Robust H_∞ observer-based control design for discrete-time nonlinear systems with time-varying delay

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Abstract: This paper investigates the problem of robust H_∞ observer-based control for a class of discrete-time nonlinear systems with time-varying delays and parameters uncertainties. We propose an observer-based controller. By constructing an appropriate Lyapunov-Krasovskii functional, some sufficient conditions are developed to ensure the closed-loop system is robust asymptotically stable with H_∞ performance γ in terms of the linear matrix inequalities. Finally, a numerical example is given to illustrate the efficiency of proposed methods.

Key-Words: Observer-based controller; Lyapunov-Krasovskii functional; discrete-time systems; performance; time-varying delays

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1 Introduction

State feedback control is a kind of control method based on the availability of all information of system state, and good closed-loop performance can be obtained by using state feedback controller. However, in the actual system, it is usually impossible to obtain all the state variables directly, because the economic or technical problems lead to the direct measurement of all the state variables is very expensive, or even impossible. Therefore, the controller may have to be combined with the state observer, which estimates the state vector. Observer-based controller design has attracted extensive attention in the past few decades [1-7]. In [2], Chang et al. investigated robust output feedback sliding mode state and disturbance observer-based controller for nonlinear systems. In [5], an observer-based output feedback controller was obtained to exponentially stabilize the linear systems.

In practical control systems, time delay and uncertainty often occur. The existence of time delay and uncertainty will result in instability or performance degradation of the system. Furthermore, completely linear systems do not exist in practical engineering systems. Therefore, the stability analysis and control design for nonlinear system with delay and uncertainty are great significance. The observer and observer-based control design for nonlinear systems have been studied [8-10]. In [9], observer-based controller design for time-delay systems was considered and the sufficient conditions of the asymptotic stability were obtained. On observer design and observer-based controller design, most of the existing literatures focus on continuous time systems [11-16]. In [11], Li et al. studied event-triggered observer-based robust H_∞ control for networked control systems. The observer-based H_∞ control for continuous-time networked control system was studied in [14]. In [15], Chen et al. dealt with the H_∞ observer-based control for continuous time-delay systems. In [17], the observer was designed for nonlinear discrete-time singular systems with time-varying delays.

Motivated by the aforementioned observation, in this paper, we investigated the robust H_∞ observer-based control for a class of discrete-time nonlinear systems with time-varying delays. By constructing the Lyapunov-Krasovskii functional, we established new criteria for discrete-time nonlinear systems to guarantee that the closed-loop system is robust asymptotically stable with H_∞ performance. Furthermore, the controller and observer gains matrices can be obtained by solving the linear matrix inequality. Finally, a numerical example is provided to verify the validity of the stability criterion.

Notations: Throughout this note, the superscripts T and (-1) mean the transpose of a matrix and the inverse of a matrix, respectively. $P > 0$ denotes P is a symmetric positive-definite matrix, $diag\{\cdots\}$ denote a block-diagonal matrix. The symmetric term in a symmetric matrix is denoted by $*$.

2. Problem Formulation

Consider the following discrete-time systems

$$\begin{aligned} x(k+1) &= (A+\Delta A(k))x(k) + (A_\tau + \Delta A_\tau(k))x(k-\tau(k)) \\ &\quad + Ff(x(k)) + Bu(k) + C\omega(k), \\ y(k) &= Ex(k), \\ x(\theta) &= \phi(\theta), \quad \theta = \{-\tau, -\tau+1, \cdots, 0\}, \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector. $u(k) \in \mathbb{R}^m$ is the control input vector, $y(k) \in \mathbb{R}^q$ is the output measurement vector. $\omega(k) \in \mathbb{R}^n$ is the exogenous disturbance vector. A, A_τ, F, C, E, B are the constant matrices of appropriate dimensions. $\phi(\theta)$ denotes initial function. The delay $\tau(k)$ is known time-varying delay, which satisfies a positive integer which is time-varying and

$$0 \le \tau(k) \le \tau \quad , \tag{2}$$

where τ is a nonnegative integer. The parametric uncertainties $\Delta A(k)$ and $\Delta A_\tau(k)$ are unknown matrices representing time-varying parameter uncertainties, which are structures as follows:

$$\begin{aligned} \Delta A(k) &= \Psi H(k)\Phi_1, \\ \Delta A_d(k) &= \Psi H(k)\Phi_2, \end{aligned} \tag{3}$$

where Ψ, Φ_1, Φ_2 are known real constant matrices, $H(k)$ is an unknown time-varying matrix satisfying

$$H^T(k)H(k) \le \tag{4}$$

The unknown function $f(x(k))$ is nonlinear with $f(0) = 0$ and satisfy the following Lipschitz condition for all $x, \hat{x} \in \mathbb{R}^n$:

$$\| f(x) - f(\hat{x}) \| \le \| M(x - \hat{x}) \|, \tag{5}$$

where M is a known constant matrices.

Consider the following observer

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + A_\tau \hat{x}(k-\tau(k)) + Bu(k) \\ &\quad + Ff(\hat{x}(k)) + L(y(k) - E\hat{x}(k)), \end{aligned} \tag{6}$$

where $\hat{x}(k)$ is the estimated of $x(k)$, L is the observer gain matrix.

The following controller is constructed for the system (1)

$$u(k) = -K\hat{x}(k), \tag{7}$$

where K is the controller gain to be designed.

Let $e(k) = x(k) - \hat{x}(k)$ be the estimation error. Then we have

$$\begin{aligned} e(k+1) &= (A - LE)e(k) + A_\tau e(k-\tau(k)) + \Delta A x(k) \\ &\quad + \Delta A_\tau x(k-\tau(k)) + F\tilde{f}(x(k), \hat{x}(k)) + C\omega(k), \end{aligned} \tag{8}$$

where $\tilde{f}(x(k), \hat{x}(k)) = f(x(k)) - f(\hat{x}(k))$.

From (8), systems (1) can be rewritten as

$$\begin{aligned} x(k+1) &= (A - BK + \Delta A)x(k) + BKe(k) + C\omega(k) \\ &\quad + (A_\tau + \Delta A_\tau)x(k-\tau(k)) + Ff(x(k)). \end{aligned} \tag{9}$$

Combining (8) and (9), the closed-loop augmented system can be obtained

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{A}_\tau \bar{x}(k-\tau(k)) + \bar{F}\bar{f}(x(k)) + \bar{C}\omega(k), \tag{10}$$

where

$$\bar{A} = \begin{bmatrix} A - BK + \Delta A & BK \\ \Delta A & A - LE \end{bmatrix}, \bar{A}_\tau = \begin{bmatrix} A_\tau + \Delta A_\tau & 0 \\ \Delta A_\tau & A_\tau \end{bmatrix},$$

$$\bar{F} = \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix}, \bar{C} = \begin{bmatrix} C \\ C \end{bmatrix}, \bar{x}(k-\tau(k)) = \begin{bmatrix} x(k-\tau(k)) \\ e(k-\tau(k)) \end{bmatrix},$$

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \ \bar{f}(x(k), \hat{x}(k)) = \begin{bmatrix} f(x(k)) \\ \tilde{f}(x(k), \hat{x}(k)) \end{bmatrix}.$$

The aim of the paper is to design H_∞ observer-based control for system (1) such that

(1) The closed-loop system (10) with $\omega(k) = 0$ is asymptotically stable;

(2) Under zero initial conditions, for all nonzero $\omega(k)$ and a prescribed positive number γ, the following inequality holds

$$\| \bar{x}(k) \|_2 \le \gamma \| \omega(k) \|_2 \tag{11}$$

Lemma 1. ([3]) For any $\varepsilon > 0$, and known real matrices Θ_1, Θ_2 and $H(k)$ of appropriate dimensions, the inequality

$$\Theta_1 H(k)\Theta_2 + \Theta_2^T H^T(k)\Theta_1^T \le \frac{1}{\varepsilon}\Theta_1\Theta_1^T + \varepsilon\Theta_2^T\Theta_2 \tag{12}$$

holds, where $H(k)$ is a time-varying uncertain matrix fulfilling $H^T(k)H(k) \le I$,

Lemma 2. ([18]) For matrices Γ, Λ_2, X and Λ_1 with appropriate dimensions and a scalar δ, the following inequality holds

$$\Gamma + \Lambda_1^T \Lambda_2^T + \Lambda_2 \Lambda_1 < 0,$$

if the following condition is satisfied:

$$\begin{bmatrix} \Gamma & \delta\Lambda_2 + \Lambda_1^T X^T \\ * & -\delta X - \delta X^T \end{bmatrix} < 0.$$

3. Main Results

In this section, sufficient conditions for system stability are derived for augmented systems (10).

Theorem 1. Given a scalar δ, the closed-loop system (10) with $\omega(k)=0$ is robust asymptotically stable, if there exist positive scalars $\alpha_1, \alpha_2, \varepsilon$, symmetric positive definite matrices P, R, S, W, and any matrices X, Y, Z, such that the following LMI holds:

$$\Sigma = \begin{bmatrix} \Upsilon & \bar{\Sigma}_{12} \\ * & \bar{\Sigma}_{22} \end{bmatrix} < 0, \tag{13}$$

where

$$\bar{\Sigma}_{12} = \begin{bmatrix} A^T P - (BY)^T & 0 & -Y^T & 0 \\ A_\tau^T P & 0 & 0 & 0 \\ F^T P & 0 & 0 & 0 \\ (BY)^T & A^T S - E^T Z & Y^T & 0 \\ 0 & A_\tau^T S & 0 & 0 \\ 0 & F^T S & 0 & 0 \end{bmatrix},$$

$$\bar{\Sigma}_{22} = \begin{bmatrix} -P & 0 & \delta(PB - BX) & P\Psi \\ * & -S & 0 & S\Psi \\ * & * & -\delta X - \delta X^T & 0 \\ * & * & * & -\varepsilon I \end{bmatrix},$$

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & \varepsilon\Phi_1^T\Phi_2 & 0 & 0 & 0 & 0 \\ * & -R + \varepsilon\Phi_2^T\Phi_2 & 0 & 0 & 0 & 0 \\ * & * & -\alpha_1 I & 0 & 0 & 0 \\ * & * & * & \Upsilon_{44} & 0 & 0 \\ * & * & * & * & -W & 0 \\ * & * & * & * & * & -\alpha_2 I \end{bmatrix},$$

$\Upsilon_{11} = -P + \tau R + \alpha_1 M^T M + \varepsilon\Phi_1^T\Phi_1$,

$\Upsilon_{44} = -S + \tau W + \alpha_2 M^T M$.

Furthermore, the controller and observer gains are given by $K = X^{-1}Y, L = Z^{-T}S^T$.

Proof. Choose the following Lyapunov-Krasovskii functional

$$V(k) = V_1(k) + V_2(k), \tag{14}$$

where

$$\begin{aligned} V_1(k) &= x^T(k)Px(k) + e^T(k)Se(k), \\ V_2(k) &= \sum_{i=k-\tau(k)}^{k-1} x^T(i)Rx(i) + \sum_{j=-\tau+1}^{0}\sum_{i=k+j}^{k-1} x^T(i)Rx(i) \\ &\quad + \sum_{i=k-\tau(k)}^{k-1} e^T(i)We(i) + \sum_{j=-\tau+1}^{0}\sum_{i=k+j}^{k-1} e^T(i)We(i). \end{aligned}$$

Defined $\Delta V(k) = V(k+1) - V(k)$. Then, we have

$$\begin{aligned} \Delta V_1(k) &= V_1(k+1) - V_1(k) \\ &= x^T(k+1)Px(k+1) - x^T(k)Px(k) \\ &\quad + e^T(k+1)Se(k+1) - e^T(k)Se(k) \\ &= \xi^T(k)\Lambda_1^T P\Lambda_1\xi(k) - x^T(k)Px(k) \\ &\quad + \xi^T(k)\Lambda_2^T S\Lambda_2\xi(k) - e^T(k)Se(k), \end{aligned} \tag{15}$$

where

$$\Lambda_1 = \begin{bmatrix} A - BK + \Delta A & A_\tau + \Delta A_\tau & F & BK & 0 & 0 \end{bmatrix},$$

$$\Lambda_2 = \begin{bmatrix} \Delta A & \Delta A_\tau & 0 & A - LE & A_\tau & F \end{bmatrix},$$

$$\xi(k) = \Big[x^T(k), x^T(k-\tau(k)), f^T(x(k)), e^T(k), e^T(k-\tau(k)), \tilde{f}^T(x(k),\hat{x}(k)) \Big]^T.$$

We have

$$\begin{aligned} \Delta V_2(k) &= \sum_{i=k+1-\tau(k+1)}^{k} x^T(i)Rx(i) - \sum_{i=k-\tau(k)}^{k-1} x^T(i)Rx(i) \\ &\quad + \sum_{j=-\tau+1}^{0}\left[\sum_{i=k+j+1}^{k} x^T(i)Rx(i) - \sum_{i=k+j}^{k-1} x^T(i)Rx(i) \right] \\ &\quad + \sum_{i=k+1-\tau(k+1)}^{k} e^T(i)We(i) - \sum_{i=k-\tau(k)}^{k-1} e^T(i)We(i) \\ &\quad + \sum_{j=-\tau+1}^{0}\left[\sum_{i=k+j+1}^{k} e^T(i)We(i) - \sum_{i=k+j}^{k-1} e^T(i)We(i) \right] \\ &\le \sum_{i=k+1-\tau}^{k} x^T(i)Rx(i) - x^T(k-\tau(k))Rx(k-\tau(k)) \\ &\quad + \tau x^T(k)Rx(k) - \sum_{i=k-\tau+1}^{k} x^T(i)Rx(i) \\ &\quad + \sum_{i=k+1-\tau}^{k} e^T(i)We(i) - e^T(k-\tau(k))We(k-\tau(k)) \\ &\quad + \tau e^T(k)We(k) - \sum_{i=k-\tau+1}^{k} e^T(i)We(i) \\ &= \tau x^T(k)Rx(k) - x^T(k-\tau(k))Rx(k-\tau(k)) \\ &\quad + \tau e^T(k)We(k) - e^T(k-\tau(k))We(k-\tau(k)). \end{aligned} \tag{16}$$

For any positive scalars α_1, α_2, it follows from (5) that

$$\alpha_1(x^T(k)M^T Mx(k) - f^T(x(k))f(x(k))) \ge 0. \tag{17}$$

$$\alpha_2(e^T(k)M^TMe(k)-\tilde{f}^T(x(k),\hat{x}(k))\tilde{f}(x(k),\hat{x}(k)))\geq 0. \tag{18}$$

Combine (15)-(18), we get

$$\begin{aligned}\Delta V(k)\leq{}&\xi^T(k)(\Lambda_1^TP\Lambda_1+\Lambda_2^TS\Lambda_2)\xi(k)\\&+x^T(k)(-P+\tau R+\alpha_1M^TM)x(k)\\&-x^T(k-\tau(k))Rx(k-\tau(k))\\&-\alpha_1f^T(x(k))f(x(k))\\&+e^T(k)(-S+\tau W+\alpha_2M^TM)e(k)\\&-e^T(k-\tau(k))We(k-\tau(k))\\&-\alpha_2\tilde{f}^T(x(k))\tilde{f}(x(k))\\={}&\xi^T(k)\Pi\xi(k),\end{aligned}\tag{19}$$

where

$$\Pi=\Xi_{1,1}+\Lambda_1^TP\Lambda_1+\Lambda_2^TS\Lambda_2,$$

$$\Xi_{1,1}=diag\{-P+\tau R+\alpha_1M^TM,-R,-\alpha_1I,-S+\tau W+\alpha_2M^TM,-W,-\alpha_2I\}.$$

Using Schur complement, $\Pi<0$ if and only if

$$\Xi=\begin{bmatrix}\Xi_{11}&\Xi_{12}\\ *&\Xi_{22}\end{bmatrix}<0, \tag{20}$$

where

$$\Xi_{12}=\begin{bmatrix}A^TP-K^TB^TP+\Delta A^TP&\Delta A^TS\\A_\tau^TP+\Delta A_\tau^TP&\Delta A_\tau^TS\\F^TP&0\\K^TB^TP&A^TS-E^TZ\\0&A_\tau^TS\\0&F^TS\end{bmatrix},$$

$\Xi_{22}=diag\{-P,-S\},\ Z=L^TS.$

By separating the matrix for uncertain and known terms, (20) can be written as

$$\Xi=\Xi_1+\Xi_2+\Xi_3<0, \tag{21}$$

where

$$\Xi_1=\begin{bmatrix}\Xi_{11}&\Xi_{61}\\ *&\Xi_{22}\end{bmatrix},\Xi_2=\begin{bmatrix}0&\Xi_{62}\\ *&0\end{bmatrix},\Xi_3=\begin{bmatrix}0&\Xi_{63}\\ *&0\end{bmatrix},$$

$$\Xi_{61}=\begin{bmatrix}A^TP&0\\A_\tau^TP&0\\F^TP&0\\0&A^TS-E^TZ\\0&A_\tau^TS\\0&F^TS\end{bmatrix},\Xi_{62}=\begin{bmatrix}\Delta A^TP&\Delta A^TS\\\Delta A_\tau^TP&\Delta A_\tau^TS\\0&0\\0&0\\0&0\\0&0\end{bmatrix},$$

$$\Xi_{63}^T=\begin{bmatrix}-PBK&0&0&PBK&0&0\\0&0&0&0&0&0\end{bmatrix}.$$

Let

$\bar{\Theta}^T=\begin{bmatrix}0\,0\,0\,0\,0\,0\,P\,S\end{bmatrix},\zeta(k)=\begin{bmatrix}\Delta A\,\Delta A_\tau\,0\,0\,0\,0\,0\,0\end{bmatrix},$

$\bar{\Phi}=\begin{bmatrix}\Phi_1\,\Phi_2\,0\,0\,0\,0\,0\,0\end{bmatrix}.$

Using condition (3) and Lemma 1, Ξ_2 can be rewritten as

$$\begin{aligned}\Xi_2&=\zeta^T(k)\bar{\Theta}^T+\bar{\Theta}\zeta(k)\\&=(\Psi H(k)\bar{\Phi})^T\bar{\Theta}^T+\bar{\Theta}\Psi H(k)\bar{\Phi}\\&\leq\varepsilon^{-1}\bar{\Theta}\Psi(\bar{\Theta}\Psi)^T+\varepsilon\bar{\Phi}^T\bar{\Phi}.\end{aligned}\tag{22}$$

It is clear that (21) holds if

$$\begin{bmatrix}\Upsilon&\Xi_{61}\\ *&\Xi_{22}\end{bmatrix}+\varepsilon^{-1}\bar{\Theta}\Psi(\bar{\Theta}\Psi)^T+\Xi_3<0 \tag{23}$$

where

$$\Upsilon=\begin{bmatrix}\Upsilon_{11}&\varepsilon\Phi_1^T\Phi_2&0&0&0&0\\ *&-R+\varepsilon\Phi_2^T\Phi_2&0&0&0&0\\ *&*&-\alpha_1I&0&0&0\\ *&*&*&\Upsilon_{44}&0&0\\ *&*&*&*&-W&0\\ *&*&*&*&*&-\alpha_2I\end{bmatrix},$$

$\Upsilon_{11}=-P+\tau R+\alpha_1M^TM+\varepsilon\Phi_1^T\Phi_1,$

$\Upsilon_{44}=-S+\tau W+\alpha_2M^TM.$

Then we introduce a non-singular matrix X and define $K=X^{-1}Y$, it is obvious that

$$PBK=(PB-BX)X^{-1}Y+BY. \tag{24}$$

Substituting (24) into (23), we have

$$\begin{bmatrix}\Upsilon&\hat{\Xi}_{61}\\ *&\Xi_{22}\end{bmatrix}+\varepsilon^{-1}\bar{\Theta}\Psi(\bar{\Theta}\Psi)^T+\begin{bmatrix}0&\hat{\Xi}_{63}\\ *&0\end{bmatrix}<0, \tag{25}$$

where

$$\hat{\Xi}_{61}=\begin{bmatrix}A^TP-Y^TB^T&0\\A_\tau^TP&0\\F^TP&0\\Y^TB^T&A^TS-E^TZ\\0&A_\tau^TS\\0&0\end{bmatrix},\hat{\Xi}_{63}=\begin{bmatrix}-\hat{\vartheta}&0\\0&0\\0&0\\\hat{\vartheta}&0\\0&0\\0&0\end{bmatrix},$$

$\hat{\vartheta}=Y^TX^{-T}(PB-BX)^T.$

Let

$$\Gamma_1=X^{-1}\begin{bmatrix}-Y&0&0&Y&0&0&0&0\end{bmatrix},$$

$$\Gamma_2^T=\begin{bmatrix}0&0&0&0&0&0&(PB-BX)^T&0\end{bmatrix}.$$

Then, it follows that

$$\begin{bmatrix}0&\hat{\Xi}_{63}\\ *&0\end{bmatrix}=\Gamma_1^T\Gamma_2^T+\Gamma_2\Gamma_1. \tag{26}$$

By substituting (26) into (25), we can obtain

$$\begin{bmatrix} \Upsilon & \hat{\Xi}_{61} \\ * & \Xi_{22} \end{bmatrix} + \varepsilon^{-1}\bar{\Theta}\Psi(\bar{\Theta}\Psi)^T + \Gamma_1^T\Gamma_2^T + \Gamma_2\Gamma_1 < 0 \quad (27)$$

By using Lemma 2, (27) can be guaranteed by the following inequality:

$$\begin{bmatrix} \begin{bmatrix} \Upsilon & \hat{\Xi}_{6,1} \\ * & \Xi_{2,2} \end{bmatrix} + \varepsilon^{-1}\bar{\Theta}\Psi(\bar{\Theta}\Psi)^T & \delta\Gamma_2 + \Gamma_0^T \\ * & -\delta X - \delta X^T \end{bmatrix} < 0, \quad (28)$$

where $\Gamma_0 = [-Y \ 0 \ 0 \ Y \ 0 \ 0 \ 0 \ 0]$.

Finally applying Schur complement for (28), we obtain $\Pi < 0$ holds if the LMI (13) is satisfied. This completes the proof.

Next, Theorem 2 gives a criterion to ensure the closed-loop system (10) is robust asymptotically stable with H_∞ performance γ.

Theorem 2. For given scalars $\delta, \gamma,$ the closed-loop system (10) is robust asymptotically stable with H_∞ performance γ, if there exist positive scalars $\alpha_1, \alpha_2, \varepsilon,$ symmetric positive definite matrices $P, R, S, W,$ and any matrices $X, Y, Z,$ such that the following LMI holds:

$$\bar{\Sigma} = \begin{bmatrix} \bar{\Upsilon} & \tilde{\Sigma}_{12} \\ * & \bar{\Sigma}_{22} \end{bmatrix} < 0, \quad (29)$$

where

$$\tilde{\Sigma}_{12} = \begin{bmatrix} A^TP - (BY)^T & 0 & -Y^T & 0 \\ A_\tau^TP & 0 & 0 & 0 \\ F^TP & 0 & 0 & 0 \\ (BY)^T & A^TS - E^TZ & Y^T & 0 \\ 0 & A_\tau^TS & 0 & 0 \\ 0 & F^TS & 0 & 0 \\ C^TP & C^TS & 0 & 0 \end{bmatrix},$$

$$\bar{\Sigma}_{22} = \begin{bmatrix} -P & 0 & \delta(PB - BX) & P\Psi \\ * & -S & 0 & S\Psi \\ * & * & -\delta X - \delta X^T & 0 \\ * & * & * & -\varepsilon I \end{bmatrix},$$

$$\bar{\Upsilon} = \begin{bmatrix} \bar{\Upsilon}_{11} & \varepsilon\Phi_1^T\Phi_2 & 0 & 0 & 0 & 0 & 0 \\ * & \bar{\Upsilon}_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\alpha_1 I & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\Upsilon}_{44} & 0 & 0 & 0 \\ * & * & * & * & -W & 0 & 0 \\ * & * & * & * & * & -\alpha_2 I & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\bar{\Upsilon}_{11} = -P + \tau R + \alpha_1 M^TM + I + \varepsilon\Phi_1^T\Phi_1,$$

$$\bar{\Upsilon}_{22} = -R + \varepsilon\Phi_2^T\Phi_2,$$

$$\bar{\Upsilon}_{44} = -S + \tau W + I + \alpha_2 M^TM.$$

Furthermore, the controller and observer gains are given by $K = X^{-1}Y, L = Z^{-T}S^T$.

Proof. Choosing the Lyapunov functional (14) and defining the following index:

$$J = \sum_{k=0}^{\infty}\left[\bar{x}^T(k)\bar{x}(k) - \gamma^2\omega^T(k)\omega(k)\right] \quad (30)$$

Under zeros-initial conditions, we have

$$\begin{aligned} J &= \sum_{k=0}^{\infty}\left[\bar{x}^T(k)\bar{x}(k) - \gamma^2\omega^T(k)\omega(k) + \Delta V(k)\right] - V(\infty) \\ &\le \sum_{k=0}^{\infty}\left[\bar{x}^T(k)\bar{x}(k) - \gamma^2\omega^T(k)\omega(k) + \Delta V(k)\right] \end{aligned} \quad (31)$$

Similar to the proof of Theorem 1, we can obtain

$$\begin{aligned} &\bar{x}^T(k)\bar{x}(k) - \gamma^2\omega^T(k)\omega(k) + \Delta V(k) \\ &\le \bar{\xi}^T(k)\bar{\Pi}\bar{\xi}(k), \end{aligned} \quad (32)$$

where

$$\bar{\xi}^T(k) = \left[\xi^T(k), \omega^T(k)\right],$$

$$\bar{\Pi} = \bar{\Xi}_{11} + \bar{\Lambda}_1^TP\bar{\Lambda}_1 + \bar{\Lambda}_2^TS\bar{\Lambda}_2,$$

$$\begin{aligned} \bar{\Xi}_{11} = diag\{&-P + \tau R + \alpha_1 M^TM + I, -R, -\alpha_1 I, \\ &-S + \tau W + \alpha_2 M^TM + I, -W, -\alpha_2 I, -\gamma^2 I\}, \end{aligned}$$

$$\bar{\Lambda}_1 = [\Lambda_1 \quad C], \bar{\Lambda}_2 = [\Lambda_2 \quad C].$$

So $\bar{\Pi} < 0$ if and only if

$$\bar{\Xi} = \begin{bmatrix} \bar{\Xi}_{11} & \bar{\Xi}_{12} \\ * & \Xi_{22} \end{bmatrix} < 0, \quad (33)$$

where

$$\bar{\Xi}_{12} = \begin{bmatrix} A^TP - K^TB^TP + \Delta A^TP & \Delta A^TS \\ A_\tau^TP + \Delta A_\tau^TP & \Delta A_\tau^TS \\ F^TP & 0 \\ K^TB^TP & A^TS - E^TZ \\ 0 & A_\tau^TS \\ 0 & F^TS \\ C^TP & C^TS \end{bmatrix},$$

$\Xi_{22} = diag\{-P, -S\}, \ Z = L^TS.$

By separating the matrix for uncertain and known terms, (33) can be written as

$$\bar{\Xi} = \bar{\Xi}_1 + \bar{\Xi}_2 + \bar{\Xi}_3 < 0, \quad (34)$$

where

$$\bar{\Xi}_1 = \begin{bmatrix} \bar{\Xi}_{11} & \Xi_{71} \\ * & \Xi_{22} \end{bmatrix}, \bar{\Xi}_2 = \begin{bmatrix} 0 & \Xi_{72} \\ * & 0 \end{bmatrix}, \bar{\Xi}_3 = \begin{bmatrix} 0 & \Xi_{73} \\ * & 0 \end{bmatrix},$$

$$\Xi_{71} = \begin{bmatrix} A^T P & 0 \\ A_\tau^T P & 0 \\ F^T P & 0 \\ 0 & A^T S - E^T Z \\ 0 & A_\tau^T S \\ 0 & F^T S \\ C^T P & C^T S \end{bmatrix}, \Xi_{72} = \begin{bmatrix} \Delta A^T P & \Delta A^T S \\ \Delta A_\tau^T P & \Delta A_\tau^T S \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Xi_{73}^T = \begin{bmatrix} -PBK & 0 & 0 & PBK & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Let

$$\hat{\Theta} = \begin{bmatrix} 0\,0\,0\,0\,0\,0\,0\,P\,S \end{bmatrix}^T,$$
$$\bar{\zeta}(k) = \begin{bmatrix} \Delta A\, \Delta A_\tau\, 0\,0\,0\,0\,0\,0\,0 \end{bmatrix},$$
$$\hat{\Phi} = \begin{bmatrix} \Phi_1\, \Phi_2\, 0\,0\,0\,0\,0\,0\,0 \end{bmatrix}.$$

Using condition (3) and Lemma 1, $\bar{\Xi}_2$ can be rewritten as

$$\begin{aligned} \bar{\Xi}_2 &= \bar{\zeta}^T(k)\hat{\Theta}^T + \hat{\Theta}\bar{\zeta}(k) \\ &= (\Psi H(k)\hat{\Phi})^T \hat{\Theta}^T + \hat{\Theta}\Psi H(k)\hat{\Phi} \\ &\le \varepsilon^{-1}\hat{\Theta}\Psi(\hat{\Theta}\Psi)^T + \varepsilon\hat{\Phi}^T\hat{\Phi}. \end{aligned} \tag{35}$$

It is clear that (34) holds if

$$\begin{bmatrix} \bar{\Upsilon} & \Xi_{71} \\ * & \Xi_{22} \end{bmatrix} + \varepsilon^{-1}\hat{\Theta}\Psi(\hat{\Theta}\Psi)^T + \bar{\Xi}_3 < 0, \tag{36}$$

where

$$\bar{\Upsilon} = \begin{bmatrix} \bar{\Upsilon}_{11} & \varepsilon\Phi_1^T\Phi_2 & 0 & 0 & 0 & 0 & 0 \\ * & \bar{\Upsilon}_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\alpha_1 I & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\Upsilon}_{44} & 0 & 0 & 0 \\ * & * & * & * & -W & 0 & 0 \\ * & * & * & * & * & -\alpha_2 I & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$\bar{\Upsilon}_{11} = -P + \tau R + \alpha_1 M^T M + \varepsilon\Phi_1^T\Phi_1 + I,$

$\bar{\Upsilon}_{22} = -R + \varepsilon\Phi_2^T\Phi_2,$

$\bar{\Upsilon}_{44} = -S + \tau W + \alpha_2 M^T M + I.$

Then introducing a non-singular matrix X and define $K = X^{-1}Y$, it is obvious that

$$PBK = (PB - BX)X^{-1}Y + BY \tag{37}$$

From (36) and (37), we can obtain

$$\begin{bmatrix} \bar{\Upsilon} & \tilde{\Xi}_{71} \\ * & \Xi_{22} \end{bmatrix} + \varepsilon^{-1}\hat{\Theta}\Psi(\hat{\Theta}\Psi)^T + \begin{bmatrix} 0 & \tilde{\Xi}_{73} \\ * & 0 \end{bmatrix} < 0, \tag{38}$$

where

$$\tilde{\Xi}_{71} = \begin{bmatrix} A^T P - Y^T B^T & 0 \\ A_\tau^T P & 0 \\ F^T P & 0 \\ Y^T B^T & A^T S - E^T Z \\ 0 & A_\tau^T S \\ 0 & F^T S \\ C^T P & C^T S \end{bmatrix}, \tilde{\Xi}_{73} = \begin{bmatrix} -\hat{\vartheta} & 0 \\ 0 & 0 \\ 0 & 0 \\ \hat{\vartheta} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$\hat{\vartheta} = Y^T X^{-T}(PB - BX)^T.$

Let

$$\bar{\Gamma}_1 = X^{-1}\begin{bmatrix} -Y & 0 & 0 & Y & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\bar{\Gamma}_2^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & (PB - BX)^T & 0 \end{bmatrix}.$$

Then, we have

$$\begin{bmatrix} 0 & \tilde{\Xi}_{73} \\ * & 0 \end{bmatrix} = \bar{\Gamma}_1^T\bar{\Gamma}_2^T + \bar{\Gamma}_2\bar{\Gamma}_1. \tag{39}$$

By substituting (39) into (38), it follows that

$$\begin{bmatrix} \bar{\Upsilon} & \tilde{\Xi}_{71} \\ * & \Xi_{22} \end{bmatrix} + \varepsilon^{-1}\hat{\Theta}\Psi(\hat{\Theta}\Psi)^T + \bar{\Gamma}_1^T\bar{\Gamma}_2^T + \bar{\Gamma}_2\bar{\Gamma}_1 < 0. \tag{40}$$

By using Lemma 2, (40) holds if

$$\begin{bmatrix} \begin{bmatrix} \bar{\Upsilon} & \tilde{\Xi}_{71} \\ * & \Xi_{22} \end{bmatrix} + \varepsilon^{-1}\hat{\Theta}\Psi(\hat{\Theta}\Psi)^T & \delta\bar{\Gamma}_2 + \bar{\Gamma}_0^T \\ * & -\delta X - \delta X^T \end{bmatrix} < 0, \tag{41}$$

where $\bar{\Gamma}_0 = \begin{bmatrix} -Y & 0 & 0 & Y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Finally applying Schur complement, (29) implies $\bar{\Pi} < 0$, Therefore, we can have $J < 0$, and

$$\sum_{k=0}^{\infty} \bar{x}^T(t)\bar{x}(t) \le \gamma^2 \sum_{k=0}^{\infty} \omega^T(k)\omega(k). \tag{42}$$

Thus, the H_∞ performance is satisfied. This completes the proof.

When $\tau(k) = 0$, consider the following systems

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{F}\bar{f}(x(k)) + \bar{C}\omega(k). \tag{43}$$

where

$$\bar{A} = \begin{bmatrix} A - BK + \Delta A & BK \\ \Delta A & A - LE \end{bmatrix}, \bar{F} = \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} C \\ C \end{bmatrix}, \ \bar{x}(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \breve{x}(k - \bar{\tau}) = \begin{bmatrix} x(k - \bar{\tau}) \\ e(k - \bar{\tau}) \end{bmatrix},$$

$$\bar{f}(x(k)) = \begin{bmatrix} f(x(k)) \\ \tilde{f}(x(k), \ \hat{x}(k)) \end{bmatrix}.$$

Corollary 1. For given scalars $\delta,\gamma,$ the augmented system (43) is robust asymptotically stable with H_∞ performance γ, if there exist positive scalars α_1,α_2, symmetric positive definite matrices $P,S,$ and any matrices $X,Y,Z,$ such that the following LMI holds:

$$\underline{\Sigma} = \begin{bmatrix} \underline{\Upsilon} & \underline{\Sigma}_{12} \\ * & \bar{\Sigma}_{22} \end{bmatrix} < 0, \quad (44)$$

where

$$\underline{\Upsilon} = diag\{-P+\alpha_1 M^T M + I + \varepsilon\Phi_1^T\Phi_1, -\alpha_1 I, -S + I + \alpha_2 M^T M, -\alpha_2 I, -\gamma^2 I\},$$

$$\underline{\Sigma}_{12} = \begin{bmatrix} A^T P - (BY)^T & 0 & -Y^T & 0 \\ F^T P & 0 & 0 & 0 \\ (BY)^T & A^T S - E^T Z & Y^T & 0 \\ 0 & F^T S & 0 & 0 \\ C^T P & C^T S & 0 & 0 \end{bmatrix},$$

$$\bar{\Sigma}_{22} = \begin{bmatrix} -P & 0 & \delta(PB - BX) & P\Psi \\ * & -S & 0 & S\Psi \\ * & * & -\delta X - \delta X^T & 0 \\ * & * & * & -\varepsilon I \end{bmatrix}.$$

Furthermore, the controller and observer gains are given by $K = X^{-1}Y, L = Z^{-T}S^T$.

When $\Delta A = \Delta A_\tau = \Delta C = 0,$ (10) can be rewritten as

$$\bar{x}(k+1) = \tilde{A}\bar{x}(k) + \tilde{A}_\tau \bar{x}(k-\tau(k)) + \bar{F}\bar{f}(x(k)) + \tilde{C}\omega(k), \quad (45)$$

where

$$\tilde{A} = \begin{bmatrix} A - BK & BK \\ 0 & A - LE \end{bmatrix}, \tilde{A}_\tau = \begin{bmatrix} A_\tau & 0 \\ 0 & A_\tau \end{bmatrix},$$

$$\bar{F} = \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix}, \ \bar{C} = \begin{bmatrix} C \\ C \end{bmatrix}, \ \bar{x}(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix},$$

$$\bar{f}(x(k), \hat{x}(k)) = \begin{bmatrix} f(x(k)) \\ \tilde{f}(x(k), \hat{x}(k)) \end{bmatrix}.$$

Corollary 2. For given scalars $\delta,\gamma,$ the augmented system (45) is robust asymptotically stable with H_∞ performance γ, if there exist positive scalars α_1,α_2, symmetric positive definite matrices $P,R,S,W,$ and any matrices $X,Y,Z,$ such that the following LMI holds:

$$\breve{\Sigma} = \begin{bmatrix} \breve{\Upsilon} & \breve{\Sigma}_{12} \\ * & \breve{\Sigma}_{22} \end{bmatrix} < 0, \quad (46)$$

where

$$\breve{\Upsilon} = diag\{-P + \tau R + \alpha_1 M^T M + I, -R, -\alpha_1 I, -S + \tau W + \alpha_2 M^T M + I, -W, -\alpha_2 I, -\gamma^2 I\},$$

$$\breve{\Sigma}_{12} = \begin{bmatrix} A^T P - (BY)^T & 0 & -Y^T \\ A_\tau^T P & 0 & 0 \\ F^T P & 0 & 0 \\ (BY)^T & A^T S - E^T Z & Y^T \\ 0 & A_\tau^T S & 0 \\ 0 & F^T S & 0 \end{bmatrix},$$

$$\breve{\Sigma}_{22} = \begin{bmatrix} -P & 0 & \delta(PB - BX) \\ * & -S & 0 \\ * & * & -\delta X - \delta X^T \end{bmatrix},$$

Furthermore, the controller and observer gains are given by $K = X^{-1}Y, L = Z^{-T}S^T$.

4. Numerical example

In this section, we present one example to demonstrate the effectiveness of our results.

Example 1. Consider the system (1) with the following parameters

$$A = \begin{bmatrix} 0.1 & -0.6 \\ 0.1 & 0.51 \end{bmatrix}, \quad A_\tau = \begin{bmatrix} 0.15 & 0.2 \\ 0.1 & -0.2 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & -0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 0.2 & 0.3 \\ 0.12 & 0.1 \end{bmatrix},$$

$$F = \begin{bmatrix} -0.01 & 0.03 \\ 0.01 & 0.01 \end{bmatrix}, E = \begin{bmatrix} -0.21 & -0.3 \end{bmatrix},$$

$$H(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Psi = \begin{bmatrix} 0.01 & 0.03 \\ 0.02 & 0.02 \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix} 0.01 & 0.02 \\ 0.11 & 0.02 \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} 0.12 & 0.02 \\ 0.12 & 0.13 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \quad \omega(k) = \begin{pmatrix} e^{-0.1k}\sin(k) \\ e^{-0.1k}\cos(k) \end{pmatrix},$$

$$f(x(k)) = \begin{pmatrix} 0.01\sin(x_1(k)) \\ 0.01\sin(x_1(k)) \end{pmatrix},$$

$$d(k) = 2 + \sin(k\pi), \ \gamma = 2, \ \delta = 1.$$

When $\omega(k) = 0,$ by using Matlab to solve LMIs (13), we have

$$P=\begin{bmatrix}0.3164 & 0.1009\\ 0.1009 & 0.3902\end{bmatrix}, R=\begin{bmatrix}0.1018 & 0.0322\\ 0.0322 & 0.0969\end{bmatrix},$$

$$S=\begin{bmatrix}0.3680 & 0.0320\\ 0.0320 & 0.4202\end{bmatrix}, W=\begin{bmatrix}0.1057 & 0.0213\\ 0.0213 & 0.1335\end{bmatrix},$$

$$X=\begin{bmatrix}0.1350 & -0.0034\\ -0.0046 & 0.1392\end{bmatrix}, Y=\begin{bmatrix}0.0031 & 0.0064\\ 0.0282 & -0.0373\end{bmatrix},$$

$$Z=\begin{bmatrix}0.3855 & -0.5029\end{bmatrix}, \varepsilon=0.2475.$$

The controller and observer gains are given by

$$K=\begin{bmatrix}0.0282 & 0.0409\\ 0.2039 & -0.2670\end{bmatrix}, L=\begin{bmatrix}2.7344\\ -3.5474\end{bmatrix}.$$

According to Theorem 1, the closed-loop system (10) is robustly robust asymptotically stable. The closed-loop system states, observer states and the estimation error are shown in Fig. 1 and Fig. 2.

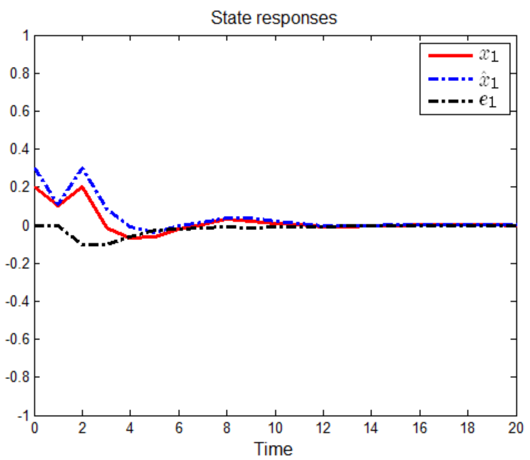

Fig. 1 The state trajectory of $x_1, \hat{x}_1, e_1$

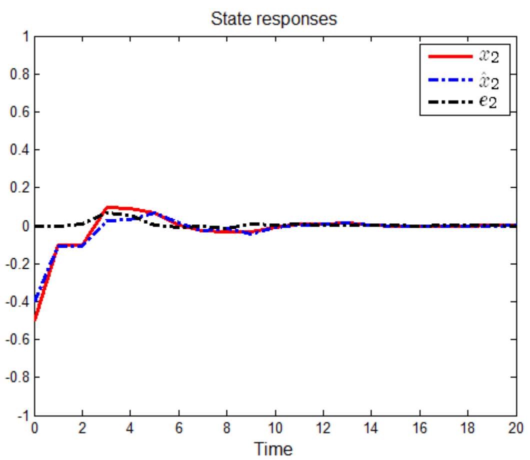

Fig. 2 The state trajectory of $x_2, \hat{x}_2, e_2$

When $\omega(k) \neq 0$, by solving LMIs (29), we have

$$P=\begin{bmatrix}6.4334 & 0.9926\\ 0.9926 & 13.8294\end{bmatrix}, R=\begin{bmatrix}1.7958 & 0.1575\\ 0.1575 & 2.7466\end{bmatrix},$$

$$S=\begin{bmatrix}6.9367 & -1.8063\\ -1.80630 & 16.9223\end{bmatrix}, W=\begin{bmatrix}1.6771 & 0.1342\\ 0.1342 & 4.2847\end{bmatrix},$$

$$X=\begin{bmatrix}4.9288 & -0.8495\\ 0.1616 & 4.6855\end{bmatrix}, Y=\begin{bmatrix}0.1222 & 0.4958\\ 2.1715 & -1.7698\end{bmatrix},$$

$$Z=\begin{bmatrix}6.6404 & -19.1114\end{bmatrix}, \varepsilon=6.7301.$$

The controller and observer gains are given by

$$K=\begin{bmatrix}0.1040 & 0.0353\\ 0.4599 & -0.3789\end{bmatrix}, L=\begin{bmatrix}1.4722\\ -3.8119\end{bmatrix}.$$

According to Theorem 2, the closed-loop system (10) is robustly robust asymptotically stable with H_∞ performance γ. The closed-loop system states, observer states and the estimation error are shown in Fig. 3 and Fig. 4.

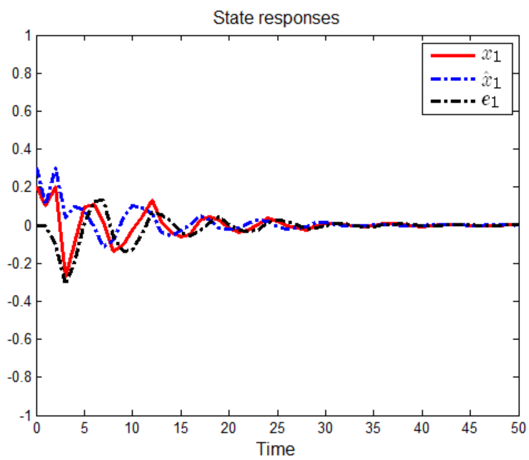

Fig. 3 The state trajectory of $x_1, \hat{x}_1, e_1$

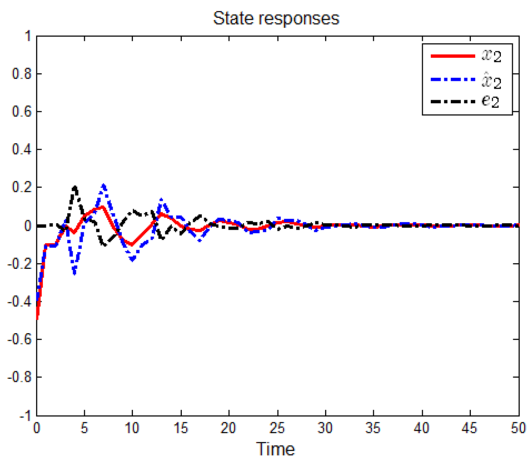

Fig. 4 The state trajectory of $x_2, \hat{x}_2, e_2$

5 Conclusion

The paper studies the robust observer-based control for the discrete-time nonlinear systems with time-varying delays and norm bounded uncertainty. Firstly, observer-based controller is proposed, then according to the linear matrix inequality technique, some sufficient conditions are obtained by building Lyapunov-Krasovskii functionals to guarantee the closed-loop augment system is the robust asymptotically stable with performance γ . Furthermore, the controller and observer gains can be calculated basing on Matlab LMI toolbox. At the end of this paper, an example is given to verify the validity of the stability criterion. The problem of observer-based finite-time control for the discrete-time stochastic singular systems with time-varying delay is very meaningful topic that deserves further exploration.

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