

Design of State-Space Controllers with the Help of Signal Flow Graphs Shown for a Buck Converter

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Abstract: - When consulting the text books on control engineering, state space controllers are described and designed with the help of matrix calculus. This implies good knowledge of linear algebra. Especially in Europe many Universities of Applied Sciences have extremely reduced mathematics in their curricula. Here a teaching concept with the help of signal flow graphs can help to explain and design the controller without solving matrix equations. As example, the bidirectional Buck converter is used. The model of the plant is derived and a simple state space controller is designed. The plant model is linearized around a working point. Therefore, the simple state space controller leads to correct results only at this point. Combination with an additional controller or using the error between the desired value and the actual value as a third state variable improves the quality of the control. The signal flow graphs for these concepts are given and the controllers designed. With the help of LTSpice the designs are checked.

Key-Words: - DC/DC converter, Buck converter, modelling, simulation, state space controller, signal flow graph

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1 Introduction

State space controllers lead to a very efficient control. Perusing the textbooks, the design of these controllers is described with linear algebra and matrix calculus. Furthermore, the design is described for systems of any order. If an example is given, it uses only simple numbers. Especially in Europe, many Universities of Applied Sciences have reduced mathematics extremely in their curricula. Another way of teaching is therefore necessary. Here a way to teach using signal-flow graphs and Mason's equation [1] is described. The use of signal flow graphs is very intuitive and helps the students to a better understanding and the possibility to design a state space controller. In this paper the state space controller concept is applied to a step-down DC/DC converter. An early study of applying the space controller concept to converters can be found in [2]. First the model of the plant is derived and the signal flow graph is drawn. Then the concept of the state space controller is included into the graph of the plant and a simple state space controller designed. The converter is a nonlinear system and the model changes with the working point. Therefore, the control has to be improved. Two concepts, first with an additional PI-controller

and second with the error as additional state variable are explained and designed with the help of the signal flow graphs. A comprehensive treatment of Power Electronics is [5], other valuable textbooks are e.g. [3-6, 9]. It should be mentioned that the here-used methodology can be applied to other DC/DC converters, or also to other plants which are described by a state-space model.

2 Model of the bidirectional Buck converter

The Buck converter with synchronous rectification (an active switch is used instead of the diode) consists of two active switches S1 and S2, an inductor L1 and a capacitor C1 (Fig. 1). A pair of connectors is used for the input voltage and a second pair serves to apply the load. The two active switches (S1 & S2) are controlled in push-pull mode, always one of the switches is turned on (with a small dead-time between).

In [7] the idealized model for the same Buck converter is used to design simple P- and PI-controllers, a compensation controller, and a feedforward controller, and in [8] the two-loop control is treated. The idealized model is derived in

[7], and a model with parasitic resistances is used in [8]. These models are the starting point for our design procedures.

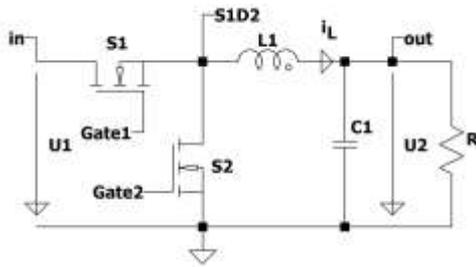


Fig. 1. Buck converter with two active switches.

In the CCM (continuous conduction mode) two modes take place (sometimes also called stages). In mode M1 the active switch S1 is turned on and switch S2 is off, and in mode M2 S1 is off and S2 is on. Including the parasitic resistors of the switches R_{S1} , R_{S2} , of the inductor R_L and of the capacitor R_C a precise model of the converter can be derived according to

$$\frac{d}{dt} \begin{pmatrix} i_L \\ u_C \end{pmatrix} = \begin{bmatrix} -\frac{R_L + R // R_C + R_S}{L} & -\frac{R}{L(R + R_C)} \\ \frac{R}{C(R + R_C)} & -\frac{1}{C(R + R_C)} \end{bmatrix} \begin{pmatrix} i_L \\ u_C \end{pmatrix} + \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix} (u_1) \quad (1)$$

This model is a nonlinear one. Linearization around a working point U_{10} , D_0 , I_{L0} , U_{C0} leads to the small signal model (all variables are written as a combination of the working point value, written as capital letters with the index 0, and the disturbance of the variable, written with small letters with a roof on top)

$$\frac{d}{dt} \begin{pmatrix} \hat{i}_L \\ \hat{u}_C \end{pmatrix} = \begin{bmatrix} -\frac{R_L + R // R_C + R_S}{L} & -\frac{R}{L(R + R_C)} \\ \frac{R}{C(R + R_C)} & -\frac{1}{C(R + R_C)} \end{bmatrix} \begin{pmatrix} \hat{i}_L \\ \hat{u}_C \end{pmatrix} + \begin{bmatrix} \frac{D_0}{L} & \frac{U_{10}}{L} \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \hat{d} \\ \hat{u}_1 \end{pmatrix} \quad (2)$$

The output equation can be calculated according to

$$u_2 = R // R_C \cdot i_L + \frac{R}{R + R_C} u_C \quad (3)$$

For simpler writing we use abbreviations for the matrix elements and the coefficients of the output equation

$$\frac{d}{dt} \begin{pmatrix} \hat{i}_L \\ \hat{u}_C \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{pmatrix} \hat{i}_L \\ \hat{u}_C \end{pmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \hat{d} \\ \hat{u}_1 \end{pmatrix} \quad (4)$$

$$u_2 = C_{11} \cdot i_L + C_{12} u_C \quad (5)$$

For getting the matrix description of the plant we need only the basic equations of electronics: Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL), and the voltage-current equations for the resistor, the inductor, and the capacitor. The matrix is only used to make the description concise. So no knowledge of linear algebra is necessary. (2, 3) are linear equations and can be used for small signal calculations. The complete system can now be drawn as a signal flow graph in the Laplace domain. The graph can be found immediately from (4, 5). For a system of second order we need two integrators $1/s$. On the left side the branches which form the derivative of the equation are connected. The output variable U_2 is achieved according to the output equation with the branches C_{11} and C_{12} .

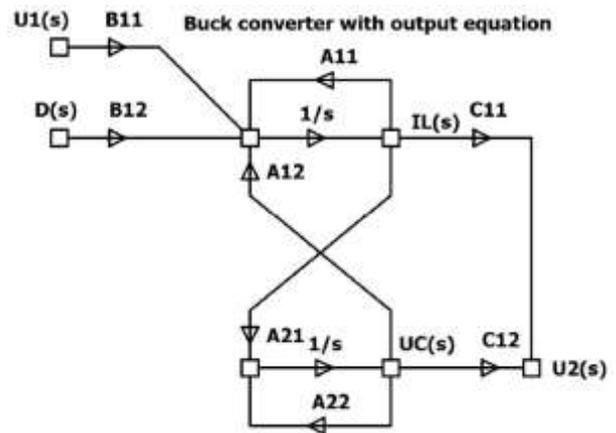


Fig. 3. Signal flow graph of the synchronous Buck converter.

It should be mentioned that all second order power converters have the same signal flow graph as shown in Fig. 3 but different coefficients.

First we calculate the two transfer functions between the current through the coil and the duty ratio and between the capacitor voltage and the duty ratio. We must calculate the forward paths

$$F_I = \frac{B_{12}}{s} \quad F_U = \frac{B_{12} A_{21}}{s^2} \quad (6)$$

and the loops

$$L_1 = \frac{A_{11}}{s}, \quad L_2 = \frac{A_{12} A_{21}}{s^2}, \quad L_3 = \frac{A_{22}}{s} \quad L_1 L_3 = \frac{A_{11} A_{22}}{s^2} \quad (7)$$

With the help of Mason's equation (a short explanation can be found in the appendix)

$$\frac{U_C(s)}{D(s)} = \frac{F_U}{1 - (L_1 + L_2 + L_3) + L_1 L_3} \quad (8)$$

one can calculate the transfer functions between the capacitor voltage, the current through the coil, and

the output voltage in dependence of the duty cycle, respectively according to

$$\frac{U_C(s)}{D(s)} = \frac{A_{21}B_{12}}{s^2 - s(A_{11} + A_{22}) + A_{11}A_{22} - A_{12}A_{21}} \quad (9)$$

$$\frac{I_L(s)}{D(s)} = \frac{B_{12}(s - A_{22})}{s^2 - s(A_{11} + A_{22}) + A_{11}A_{22} - A_{12}A_{21}} \quad (10)$$

$$\frac{U_2(s)}{D(s)} = \frac{B_{12}(C_{11}(s - A_{22}) + A_{21}C_{12})}{s^2 - s(A_{11} + A_{22}) + A_{11}A_{22} - A_{12}A_{21}} \quad (11)$$

In [7] a simplified model of the Buck converter is used. The parasitic resistors are omitted. In this case the plant is a less damped one, so the derived controller will be slower. The large signal model is now described by

$$\frac{d}{dt} \begin{pmatrix} i_L \\ u_C \end{pmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{CR} \end{bmatrix} \begin{pmatrix} i_L \\ u_C \end{pmatrix} + \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix} u_1 \quad (12)$$

and for the linearized model one gets

$$\frac{d}{dt} \begin{pmatrix} \hat{i}_L \\ \hat{u}_C \end{pmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{CR} \end{bmatrix} \begin{pmatrix} \hat{i}_L \\ \hat{u}_C \end{pmatrix} + \begin{bmatrix} \frac{D_0}{L} & \frac{U_{10}}{L} \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \hat{u}_1 \\ \hat{d} \end{pmatrix} \quad (13)$$

For a rough design of the controller this plant model is sufficient. The signal flow graph is now also simpler and shown in Fig. 4.

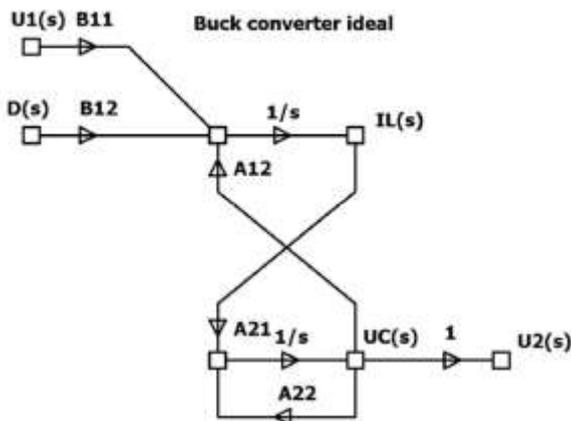


Fig. 4. Signal flow graph of the idealized synchronous Buck converter.

The branches A11 (consists only out of parasitic resistances) and C11 ($R_C \ll R$) are now zero and C12 is equal to one. The transfer functions (9-11) can now accordingly be simplified. For the design of the controller these simplified equations can be used for a first design.

3 Basic state space controller

The input variable of the state spaced controlled Buck converter consists of two P-controllers R1 and R2 which feedback the state variables IL and UC

and a so-called input filter K which scales the reference value (the desired output voltage). The signal flow graph is shown in Fig. 5.

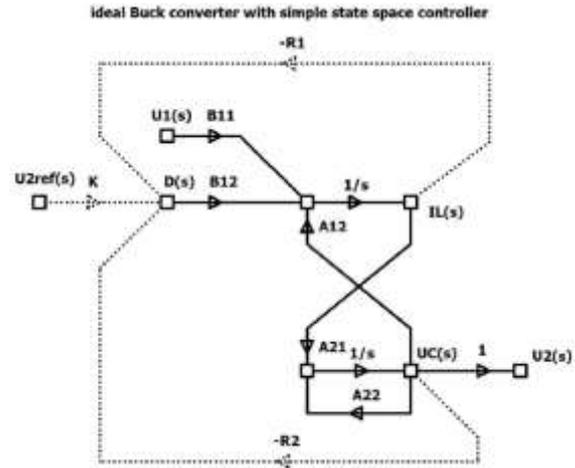


Fig. 5. Simple state space controller for the idealized plant.

When using the Mason equation, one has first to find the forward paths (all the series of branches which are starting at the input (the independent variable in our case U2ref) and end at the output (the dependent variable in our case U2) and the loops. A first-order loop is defined as the product of branches encountered in a round trip, when one moves from one node in the direction of the arrows back to the node where one has started. Furthermore, we have to find all second order loops which consist of the product of any first-order loops which are not touching each other.

We start from the input variable D. There is only one forward path

$$F_1 = \frac{B_{12}A_{21}}{s^2} \quad (14)$$

The first-order loops are

$$L_1 = \frac{A_{12}A_{21}}{s^2}, L_2 = \frac{A_{22}}{s}, L_3 = -\frac{B_{12}R_1}{s}, L_4 = -\frac{B_{12}A_{21}R_2}{s^2} \quad (15)$$

The loops L_2 and L_3 are not touching each other, therefore we have also a second-order loop

$$L_2L_3 = -\frac{A_{22}B_{12}R_1}{s^2} \quad (16)$$

All loops touch the forward path. Applying Mason's rule, the transfer function between the output voltage and the duty cycle can be calculated according to

$$\frac{U_2(s)}{D(s)} = \frac{F_1}{1 - (L_1 + L_2 + L_3 + L_4) + (L_2L_3)} \quad (17)$$

Including the pre-filter K , the transfer function between the output voltage and the reference value can now be written according to

$$\frac{U_2(s)}{U_{2ref}(s)} = \frac{B_{12}A_{21}K}{s^2 + (-A_{22} + B_{12}R_1)s + (B_{12}A_{21}R_2 - A_{12}A_{21} - A_{22}B_{12}R_1)} \quad (18)$$

This transfer function has to be compared with the desired transfer function and by comparing the coefficients the values K , R_1 and R_2 of the state space controller can be found.

One can now choose the poles. A typical choice would be a conjugate complex pole pair

$$s_{1,2} = -\sigma \pm j\omega \quad (19)$$

or a double pole on the negative real axis. The denominator of the desired transfer functions is therefore

$$Den_{complex} = (s + \sigma + j\omega)(s + \sigma - j\omega) = s^2 + 2\sigma \cdot s + \sigma^2 + \omega^2 \quad (20)$$

$$Den_{real} = (s + x)^2 = s^2 + 2x \cdot s + x^2 \quad (21)$$

To avoid a steady state control error, the numerator must have the same value as the constant coefficient of the denominator. The desired transfer functions are therefore

$$G_{complex} = \frac{\sigma^2 + \omega^2}{s^2 + 2\sigma \cdot s + \sigma^2 + \omega^2} \quad (22)$$

$$G_{real} = \frac{x^2}{s^2 + 2x \cdot s + x^2} \quad (23)$$

For the idealized converter one gets for the real double pole

$$K = \frac{x^2}{B_{12}A_{21}} \quad R_1 = \frac{2x + A_{22}}{B_{12}} \quad (24)$$

$$R_2 = \frac{x^2 + A_{22}B_{12}R_1 + A_{12}A_{21}}{B_{12}A_{21}}$$

and for the conjugate pole pair

$$K = \frac{\sigma^2 + \omega^2}{B_{12}A_{21}} \quad (25)$$

$$-A_{22} + B_{12}R_1 = 2\sigma \quad R_1 = \frac{2\sigma + A_{22}}{B_{12}} \quad (26)$$

$$B_{12}A_{21}R_2 - A_{12}A_{21} - A_{22}B_{12}R_1 = \sigma^2 + \omega^2$$

$$R_2 = \frac{\sigma^2 + \omega^2 + A_{22}B_{12}R_1 + A_{12}A_{21}}{B_{12}A_{21}} \quad (27)$$

For the converter with included losses the comparison of the coefficients leads for the real pole pair to

$$K = \frac{x^2}{A_{21}B_{12}C_{12}} \quad (28)$$

$$R_1 = \frac{2x + A_{11} + A_{22}}{B_{12}} \quad (29)$$

$$R_2 = \frac{x^2 + A_{22}B_{12}R_1 + A_{12}A_{21} - A_{11}A_{22}}{B_{12}A_{21}} \quad (30)$$

For the conjugate complex pole pair one gets

$$K = \frac{\sigma^2 + \omega^2}{B_{12}A_{21}C_{12}} \quad (31)$$

$$R_1 = \frac{2\sigma + A_{11} + A_{22}}{B_{12}} \quad (32)$$

$$R_2 = \frac{\sigma^2 + \omega^2 + A_{22}B_{12}R_1 + A_{12}A_{21} - A_{11}A_{22}}{B_{12}A_{21}} \quad (33)$$

The simulation is done by implementing (Fig. 6) the model of the converter according the nonlinear model (12). The integration is done with the help of the voltage controlled voltage sources E1, E3. The derivative of the state variables is realized with the arbitrary voltage sources B1, B2. The arbitrary voltage source B3 calculates the duty cycle from the state variables and the reference value. The input voltage u_1 and the reference value U_{ref} are given by the voltage sources V2 and V1, respectively.

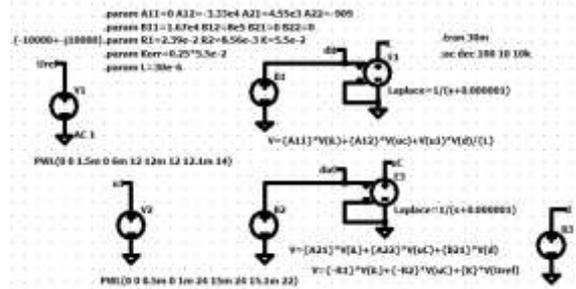


Fig. 6. Simulation of the simple state space controller.

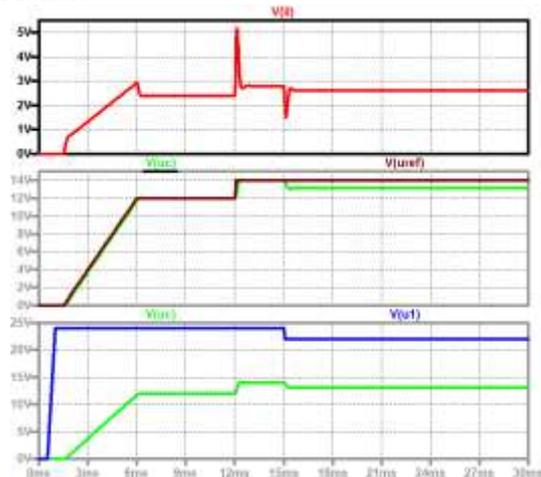


Fig. 7. Simple state space controller with ideal Buck with poles at $s_{1,2} = -1000 \pm j1000$: current through the coil (red); reference value (black), voltage across the capacitor (green); input voltage (blue), capacitor voltage (green).

Fig. 7 show the current through the coil, the voltage across the capacitor, the reference value, and the

input voltage. The controller was designed for the linearized system; the model of the converter in the simulation, however, is the nonlinear one. Therefore, a steady-state error occurs when the input voltage is changed (at 15 ms).

To achieve a Bode plot (Fig. 9) of the controlled system one can use a simulation according to Fig. 8. The desired closed loop transfer function (23) is calculated with the voltage controlled voltage source E1.

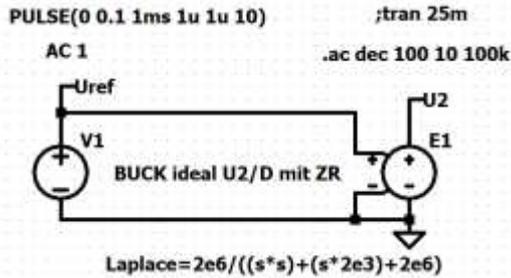


Fig. 8. Simulation circuit for generating the closed loop transfer function.

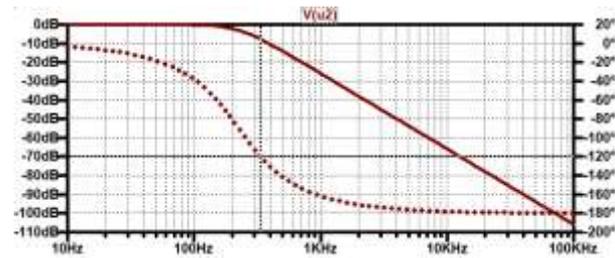


Fig. 9. Closed loop transfer function.

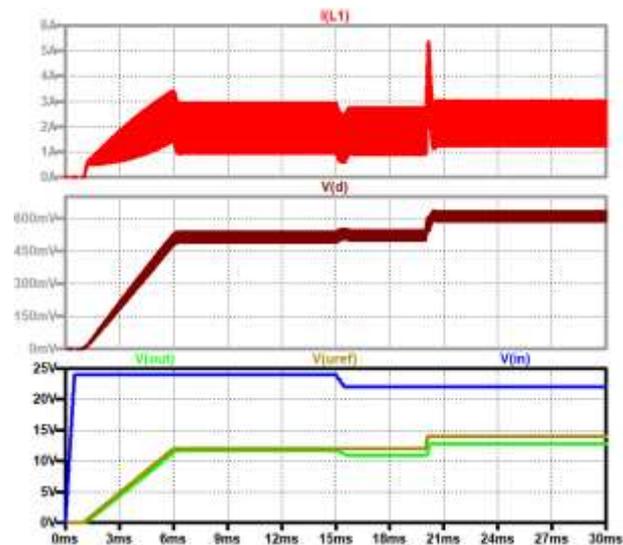


Fig. 10. Circuit simulated Buck converter with simple state space controller, up to down: current through the coil (red); duty cycle (black); input voltage (blue), reference value (brown), output voltage (green).

With the program LTSpice one can implement the state space controller into the circuit simulation. The used simulation circuit is shown in the appendix.

Using a circuit simulation of the Buck converter gives a very good possibility to prove the controller, not only around the working point, but in a “real” surrounding. In Fig. 10 one can see the soft-start by increasing the reference value and a reference value step and a step of the input voltage. The current through the coil and the duty cycle are also depicted. Because of the nonlinearity of the converter a steady-state error occurs especially after the input voltage and the reference value steps.

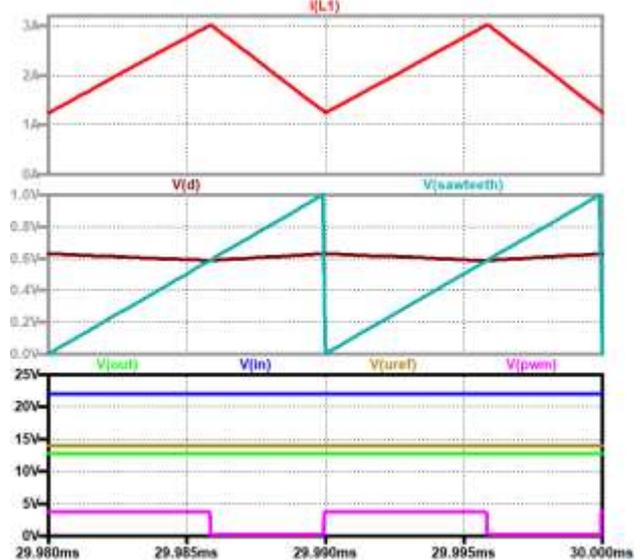


Fig. 11. Circuit simulated Buck converter with simple state space controller, up to down: current through the coil (red); duty cycle (black), sawtooth for the pwm-generator (turquoise); input voltage (blue), reference value (brown), output voltage (green), pwm-signal (violet).

Fig. 11 shows the steady-state signals: the current through the coil, the analog duty cycle, and the sawtooth of the pwm-modulator. In the picture at the bottom the input voltage, the reference value, the output voltage, and the output of the pwm-modulator are depicted. The analog duty cycle signal changes with the current of the coil. With a comparator this signal is compared with a saw-teeth signal to generate the pulse width signal to control the two active switches of the converter. When the analog duty cycle signal is lower than the saw-teeth, the comparator output signal is high, otherwise it is low. The steady state error of the output voltage can be seen clearly.

4 Improvements of the state space controller

The state space controller only works precisely, when the system is linear. When the controller is designed for a linearized plant (as in the case of the Buck converter which is a nonlinear plant), it works precisely only at the working point where the linearization was done. To improve the control, we have two possibilities. First we control the state-spaced controlled plant by an additional controller with an integral part, or second another state variable representing the error is included and a state space controller with increased order is designed.

4.1 Improved state space controller type 1

The concept is shown with the help of the signal flow graph (Fig. 12). Instead of the pre-filter, a simple linear controller is used. To avoid a steady-state error, we choose a PI-controller.

Ideal Buck converter with simple state space controller and overlaid standard controller

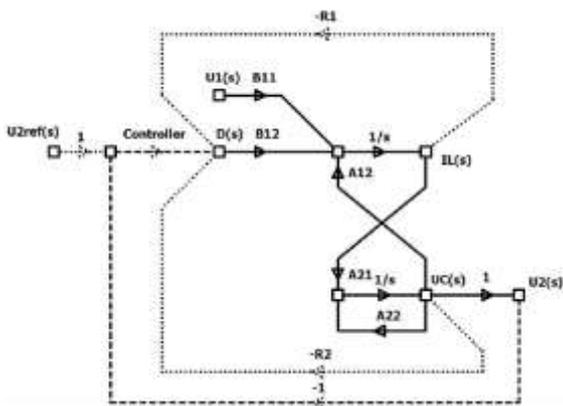


Fig. 12. Simple state space controller with additional linear controller.

With the transfer function of the PI-controller in Bode nominal form

$$R(s) = \frac{K(sT + 1)}{s} \quad (34)$$

and choosing the zero of the controller equal to the real part of the poles of the plant, the open loop system can be written according to

$$G_O = K \frac{(s0.001+1)}{s} \frac{2e6}{s^2 + 2e3 + 2e6} \quad (35)$$

Now one can draw a Bode plot for the open loop system with K=1 (Fig. 13).

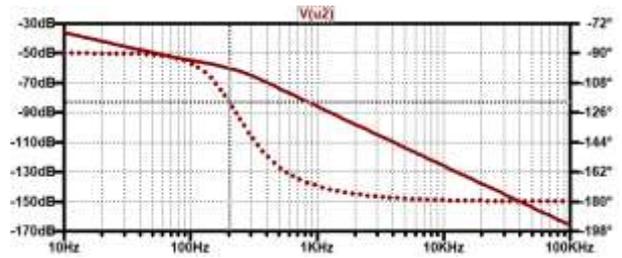


Fig. 13. Bode plot of the simple state space controlled system with additional PI-controller.

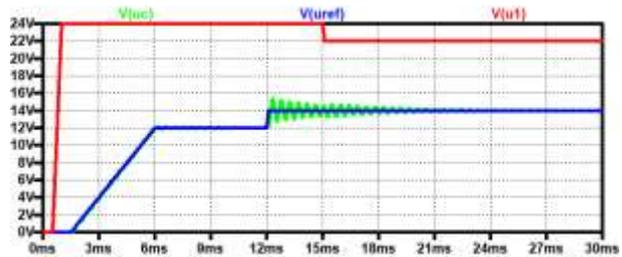


Fig. 14. State space controlled Buck converter with additional PI-controller, controller gain 1000, input voltage (red), reference value (blue), capacitor voltage (green).

For a phase margin of 60 degrees the gain is -60 dB. Therefore, the controller gain can be set at 1000. The start-up, a reference value step, and an input voltage step are shown in Fig. 14. The additional PI-controller leads to an additional ringing at the reference value step, but it compensates the input voltage step! Reducing the controller gain (Fig. 15) avoids the ringing and the control speed is still satisfactory.

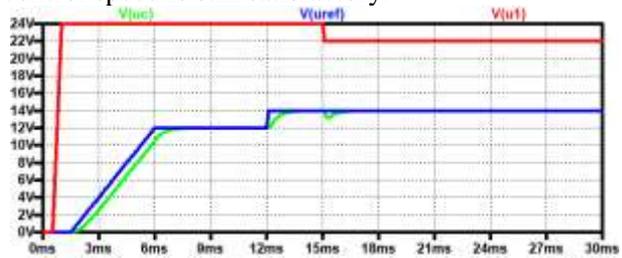


Fig. 15. State space controlled Buck converter with additional PI-controller, controller gain 100, input voltage (red), reference value (blue), capacitor voltage (green).

The simulations in Figs. 14, 15 were done with using integrators to calculate the nonlinear model of the converter. Fig. 16 shows the results of the simple state space controlled system with additional PI-controller. The plant is circuit-simulated as synchronous rectified Buck converter.

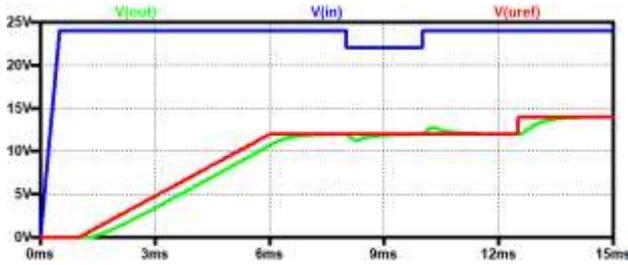


Fig. 16. State space controlled Buck converter (modelled as a circuit) with additional PI-controller, controller gain 100: input voltage (blue), reference value (brown), output voltage (green).

The improvement of the state space controller by an additional controller leads to a very satisfying result. With the state space controller, the poles of the system to be controlled are committed to a desired value. This system is subsequently controlled by an additional normal linear controller. The design can easily be done with the help of a Bode diagram of the open loop with a controller gain of one. Choosing a phase margin and shifting the open loop diagram, so that at a phase shift of -120° the gain is 1 (0 dB), leads to a fast and stable controller. To avoid ringing one has to reduce the gain. Optimizing can be done easily using some simulations. By including the controllers into a circuit simulation of the converter one gets the final check of the design.

4.2 Improved state space controller type 2

A second method to improve the state space controller is to add the integral of the error between the output variable and the reference value as an additional state variable. This concept is also called extended state space controller. Fig. 17 shows the signal flow graph of this concept.

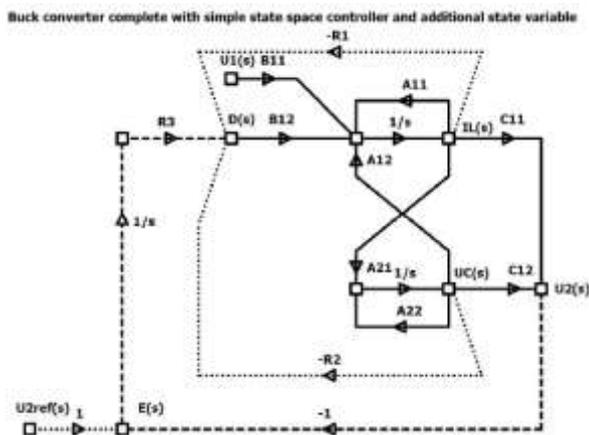


Fig. 17. Signal flow graph of the extended state space controller.

We find two forward paths between the output node U2 and the input node U2ref

$$F_1 = \frac{R_3 B_{12} C_{11}}{s^2}, \quad F_2 = \frac{R_3 B_{12} A_{21} C_{12}}{s^3}.$$

(36)

We find seven first order loops

$$L_1 = \frac{A_{11}}{s}, \quad L_2 = \frac{A_{12} A_{21}}{s^2}, \quad L_3 = \frac{A_{22}}{s}, \quad L_4 = -\frac{B_{12} R_1}{s}$$

$$L_5 = -\frac{B_{12} A_{21} R_2}{s^2}, \quad L_6 = -\frac{R_3 B_{12} C_{11}}{s^2},$$

$$L_7 = -\frac{R_3 B_{12} A_{21} C_{12}}{s^3}. \quad (37)$$

The loops L_3 and L_4 , the loops L_1 and L_3 and the loops L_3 and L_6 do not touch each other, therefore we have also three second-order loops

$$L_1 L_3 = \frac{A_{11} A_{22}}{s^2}, \quad L_3 L_4 = -\frac{A_{22} B_{12} R_1}{s^2}$$

$$L_3 L_6 = -\frac{A_{22} B_{12} C_{11} R_3}{s^3}. \quad (38)$$

The loop L_3 does not touch the forward path F_1 , but all loops touch the second forward path F_2 . The transfer function can now be calculated according to

$$\frac{U_2(s)}{D(s)} = \frac{F_1(1-L_3)+F_2}{1-(L_1+L_2+L_3+L_4+L_5+L_6+L_7)+\frac{(L_1 L_3+L_3 L_4+L_3 L_6)}{s^3}}. \quad (39)$$

$$\frac{U_2(s)}{D(s)} = \frac{B_{12} R_3 [(C_{11})s + (-A_{22} C_{11} + A_{21} C_{12})]}{s^3 + (-A_{11} - A_{22} + B_{12} R_1)s^2 + \left[\begin{array}{l} -A_{12} A_{21} + B_{12} A_{21} R_2 + \\ R_3 B_{12} C_{11} + A_{22} (A_{11} - B_{12} R_1) \end{array} \right] s + R_3 (B_{12} A_{21} C_{12} - A_{22} B_{12} C_{11})}. \quad (40)$$

Now one has to choose the poles of the closed loop system. This can be done arbitrarily, but one has to have in mind that a too fast control leads to high signal values which may destroy the circuit. It is ingenious to choose a triple pole on the real axis, or one real pole and a conjugate complex pole pair. A triple pole at minus x leads to a desired denominator of

$$Den3 = (s+x)^3 = s^3 + 3x \cdot s^2 + 3x^2 \cdot s + x^3. \quad (41)$$

We can also use one real pole and a conjugate complex pole-pair. This leads to the denominator

$$Dencomplex3 = (s^2 + 2\sigma \cdot s + \sigma^2 + \omega^2)(s+x) = s^3 + (2\sigma+x)s^2 + (\sigma^2 + \omega^2 + 2\sigma x)s + (\sigma^2 + \omega^2)x. \quad (42)$$

The desired transfer function with zero steady state error is therefore

$$G_3(s) = \frac{x^3}{s^3 + 3x \cdot s^2 + 3x^2 \cdot s + x^3} \quad (43)$$

$$G_{3comp} = \frac{(\sigma^2 + \omega^2)x}{s^3 + (2\sigma+x)s^2 + (\sigma^2 + \omega^2 + 2\sigma x)s + (\sigma^2 + \omega^2)x}. \quad (44)$$

4.2.1 Ideal plant with triple pole at minus x

To achieve the controller values, the desired transfer function (43) and the transfer function of the system (40) must be compared. A_{11} and C_{11} are very small, so they can be omitted ($A_{11}=0$, $C_{11}=0$). The transfer function can be reduced to

$$\frac{U_2(s)}{D(s)} = \frac{A_{21}B_{12}C_{12}R_3}{s^3 + (-A_{22} + B_{12}R_1)s^2 + \begin{bmatrix} -A_{12}A_{21} + B_{12}A_{21}R_2 \\ + A_{22}(-B_{12}R_1) \end{bmatrix} s + A_{21}B_{12}C_{12}R_3} \quad (45)$$

Comparison of the coefficients leads to

$$-A_{22} + B_{12}R_1 = 3x \quad R_1 = \frac{3x + A_{22}}{B_{12}} \quad (46)$$

$$-A_{12}A_{21} + B_{12}A_{21}R_2 + A_{22}(-B_{12}R_1) = 3x^2$$

$$R_2 = \frac{3x^2 + A_{12}A_{21} + A_{22}B_{12}R_1}{B_{12}A_{21}} \quad (47)$$

$$A_{21}B_{12}C_{12}R_3 = x^3 \quad R_3 = \frac{x^3}{A_{21}B_{12}C_{12}} \quad (48)$$

4.2.2 Ideal controller with a complex pole pair and a real pole

The complex pole pair is chosen at $-\sigma \pm j\omega$ and the real pole at $-x$. To get the controller parameters the transfer functions (44) and (45) have to be compared. This leads to

$$-A_{22} + B_{12}R_1 = 2\sigma + x \quad R_1 = \frac{2\sigma + x + A_{22}}{B_{12}} \quad (49)$$

$$-A_{12}A_{21} + B_{12}A_{21}R_2 - A_{22}B_{12}R_1 = \sigma^2 + \omega^2 + 2\sigma x$$

$$R_2 = \frac{\sigma^2 + \omega^2 + 2\sigma x + A_{12}A_{21} + A_{22}B_{12}R_1}{B_{12}A_{21}} \quad (50)$$

$$A_{21}B_{12}C_{12}R_3 = \sigma^2 + \omega^2 \quad R_3 = \frac{\sigma^2 + \omega^2}{A_{21}B_{12}C_{12}} \quad (51)$$

The second concept (4.2) is equivalent to (4.1) when a simple I-controller is taken. The I-controller leads to a slower system. Therefore, it is better to use concept (4.1).

5 Conclusion

The design of state controllers for a second order system without using linear algebra and matrix operation was shown for a step-down DC/DC converter. Now only some knowledge about the signal flow graph and the equation of Mason is necessary. The system has to be described in state-space form which is very common and does not need knowledge about matrix calculus. The design equations for the controller follow immediately from the signal flow graph. The Buck converter used as an example is a nonlinear system, therefore the simple state space controller works only correctly for the chosen working point. Two methods to overcome this limitation are shown. The better way is to use an additional linear controller e.g. a PI-controller. The second concept to include the error as a third state variable is also possible and is similar, when in the first concept an I-controller is used instead of a PI-controller. The I-controller leads however to slower control and is therefore not so useful. The free simulation tool LTSpice helps in the design and makes possible to check the system with a circuit simulation of the plant.

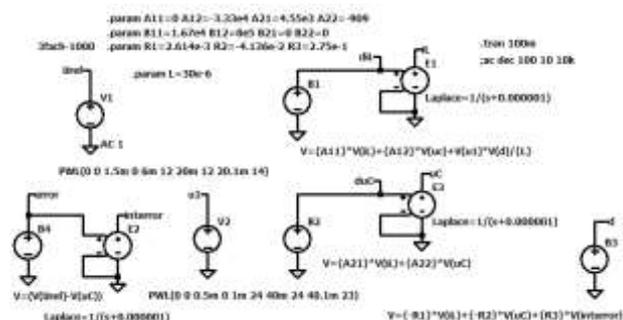


Fig. 18. Simulation circuit: idealized plant of the synchronous Buck converter with extended state space controller with triple pole at -1000.

Fig. 18 shows the simulation program. The converter is modelled with its state space description.

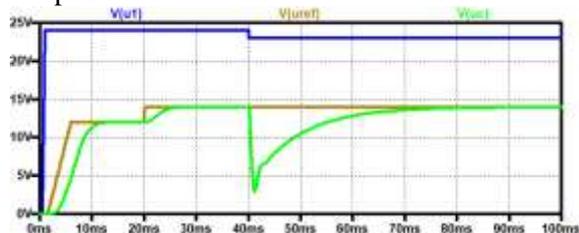


Fig. 19. State space controlled Buck converter (modelled as a non-linear system) with extended state space controller with triple pole at -1000: input voltage (blue), reference value (brown), output voltage (green).

In Fig. 19 the start-up, a reference value step, and an input voltage step for the Buck with extended state control is shown. The system is slower and the reaction to the input voltage change is much more pronounced.

References:

- [1] G. Gonzalez, *Microwave Transistor Amplifiers*, Prentice-Hall, 1984.
- [2] F. A. Himmelstoss, and F. C. Zach, State Space Control for Switched Mode Power Supplies, *Proceedings of the International Power Electronics Conference IPEC90-93*, Tokyo, April 2-6, 1990, pp.1157-1164.
- [3] F. Zach, in German: *Leistungselektronik*, Springer, 6th ed., 2022.

- [4] N. Mohan, T. Undeland and W. Robbins, *Power Electronics, Converters, Applications and Design*, 3rd ed. New York: W. P. John Wiley & Sons, 2003
- [5] R.W. Erickson & D. Maksimovic, *Fundamentals on Power Electronics*, Springer 2020.
- [6] Y. Rozanov, S. Rylvkin, E. Chaplygin, P. Voronin, *Power Electronics Basics*, CRC Press, 2016.
- [7] F. A. Himmelstoss, Controller design of a Buck converter with the help of LTSpice, International Asian Congress on Contemporary Sciences-VI, 2022, pp.195-202, ISBN-978-625-8323-27-6.
- [8] F. A. Himmelstoss, Cascaded control of a Buck converter designed and simulated with the help of LTSpice, 6th International European Congress on Interdisciplinary Scientific Research, Bucharest, pp. 1086-1096, ISBN: 978-625-8213-38-6.
- [9] R. H. Bishop (editor), *The Mechatronics Handbook*, CRC Press, 2008.

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Appendix

A1. Signal flow graphs - a short summery

The signals are nodes and over the branches which connect the nodes the transfer coefficient or function are written. We distinguish between forward paths and loops. To get the value of a forward path (between an input node and an output node), one has to multiply the values of the connecting branches. A loop starts at a node and ends at the same node. The value of the loop is the multiplication of the values of all branches which form the loop. Second-order loops are formed by the product of two loops which are not touching each other. Mason’s equation enables us to find transfer functions. The denominator is 1 minus the sum of

all first-order loops plus the sum of all second-order loops minus etc. The numerator is the sum of all forward paths multiplied by the denominator, where all loops which are touching the forward path are deleted.

A2. Circuit simulation

Fig. A shows the used simulation circuit for the simple state space controller. The synchronous rectified Buck converter is modelled with the active switches (MOSFETs) S1 and S2, the inductor L1, and the capacitor C1. The input voltage is built with the voltage source V1 and the load by the resistor R1. The electronic switches are controlled by the voltage controlled voltage sources E1 and E2. E1 is necessary, because S1 is a high-side switch, S2 is a low-side switch, therefore no floating driver is necessary, but E2 is used to realize the same propagation delay. The dead-time of the switches is realized with the capacitors C2, C3, the resistors R2 and R3, the diodes D2 and D3, and the AND-gates A1 and A2 (which can also be used for connecting an enable signal). S1 is controlled by the pwm output of the comparator U1, and the second switch S2 by the inverted output pwm_q. The pwm-modulation is achieved by the saw-teeth generator V2 and the output of the arbitrary voltage source B1, which calculates the simple state space controller. With the voltage source V5 the desired value Uref is produced. The comparator U1 is double side supplied with the voltage sources V3 and V4. Only 5 V is allowed for this device. The voltage source V6 with the value zero in series to the inductor L1, shows the current measurement device. This current controls the current-controlled voltage source H1 which is used in the state space controller B1.

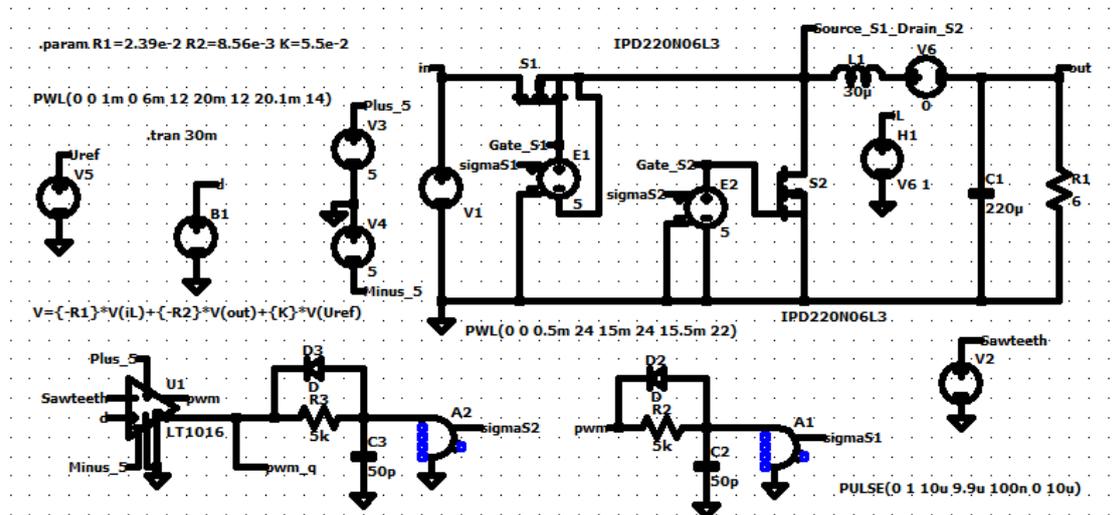


Fig. A. Circuit orientated simulation of the synchronous Buck converter with simple state space controller.