Full-State Observer Synthesis for a Class of One-Sided Lipschitz Nonlinear Fractional-Order Systems

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Abstract: - A full-order observer design for a large class of fractional-order continuous-time nonlinear systems that satisfies the one-sided Lipschitz and quadratic inner boundedness conditions problem is addressed. By employing the fractional-order extension of the Lyapunov direct approach, a sufficient condition for the existence of the proposed observer is established in the form of a Nonlinear Matrix Inequality (NMI), guaranteeing the asymptotic convergence of the observation error to the origin. The effectiveness and distinct advantages of the proposed design are validated through numerical simulations on a representative fractional-order system.

Key-Words: - Fractional-order observers, One-sided Lipschitz systems, Quadratic inner boundness, Riccati equation, Lyapunov direct approach extension, Nonlinear systems, Nonlinear Matrix Inequality (NMI).

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1 Introduction

In the last few decades, state estimation of nonlinear systems has emerged as a vital research focus for a broad community of researchers, [1], [2], [3], particularly in the control systems field, [4], [5], [6], where state measurements are essential for designing robust and effective control strategies. Despite significant advancements, designing a state observer for a general class of nonlinear systems remains a challenging and open topic.

A review of the literature reveals that established observer design techniques can be categorized into two distinct approaches. The first involves applying a coordinate transformation to the observation error dynamical system, thus converting it into a linear form, [7]. This transformation facilitates the implementation of well-known techniques applicable to linear systems; however, it is often constrained by the complexities of establishing such transformations. In contrast, the second approach does not require any state transformation, directly utilizing the system dynamics in the observer design, [8]. The most widely adopted strategy in this context is predicated on solving a Riccati-like equation for specific classes of nonlinear systems that satisfy the Lipschitz continuity condition, [9], [10], [11], [12]. Notably, most existing results are grounded in the principle that the linear component of the observation error dynamics predominates over the nonlinear component. This approach leverages the Lipschitz property of nonlinearity, allowing for the substitution of the nonlinear term with a linear positive term, thereby simplifying the observer design process, [13], [14]. However, for nonlinear systems characterized by large Lipschitz constants, this methodology may become ineffective, as the associated Riccati-like equations may no longer be solvable. To expand the class of nonlinear systems that can be considered and surmounting the drawbacks of the aforementioned techniques, a more general condition for observer design is introduced, referred to as the one-sided Lipschitz continuity condition. Up to date, many interesting observation schemes for this class of system have been developed; readers can refer to [15], [16].

Over the last three centuries, fractional-order calculus has made substantial contributions to the mathematical literature. Its fundamental concept lies in generalizing the traditional definitions of derivatives and integrals, [17]. With these powerful tools, dynamical systems modeling, analysis, and control have undergone considerable development in various scientific fields, including viscoelastic [18], electromagnetism [19], biology [20], mechanics [21], robotics [22], aerodynamics [23], renewable energy [24], and many others. Nevertheless, the design of observers for fractional-order systems, particularly those with one-sided Lipschitz conditions, remains an underexplored area, with few existing results in the current literature [25], [26].

Motivated by the aforementioned discussion, the primary contribution of this paper is the development of a novel NMI-based observer for fractional-order one-sided Lipschitz nonlinear systems. Utilizing the one-sided Lipschitz property and quadratic inner boundness, along with the fractional-order extension of the Lyapunov direct method, we derive sufficient conditions for the observer's existence and the asymptotic convergence of the observation error, expressed in the form of an NMI.

The rest of this paper is organized as follows: Section 2 introduces the foundational concepts and pertinent results related to fractional-order calculus. In Section 3, the observation problem for fractional-order one-sided Lipschitz systems is comprehensively detailed. The primary contributions and main results are presented in Section 4. To validate the efficacy of the proposed observation technique, simulation results for a fractional-order nonlinear system are provided in Section 5. Finally, conclusions are drawn in Section 6.

2 Preliminaries

Fractional calculus is concerned with the integrals and derivatives of orders that might have real or complex values. One of the fundamental notions of fractional-order calculus is the Riemann-Liouville fractional integral, which is stated in **Definition 1.**

Definition 1, [27]. The orderα^{th}order Riemann-Liouville fractional integral of a continuous function $f(t)$on the left half-axis of the real numbers is defined by:

$$I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \tag{1}$$

Where$\Gamma(.)$ is the gamma function.

Definition 2, [27]. The Riemann-Liouville fractional derivative of order $\alpha \in \mathbb{R}^{+}$of a function $f(t)$ is defined by:

$${}^{RL}D_t^{\alpha} f(t) = \left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^{n} I^{n-\alpha} f(t) \tag{2}$$

Where $\mathrm{n}-1 < \alpha < n, n \in \mathbb{N}$

Definition 3, [27]. The Caputo fractional Derivatives of order $\alpha \in \mathbf{R}^{+}$of a function $f(t)$ is defined as follows:

$${}_{t_0}^{C}D_t^{\alpha} f(t) = I^{n-\alpha} f^{(n)}(t) \tag{3}$$

It is the one most frequently used in engineering problems and the one used in this paper.

Lemma 1, [28]. Let$f \in \mathbf{R}^{n}$ be a derivable functions vector. Then for any given time instant $\geq t_0$:

$$\frac{1}{2} \; {}^{C}D_t^{\alpha} f^{2}(t) \leq f(t) \, {}^{C}D_t^{\alpha} f(t) \tag{4}$$

Theorem 1, [29]. Let$x = 0$ be an equilibrium point for the Caputo fractional non-autonomous system

$$D^{\alpha} x(t) = h(x, t) \tag{5}$$

where $h(x, t)$ satisfies the Lipschitz condition with $l > 0$ as Lipschitz constant and $\alpha \in (0,1)$.

Assume that there exists a Lyapunov functional$V(t, x)$satisfying:

$$\begin{gathered} \alpha_1 \parallel x \parallel^{a_1} \leq V(t, x) \leq \alpha_2 \parallel x \parallel^{a_1 a_2} \\ D^{\beta} V(t, x) \leq -\alpha_3 \parallel x \parallel^{a_1 a_2} \end{gathered} \tag{6}$$

where $\beta \in (0,1), \alpha_1, \alpha_2, \alpha_3, a_1, a_2$ are positive constants and $||.||$ denotes an arbitrary norm. Then the equilibrium point of the system (.) is Miattag-Leffler stable.

Lemma 2, [30]. Consider a given matrix $\zeta = \begin{bmatrix} \zeta_{11} & \zeta_{12} \\ \zeta_{21} & \zeta_{22} \end{bmatrix}$ knowing that $\zeta_{11}^{T} = \zeta_{11}$ and $\zeta_{22}^{T} = \zeta_{22}$ In this case, the criteria set out below are equivalent:

1 $\zeta < 0$
2 $\zeta_{11} < 0, \zeta_{22} - \zeta_{12}^{T} \zeta_{11}^{1} \zeta_{12} < 0$
3 $\zeta_{22} < 0, \zeta_{11} - \zeta_{12}^{T} \zeta_{22}^{1} \zeta_{12} < 0$

3 Problem Statement

In this study, we are interested in the class of fractional-order nonlinear systems, modeled by:

$$\begin{cases} D^{\alpha} x(t) = Ax(t) + \Phi(x(t), u) \\ y(t) = Cx(t) \end{cases} \tag{7}$$

Where $x \in \mathbf{R}^n$ is the state vector, $y \in \mathbf{R}^m$ is the output, $u \in \mathbf{R}^p$ denotes the input, $A \in \mathbf{R}^{n\times n}, C \in \mathbf{R}^{m\times n}$, and $\Phi(x,u)$ represents a one-sided Lipschitz nonlinear functions vector.

Before proceeding with our problem statement, we require definitions of Lipschitz and one-sided Lipschitz nonlinear functions.

Definition 4. The function $\Phi(x,u)$ is said to be locally Lipschitz with respect to x in a region $\mathbb{D}$ if there exists a constant $l \geq 0$ such that the following condition holds:

$$\| \Phi(x,u) - \Phi(y,u) \| \leq l \| x - y \|, \quad \forall x,y \in \mathbb{D} \tag{8}$$

Where l is the Lipschitz constant and u is an admissible control law. If $\mathbb{D} = \mathbb{R}^n$,then $\Phi(x,u)$ is said to be globally Lipschitz.

Definition 5. The function $\Phi(x,u)$ is said to be one-sided Lipschitz if $\forall x,y \in \mathbb{D}$, there exists a constant $\rho \in \mathbb{R}$ satisfiying:

$$< \Phi(x,\bar{u}) - \Phi(y,\bar{u}), x - y > \leq \rho \| x - y \|^2 \tag{9}$$

Where ρ is called the one-sided Lipschitz constant and it corresponds to the Jacobian's logarithmic matrix norm:

$$\rho = \lim_{t\to\infty} \sup \left[\mu\left(\frac{\partial \Phi}{\partial x}\right)\right], \quad \forall x \in \mathbb{D} \tag{10}$$

Definition 6. The function $\Phi(x,u)$ is quadratically inner bounded in $\mathbb{D}$ if there exsit $\beta, \gamma \in \mathbb{R}$ such that the following inequality holds $\forall x,y \in \mathbb{D}$.

$$\begin{aligned}(\Phi(x,\bar{u}) - \Phi(y,\bar{u}))^T(\Phi(x,\bar{u}) - \Phi(x,\bar{u})) \leqslant \\ \beta \| x - y \|^2 + \gamma\langle x - y, \Phi(x,\bar{u}) - \Phi(y,\bar{u})\rangle\end{aligned} \tag{11}$$

Assuming that the defined system includes certain states that cannot be directly measured, this research aims to devise a fractional-order observer that provides an accurate estimate of the full state vector. To overcome this latter challenge, we consider the observation scheme as follows:

$$D^\alpha \hat{x}(t) = A\hat{x}(t) + \Phi(\hat{x},u) + L(y - C\hat{x}) \tag{12}$$

Then the observation error system dynamics for $\tilde{x}(t) = x(t) - \hat{x}(t)$ are provided by:

$$D^\alpha \tilde{x}(t) = (A - LC)\tilde{x}(t) + \Delta\Phi \tag{13}$$

Where $\Delta\Phi = \Phi(x,u) - \Phi(\hat{x},u)$.

Here, the observer gain L should be designed in such a way as to guarantee the asymptotic convergence of the error system trajectories toward the origin.

4 Main Results

In this section, a sufficient condition for the existence and asymptotic convergence of the proposed observer is established. The following theorem summarizes our main findings:

Theorem 2. Assuming that system (7) satisfies the conditions (10) and (11) with constants σ, ρ and η, and if there exist scalars $c, \sigma > 0$ such that the following Riccati-like inequality has a symmetric positive definite solution P:

$$\begin{aligned}A^TP + PA + \varepsilon P^2 + 2\left(\rho + \frac{2\beta}{\varepsilon} + \frac{(2\gamma-\varepsilon)^2}{4\varepsilon}\right)I_n \\ -\sigma C^TC < 0\end{aligned} \tag{14}$$

And the observer holds the form (13), with:

$$L = \frac{\sigma}{2}P^{-1}C^T \tag{15}$$

Then it can be assured that observer error dynamicsare asymptotically stable.

Proof: Examining the following Lyapunov functional candidate:

$$V(t,\tilde{x}) = \tilde{x}^T(t)P\tilde{x}(t) \tag{16}$$

Applying the fractional derivative of order$\alpha$to expression (19) and referencing **Lemma 1** yields:

$$D^\alpha V \leq 2\tilde{x}^TP(D^\alpha \tilde{x}) \tag{17}$$

By substituting Equation (14) into Equation (17), we obtain:

$$\begin{aligned}D^\alpha V \leq \tilde{x}^T((A - LC)^TP + P(A - LC))\tilde{x}(t) \\ +2\tilde{x}^TP\Delta\Phi\end{aligned} \tag{18}$$

The following property holds for all scalar values of $\epsilon > 0$:

$$2\tilde{x}^TP\Delta\Phi \leq \varepsilon\tilde{x}^TP^2\tilde{x} + \frac{1}{\varepsilon}\Delta\Phi^T\Delta\Phi \tag{19}$$

Since $\Phi(x,u)$ is quadratically inner bounded, it follows from (-) that one can derive:

$$\begin{aligned}2\bar{x}^TP\Delta\Phi \leq \epsilon\bar{x}^TP^2\bar{x} + \\ \frac{1}{\varepsilon}(2(\beta\bar{x}^T\bar{x} + \gamma\bar{x}^T\Delta\Phi) - \Delta\Phi^T\Delta\Phi)\end{aligned} \tag{20}$$

From the one sided Lipschitz definition, it is evident that:

$$\rho\bar{x}^Tx - \bar{x}^T\Delta\Phi \geq 0 \tag{21}$$

On the basis of inequalities (20) and (21), we can express equation (18) in the following reformulated form:

$$D^{\alpha}V \leq \begin{bmatrix} \tilde{x}^T \\ \Delta\Phi^T \end{bmatrix}^T \times \begin{bmatrix} (A-LC)^T P + P(A-LC) + \epsilon P^2 + \left(\rho + \frac{2\beta}{\varepsilon}\right) I_n & \left(\frac{2\gamma-\varepsilon}{2\varepsilon}\right) I_n \\ \left(\frac{2\gamma-\varepsilon}{2\varepsilon}\right) I_n & -\frac{1}{\varepsilon} I_n \end{bmatrix} \times \begin{bmatrix} \tilde{x} \\ \Delta\Phi \end{bmatrix} \tag{22}$$

To achieve $D^{\alpha}V \leq -\alpha_3 \parallel x \parallel^{a_1 a_2}$, the following condition should be accomplished:

$$\begin{bmatrix} (A-LC)^T P + P(A-LC) + \epsilon P^2 + \left(\rho + \frac{2\beta}{\varepsilon}\right) I_n & \left(\frac{2\gamma-\varepsilon}{2\varepsilon}\right) I_n \\ \left(\frac{2\gamma-\varepsilon}{2\varepsilon}\right) I_n & -\frac{1}{\varepsilon} I_n \end{bmatrix} < 0 \tag{23}$$

By referring to Lemma 2 the condition in (23) can be expressed as :

$$(A-LC)^T P + P(A-LC) + \epsilon P^2 + \left(\rho + \frac{2\beta}{\varepsilon} + \frac{(2\gamma-\varepsilon)^2}{4\varepsilon}\right) I_n < 0 \tag{24}$$

Let $L = \frac{\sigma}{2} P^{-1} C^T$, thenconditions (24), (14) are equivalent.

5 Simulation Results

This section presents a numerical simulation example demonstrating the effectiveness of the fractional-order observer design technique introduced in Section 3. We consider the simulation example provided in [31]. Consider a fractional-order nonlinear dynamical system represented by Eq (1), defined as follows:

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -6 \end{bmatrix};\ C = [1\ \ 0] \tag{25}$$

$$\Phi(x,u) = \begin{bmatrix} \sin(x_1) - 2x_1 \\ -2x_2 + \cos(x_2) \end{bmatrix} \tag{26}$$

By applying the mean value theorem one can derive:

$$\Delta\Phi^T\, \tilde{x}(t) \leq -\|\tilde{x}(t)\|; \tag{27}$$

$$\Delta\Phi^T \Delta\Phi \leq 9\|\tilde{x}(t)\|^2; \tag{28}$$

Consequently, the system's nonlinearity satisfies the one-sided Lipschitz continuity condition (.) and the quadratic inner boundness(.), with $\rho = -1, \beta = 9$, and $\gamma = 0$. In turn, **Theorem 2** can be examined to design a full-state observer for this system. By setting $\sigma = 10$ and utilizing the YALMIP toolbox, the resolution of the NMI (−) provides the following result:

$$P = \begin{bmatrix} 1.7229 & 0.2195 \\ 0.2195 & 2.6591 \end{bmatrix} = P^T > 0 \tag{29}$$

Thus, the observer gain matrix:

$$L = \begin{bmatrix} 2.9329 \\ -0.2421 \end{bmatrix} \tag{30}$$

The system's initial conditions are selected as $x_0 = [2, -1]$, whereas the designed observer is initialized with zero initial conditions.

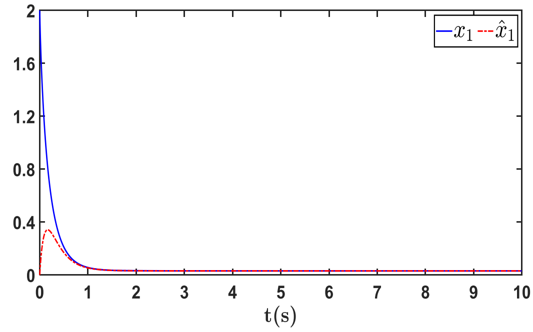

Fig. 1: Actual state x_1 and its estimation time history

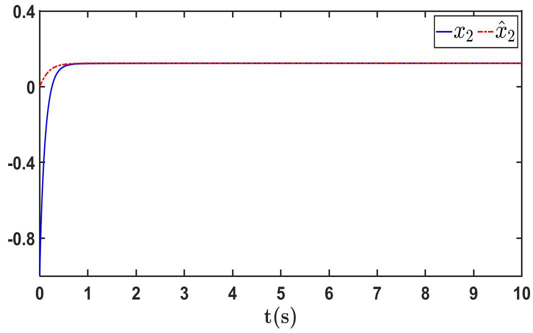

Fig. 2: Actual state x_2 and its estimation time evolution

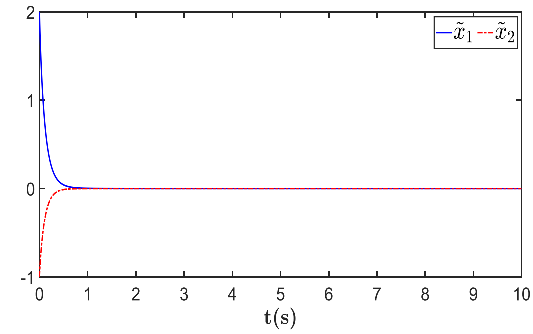

Fig. 3: The observation error time evolution

The observation results of the system states x_1 and x_2 are illustrated in Figure 1 and Figure 2, respectively.The proposed observer demonstrates significant effectiveness in accurately reconstructing the system's full state vector, which. This effectiveness is further corroborated by the observation error time history Figure 3, which exhibits asymptotic convergence to zero.

6 Conclusion

An innovative NMI-based observer design for a broader class of nonlinear systems is presented in this study. Leveraging the one-sided Lipschitz condition and the quadratic inner-boundedness property, combined with the fractional-order extension of Lyapunov's direct method, sufficient conditions for the applicability of the proposed observer are established. The stability analysis ensures that the observation error converges asymptotically to the origin. The effectiveness of the proposed technique is validated through the state estimation of a fractional-order nonlinear system.

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