Magnetic Torquers Only Attitude Control of a 3U Cube Satellite

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Abstract: - In this paper, attitude control of a 3U CubeSat is discussed using magnetic torquers only. As the capabilities of attitude control systems increase, the use of magnetic torques in the CubeSat is becoming more popular. Attitude control using only magnetic torquers has been researched for years and is still a current problem to study. This paper presents the use of an LQR controller for a 3U CubeSat without full gravity gradient stability of the model in a deterministic approach.

Key-Words: - Magnetic control, LQR, CubeSat, magnetorquer

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1 Introduction

One of the first usage of nanosatellite term can be found in [1]. It is mentioned as a new class of satellite with a mass less than 10 kg. Currently, the definition is mostly used for satellites with masses between 10 kg and 1 kg. Cost-efficient and easy to develop feature of this satellite platform makes it more popular as can be seen in Fig. 1. Total number of launched nanosatellites reached 1800 at the end of 2021 [2]. They can be used for different purposes such as communication to observation.

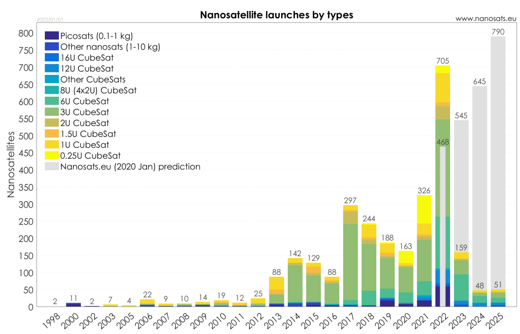

Figure 1: Nanosatellite launches by types [2].

The nanosatellites have both low mass and low volume. As a result of low volume, the capabilities of nanosatellites have some limits. So that, usage of highly reliable and capable subsystems are more important. In terms of attitude control systems (ACS), magnetorquers are one of the most suitable options for nanosatellites. They have high reliability, long life, and less power requirement within lowered volume with respect to its alternatives.

Interaction of Earth's magnetic field and created magnetic dipole by magnetorquer generates a magnetic torque. With the controlling generated magnetic dipole, this magnetic torque can be used to orient satellite. They can only be used effectively on LEO as they are directly affected by the Earth's magnetic field.

While the first satellites carry magnetometers to measure magnetic field, idea of using this natural phenomena as an assistant for stabilizing the satellite is came up. TIROS-2 is the first satellite that used active magnetic control in 1960 [3]. Used magnetic actuator of TIROS-2 is just wounded wires around satellite. However, that was the proof of concept, and capabilities of magnetic attitude control is discovered rapidly.

On the other hand, it took about 40 more years to use magnetic coils in a nanosatellite. According to research on [4, 5], TUBSAT-N, launched in 1998, appears to be the first nanosatellite with magnetic coils. When the low volume and power of nanosatellites is considered, meeting the needs of three-axis attitude control for a nanosatellite becomes a challenge. Here magnetic torquers become a viable solution to this problem.

To apply this solution, different control methods are implemented. In [6], a fully automatic attitude control system has been released for a high inclined, momentum-biased LEO satellite is published. For a rigid satellite with near circular and near polar orbit, attitude control of satellite by only magnetorquers is searched in [7]. An LQR controller is used in the paper. Also, a new type of asymptotic periodic LQR controller is used in [8].

In this study, attitude of a 3U CubeSat model is controlled using LQR with magnetic torquers only.

2 Magnetic Actuation and Satellite Model

Magnetic torquers are simple wires with current flowing through them. The amount of dipole moment produced depends on the total number of wire turns (N), current (I), and wire loop area (a) as in the (1). However, producing required magnetic dipole or required direction for any time is not possible. The direction of magnetic torque can be found by cross product of magnetic dipole moment of torquer and magnetic field. So that, while the angle between magnetic field and magnetic dipole moment getting parallel, produced torque moves away from required.

$$m = NIa \tag{1}$$

To solve this problem, a projection based method is mentioned in [9]. Using (3), where $\boldsymbol{u}$ is required magnetic control moment and $\boldsymbol{B}$ is magnetic field of Earth, gives applicable magnetic dipole $\boldsymbol{m}$. Then, it is easy to calculate applicable magnetic torque $\boldsymbol{\tau}_{\text{applicable}}$ as in (4). Figure 2 can be given as an explanation of the calculations.

$$\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}_{\text{Earth}} \tag{2}$$

$$\boldsymbol{m} = \frac{\boldsymbol{B} \times \boldsymbol{u}}{||\boldsymbol{B}||^2} \tag{3}$$

$$\boldsymbol{\tau}_{\text{applicable}} = \boldsymbol{m} \times \boldsymbol{B} \tag{4}$$

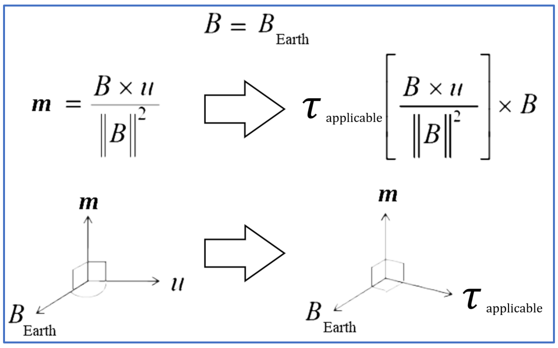

Figure 2: Projection based control torque.

A basic 3U CubeSat model is used with an inertia as (5).

$$I = \begin{bmatrix} 0.06297 & 0 & 0 \\ 0 & 0.06297 & 0 \\ 0 & 0 & 0.01000 \end{bmatrix} \left[\text{kg.m}^2\right] \tag{5}$$

3 Attitude Kinematics and Dynamics

Orientation of satellite can be represented with using quaternions. Quaternions has a vector and a scalar part as in (6). To calculate the change of a quaternion, (7) can be used.

$$\boldsymbol{q} = [\boldsymbol{q}_{1:3}, q_4]' = [\ q_1 \quad q_2 \quad q_3 \quad q_4\ ]' \tag{6}$$

In (7), it is possible to rewrite $\Omega(\boldsymbol{w})$ as (8), where $\boldsymbol{w}$ is angular velocity of satellite with respect to orbital frame.

$$\dot{\boldsymbol{q}} = \frac{1}{2}\Omega(\boldsymbol{w})\boldsymbol{q} \tag{7}$$

$$\Omega(\boldsymbol{w}) = \begin{bmatrix} -\boldsymbol{w}^{\times} & \boldsymbol{w} \\ -\boldsymbol{w}^T & 0 \end{bmatrix} = \begin{bmatrix} 0 & w_3 & -w_2 & w_1 \\ -w_3 & 0 & w_1 & w_2 \\ w_2 & -w_1 & 0 & w_3 \\ -w_1 & -w_2 & -w_3 & 0 \end{bmatrix} \tag{8}$$

Skew symmetric matrix of $\boldsymbol{w}$ is represented as $\boldsymbol{w}^{\times}$ and open form of it is given in (9).

$$\boldsymbol{w}^{\times} = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix} \tag{9}$$

Then, solution of (7) is rearranged as (10).

$$\dot{\boldsymbol{q}} = \frac{1}{2}\begin{bmatrix} q_2 w_3 - q_3 w_2 + q_4 w_1 \\ q_3 w_1 - q_1 w_3 + q_4 w_2 \\ q_1 w_2 - q_2 w_1 + q_4 w_3 \\ -q_1 w_1 - w_2 q_2 - w_3 q_3 \end{bmatrix} \tag{10}$$

The change of angular velocity is searched within the attitude dynamics. Attitude dynamics can be written as a nonlinear system in (11). Only the gravity gradient torque and magnetic torque caused by magnetorquers are considered as disturbance torque in the equation.

$$\dot{\boldsymbol{w}} = -I^{-1}\boldsymbol{w} \times I\boldsymbol{w} + I^{-1}\boldsymbol{\tau}_{\text{gg}} + I^{-1}\boldsymbol{\tau}_{\text{mag}} \tag{11}$$

The nonlinear attitude dynamics can be linearized around the small angle assumption and (13) can be used as generalized linear attitude dynamic model. In here, matrix $\boldsymbol{A}$ and matrix $\boldsymbol{B}$ is written in (14) and (15) by following [10].

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{12}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} = \boldsymbol{A}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} + \boldsymbol{B}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{13}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -4n^2\sigma_1 & 0 & 0 & 0 & 0 & -n-n\sigma_1 \\ 0 & 3n^2\sigma_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & n^2\sigma_3 & -n-n\sigma_3 & 0 & 0 \end{bmatrix} \quad (14)$$

$$\boldsymbol{B} = \begin{bmatrix} \mathbf{0}_{3\times3} \\ I^{-1} \end{bmatrix} \quad (15)$$

Where,

$$n = \frac{\mu}{a^3} \quad (16)$$

$$\sigma_1 = \frac{J_2-J_3}{J_1}, \quad \sigma_2 = \frac{J_3-J_1}{J_2}, \quad \sigma_3 = \frac{J_1-J_2}{J_3} \quad (17)$$

Initial attitude of the satellite are given in (18) and (19). Both of angular velocity and quaternion vectors of body are defined in orbital frame.

$$\boldsymbol{w}_{BO} = \begin{bmatrix} 0.01967 \\ 0.02010 \\ 0.02011 \end{bmatrix} \text{[deg/sec]} \quad (18)$$

$$\boldsymbol{q}_{BO} = \begin{bmatrix} 0.050230 \\ 0.055884 \\ 0.050230 \\ 0.995907 \end{bmatrix} \quad (19)$$

Also the orbit of satellite is given in Table 1. The parameters are semi-major axis ($\boldsymbol{a}$), eccentricity ($\boldsymbol{e}$), inclination ($\boldsymbol{i}$), right ascension of ascending node ($\boldsymbol{\Omega}$), argument of perigee ($\boldsymbol{w}$), and true anomaly ($\boldsymbol{\theta}$) respectively.

Table 1: Orbit of the satellite.

Parameter	Value	Unit
$\boldsymbol{a}$	6906.13	[km]
$\boldsymbol{e}$	1.126×10^{-3}	-
$\boldsymbol{i}$	97.51	[deg]
$\boldsymbol{\Omega}$	89.44	[deg]
$\boldsymbol{w}$	227.1	[deg]
$\boldsymbol{\theta}$	132.9	[deg]
Epoch	25 Jan 2021 18:58:30	

4 LQR Controller

In this paper, LQR controller is used with its constant coefficients or constant gain. To calculate gain of controller, a cost function should be minimized as in (20). State vector of satellite, $\boldsymbol{x}$, is the same with (12). On the other hand, matrix Q is positive definite and matrix R is non-negative definite matrices as stated in [7].

$$\begin{aligned} J(\boldsymbol{u}) &= \lim_{t_f\to\infty} \tfrac{1}{2}\boldsymbol{x}^T(t_f)\, S_f \boldsymbol{x}(t_f) \\ &+ \tfrac{1}{2}\int_{t_0}^{t_f} \left(\boldsymbol{x}^T(t)\, Q\boldsymbol{x}(t) + \boldsymbol{u}^T R\boldsymbol{u}\right) dt \end{aligned} \quad (20)$$

Now, (13) is rewritten in (21).

$$\dot{\boldsymbol{x}}(t) = \mathrm{A}\boldsymbol{x}(t) + B(t)\,\boldsymbol{u}(t) \quad (21)$$

It has a common solution as (22), where $S(t)$ satisfies Riccati differential equation in (23) with the help of latest status $S(t_f) = S_f$ [7].

$$\boldsymbol{u}(t) = -R^{-1}B^T(t)\,S(t)\,\boldsymbol{x}(t) \quad (22)$$

$$\begin{aligned} \dot{S}(t) &= -S(t)\,\mathrm{A} - \mathrm{A}^T S(t) \\ &+ S(t)\,B(t)\,R^{-1}B^T(t)\,S(t) - Q \end{aligned} \quad (23)$$

Finally, a backward integration of the equation can give gain matrix (24). While the time increasing, dependency of $S(t)$ on S_f decreases and becomes unimportant.

$$\boldsymbol{K}(t) \equiv -R^{-1}B^T(t)\,S(t) \quad (24)$$

The process is run once before the run simulation to obtain the gain matrix. Obtained gain never changes during simulations and stays steady. LQR function of MATLAB is used to calculate constant gain matrix for this thesis. N term is neglected and assumed as zero matrix while R matrix assumed as 3 by 3 identity matrix.

5 Simulation Results

It is better to first consider the torque free motion of the satellite to obtain a comparison reference point. Figure 3 is the change of Euler angles of the satellite without any control. Pitch and yaw angles of the CubeSat oscillates up to 15 degrees during simulation. Yaw angle of the satellite has a large frequency to complete its one period, but there is no limit for its oscillation.

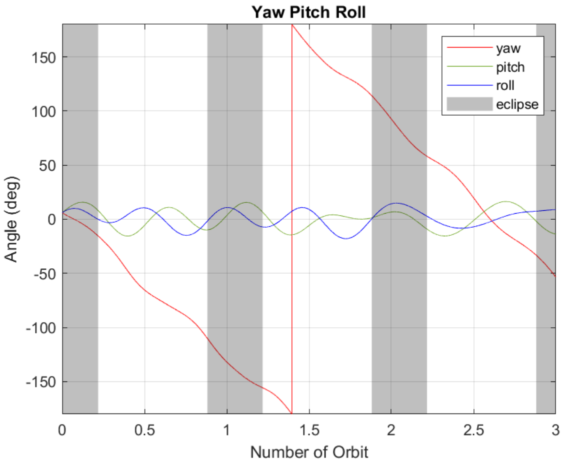

Figure 3: Euler angles without attitude control.

Quaternions of the CubeSat have similar motion as can be seen in Figure 4. While q_1 and q_2 are oscillating approximately 2.5 times in one orbit, q_3 and q_4 are change a lot without oscillating rapidly.

Also, change of the angular velocity in torque free motion is given in Figure 5. All the variables are oscillates. However, pay attention to third element of the angular velocity vector. It never reaches 0 deg/sec during propagation. This is the reason of large change on the yaw axis.

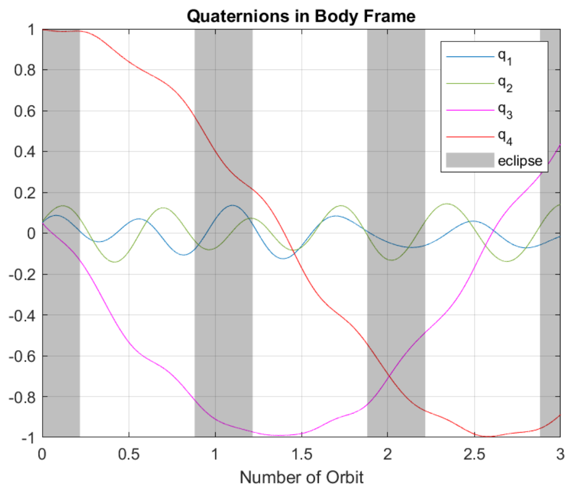

Figure 4: Quaternions without attitude control.

For the torque free motion, mean error on the Euler axes can be seen in Figure 3 and Table 2. Mean error is the total error in degree per time passed through simulation.

Table 2: Mean error of Euler angels for torque free motion.

Error	Value
Mean yaw	80.2660 [deg]
Mean pitch	8.04858 [deg]
Mean roll	7.16065 [deg]

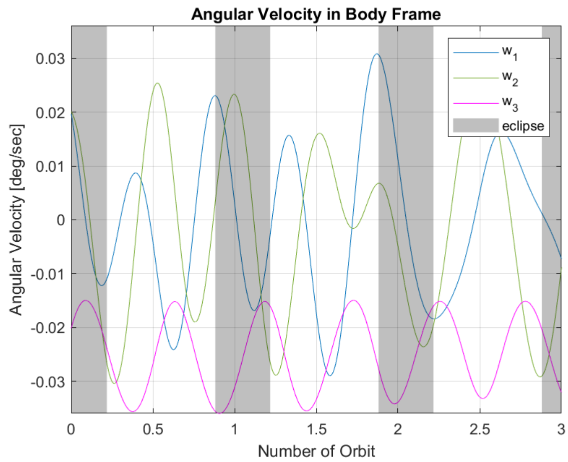

Figure 5: Angular velocities without attitude control.

Then, Q matrix of LQR is set to (25) to compute constant gain of LQR.

$$Q = \begin{bmatrix} 3.55181 * I_{3\times3}10^{-14} & 0_{3\times3} \\ 0_{3\times3} & 9.65961 * I_{3\times3}10^{-14} \end{bmatrix} \quad (25)$$

Obtained LQR gain gives following results in Figure 6. The error on Euler angles for each axes decreases below 1 degree after almost 1.5 orbit.

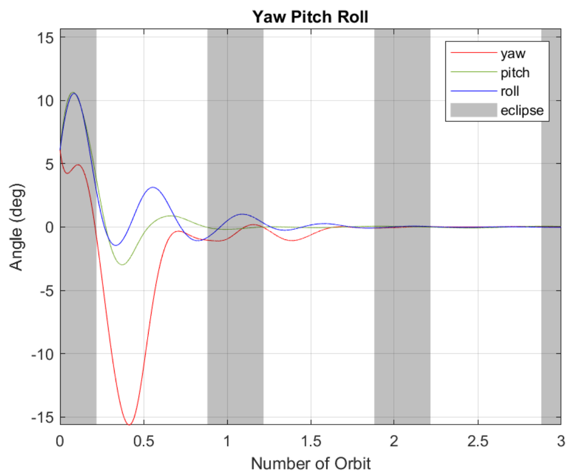

Figure 6: Euler angles of LQR controller simulation.

Similarly, quaternions reach very close values to their target within 1 orbit as can be seen in Figure 7.

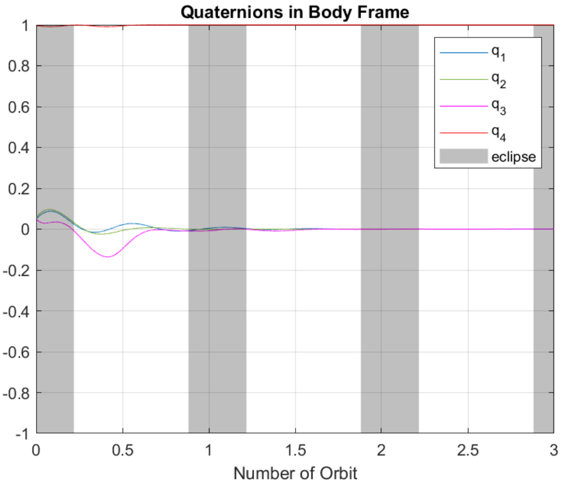

Figure 7: Quaternions of LQR controller simulation.

Figure 8 shows the change of angular velocity change of simulation. It reaches below 0.005 deg/sec before 1 orbit.

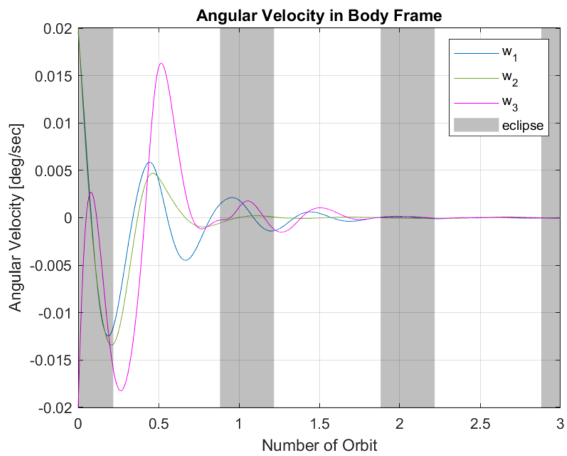

Figure 8: Angular velocities of LQR controller simulation.

When LQR controller is used with a good gain it is possible to obtain accurate pointing as presented during this section. In terms of the Euler angles, mean error decreases to 2 degree for the simulation as given in Table 3.

Table 3: Mean error on Euler angles of controlled satellite.

Error	Value
Mean yaw	1.78644 [deg]
Mean pitch	0.895059 [deg]
Mean roll	1.00956 [deg]

6 Conclusion

A 3U CubeSat model which only uses magnetic actuators as attitude actuator is controlled by an LQR controller in this study. Q matrix of the LQR controller is well chosen to improve the accuracy of the satellite attitude. As a future work, adding a determination algorithm as SVD or TRIAD and then applying a Kalman Filter as in [11, 12] can be done to make the simulations more realistic.

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