional simulations of 500 stimuli at a BCL of 300ms. In [54], the dynamic nature of the nonlinear system switches between a stable equilibrium point and a stable limit cycle has been presented. In [55], the stable limit cycle has been observed in an autocatalytic system through the characteristics of the Hopf bifurcation. In [56], the existence of limit cycling oscillations has been observed in Biological Oscillators having both positive and negative feedback. The researchers, [57], have observed in natural systems a closed loop as in a stable limit cycle by reviewing empirical dynamic modeling.

Scanty literature is available, which addresses 3X3 nonlinear MIMO systems in the last two decades only [6], [38], [40], [41], [42].

However, several industrial problems with two or more higher dimensional configurations, [14], and prediction of limit cycles via the describing function method prove to be quite essential, [4], [5], [10], [13], [14], [16], [23], [49]. Hence the prediction of limit cycles in three dimensional nonlinear multivariable systems which can fit the structure of general 3X3, [6], [27], nonlinear systems has been addressed.

In the event of the existence of limit cycling oscillations, the possibility, of quenching the sustained oscillations using the method of signal stabilization has been investigated, [5], [30], [31], [49], [50], in 2X2 nonlinear systems with non-memory type nonlinear elements and memory type nonlinear elements in [46] using deterministic signals. Until, [47] signal stabilization with random signal for memory type multivariable nonlinear systems was not available even for 2X2 systems.

The authors in [47], focused on robust and non-fragile stabilization of nonlinear systems described by the multivariable Hammerstein model. The method illustrates a general procedure that addresses the general multi-variable nonlinear systems. Of course, the method considers uncertainties and most importantly control is adopted for stabilization which fails to project insight into the problem. However, the present work shows a simple method to quench the limit cycling oscillations exhibited in a class of 3x3 nonlinear systems and stabilize the systems using random signals in particular a Gaussian Signal. The signal stabilization refers to the possibility of quenching the self-sustained oscillations by injecting a suitable high frequency preferably more than ten times of ω_s (the frequency of LC) signal at any point of the system, [5]. The random signal having Gaussian distribution contains infinite components of frequency. The Gaussian Signals are passed through a high pass filter so that the high frequency signal quenches the limit cycles and stabilizes the system. This has been illustrated through examples 1 and 2 revisited. This method projects a clear and lucid insight into the problem.

Prediction and suppression of limit cycles oscillations in 2X2 memory type nonlinear systems using arbitrary pole placement has been discussed in [8], [32], [43], [44], [58] and pole placement by optimal selection using Riccati equation, [48], [59]. The suppression of limit cycle oscillations using the state feedback approach has been dealt with to an extent [8].

The proposed work follows the dynamics of general 3X3 nonlinear systems shown in Figure 2, Figure 3, [6], which can also be taken as an equivalent representation of the general multivariable system considered in [42].

Having realized the importance of quenching/suppression of limit cycle oscillations the proposed work first establishes the exhibitions of limit cycles in 3X3 nonlinear systems following a similar procedure as depicted/illustrated, [6].

2 Prediction of Limit Cycles in a General 3X3 Nonlinear Systems

To avoid the complexity involved in the structure, [6], [49], a graphical method is opted for investigation of the existence of limit cycling oscillations in 3X3 nonlinear systems.

2.1 Graphical Method

A graphical method has been adapted, [6], for the prediction of limit cycling oscillations in a 3x3 nonlinear system. The steps as depicted in [6], have been followed for the establishment of existing limit cycling oscillations in a 3X3 nonlinear system which has been; illustrated through numerical examples and validated by (i) digital simulation, (ii) by use of SIMULINK Toolbox of MATLAB software.

Consider a system of Figure 1, a class of 3X3 nonlinear systems for simplicity it is assumed that the whole 3X3 system exhibits the LC predominantly of a single frequency sinusoid and harmonic linearization/harmonic balance leading to the use of describing function methods have been opted.

The normalized phase diagrams, [28], [46], are drawn with three combinations such as:

Combination 1: For subsystems S1 & S2: C1 (+ve), C2 (-ve) and C3 (+ve)

Combination 2: For subsystems S3 & S2: C2 (+ve), C3 (-ve) and C1 (+ve)

Combination 3: For subsystems S1 & S3: C3 (+ve), C1 (-ve) and C2 (+ve)

Example 1 and 2 are used for illustration of procedures of Normalized phase diagrams.

The linear elements are represented by $G_1(s) = \frac{2}{s(s+1)^2}$; $G_2(s) = \frac{1}{s(s+4)}$; $G_3(s) = \frac{1}{s(s+2)}$ and Nonlinear elements are taken, Ideal relays as shown in Figure 2(a) and ideal saturations as shown in Figure 2(b) for Example 1 and Example 2 respectively

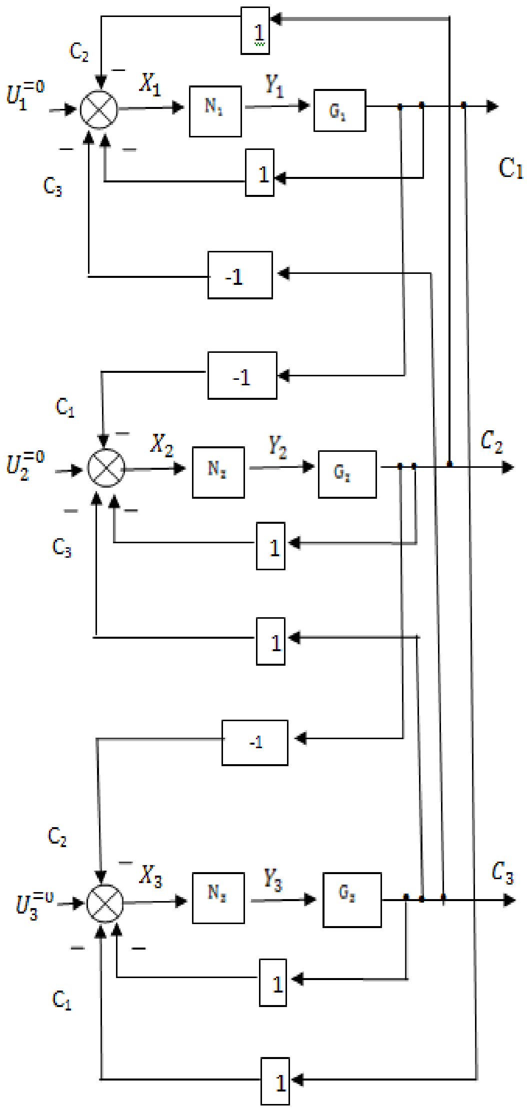


Fig. 1: An equivalent 3X3 multivariable nonlinear system of Figure 2 in [6]

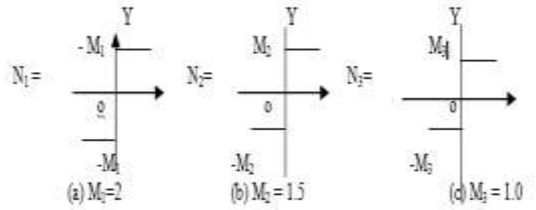


Fig. 2(a): All Ideal Relays

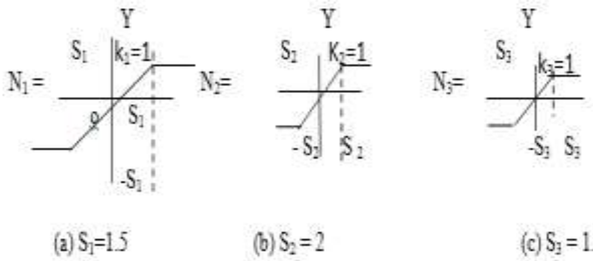


Fig. 2(b): All Ideal Saturation type nonlinear elements (with slopes k_1, k_2, k_3)

In examples 1 and 2 non-memory type nonlinear elements are used. Assuming harmonic linearization these nonlinear elements can be equivalently represented by their describing functions, [28], which are real functions in these two examples and do not contribute any phase angles to the system. Hence the phase angles of the system are due to linear functions) $G_1(s)$, $G_2(s)$, $G_3(s)$ which are complex functions of complex variable s, the Laplace operator. *It may be noted that for frequency response, input is sinusoidal and outputs are steady state values considered, so that s (Laplace Operator) is replaced by $j\omega$*, [6].

X_1, X_2 & X_3 are the amplitudes of respective sinusoidal inputs to the nonlinear elements. C_1, C_2 & C_3 are the amplitudes of sinusoidal output of subsystems S_1, S_2 & S_3 respectively. G_1, G_2 & G_3 are the magnitudes/absolute values of linear elements represented by their transfer functions of subsystems S_1, S_2 & S_3 respectively. N_1, N_2 & N_3 are the magnitudes/absolute values of linear elements represented by their describing functions of subsystems S_1, S_2 & S_3 respectively.

$\theta_{L1} = \text{Arg}.\ (G_1(j\omega)) = -90 - 2\tan^{-1}(\omega)$:

$\theta_{L2} = \text{Arg}.\ (G_2(j\omega)) = -90 - \tan^{-1}(\frac{\omega}{4})$:

$\theta_{L3} = \text{Arg}.\ (G_3(j\omega)) = -90 - \tan^{-1}(\frac{\omega}{2})$:

$$N_2 = (11-3\omega^2)\omega^2 \pm \sqrt{(11-3\omega^2)\omega^4 - 8(\omega^2+16)(1-\omega^2)^2\omega^2} \quad (1)$$

$$N_1 = \frac{(\omega^2-1)}{8} N_2 + \frac{9\omega^2-\omega^4}{8} \quad (2)$$

$$\frac{X_1}{X_2} = \frac{(1+\omega^2)\sqrt{[\omega^2(\omega^2+16-2N_2)+N_2^2]}}{2N_1\sqrt{\omega^2+16}} \quad (3)$$

$$\frac{X_1}{X_2} = \frac{BD_i}{AD_i} = \sqrt{\frac{(1-u_i)^2+(u_i)^2}{(1+u_i)^2+(u_i)^2}} \quad (4)$$

For a fixed value of ω the Combinations of Subsystems 1, 2, and 3, Normalised Phase Diagrams are shown in Figure 3(a), (b), and (c) respectively. However, any one of these combinations can be used for the determination of limit cycling conditions and the related quantities of interest.

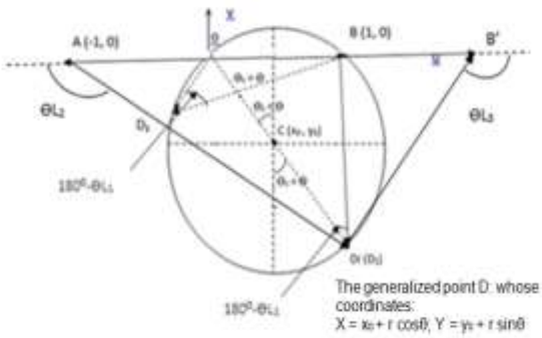


Fig. 3(a): Normalised Phase Diagram with C_1, C_2 & C_3 for the combination 1, where C_1 (+ve), C_2 (-ve) and C_3 (+ve)

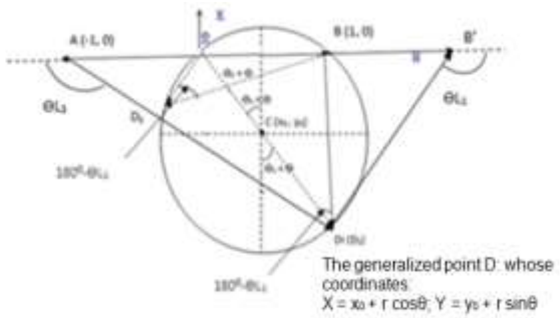


Fig. 3(b): Normalised Phase Diagram with C_1, C_2 & C_3 for the combination 2, where C_2 (+ve), C_3 (-ve) and C_1 (+ve)

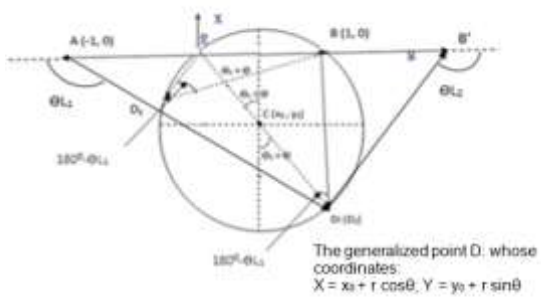


Fig. 3(c): Normalised Phase Diagram with C_1, C_2 & C_3 for the combination 3, C_3 (+ve), C_1 (-ve) and C_2 (+ve)

With reference to a normalized phase diagram [28], [46], the phase representing X_2 would lie along a straight line drawn at an angle θ_{L2} with the phase C_2 ($C_2 = - R_1$). The intersections of this straight line with the circle drawn concerning θ_{L1} would represent possible self-oscillations [28]. The concept has been extended for 3 x 3 as:

(i) Consider Figure 3(a) the phase representing X_2 and X_3 would lie along straight lines drawn at angles θ_{L2} and Θ_{L3} with the phase C_2 ($C_2 = - R_1$) and C_3 ($C_3=R_1$) respectively. The intersections of these straight lines with the circle drawn concerning θ_{L1} would represent possible self-oscillations.

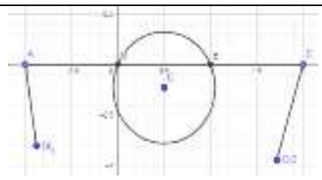
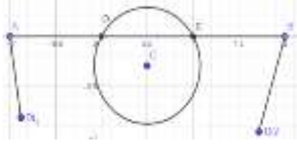
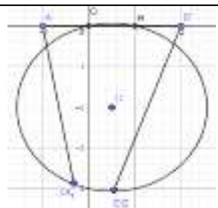
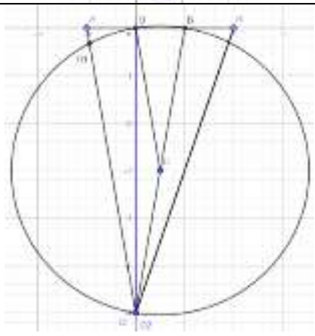
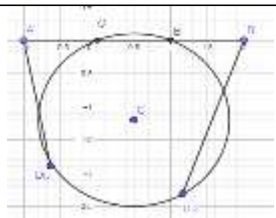
(ii) Consider Figure 3(b), the phase representing X_3 and X_1 would lie along straight lines drawn at angles

θ_{L3} and θ_{L1} with the phase C_3 ($C_3= - R_2$) and C_1 ($C_1=R_2$) respectively. The intersections of these straight lines with the circle drawn concerning θ_{L2} would represent possible self-oscillations.

(iii) Consider Figure 3(c) the phase X_1 and X_2 would lie along straight lines drawn at angles θ_{L1} and θ_{L2} with phase C_1(C_1=-R_3) and C_2(C_2 = R_3) respectively. The intersections of these straight lines with the circle drawn concerning θ_{L3} would represent possible self-oscillations.

Table 1 shows the θ_{L1}, θ_{L2}, θ_{L3}, r (radius), and the intersection points of the straight lines and circle for combination 1 corresponding to example 1. It may be noted that Table 1 Contains $\frac{X_1}{X_2}$ obtained from Eqn.3 and Eqn.4 are matched at a limit cycling frequency.

Table 1. θ_{L1}, θ_{L2},θ_{L3}, r (radius), and the intersection points of the straight lines and circles for combination 1 corresponding to Example 1

ω	**θ_{L1}**	**θ_{L2}**	**θ_{L3}**	**r**	**X_1/X_2 from eqn. 3**	**X_1/X_2 from eqn. 4**	**Normalized Phase Diagrams**	**Remark**
0.600	-151.93	-98.531	-106.7	-0.55257	-	-		No intersection of straight lines and circle
0.650	-156.05	-99.23	-108	0.58256	-	-		No intersection of straight lines and circle
0.700	-159.98	-99.926	-109.29	-2.128	-	-		No intersection of straight lines and circle
0.701	**-160.06**	**-99.94**	**-109.32**	**-3.1323**	**1.0**	**1.02 (matched)**		The intersection of st. lines & circle found: Confirms the occurrence of limit cycles ω=0.701, C1 = OD2 = 6 C2 = 1 C3 = 1 X1=BD2=6.08 X2=AD2=6.08 X3=B'D2= 6.32
0.750	-163.74	-100.62	-110.56	-1.3583	-	-		No intersection of straight lines and circle

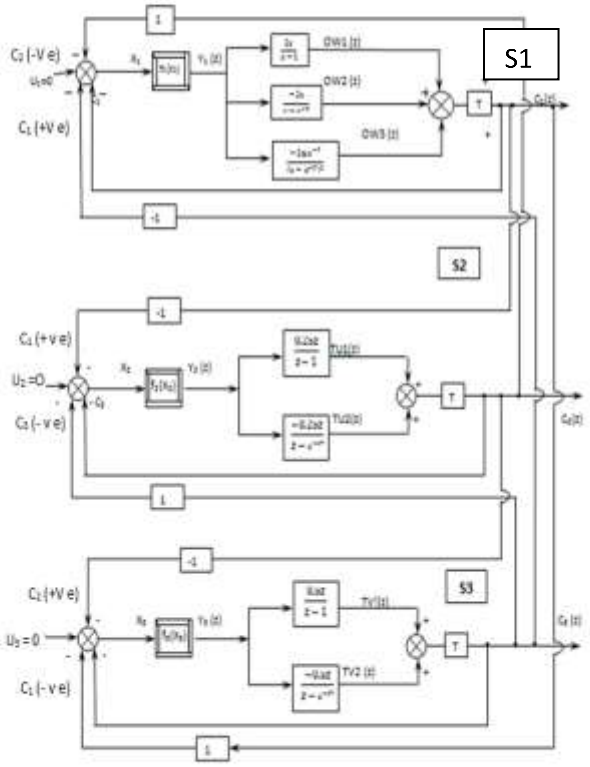


Fig. 4(a): Equivalent Canonical form of Figure 1 for Ex1 & 2

2.2 Digital Simulation

Example 1 and Example 2 are revisited.
A program has been developed [6] with the use of MATLAB code for digital simulation.
The equivalent canonical form of Figure 1 for example 1 and 2 is shown in Figure 4(a) and digital representation is shown in Figure 4(b).

Numerical results obtained from different methods are compared in Table 2(a) and Table 2(b) for example 1 and 2 respectively.

The results/images for example 1 and 2 are shown in Figure 5 and Figure 6 respectively. These are also compared with those of obtained using the SIMULINK Toolbox of MATLAB software.

Table 2(a). Results obtained using different methods corresponding to Ideal Relay Example-1

Sl. No	Methods	C_1	C_2	C_3	X_1	X_2	X_3	ω
1	Graphical	6.0	1.0	1.0	6.08	6.08	6.32	0.701
2	Digital Simulation	4.83	0.74	0.95	4.72	4.91	5.23	0.70
3	Using SIMULINK TOOL BOX OF MATLAB	5.95	1.01	0.96	4.84	5.12	5.62	0.70

Table 2(b). Results obtained using different methods corresponding to Example2: (Saturation)

Sl. No	Methods	C_1	C_2	C_3	X_1	X_2	X_3	ω
1	Digital Simulation	4.345	1.06	1.06	4.464	4.581	4.762	0.628
2	Use of SIMULINK TOOL BOX OF MATLAB	4.30	1.05	1.05	4.425	4.534	4.74	0.6283

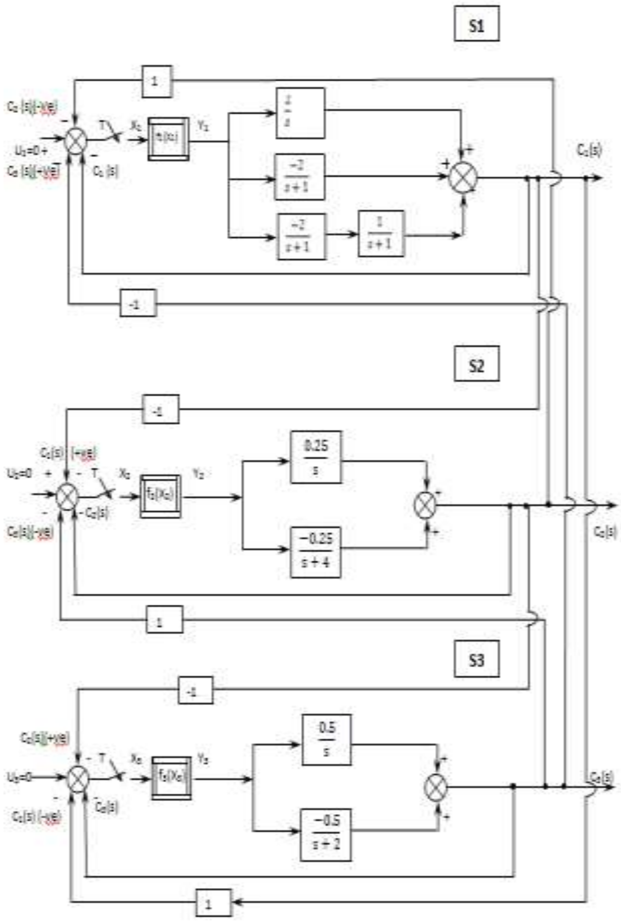


Fig. 4(b): The Digital representation of Figure 1 for Ex.1 & 2

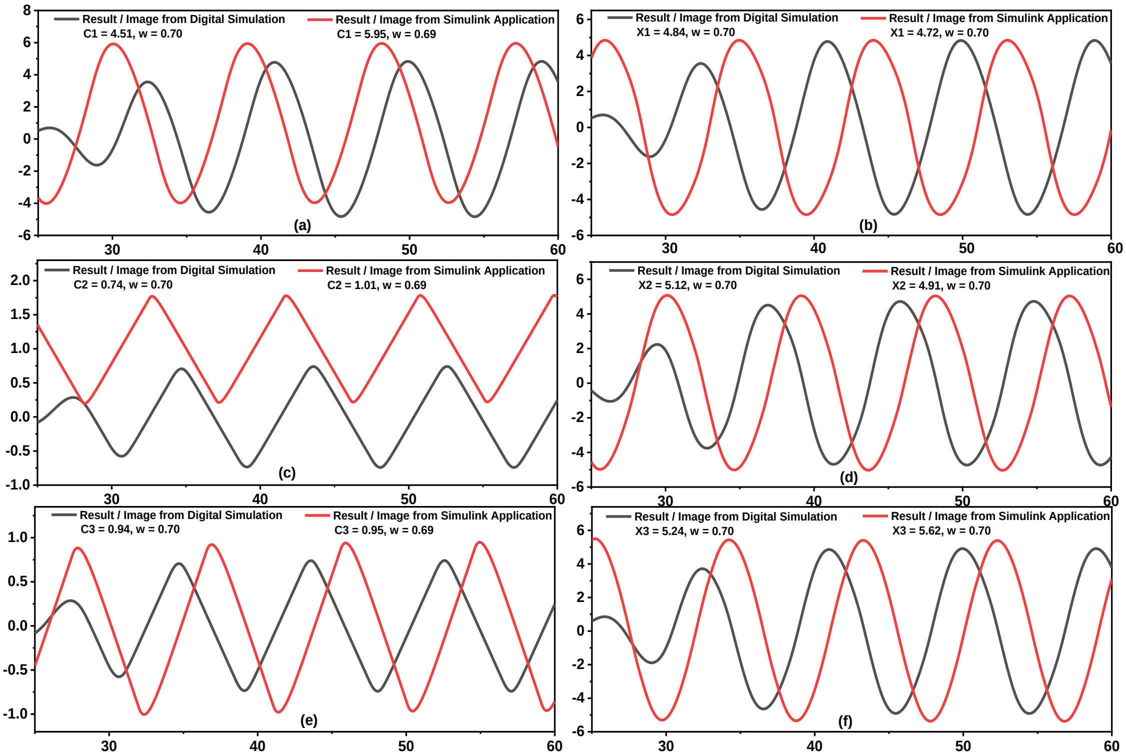


Fig. 5: Results/Images from digital simulation and SIMULINK for C1, C2, C3, X1, X2 and X3 of Example 1 (relay type nonlinearities)

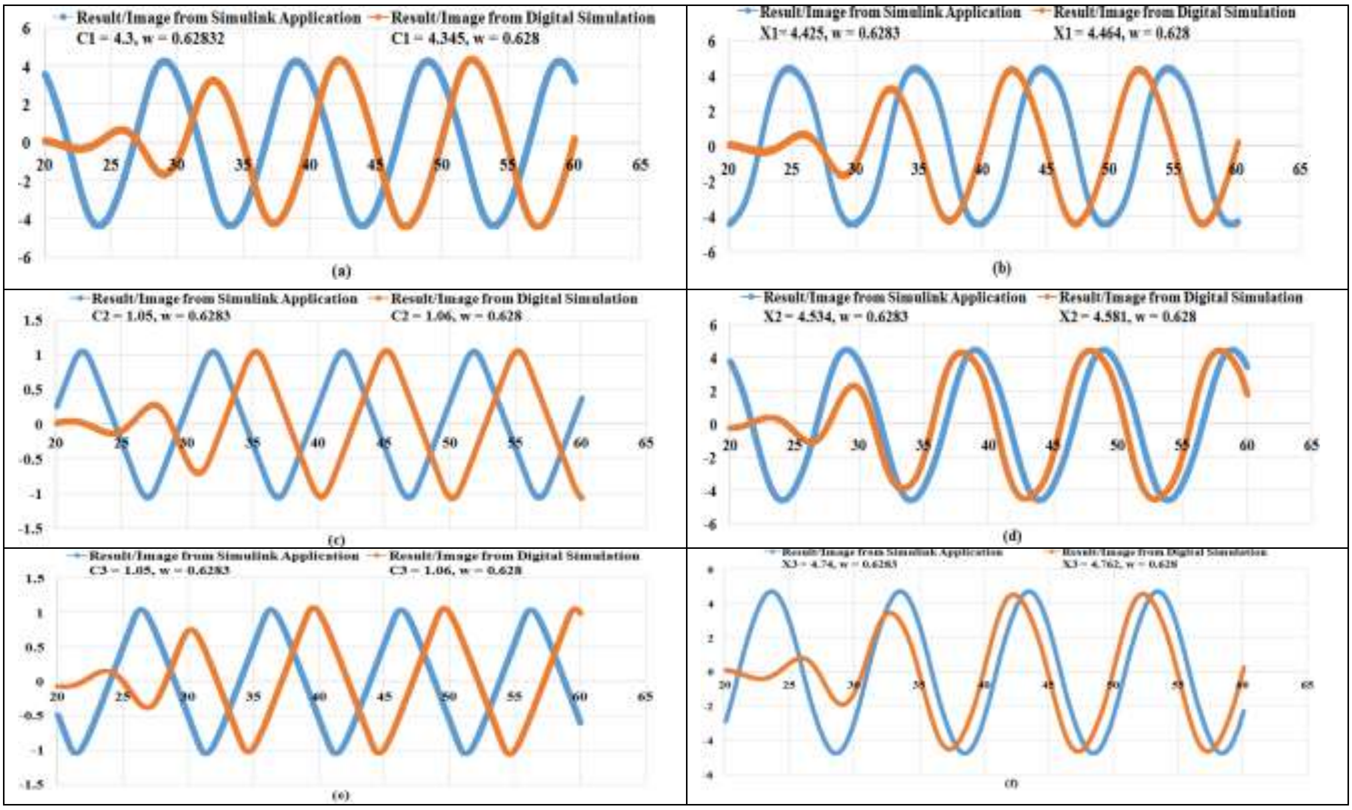


Fig. 6: Results/Images from digital simulation and SIMULINK for C1, C2, C3, X1, X2, and X3 of Example 2 (saturation type nonlinearities)

3 Signal Stabilization in a 3X3 Nonlinear System

3.1 Using Deterministic Signal

In the case of 2x2 nonlinear systems the concept of Signal Stabilisation is as under:

If a 2x2 Nonlinear system exhibits limit cycles (L.C.), in the autonomous state, the possibility of quenching the limit cycling oscillations by injecting a suitable high frequency signal, preferably at least 10 times of limit cycling frequency [5], [30], [46], [49], [50]. The process is also termed forced oscillations which have also been extensively discussed by several researchers [30], [33]. Under the forced oscillation process the phenomena of Synchronization and De-synchronization have been addressed thoroughly in [46] for 2x2 nonlinear systems. Such phenomena have been realized /observed by injecting a sinusoidal input $B_1 \sin \omega_f t$ /$B_2 \sin \omega_f t$ at any one or both input points of two subsystems S_1 and S_2 respectively. When the amplitude B_1 of forcing signal $B_1 \sin \omega_f t$ gradually increased keeping the amplitude B_2 of forcing signal $B2 \sin \omega_f t$ fixed, the system would continue to exhibit a limit cycle. The variables at various points in the system would be composed of signals of the input frequency (ω_f), the frequency of self-oscillations (ω_s), and the combination of frequencies, $k_1\omega_f \pm k_2\omega_s$ where k_1, k_2 assume various integer values. At this condition the system exhibits complex oscillations. In the process of gradual increase of B_1, the frequency of oscillations ωs would also gradually change and for a certain value of B_1, the synchronization would occur, the self-oscillation would be quenched and the system would exhibit forced-oscillation at frequency ωf. On the other hand, if subsequently the magnitude B_1 is reduced gradually, a point would arrive at which the self-oscillations would reappear which is termed a de-synchronization phenomenon. It may be noted that the synchronization value of B_1 is larger than the De-synchronization value of B_1 [46].

Similar facts have been observed in 3x3 nonlinear systems. The forced oscillation can be realized by feeding deterministic or random signals of high frequency, at least greater than 10 times the limit cycling frequency at any one / all input points of subsystems S_1, S_2 and S_3.

If the amplitude B of the high frequency signal is gradually increased, the system would exhibit complex oscillations before the synchronization takes place. On the reverse operation, if the amplitude B is gradually reduced at a certain value of B the self-oscillations i.e. the Limit cycle would reappear and the system would reappear and the system would exhibit complex oscillations again which can be called de-synchronisation. The phenomena of synchronization and de-synchronization can be observed/identified analytically using the Incremental Input Describing function (IDF).

However, the forced oscillation can also be analyzed using the Equivalent Gain/Dual input Describing Function (DIDF), [33], [49], in the case of a deterministic forcing signal in particular with a sinusoidal signal. Similarly, Equivalent Gain (Random input Describing Function-RIDF), [60], [61], [62] in case of random forcing signal, in particular with Gaussian Signal.

The complexity arises in the structures, [6], particularly for the implicit non-memory type or memory type nonlinearities, it may be extremely difficult to formulate and simplify the expressions even using the harmonic linearization method [48]. Hence an attempt has been made to develop a graphical technique using the harmonic linearization / harmonic balance method for prediction of limit cycles in 3x3 nonlinear systems by extension of the procedure as presented in [28]. The method uses the simultaneous intersection of two straight lines and one circle in three combinations.

The analytical/mathematical observation of synchronization and de-synchronization of complex oscillation in the process of signal stabilization would be quite involved and time-consuming. Hence the digital simulation (Using our developed program) opted for the demonstration of signal stabilization with deterministic/random (Gaussian) signals which have been validated through the use of the SIMULINK Toolbox of MATLAB Software.

It is established that the system shown in Figure 1 with Numerical Example 1 and Example 2 exhibits a limit cycle in the autonomous state. The possibility of quenching the self-sustained oscillations has been explored by injecting suitable high frequency preferably more than ten times of ω_s signals, [5], at any *one/all* three input points (U_1, U_2, U_3) .

However taking the second option i.e. all three inputs are the same as $B \sin\omega_f t$ at 3 input points U_1, U_2, & U_3, shown in Figure 7. Amplitude B is gradually increased, the frequency of self-oscillation, ω_s would gradually change, the system will synchronize to forcing frequency i.e. the self-oscillation would be quenched and the system would exhibit forced oscillations at frequency ω_f.

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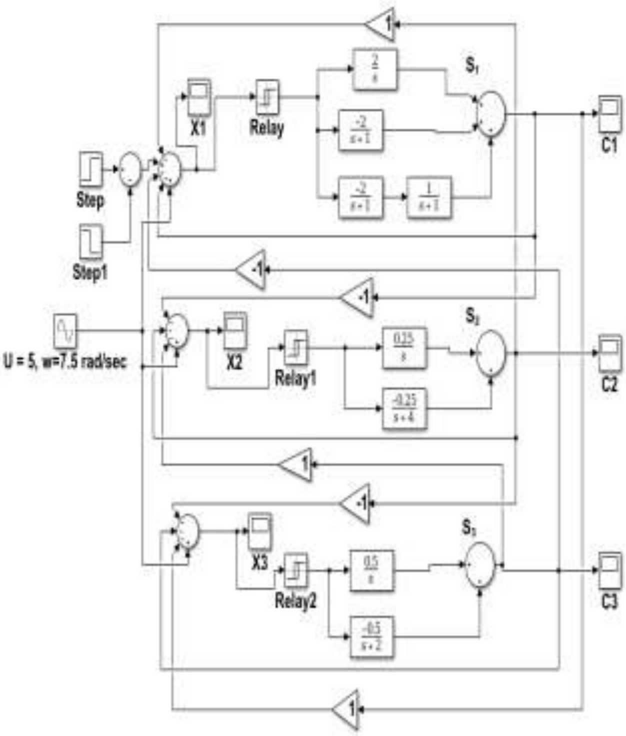

Fig. 7(a): Equivalent System of Figure 1 for forced oscillations (Signal Stabilization) with deterministic signal for Example 1.

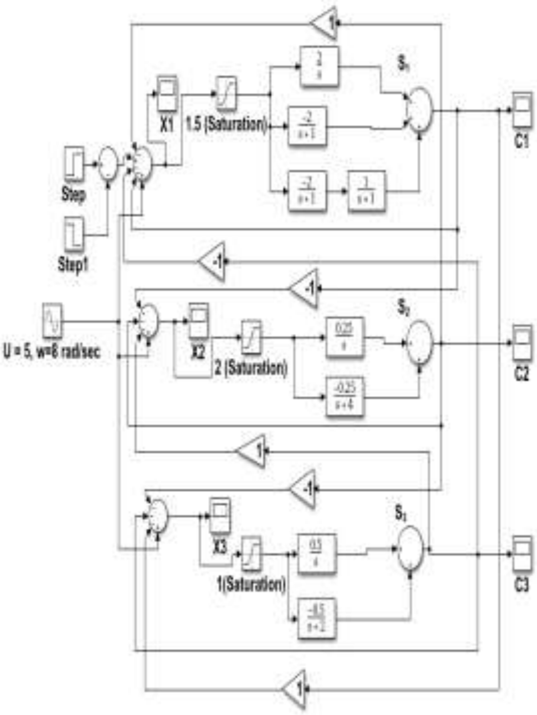

Fig 7(b): Equivalent System of Figure 1 for forced oscillations (Signal Stabilization) with deterministic signal for Example 2

The results/ images from digital simulation for signal stabilization with deterministic signals in Examples 1&2 are shown in Figure 8 & Figure 9 respectively.

The steady state values are represented as C_{1ss}, C_{2ss}, C_{3ss} and X_{1ss}, X_{2ss}, and X_{3ss} with their frequencies,ω, which are almost equal to ω_f.

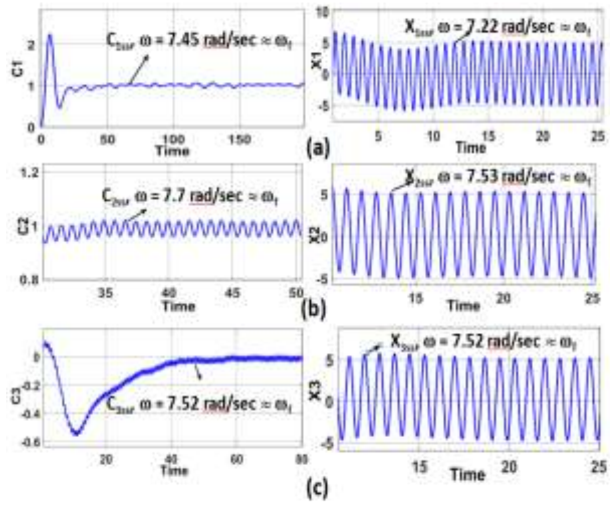

Fig. 8: Forced Oscillations by Signal Stabilization with deterministic signal for Example 1
Forcing Signal $U = 5\sin\omega_f t$ ($\omega_f = 7.5$ rad / sec)

3.2 Using Gaussian Signal

The concept of signal stabilization with random inputs for SISO nonlinear systems was discussed, [60], [61], [62]. Current research gives importance to robust design and analysis which considers uncertainty/ randomness. Until [47], signal stabilization with random signal for multivariable nonlinear systems was not available even for 2X2 systems.

The authors in [47], focused on robust and non-fragile stabilization of nonlinear systems described by the multivariable Hammerstein model. The method illustrates a general procedure that addresses the general multi variable nonlinear systems. Of course, the method considers uncertainties and most importantly control is adopted for stabilization which fails to project insight into the problem. However, the present work shows a simple method to quench the limit cycling oscillations exhibited in a class of 3x3 nonlinear systems and stabilize the systems using random signals in particular a Gaussian Signal. The signal stabilization refers to the possibility of quenching the self-sustained oscillations by injecting a suitable high frequency preferably more than ten times of ω_s (the frequency of LC) signal at any point of the system, [5]. The random signal having Gaussian distribution contains infinite components of frequency. The Gaussian Signals are passed through a high pass filter so that the high frequency signal quenches the limit cycles and stabilizes the system. This has been illustrated through examples 1 and 2 revisited. This method projects a clear and lucid insight into the problem.

Consider the Examples 1 and 2 again. The system is exhibiting LC under an autonomous state, A Gaussian signal with specified *mean* and *variance* is injected at U_1, U_2 & U_3 of subsystems for stabilizing the system / quenching the self-sustained oscillations. At a suitable value of mean (μ) and variance (φ), the self-sustained oscillations vanish / the system is synchronised to high frequency forcing input.

The results/ images are shown in Figure 10 and Figure 11 for Examples 1 & 2 respectively, which are obtained from digital simulation by signal stabilization with Gaussian signals in examples 1 and 2 replacing B sinω_f t using a suitable random signals in Figure 7(a) & Figure 7(b).

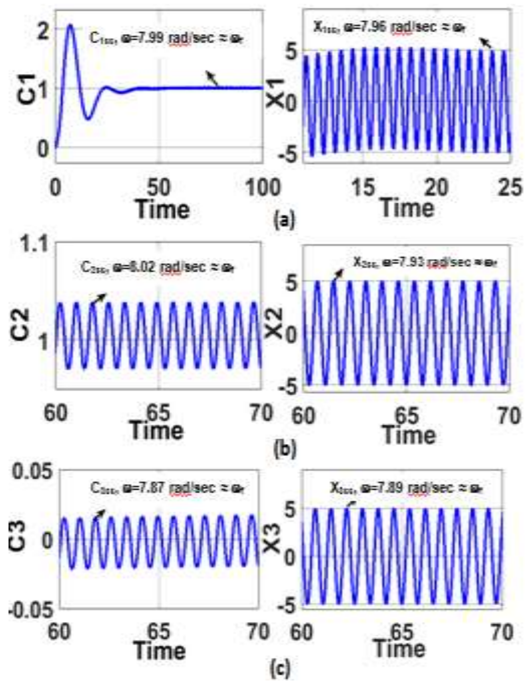

Fig. 9: Forced Oscillations by Signal Stabilization with deterministic signal for Example 2
Forcing signal U = 5 sin $\omega_f t$, (ω_f = 8 rad/sec)

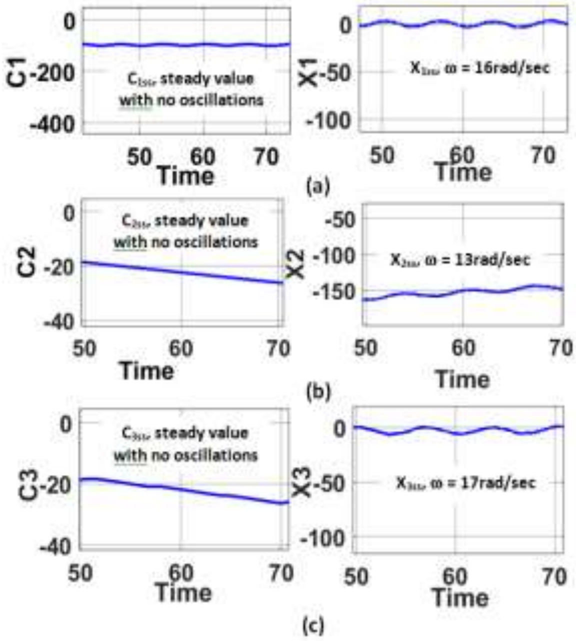

Fig. 10: Forced Oscillations by signal stabilization with Gaussian Signal of mean 50 and variance 0.05 for Example 1

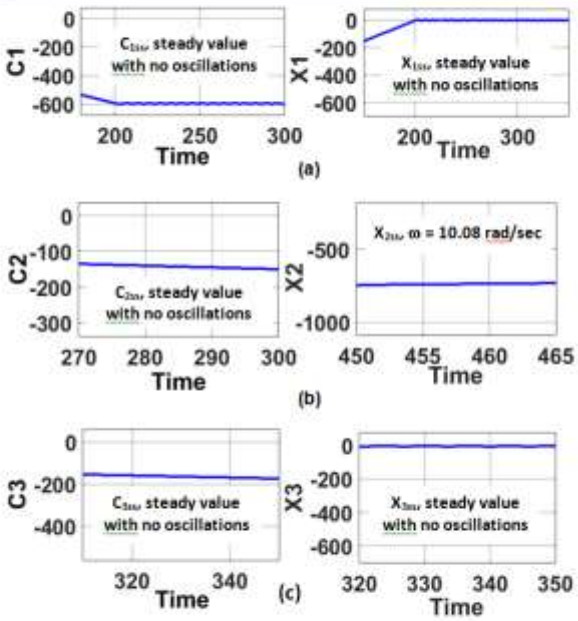

Fig. 11: Forced Oscillations by signal stabilization with Gaussian Signal of mean 300 and variance 0.025 for Example 2

Example 3: Consider a system where linear elements are represented by their transfer functions G(s) and the nonlinear elements are dead zone with saturation whose input output characteristics is shown in Figure 12 and represented by their describing functions N: where $G_1(s) = \frac{2}{s(s+1)^2}$; $G_2(s) = \frac{1}{s(s+4)}$; $G_3(s) = \frac{1}{s(s+2)}$

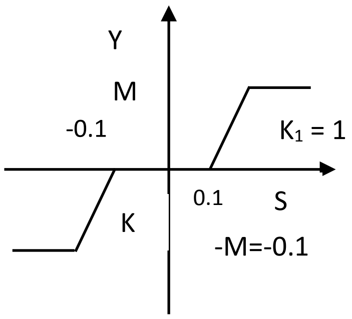

Fig. 12: The nonlinear elements used in the system of example 3 (the same nonlinear elements are used for three sub systems, S_1, S_2, and S_3.)

The simulation diagram of Ex 3 is shown in Figure 13(a).

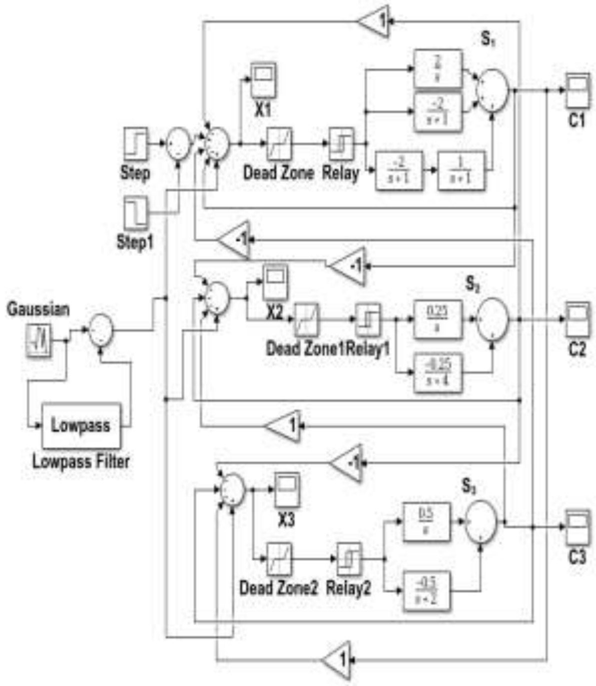

Fig. 13(a): Equivalent System of Figure 1 for forced oscillations (Signal Stabilization) with random/Gaussian signal for Example 3

The result / images obtained from digital simulation, using Gaussian signals are shown in Figure 13 (a).

Figure 13 (b) shows the limit cycling oscillation in the absence of the forcing signal and Figure 13(c) shows the forced oscillation with a Gaussian signal of mean 70 and variance 0.025 for Example 3.

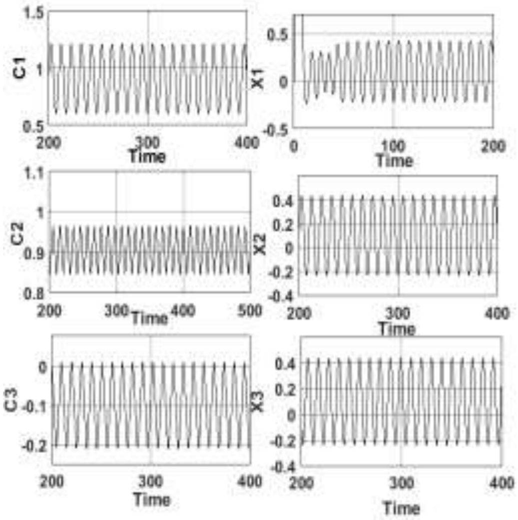

Fig. 13(b): The limit cycle oscillations in the absence of forcing signals for Example 3. (ω_s=0.647 rad/sec)

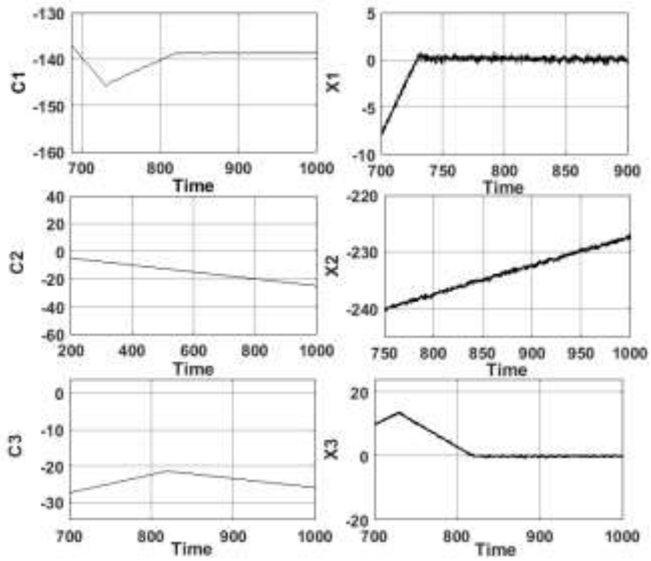

Fig. 13(c): Forced oscillations with Gaussian signal of mean 70 and variance 0.025 for the Example 3

4 Suppression of Limit Cycle in 3x3 Nonlinear System using Pole Placement Technique

Limit cycles or self-sustained oscillations of a 2X2 system can be suppressed by pole placement technique, [8]. The problem of placing the closed loop poles or Eigen values of the closed loop systems at the desired location using state feedback through an appropriate state feedback gain matrix K [k_1, k_2, k_3]. Necessary and sufficient condition for arbitrary pole placement is that the system be completely state controllable, [58]. This can also be done by optimal selection of feedback gain matrix K using Riccati Equation, [48], [59].

4.1 Suppression of Limit Cycles in 3X3 Nonlinear System using Arbitrary Pole Placement by State Feedback

Pole placement technique by state feedback is done by determining the Eigen values or poles of the system. These Eigen values cause the limit cycles in the system, and as the complete removal of these self-oscillations may not be possible, the location of the poles must be changed from its original position to bring about suppression of the limit cycle. The most general multivariable nonlinear system, [6], is shown in Figure 14(a). For the existence of limit cycles, an autonomous system (input U=0) Figure 12(a) can be represented in a simplified form as shown in Figure 14(b). Making use of the first harmonic linearization of the nonlinear elements, the matrix equation for the system of Figure 14(b) can be expressed as

X = -HC, where C = GN(x) X. Hence,

$$X = -HGN(x) = AX \tag{5}$$

Where, A = -HGN(x)

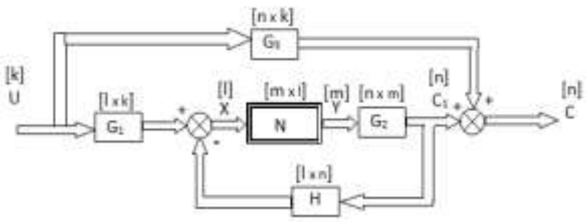

Fig. 14(a): Block diagram representation of a most general nonlinear multivariable system

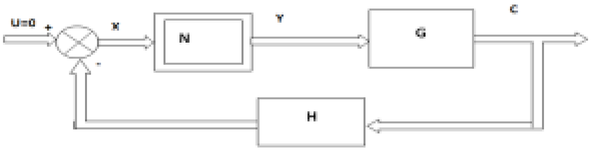

Fig. 14(b): Equivalent of the system of Figure 12 (a) with input U= 0

Realizing Eqn. (5) As a transformation of the vector X onto itself, it is noted that for a limit cycle to exist the following two conditions should be satisfied, [6]. For every non-trivial solution of X, the matrix A must have an Eigen value λ equal to unity, and

(i) The Eigen vector of "A" corresponding to this unity Eigen value must be coincident with X.

4.1.1 Arbitrary Pole Placement for Suppression of Limit Cycles in Example 1 with All Ideal Relays

To suppress the limit cycles, arbitrary pole placements may be possible if the system is completely state controllable [58].

The controllability matrix $S = [B \quad AB\, A^2B \ldots \ldots]$ (6)

Where,

$$A = \begin{bmatrix} -N_1G_1 & -N_2G_2 & N_3G_3 \\ N_1G_1 & -N_2G_2 & -N_3G_3 \\ -N_1G_1 & N_2G_2 & -N_3G_3 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};$$

$$H = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix};$$

$$G(\omega) = \begin{bmatrix} G_1(\omega) & 0 & 0 \\ 0 & G_2(\omega) & 0 \\ 0 & 0 & G_3(\omega) \end{bmatrix};$$

$$N(X) = \begin{bmatrix} N_1(X_1) & 0 & 0 \\ 0 & N_2(X_2) & 0 \\ 0 & 0 & N_3(X_3) \end{bmatrix};$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}; C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

From Table 1 for Example 1,

$\omega = 0.701$ radian/sec, X_1= 6.08, X_2= 6.08, X_3 =6.32

$N_1(X_1) = \frac{4M_1}{\pi X_1} = \frac{4x2}{\pi x6.08} = 0.419$; $N_2(X_2) = \frac{4M_2}{\pi X_2} = \frac{4x1.5}{\pi x6.08} = 0.314$, $N_3(X_3) = \frac{4M_3}{\pi X_3} = \frac{4x1}{\pi x6.32} = 0.202$

$$|G_1(j\omega)| = \frac{2}{\sqrt{(\omega-\omega^3)^2+(2\omega^2)^2}} = \frac{2}{\omega(\omega^2+1)} = 1.913$$

$$|G2(j\omega)| = \frac{1}{\sqrt{(\omega^2)^2+(4\omega)^2}} = \frac{1}{\omega\sqrt{16+\omega^2}} = 0.351$$

$$|G_3| = \frac{1}{\omega\sqrt{\omega^2+4}} = 0.673$$

On substitution of the numerical values:

$-N_1G_1$ = -0.419 X 1.913 = - 0.802,

$-N_2G_2$ = - 0.314 X 0.351 = -0.110,

$-N_3G_3$ = -0.202 X 0.673 = -0.136

$$A = \begin{bmatrix} -0.802 & -0.110 & 0.136 \\ 0.802 & -0.110 & -0.136 \\ -0.802 & 0.110 & -0.136 \end{bmatrix}; \quad AB = \begin{bmatrix} -0.802 & -0.110 & 0.136 \\ 0.802 & -0.110 & -0.136 \\ -0.802 & 0.110 & -0.136 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.136 \\ -0.136 \\ -0.136 \end{bmatrix};$$

$$A^2B = \begin{bmatrix} -0.802 & -0.110 & 0.136 \\ 0.802 & -0.110 & -0.136 \\ -0.802 & 0.110 & -0.136 \end{bmatrix} \begin{bmatrix} 0.136 \\ -0.136 \\ -0.136 \end{bmatrix} = \begin{bmatrix} -0.1125 \\ 0.1425 \\ -0.1055 \end{bmatrix}$$

Hence $S = \begin{bmatrix} 0 & 0.136 & -0.1125 \\ 0 & -0.136 & 0.1425 \\ 1 & -0.136 & -0.1055 \end{bmatrix} = 0.0215 \neq 0$

(*The system is completely state controllable*)

Hence arbitrary pole placement is possible [58]

$$\frac{d}{dt}[x(t)] = AX + Bu \qquad (7)$$

The system under autonomous state is represented as shown in Figure 15.

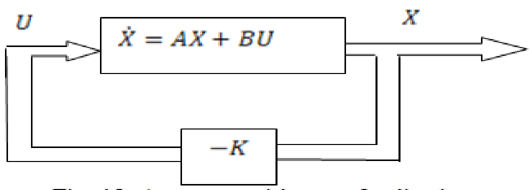

Fig. 15: A system with state feedback

Consider Figure 15:

The control law u = -KX (8)

Where K= [k_1 k_2 k_3] is the feedback matrix.

Replacing K in Eqn. (7) by Eqn. (8), we get:

$$\frac{d}{dt}[x(t)] = (A\text{-}BK)\,X \qquad (9)$$

Substituting the values of A, B and K, we get: The Characteristic Equation as:

$[\![\lambda I - (A - BK)]\!] = 0$ *or*

$$\left| \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -N_1G_1 & -N_2G_2 & N_3G_3 \\ N_1G_1 & -N_2G_2 & -N_3G_3 \\ -N_1G_1 & N_2G_2 & -N_3G_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [\,k_1 \quad k_2 \quad k_3\,] \right| = 0$$

Hence,

$$\begin{bmatrix} (\lambda + N_1G_1) & N_2G_2 & -N_3G_3 \\ -N_1G_1 & (\lambda + N_2G_2) & N_3G_3 \\ N_1G_1 + k_1 & -N_2G_2 - k_2 & \lambda + N_3G_3 + k_3 \end{bmatrix}$$

$$= (\lambda + N_1G_1) \begin{vmatrix} \lambda + N_2G_2 & N_3G_3 \\ -N_2G_2 - k_2 & \lambda + N_3G_3 + k_3 \end{vmatrix}$$
$$-N_2G_2 \begin{vmatrix} -N_1G_1 & N_3G_3 \\ N_1G_1 + k_1 & \lambda + N_3G_3 + k_3 \end{vmatrix} - N_3G_3 \begin{vmatrix} -N_1G_1 & (\lambda + N_2G_2) \\ N_1G_1 + k_1 & -N_2G_2 - k_2 \end{vmatrix} =$$

$=(\lambda + N_1G_1)\{((\lambda + N_2G_2)(\lambda + N_3G_3 + k_3) + N_3G_3(N_2G_2 + k_2)\}$
$-N_2G_2\{(-N_1G_1(\lambda + N_3G_3 + k_3) - N_3G_3(N_1G_1 + k_1)\}$
$-N_3G_3\{N_1G_1(N_2G_2 + k_2) - (\lambda + N_2G_2)(N_1G_1 + k_1)\}$
$=\{\lambda^3 + \lambda^2N_3G_3 - \lambda^2k_3 + \lambda^2N_2G_2 + \lambda N_2N_3G_2G_3 - \lambda k_3N_2G_2 + \lambda N_2N_3G_2G_3 + \lambda k_2N_3G_3 + \lambda^2N_1G_1 + \lambda N_1N_3G_1G_3 - \lambda k_3N_1G_1 + \lambda N_1N_2G_1G_2 + N_1N_2N_3G_1G_2G_3 - k_3N_1N_2G_1G_2 + N_1N_2N_3G_1G_2G_3 + k_2N_1N_3G_1G_3\} + \{\lambda N_1N_2G_1G_2 + N_1N_2N_3G_1G_2G_3 - k_3N_1N_2G_1G_2 + N_1N_2N_3G_1G_2G_3 - k_1N_2N_3G_2G_3\} + \{-N_1N_2N_3G_1G_2G_3 - k_2N_1N_3G_1G_3 + N_3G_3(\lambda N_1G_1 - k_1\lambda + N_1N_2G_1G_2 - k_1N_2G_2)$
$=\lambda^3 + \lambda^2(N_3G_3 + k_3 + N_2G_2 + N_1G_1) + \lambda(N_2N_3G_2G_3 + k_3N_2G_2 + N_2N_3G_2G_3 + k_2N_3G_3 + N_1N_3G_1G_3 + k_3N_1G_1 + N_1N_2G_1G_2 + N_1N_2G_1G_2 + N_1N_3G_1G_3 - k_1N_3G_3) + N_1N_2N_3G_1G_2G_3 - k_3N_1N_2G_1G_2G_3 + k_2N_1N_3G_1G_3 + N_1N_2N_3G_1G_2G_3 - k_3N_1N_2G_1G_2 + N_1N_2N_3G_1G_2G_3 - k_1N_2N_3G_2G_3 - N_1N_2N_3G_1G_2G_3 - k_2N_1N_3G_1G_3 + N_1N_2N_3G_1G_2G_3 - k_1N_2N_3G_2G_3$
$=\lambda^3 + \lambda^2(N_1G_1 + N_2G_2 + N_3G_3 + k_3) + \lambda(2N_1N_2G_1G_2 + 2N_1N_3G_1G_3 + 2N_2N_3G_2G_3 + k_1N_3G_3 + k_3N_1G_1 + k_3N_2G_2 + k_2N_3G_3) + (4N_1N_2N_3G_1G_2G_3 + 2k_3N_1N_2G_1G_2 + 2k_1N_2N_3G_2G_3) = 0 \cdots \cdots$ (10)

(***Ch. Equation***)

On substitution of the values of N_1, G_1, N_2, G_2 and N_3, G_3 in Eqn. (10), we get,

$\lambda^3 + \lambda^2(0.136 + 0.11 + 0.802 + k_3) + \lambda\{0.177 + 0.218 + 0.030 + k_1 x\, 0.136 + k_3(0.80 + 0.11) + k_2 x\, 0.136\} + (0.048 + 0.522 + k_1 x 0.03) = 0$ Or

$\lambda^3 + \lambda^2(1.048 + k_3) + \lambda(0.425 + k_1 x 0.136 + k_2 x\, 0.136 + k_3 x 0.91) + (0.57 + k_1 x 0.03) = 0$ (11)

If the poles are selected arbitrarily at $\lambda_1, \lambda_2, \lambda_3 = -1, -1 \,\&\, -2$ respectively, the characteristic equation becomes:

$(\lambda + 1)\,(\lambda + 1)\,(\lambda + 2) = \lambda^3 + 4\lambda^2 + 5\lambda + 2 = 0$ (12)

Comparing Eq. (12) with Eq. (11), and equating the coefficients of like powers of λ we get:

$$4 = 1.048 + k_3, \text{whence } k_3 = 2.952 \cdots \cdots \quad (13)$$

$$2 = (0.57 + k_1 \text{ x } 0.03), \text{ whence } k_1 = 47.67 \quad (14)$$

$$5 = (0.425 + k_1 \text{ x } 0.136 + k_2 \text{ x } 0.136 + k_3 \text{ x } 0.91)$$

or

$$5 = (0.425 + 47.67\text{x } 0.136 + k_2 \text{ x } 0.136 + 2.952\text{x}91 \text{ x } 0.91) \text{ or}$$

$$5 = (0.425 + 6.48 + k_2 \text{ x } 0.136 + 2.68), \text{ whence } k_2 = -33.71 \quad (15)$$

Hence

$$K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k1 & k2 & k3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 47.67 & -33.71 & +2.952 \end{bmatrix} \cdots\cdots \quad (16)$$

From Eqn. (9), $(A - BK) = A_1$, with shifted poles for Example 1. Or

$$A_1 = \begin{bmatrix} -N_1G_1 & -N_2G_2 & N_3G_3 \\ N_1G_1 & -N_2G_2 & -N_3G_3 \\ -N_1G_1 - k_1 & N_2G_2 - k_2 & -N_3G_3 - k_3 \end{bmatrix} = \begin{bmatrix} -0.802 & -0.11 & 0.136 \\ 0.802 & -0.11 & -0.136 \\ -48.472 & 33.6 & -3.088 \end{bmatrix} \cdots\cdots \quad (17)$$

The images/response $C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}, X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ in the autonomous state obtained from digital simulation for A_1 of Example 1, are shown in Figure 16.

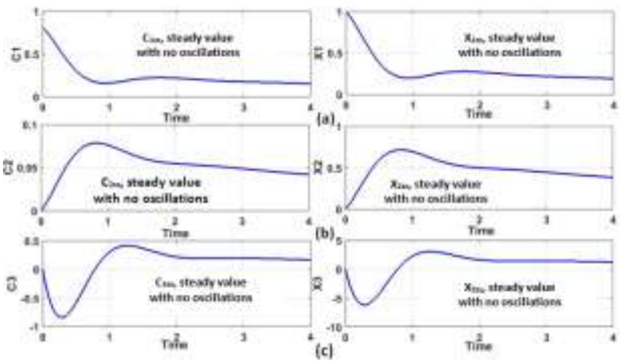

Fig. 16: Suppression of Limit Cycles by State Feedback with arbitrarily selection of feedback gain matrix for Example 1

4.1.2 Optimal Selection of Feedback gain Matrix using Riccati Equation for Example 1

The Riccati Equation is $A'P+PA- PBR^{-1}B'P+Q=0$ (18)

And K = Feedback gain matrix = $R^{-1}B'P \cdots\cdots$ (19)

Assuming $R = 1$, $B=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $A = \begin{bmatrix} -N_1G_1 & -N_2G_2 & N_3G_3 \\ N_1G_1 & -N_2G_2 & -N_3G_3 \\ -N_1G_1 & N_2G_2 & -N_3G_3 \end{bmatrix}$

Let $P=\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$, considering P to be symmetric matrix: $p_{21}=p_{12}, p_{31} = p_{13}, p_{32} = p_{23}$

Hence $P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{bmatrix}$

$$P = \begin{bmatrix} -N_1G_1 & N_1G_1 & -N_1G_1 \\ -N_2G_2 & -N_2G_2 & N_2G_2 \\ N_3G_3 & -N_3G_3 & -N_3G_3 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

$$= \begin{bmatrix} (-N_1G_1p_{11} + N_1G_1p_{21} - N_1G_1p_{31}) & (-N_1G_1p_{12} + N_1G_1p_{22} - N_1G_1p_{32}) & (-N_1G_1p_{13} + N_1G_1p_{23} - N_1G_1p_{33}) \\ (-N_2G_2p_{11} - N_2G_2p_{21} + N_2G_2p_{31}) & (-N_2G_2p_{12} - N_2G_2p_{22} + N_2G_2p_{32}) & (-N_2G_2p_{13} - N_2G_2p_{23} + N_2G_2p_{33}) \\ (+N_3G_3p_{11} - N_3G_3p_{21} - N_3G_3p_{31}) & (+N_3G_3p_{12} - N_3G_3p_{22} - N_3G_3p_{32}) & (+N_3G_3p_{13} - N_3G_3p_{23} - N_3G_3p_{33}) \end{bmatrix} \quad (20)$$

$$PA= \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} -N_1G_1 & -N_2G_2 & N_3G_3 \\ N_1G_1 & -N_2G_2 & -N_3G_3 \\ -N_1G_1 & N_2G_2 & -N_3G_3 \end{bmatrix}$$

$$= \begin{bmatrix} (-N_1G_1p_{11} + N_1G_1p_{12} - N_1G_1p_{13}) & (-N_2G_2p_{11} - N_2G_2p_{12} + N_2G_2p_{13}) & (+N_3G_3p_{11} - N_3G_3p_{12} - N_3G_3p_{13}) \\ (-N_1G_1p_{21} + N_1G_1p_{22} - N_1G_1p_{23}) & (-N_2G_2p_{21} - N_2G_2p_{22} + N_2G_2p_{23}) & (+N_3G_3p_{21} - N_3G_3p_{22} - N_3G_3p_{23}) \\ (-N_1G_1p_{31} + N_1G_1p_{32} - N_1G_1p_{33}) & (-N_2G_2p_{31} - N_2G_2p_{32} + N_2G_2p_{33}) & (+N_3G_3p_{31} - N_3G_3p_{32} - N_3G_3p_{33}) \end{bmatrix} \quad (21)$$

$$PBR^{-1}B'P=\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} =$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [(0xp_{11} + 0xp_{21} + 1xp_{31}) \quad (0xp_{12} + 0xp_{22} + 1xp_{32}) \quad (0xp_{13} + 0xp_{23} + 1xp_{33})]$$

$$= \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} 0xp_{31} & 0xp_{32} & 0xp_{33} \\ 0xp_{31} & 0xp_{32} & 0xp_{33} \\ 1xp_{31} & 1xp_{32} & 1xp_{33} \end{bmatrix} =$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} p_{13}p_{31} & p_{13}p_{32} & p_{13}p_{33} \\ p_{23}p_{31} & p_{23}p_{32} & p_{23}p_{33} \\ p_{33}p_{31} & p_{33}p_{32} & p_{33}p_{33} \end{bmatrix} \quad (22)$$

On substitution of numerical values, Eqn. 20 can be written as:

$$\begin{bmatrix} (-0.802p_{11}+0.802p_{21}-0.802p_{31}) & (-0.802p_{12}+0.802p_{22}-0.802p_{32}) & (-0.802p_{13}+0.802p_{23}-0.802p_{33}) \\ (-0.11p_{11}-0.11p_{21}+0.11p_{31}) & (-0.11p_{12}-0.11p_{22}+0.11p_{32}) & (-0.11p_{13}-0.11p_{23}+0.11p_{33}) \\ (+0.136p_{11}-0.136p_{21}-0.136p_{31}) & (+0.136p_{12}-0.136p_{22}-0.136p_{32}) & (+0.136p_{13}-0.136p_{23}-0.136p_{33}) \end{bmatrix} \quad (23)$$

On substitution of numerical values, Eqn. 21 can be written as:

$$\begin{bmatrix} (-0.802p_{11}+0.802p_{12}-0.802p_{13}) & (-0.11p_{11}-0.11p_{12}+0.11p_{13}) & (+0.136p_{11}-0.136p_{12}-0.136p_{13}) \\ (-0.802p_{21}+0.802p_{22}-0.802p_{23}) & (-0.11p_{21}-0.11p_{22}+0.11p_{23}) & (+0.136p_{21}-0.136p_{22}-0.136p_{23}) \\ (-0.802p_{31}+0.802p_{32}-0.802p_{33}) & (-0.11p_{31}-0.11p_{32}+0.11p_{33}) & (+0.136p_{31}-0.136p_{32}-0.136p_{33}) \end{bmatrix} \quad (24)$$

On substitution of these values of Eqns. (22), (23), (24) and the assumed value of Q in Riccati Eqn. 18 yields:

$$(-1.604p_{11} + 1.604p_{12} - 1.604p_{13}) + 1 - p^2 13 = 0 \quad (25)$$

$$(-0.912p_{12} + 0.802p_{22} - 0.802p_{23} - 0.11p_{11} + 0.11p_{13} - p_{13}\, p_{23} = 0 \quad (26)$$

$$(-0.938p_{13} + 0.802\, p_{23} - 0.802\, p_{33} + 0.136\, p_{11} - 0.136\, p_{12} - p_{13}\, p_{33} = 0 \quad (27)$$

$$(-0.22\, p_{12} - 0.22\, p_{22} + 0.22\, p_{23} - p^2_{23} = 0 \quad (28)$$

$$(-0.11\, p_{13} - 0.246\, p_{23} + 0.11\, p_{33} + 0.136\, p_{12} - 0.136\, p_{22} - p_{23}\, p_{33} = 0 \quad (29)$$

$$(0.272\, p_{13} - 0.272\, p_{23} - 0.272\, p_{33} - p_{33}\, p_{23}) = 0 \quad (30)$$

Further, subtracting Eqn. (29) from Eqn. (30), we get,

$$0.382\, p_{13} - 0.026\, p_{23} - 0.382\, p_{33} - 0.136\, p_{12} + 0.136\, p_{22} \quad (31)$$

The solution of these simultaneous Eqns. (26),(27),(28),(29),(30) & (31) yields :
p_{11} = -116.68, p_{12} = -110.48, p_{13}=6.58, p_{22} = -93.24, p_{23} = -6.58, p_{33} = 0
From Eqn. (19), $K = R^{-1}\ B'P = 1\ [001] \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{bmatrix}$

Or $[k_1 k_2\, k_3]$=
$[(0xp_{11} + 0xp_{12} + 1xp_{13})\ (0xp_{12} + 0xp_{22} + 1xp_{23})\ (0xp_{13} + 0xp_{23} + 1xp_{33})]$

Or $[k_1 k_2\, k_3]$=$[(p_{13})\,(p_{23})\,(p_{33})]$
=$[6.58\ \ -6.58\ \ 0]$,
Whence, k_1 = 6.58, k_2= -6.58 and k_3 = 0 (32)

Hence, A – BK =

$$A_2 = \begin{bmatrix} -N_1G_1 & -N_2G_2 & N_3G_3 \\ N_1G_1 & -N_2G_2 & -N_3G_3 \\ -N_1G_1 - k_1 & N_2G_2 - k_2 & -N_3G_3 - k_3 \end{bmatrix}$$

On substitution of numerical values for Example 1, A_2 becomes:

$$A_2 = \begin{bmatrix} -0.802 & -0.11 & 0.136 \\ 0.802 & -0.11 & -0.136 \\ -802 - 6.58 & 0.11 + 6.5 & -0.136 - 0 \end{bmatrix} = \begin{bmatrix} -0.802 & -0.11 & 0.136 \\ 0.802 & -0.11 & -0.136 \\ -7.382 & 6.69 & -0.136 \end{bmatrix} \quad (33)$$

The images/ responses $C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$ and $X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ in the autonomous state, obtained from digital simulation for Example 1, are shown in Figure 17.

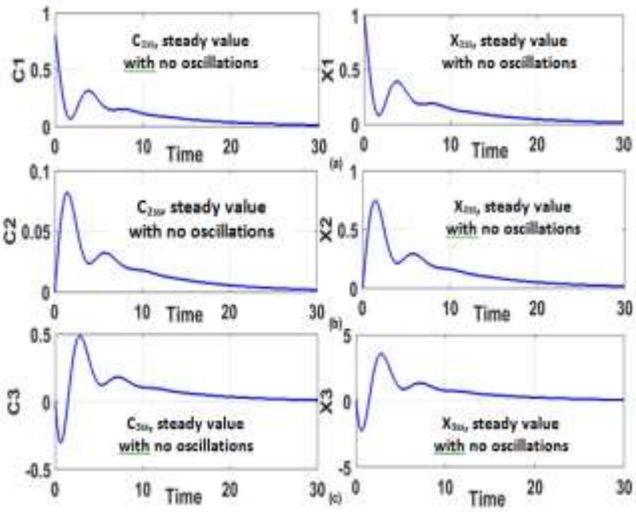

Fig. 17: Suppression of Limit Cycles by State Feedback with optimal selection of feedback gain matrix for Example 1

5 Conclusion

In today's scenario, nonlinear self-sustained oscillations or Limit Cycles are the basic feature of instability. The existence /exhibition of such phenomena limit the performance of most of the physical systems such as the speed and position control in robotics, the automation industry in particular. Quenching / complete extinction of such LC has been severe headache among researchers for several decades. There are some methods, seen in the available literatures which suggest the solution to this problem occurring in SISO or 2x2 systems. However, the present work explores the solution for 3x3 systems in the event of the existence of an LC problem and establishes the result graphically & validated by digital simulation. The novelty of the work claims in (i) Quenching of LC exhibited in nonlinear systems by Signal Stabilization with deterministic as well as random (Gaussian) signals, (ii) Suppression of limit cycles in 3x3 nonlinear systems by Pole Placement using State feedback with arbitrary selection as well as optimal selection of feedback gain matrix K.

More importantly, the poles of such 3x3 systems are shifted or placed suitably by State feedback so that the system does not exhibit limit cycles. This pole placement is done either by arbitrary selection satisfying the complete state controllability condition or by optimal selection of feedback gain matrix K using the Riccati equation which has not been attempted earlier.

The present work has the brighter future scope of adopting techniques like signal stabilization [46], [47] and suppression of limit cycles [48], in the event of the existence of limit cycling oscillations for 3x3 or higher dimensional systems through an exhaustive analysis.

Analytical/Mathematical procedures may also be developed for signal stabilization using both deterministic and random signals applying DIDF and RIDF respectively.

Backlash is one of the nonlinearities commonly occurring in physical systems which are an inherent characteristic of Governor, more popularly used for load frequency control (LFC) in power systems. The LFC shows poor performance due to the backlash characteristic of the governor. Similarly, the backlash characteristic limits the performance of speed and position control in the robotics, and automation industry. The poor performance of LFC, speed, and position control in robotics and in automation industries is happening since these systems exhibit limit cycles due to their backlash type of nonlinear characteristics. The proposed method of suppression of L.C. can be *extended and developed* for backlash type nonlinearity in 3x3 systems and used to eliminate the limit cycle to mitigate such problems.

The phenomena of synchronization and de-synchronization can be observed / identified analytically using the Incremental Input Describing function (IDF).

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- Kartik Chandra Patra has formulated the problem, methodology of analysis adopted and algorithm of computation presented.
- Asutosh Patnaik has made the validation of the results using the geometric tools and SIMULINK toolbox of MATLAB software.

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Conflict of Interest

The authors have no conflicts of interest to declare.