Approximating the ARL to Monitor Small Shifts in the Mean of an AR Fractionally Integrated with an exogenous variable Process Running on an EWMAControl Chart

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Abstract: - Control charts are used to monitor processes and detect changes in a given control scheme. The Exponential Weighted Moving Average (EWMA) control chart is a well-recognized control chart used to detect small changes in parameters. The efficiency of the chart studied is usually achieved using ARL. Approximating ARL using the Gauss-Legendre quadrature method, also known as NIE,. This approach is used to evaluate the ARL of developments, such as explicit formulas because it provides a robust way to validate their validity and accuracy. Moreover, it evaluates the performance of control charts for time series under exponential white noise. Exponential white noise is obtained from a long-memory fractionally integrated AR with exogenous variables or the long-memory ARFIX process. Under the long-memory ARFIX model, the proposed technique compares the control chart's performance to an explicit formula using the criterion of percentage accuracy. The results of the comprehensive numerical study include investigations into a wide range of out-of-control processes and situations. Specifically, the results from the accuracy percentage in all cases are more than 95%, which means that the proposed technique is accurate and completely consistent with the well-defined explicit formula. Therefore, it is recommended that it be used in this situation. There are examples from real data that were found to be consistent with the research results.

Key-Words: - exponentially weighted moving average (EWMA) control chart, long-memory, fractionally integrated autoregressive process with an exogenous variable process, Gauss-Legendre quadrature, explicit formulas, exponential white noise.

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1 Introduction

Control charts are statistical tools used to monitor a process and indicate when it goes out of control. Since the introduction of the Shewhart control chart, [1], it has become common practice to make use of control charts to monitor changes that can occur in various manufacturing and production processes, [2]. The Shewhart control charts are frequently referred to as memoryless because information from the past is not utilized in its derivation, and thus it is only suitable for monitoring large process parameter shift sizes (i.e., location and/or dispersion). On the other hand, memory control charts are extremely useful for monitoring small-to-moderate changes in a process parameter. An example of this is the exponentially weighted moving average (EWMA) control chart, [3], which is of interest in the present study.

Real-world scenarios often contain serially correlated data in the underlying observations. One technique to deal with autocorrelation in the observations of a process running on a control chart is to examine the correlation between subsequent data points. A comprehensive elucidation of long-memory processes is presented in [4], [5]. According to [6], long-memory processes require differencing parameter d in an autoregressive fractionally integrated moving average order (p, d, q), abbreviated as (ARFIMA(p, d, q) process to fall within the range of 0 to 0.5. In addition to the primary time series data, there can be auxiliary or exogenous variables that are either already accessible or can be easily obtained. These variables can be significantly correlated with the primary time series. As stated by [7], the inclusion of these exogenous variables in time series models enhances their performance and improves the forecasting accuracy. The fractionally integrated AR model with an exogenous variable (ARFIX) is of special interest in the present study.

In a time series model, the difference between the actual and estimated values is referred to as the error (also known as white noise). This should be minimized to achieve the highest possible[level of accuracy with the model. The white noise produced by autocorrelated data following a normal distribution is commonly referred to as Gaussian white noise, [8]. Nevertheless, many phenomena such as wind speed, oxygen content, and water flow rate have been studied using non-Gaussian white noise, with exponentially distributed white noise being particularly intriguing, [9].

The effectiveness of a control chart is frequently evaluated utilizing the average run length (ARL) to determine the expected number of observations before a control chart signals a change in the monitored process. The ARL consists of two parts: ARL_0 and ARL_1. Separate ARL—ARL_0, which is the in-control state, and ARL—ARL_1, which is the out-of-control state. Ideally, ARL_0 should be as large as possible, while ARL_1 should be as small as possible. In the study of techniques for evaluating the performance of control charts in terms of ARL. These include the Monte Carlo simulation method, [10], the Markov chain approach, [11], the explicit formulas put forth by [12], [13], [14], [15], and the numerical integration equation (NIE) technique, which has received support from [16], [17]. The NIE technique is considered an efficient technique for the computation of ARL. Since it is a computation technique to get accurate and precise results for research, several rules have been studied, including the trapezoidal rule, the Simpsons rule, the midpoint rule, and the quadrature rule. This article focuses on quadrature rules that incorporate weighting and interpolation. This rule is proposed as a technique that was chosen from integration for approximating ARL. Moreover, it has been devised by many researchers to expand in many different fields.

The author in [18], developed a numerical technique for approximating the ARL of processes running on an EWMA control chart. In [19], the author resolved an integral equation to determine the ARL of a process running on a cumulative sum (CUSUM) chart while in the in-control process stage. Numerous studies have been devoted to assessing the performance of the NIE technique in various scenarios, including detecting changes to the mean of an autocorrelation process. In the present research, we utilized the NIE technique to approximate the ARL through an integral equation using the Gauss-Legendre quadrature for a long-memory ARFIX running on an EWMA control chart.

The remainder of the paper is organized as follows. Section 2 provides brief derivations of a long-memory ARFIX(p, d, k) process with underlying exponential white noise and an EWMA control chart. The numerical approximation of the integral equation utilizing the NIE method through the application of the Gauss-Legendre rule technique is presented in Section 3. The numerical results for the NIE technique and explicit formula are compared in Section 4. To illustrate the efficacy of the proposed technique, an example process involving real data is also provided in Section 5. Finally, conclusions and future recommendations are offered in Section 6.

2 The Long-Memory ARFIX(*p, d, k*) Model With Underlying Exponential White Noise and the EWMA Control Chart

The following subsections provide brief derivations of the EWMA chart and the model of interest.

2.1 The ARFIX(*p, d, k*) Model

Let $Y_t;\ t = 1, 2, ...$ be a sequence of fractionally integrated AR models with exogenous variables of order (p, d, k), where p is the order of the AR process, d is the fractional integration parameter, and k is the exogenous variable in the model. The $\text{ARFIX}(p, d, k)$ model with exponential white noise can be written as

$$(1-\sum_{i=1}^{p}\phi_i B^i)(1-B)^d Y_t = \sum_{j=1}^{k}\beta_j X_{jt} + \varepsilon_t, \tag{1}$$

where ϕ_i is the *i-th* AR coefficient, β_j the *j-th* coefficients corresponding to k, ε_t is a white noise process assumed to follow exponential distribution $\varepsilon_t \sim Exp(\alpha)$ when shift parameter $\alpha > 0$, and $(1- B)^d$ is the fractional differencing operator in which B is the backward-shift operator and d is the degree of the differencing parameter. Since the focus of the current investigation is on long-term memory processes, d was limited to between 0 and 0.5.

Operator $(1-B)^d$ can be expanded using a binomial series expansion in the following manner:

$$(1-B)^d = \sum_{r=0}^{\infty}\binom{d}{r}(-B)^r$$
$$=1-dB-\frac{1}{2}d(1-d)B^2-\frac{1}{6}d(1-d)(2-d)B^3-\dots, \quad (2)$$

For any real value of d, the fractionally integrated white noise process $(1-B)^d Y_t=\varepsilon_t$, can be defined as

$$Y_t-dY_{t-1}-\frac{1}{2}d(1-d)Y_{t-2}-\frac{1}{6}d(1-d)(2-d)Y_{t-3}-\dots=\varepsilon_t, \quad (3)$$

Note that $B^r Y_t = Y_{t-r}$ for order r.

Equation (1) can be reformulated to solve for Y_t in the generalized long-memory ARFIX(p, d, k) process running on an EWMA control chart as follows:

$$Y_t=(1-\sum_{i=1}^{p}\phi_i B^i)^{-1}(1-B)^{-d}\sum_{j=1}^{k}\beta_j X_{jt}+\varepsilon_t, \text{ or}$$
$$\begin{aligned} Y_t = {} & \phi_1 Y_{t-1}+\phi_2 Y_{t-2}+\dots+\phi_p Y_{t-p} \\ & +d(Y_{t-1}-\phi_1 Y_{t-2}-\phi_2 Y_{t-3}-\dots-\phi_p Y_{t-p-1}) \\ & +\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1 Y_{t-3}+\phi_2 Y_{t-4}+\dots+\phi_p Y_{t-p-2}) \\ & +\dots+\beta_1 X_{1t}+\beta_2 X_{2t}+\dots+\beta_k X_{kt}+\varepsilon_t \end{aligned} \quad (4)$$

where the initial value of $Y_{t-1}, Y_{t-2},\dots,Y_{t-p},Y_{t-(p+1)},\dots,$ $X_{1t},X_{2t},\dots,X_{kt}$ are equal to 1 and the initial value of exponential white noise $\varepsilon_t=1$.

2.2 The EWMA Control Chart

The EWMA control chart is highly effective for rapidly detecting small changes in a process parameter by appropriately assigning weights to both the current and previous observations. According to M_t, the plotting statistic for the EWMA control chart is defined in the following form:

$$M_t=(1-\lambda)M_{t-1}+\lambda Y_t, \text{ for } t=1,2,\dots,$$

where Y_t is the sequence of the long-memory ARFIX(p, d, k) process with underlying exponential white noise, M_t represents the MA at time t, M_{t-1} represents the past values, and M_0 represents the initial value. If the process parameters are known, the target or in-control mean (Y_0) is assumed to be M_0 while the smoothing parameter (or weighting parameter) λ is constrained by $0<\lambda\le 1$.

Note that although the value of λ can range from zero to one inclusively, it is typically selected to be between 0.01 and 0.05 because the EWMA control chart is designed to detect small changes in a process parameter. When $\lambda=1$, it becomes the same as the Shewhart chart. In addition, the smoothing parameter has an inverse relationship with the chart's sensitivity to slight shifts.

Assuming that observations Y_t are independent random variables with mean (μ_0) and variance (σ^2), the respective mean and variance of the EWMA statistic for the in-control process are $E(M_t)=\mu_0$ and

$$V(M_t)=\sigma^2(\lambda/(2-\lambda))\left[1-(1-\lambda)^{2t}\right]. \quad (5)$$

The upper control limit (UCL) and lower control limit (LCL) of the EWMA chart can be derived as

$$(UCL,\ LCL)=\mu_0\pm L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2t}\right]}, \quad (6)$$

where constant L determines the control limits' width, the value of which is determined by the desired in-control ARL (ARL_0). For sufficiently large values of t, the control limits can be expressed as

$$(UCL,\ LCL)=\mu_0\pm L\sigma\sqrt{\frac{\lambda}{2-\lambda}}, \quad (7)$$

where L is the coefficient of the control chart for a predetermined rate of false alarms. EWMA statistic M_t is plotted to fall between the UCL and the LCL when the process is in control. On the contrary, the process is considered to be out of control when M_t is less than the LCL or more than the UCL.

The long-memory ARFIX(p, d, k) process in Equation (4) can be replaced with Equation (5). Consequently, the EWMA statistic can be expressed as

$$\begin{aligned} M_t = {} & (1-\lambda)M_{t-1}+\lambda\big(\phi_1 Y_{t-1}+\phi_2 Y_{t-2}+\dots+\phi_p Y_{t-p} \\ & +d(Y_{t-1}-\phi_1 Y_{t-2}-\phi_2 Y_{t-3}-\dots-\phi_p Y_{t-p-1}) \\ & +\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1 Y_{t-3}+\dots+\phi_p Y_{t-p-2}) \\ & +\dots+\beta_1 X_{1t}+\beta_2 X_{2t}+\dots+\beta_k X_{kt}+\varepsilon_t\big) \end{aligned} \quad (5)$$

$$
\begin{aligned}
&= (1-\lambda)M_{t-1} + \lambda\phi_1 Y_{t-1} + \lambda\phi_2 Y_{t-2} + \ldots + \lambda\phi_p Y_{t-p} \\
&\quad + \lambda d(Y_{t-1} - \phi_1 Y_{t-2} - \phi_2 Y_{t-3} - \ldots - \phi_p Y_{t-p-1}) \\
&\quad + \lambda \frac{1}{2} d(d-1)(-Y_{t-2} + \phi_1 Y_{t-3} + \ldots + \phi_p Y_{t-p-2}) \\
&\quad + \ldots + \lambda\beta_1 X_{1t} + \lambda\beta_2 X_{2t} + \ldots + \lambda\beta_k X_{kt} + \lambda\varepsilon_t) \qquad (8)
\end{aligned}
$$

Hence, the stopping time for the EWMA control chart (τ_b) can be written as

$$\tau_b = \inf\{t > 0;\, M_t > b\}, \text{for } \psi \le b, \qquad (9)$$

where b is a constant equivalent to the UCL.

Assume that the process is in control at time t if the EWMA statistics M_t is in range $0 < M_t < b$ and the process is out-of-control if $M_t > b.$ Let us establish that the lower limit is $L = 0,$ and the upper limit is $U = b.$ Concerning the EWMA statistics M_1 while the process is in an in-control state:

$$
\begin{aligned}
0 &< (1-\lambda)M_{t-1} + \lambda\phi_1 Y_{t-1} + \lambda\phi_2 Y_{t-2} + \ldots + \lambda\phi_p Y_{t-p} \\
&\quad + \lambda d(Y_{t-1} - \phi_1 Y_{t-2} - \phi_2 Y_{t-3} - \ldots - \phi_p Y_{t-p-1}) \\
&\quad + \lambda \frac{1}{2} d(d-1)(-Y_{t-2} + \phi_1 Y_{t-3} + \ldots + \phi_p Y_{t-p-2}) \\
&\quad + \ldots + \lambda\beta_1 X_1 + \lambda\beta_2 X_2 + \ldots + \lambda\beta_r X_r + \lambda\varepsilon_1) < b
\end{aligned}
$$

Let $\Upsilon(\varphi)$ denote the ARL to monitor small shifts in the mean of the long-memory ARFIX(p, d, k) process with an initial value $(M_0 = \varphi)$. Now, we define the function $\Upsilon(\varphi)$ as follows:

$$\text{ARL} = \Upsilon(\varphi) = E_\infty(\tau_b) \ge \gamma,\ 0 < \varphi < b. \qquad (10)$$

where $E_\infty(\tau_b)$ is the expectation under the density function $f(\varepsilon_t, \alpha).$

Let $L = \Big((1-\lambda)\varphi - \lambda\big(\phi_1 Y_0 + \ldots + \phi_p Y_{t-p} + d(Y_0 - \ldots - \phi_p Y_{t-p-1}) + \frac{1}{2} d(d-1)(-Y_{t-2} + \ldots + \phi_p Y_{t-p-2}) + \ldots + \beta_1 X_1 + \ldots + \beta_k X_k\big)\Big)/\lambda$

$$U = \Big(b - (1-\lambda)\varphi - \lambda\big(\phi_1 Y_0 + \ldots + \phi_p Y_{t-p} + d(Y_0 - \ldots - \phi_p Y_{t-p-1}) + \frac{1}{2} d(d-1)(-Y_{t-2} + \ldots + \phi_p Y_{t-p-2}) + \ldots + \beta_1 X_1 + \ldots + \beta_k X_k\big)\Big)/\lambda$$

For a probability distribution function $\varepsilon_1,$, the probability that $f(\varepsilon_1)$ satisfies the constraints in the previous equation can be rewritten as follows.

$$P(L < \varepsilon_1 < H) = \int_L^U f(z)\, dz,$$

where $f(z)$ is the probability density function of $z.$

3 Approximating the ARL to Monitor Small Shifts in the Mean of an ARFIX Process Running on an EWMAControl Chart

The numerical approximation of the integral equation utilizing the NIE method through the application of the Gauss-Legendre rule technique is proposed in this section.

Let $M_0 = \varphi$ represent an initial value of the EWMA statistics, and replace z with ε_t where $\varepsilon_t \sim Exp(\alpha)$ represents white noise error terms. According to [20], we propose a method where in the function $\Upsilon(\varphi)$ can be reformulated as follows:

$$
\begin{aligned}
\Upsilon(\varphi) &= \Big\{1 - P\Big((1-\lambda)\varphi - \lambda\phi_1 Y_0 - \ldots - \lambda\phi_p Y_{t-p} \\
&\quad - \lambda d(Y_0 - \phi_1 Y_{t-2} \ldots - \phi_p Y_{t-p-1}) \\
&\quad - \frac{\lambda}{2} d(d-1)(-Y_{t-2} + \phi_1 Y_{t-3} + \ldots + \phi_p Y_{t-p-2}) \\
&\quad - \ldots - \lambda\beta_1 X_1 - \ldots - \lambda\beta_k X_k\Big)/\lambda \\
&< \varepsilon_1 < \Big(b - (1-\lambda)\varphi - \lambda\phi_1 Y_0 - \ldots - \lambda\phi_p Y_{t-p} \\
&\quad - \lambda d(Y_0 - \phi_1 Y_{t-2} \ldots - \phi_p Y_{t-p-1}) \\
&\quad - \frac{\lambda}{2} d(d-1)(-Y_{t-2} + \phi_1 Y_{t-3} + \ldots + \phi_p Y_{t-p-2}) \\
&\quad - \ldots - \lambda\beta_1 X_1 - \ldots - \lambda\Big\} \\
&+ \int_L^U (1 + \Upsilon(1-\lambda)\varphi + \lambda\phi_1 Y_0 + \ldots + \lambda\phi_p Y_{t-p} \\
&\quad + \lambda d(Y_0 - \phi_1 Y_{t-2} \ldots - \phi_p Y_{t-p-1}) \\
&\quad + \frac{\lambda}{2} d(d-1)(-Y_{t-2} + \phi_1 Y_{t-3} + \ldots + \phi_p Y_{t-p-1}) \\
&\quad + \ldots + \lambda\beta_1 X_1 + \ldots + \lambda\beta_r X_r + \lambda\varepsilon_1\Big) f(y)dy \\
&= 1 + \int_L^U \Upsilon(1-\lambda)\varphi + \lambda\phi_1 Y_0 + \ldots + \lambda\phi_p Y_{t-p} \\
&\quad + \lambda d(Y_0 - \phi_1 Y_{t-2} - \ldots - \phi_p Y_{t-p-1})
\end{aligned}
$$

$$+\lambda\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1Y_{t-3}+...+\phi_pY_{t-p-2})$$
$$+...+\lambda\beta_1X_1+...+\lambda\beta_kX_k+\lambda\varepsilon_1\big)f(y)dy$$

As a result of changing the integral variable, the integral equation can be expressed:

$$\begin{aligned}\Upsilon(\varphi)=1+\frac{1}{\lambda}\int_0^b\Upsilon(z)f\Big(&\frac{z-(1-\lambda)\varphi}{\lambda}-\phi_1Y_{t-1}-...-\phi_pY_{t-p}\\&-d(Y_{t-1}-\phi_1Y_{t-2}-...-\phi_pY_{t-p-1})\\&-\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1Y_{t-3}+...+\phi_pY_{t-p-2})\\&-...-\beta_1X_{1t}-...-\beta_kX_{kt}\Big)dz\end{aligned}$$

$$=1+\frac{1}{\lambda}\int_0^b\Upsilon(z)\left(\frac{1}{\alpha}\exp-\frac{1}{\alpha}\left\{\begin{array}{l}(\frac{z-(1-\lambda)\varphi}{\lambda}-\phi_1Y_{t-1}-...-\phi_pY_{t-p}\\-d(Y_{t-1}-\phi_1Y_{t-2}-...-\phi_pY_{t-p-1})\\-\frac{1}{2}d(d-1)(-Y_{t-2}+..+\phi_pY_{t-p-2})\\-...-\beta_1X_{1t}-...-\beta_kX_{kt})\end{array}\right\}\right)dz$$

Thus, we get the integral equation in the following manner:

$$\Upsilon(\varphi)=1+\frac{1}{\lambda\alpha}\int_0^b\Upsilon(z).\exp\left\{\frac{-z}{\lambda\alpha}\right\}.\exp\left\{\frac{(1-\lambda)\varphi}{\lambda\alpha}\right\}$$
$$.\exp\left\{\begin{array}{l}\frac{1}{\alpha}\big(\phi_1Y_{t-1}+\phi_2Y_{t-2}+...+\phi_pY_{t-p}\\+d(Y_{t-1}-\phi_1Y_{t-2}-...-\phi_pY_{t-p-1})\\+\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1Y_{t-3}+...+\phi_pY_{t-p-2})\\+...+\beta_1X_{1t}+...+\beta_kX_{kt}\big)\end{array}\right\}dz \quad (11)$$

Equation (11) is represented as a linear Fredholm integral equation of the second kind, [21]. By using the Quadrature Rule, we can approximate the integral $\int_0^b f(z)dz$ as the sum of the areas of rectangles. These rectangles have bases of length b/m and heights determined by the values of f at the midpoints of intervals with a length of b/m, starting from zero. The interval [0, b] is partitioned into a sequence of points $0\le\nu_1\le\nu_2\le....\le\nu_m\le b$, where $w_j=b/m\ge0.$ represents a set of constant weights.

An integral equation from Equation (11) can be approximated using the quadrature rule as follows:

$$\int_0^b W(z)f(z)dz\approx\sum_{j=1}^m w_jf(\nu_j) \qquad (12)$$

where $W(z)$ is a weight function, $\nu_j=b/m(j-1/2)$ and $w_j=b/m;\ j=1,2,...,m.$

Solving a system of algebraic linear equations with m equations and m unknowns, can be used to approximate the solution for $\Upsilon(\varphi)$ by replacing φ with ν_i in Equation (11) as follows:

$$\begin{aligned}\hat\Upsilon(\nu_i)\approx1+\frac{1}{\lambda}\sum_{j=1}^m w_j\hat\Upsilon(\nu_j)f\Bigg(&\frac{\nu_j-(1-\lambda)\nu_i}{\lambda}-\big(\phi_1Y_{t-1}+...\\&+\phi_pY_{t-p}+d(Y_{t-1}-\phi_1Y_{t-2}-...-\phi_pY_{t-p-1})\\&+\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1Y_{t-3}+...+\phi_pY_{t-p-2})\\&+...+\beta_1X_{1t}+...+\beta_kX_{kt}+\varepsilon_t\big)\Bigg),\end{aligned}$$

for $i=1,2,...,m.$

Let $\hat\Upsilon(\varphi)$ denote the NIE method for ARL when using the interval $[0,b]$ to apply the Gauss-Legendre rule. Hence, the integral equation represented by Equation (11) consists of the set $\hat\Upsilon(\varphi)=\hat\Upsilon(\nu_1),\ \hat\Upsilon(\nu_2),\ ...,\ \hat\Upsilon(\nu_m),$ which can be approximated as

$$\begin{aligned}\hat\Upsilon(\nu_1)\approx1+\frac{1}{\lambda}\sum_{j=1}^m w_j\hat\Upsilon(\nu_j)f\Bigg(&\frac{\nu_j-(1-\lambda)\nu_1}{\lambda}-\big(\phi_1Y_{t-1}+...\\&+\phi_pY_{t-p}+d(Y_{t-1}-\phi_1Y_{t-2}-...-\phi_pY_{t-p-1})\\&+\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1Y_{t-3}+...+\phi_pY_{t-p-2})\\&+...+\beta_1X_{1t}+...+\beta_kX_{kt}+\varepsilon_t\big)\Bigg)\end{aligned}$$

$$\begin{aligned}\hat\Upsilon(\nu_2)\approx1+\frac{1}{\lambda}\sum_{j=1}^m w_j\hat\Upsilon(\nu_j)f\Bigg(&\frac{\nu_j-(1-\lambda)\nu_2}{\lambda}-\big(\phi_1Y_{t-1}+...\\&+\phi_pY_{t-p}+d(Y_{t-1}-\phi_1Y_{t-2}-...-\phi_pY_{t-p-1})\\&+\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1Y_{t-3}+...+\phi_pY_{t-p-2})\\&+...+\beta_1X_{1t}+...+\beta_kX_{kt}+\varepsilon_t\big)\Bigg)\end{aligned}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\hat{\Upsilon}(\nu_m) \approx 1+\frac{1}{\lambda}\sum_{j=1}^{m} w_j \hat{\Upsilon}(\nu_j) f\left(\frac{\nu_j-(1-\lambda)\nu_m}{\lambda}-\left(\phi_1 Y_{t-1}+\ldots\right.\right.$$
$$+\phi_p Y_{t-p}+d(Y_{t-1}-\phi_1 Y_{t-2}-\ldots-\phi_p Y_{t-p-1})$$
$$+\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1 Y_{t-3}+\ldots+\phi_p Y_{t-p-2})$$
$$\left.\left.+\ldots+\beta_1 X_{1t}+\ldots+\beta_k X_{kt}+\varepsilon_t\right)\right)$$

The above set of m equations in m unknowns can be rewritten in matrix form.

Let $\mathbf{L}_{m\times 1}=\left[\hat{\Upsilon}(\nu_1),\ \hat{\Upsilon}(\nu_2),\ldots,\ \hat{\Upsilon}(\nu_m)\right]'$ be a column vector of $\hat{\Upsilon}(\nu_i);\ i=1,2,\ldots,m,$ $\mathbf{1}_{m\times 1}=[1,1,\ldots,1]'$ is a column vector of ones, and $\mathbf{I}_{m\times 1}=diag(1,1,\ldots,1)$ is the unit matrix order m. Let matrix $\mathbf{R}_{m\times m}$ be a matrix with dimension $m\times m$ with element can be expanded as

$$R_{ij} \approx \frac{1}{\lambda} w_j f\left(\frac{\nu_j-(1-\lambda)\nu_i}{\lambda}-\left(\phi_1 Y_{t-1}+\ldots\right.\right.$$
$$+\phi_p Y_{t-p}+d(Y_{t-1}-\phi_1 Y_{t-2}-\ldots-\phi_p Y_{t-p-1})$$
$$+\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1 Y_{t-3}+\ldots+\phi_p Y_{t-p-2})$$
$$\left.\left.+\ldots+\beta_1 X_{1t}+\ldots+\beta_k X_{kt}+\varepsilon_t\right)\right)$$

where $R_{ij};\ i,j=1,2,\ldots,m$

If the inverse of $(\mathbf{I}_m-\mathbf{R}_{m\times m})^{-1}$ exists and is invertible, then the ARL approximation for the NIE can be expressed in a system of linear equations in matrix form as follows:

$$\mathbf{L}_{m\times 1}=(\mathbf{I}_m-\mathbf{C}_{m\times m})^{-1}\mathbf{1}_{m\times 1}, \tag{13}$$

Finally, the function $\hat{\Upsilon}(\nu_1), \hat{\Upsilon}(\nu_2), \ldots, \hat{\Upsilon}(\nu_m)$, is obtained by replacing ν_i with φ. Therefore, NIE technique for approximating the ARL of a long-memory ARFIX(p, d, k) process running on the EWMA control chart is

$$\hat{\Upsilon}(\varphi) \approx 1+\frac{1}{\lambda}\sum_{j=1}^{m} w_j \hat{\Upsilon}(\nu_j) f\left(\frac{\nu_j-(1-\lambda)\varphi}{\lambda}-\left(\phi_1 Y_{t-1}+\ldots\right.\right.$$
$$+\phi_p Y_{t-p}+d(Y_{t-1}-\phi_1 Y_{t-2}-\ldots-\phi_p Y_{t-p-1})$$
$$+\frac{1}{2}d(d-1)(-Y_{t-2}+\phi_1 Y_{t-3}+\ldots+\phi_p Y_{t-p-2})$$
$$\left.\left.+\ldots+\beta_1 X_{1t}+\ldots+\beta_k X_{kt}+\varepsilon_t\right)\right) \tag{14}$$

where $\nu_j=b/m(j-1/2)$ and $w_j=b/m;\ j=1,2,\ldots,m.$

The NIE technique is used to detect small shifts in the mean process and the accuracy of explicit formulas. Therefore, the explicit formula for the in-control ARL is

$$\mathrm{ARL}_0=1-\left[\lambda\exp\{(1-\lambda)\varphi/\lambda\alpha_0\}\left(\exp\{-b/\lambda\alpha_0\}-1\right)\right]$$
$$\cdot\left[\lambda\exp-\left\{\begin{array}{l}\left(\phi_1 Y_{t-1}+\ldots+\phi_p Y_{t-p}\right.\\ +d(Y_{t-1}-\phi_1 Y_{t-2}-\ldots-\phi_p Y_{t-p-1}\\ +\frac{1}{2}d(d-1)(-Y_{t-2}+\ldots+\phi_p Y_{t-p-2})\\ \left.+\ldots+\beta_1 X_{1t}+\ldots+\beta_k X_{kt}+\varepsilon_t\right)/\alpha_0\end{array}\right\}\right]^{-1}$$
$$+\left[\exp\{-b/\alpha_0\}-1\right]^{-1}. \tag{15}$$

On the contrary, the explicit formula for the out-of-control ARL is:

$$\mathrm{ARL}_1=1-\left[\lambda\exp\{(1-\lambda)\varphi/\lambda\alpha_1\}\left(\exp\{-b/\lambda\alpha_1\}-1\right)\right]$$
$$\cdot\left[\lambda\exp-\left\{\begin{array}{l}\left(\phi_1 Y_{t-1}+\ldots+\phi_p Y_{t-p}\right.\\ +d(Y_{t-1}-\phi_1 Y_{t-2}-\ldots-\phi_p Y_{t-p-1}\\ +\frac{1}{2}d(d-1)(-Y_{t-2}+\ldots+\phi_p Y_{t-p-2})\\ \left.+\ldots+\beta_1 X_{1t}+\ldots+\beta_k X_{kt}+\varepsilon_t\right)/\alpha_1\end{array}\right\}\right]^{-1}$$
$$+\left[\exp\{-b/\alpha_1\}-1\right]^{-1}. \tag{16}$$

4 The Numerical Study

To assess the efficacy of the proposed NIE technique in comparison with deriving the ARL using explicit formulas, the accuracy percentage between them can be expressed as

$$\%\,\mathrm{Accuracy}=100-\left|\frac{\Upsilon(\varphi)-\hat{\Upsilon}(\varphi)}{\Upsilon(\varphi))}\right|\times 100\%, \tag{17}$$

where $\hat{\Upsilon}(\varphi)$ and $\Upsilon(\varphi)$ are the ARL results for the NIE technique and explicit formulas, respectively. An accuracy percentage of greater than 95% means that the ARL results of the two methods are close to each other (i.e., the results are highly consistent).

The performances of the NIE technique and explicit formulas were assessed using λ = 0.03, 0.05, or 0.10 to compute the UCL (b) from Equation (14) to obtain ARL_0 = 370. In the experiment, we generated several long-memory ARFIX(p, d, k) processes with exponential white noise running on a EWMA control chart using Equation (8) and employed a wide range of possible changes and autocorrelation coefficient values. The white noise of the process in this investigation was exponentially distributed $(\varepsilon_t \sim Exp(\alpha))$. The in-control process $(\alpha = \alpha_0)$ has an exponential mean parameter of 1 whereas the out-of-control processes was assigned changes in the mean $(\alpha = \alpha_1)$ of 1.025,1.05, 1.075, 1.100, 1.125, 1.15, 1.20, 1.30, or 1.50. The autocorrelation coefficients were assigned values of $\phi_1 = \pm 0.1, \phi_2 = 0.2, \phi_3 = 0.3, \beta_1 = 0.1$. Eight hundred division points (m) were utilized in the system of linear equations. The calculations for the numerical results for the ARL derived by using both techniques were performed using Wolfram Mathematica.

The principal findings using the suggested NIE technique for approximating the ARL of the long-memory ARFIX processes running on a EWMA chart for each scenario are reported in Table 1 (Appendix). The smoothing parameter (λ) of the control chart was utilized to determine the optimal value for λ to compute the UCL (b). For each coefficient parameter combination in each long-memory ARFIX process, we found that the values of λ and b increased. Furthermore, upon examination of the coefficient parameters, contrasting results were obtained for the positive and negative values of ϕ_1.

Table 2 (Appendix) and Table 3 (Appendix) report the numerical results for the out-of-control ARL $(\alpha_1 > \alpha_0)$ computed using the process and parameter values in Table 1. The ARL_1 was then calculated using the NIE technique in Equation (14) and explicit formulas in Equation (16) for the long-memory ARFIX(p, d = 0.1, k = 1) process when p was varied as 1, 2, or 3 running on an EWMA control chart. To accomplish this, the NIE search algorithm was utilized to identify the corresponding values of b. The results indicate that the ARL efficacies derived from both techniques were similar for detecting small changes in the process mean. The ARL_1 results for the NIE and the explicit formulas methods decreased rapidly as the mean change magnitude was increased. When analyzing the chart's properties, it is evident that an increase in the λ value resulted in a proportional rise in ARL_1. This demonstrates that the sensitivity of the EWMA chart decreased as λ was increased.

Figure 1 shows the ARL_1 results for the NIE technique where several processes were assigned various positive and negative coefficient values. It was found that positive coefficient values resulted in a reduction in ARL_1 at every change level, which resulted in increased detection efficacy. In addition, the percentage change results were computed for various changes in mean magnitude for each scenario. The results of the calculations were greater than 95%, which indicates that the suggested technique is accurate and fully consistent with the explicit formulas method.

5 An Illustration of the Efficacy of the NIE Technique with Real Data

For this part of the study, we utilized the weekly stock market price data for iron ore futures 62% Fe CFR-(TIOc1) from January 5, 2020, to November 26, 2023, obtained from https://th.investing.com/. In addition, daily UDS/THB exchange rate data were also included as the exogenous variable. The datasets consisted of 204 observations each.

Estimation of the parameters and testing of the distribution of the white noise were performed using the statistical software packages Eviews and SPSS, respectively (Table 4 and Table 5). The p-values of all of the parameters were found to be less than 0.05 indicating that they were all statistically significant. Moreover, the value of d (0.163219, p-value < 0.5) means that this model is a long-memory process.

Table 4. Parameter estimates for the TIOc1 dataset including the UDS/THB exchange rates as the exogenous variable

Parameters:	Coefficient	t-Statistic	Prob.
UDS/THB	-3.4336	-2.9599	0.0034*
d	0.1632	2.7586	0.0063*
AR(1)	0.9998	177.3717	0.0000*
R-squared			0.96493
Adjusted R-squared			0.96458

**A significance level of 0.05.*

Table 5. The results of testing the distribution of the white noise of the TIOc1 dataset

Testing exponential white noise.	
Exponential Parameter ($\alpha = \alpha_0$)	3.6471
Kolmogorov-Smirnov	0.8572
Asymptotic Significance (2-Sided)	0.4544^{ns}

^{ns} *non-significance level of 0.05.*

The residuals (white noise) of the long-memory ARFIX model were tested to see whether they followed an exponential distribution by using a Kolmogorov-Smirnov test, which was the case (p-value > 0.05). The exponential parameter (α) was 3.6471 (Table 5). The model is defined as

$$Y_t = 1.136Y_{t-1} - 0.0949Y_{t-2} - 0.0265Y_{t-3} - 0.0418Y_{t-4}$$

$$-3.4336X_1 + \varepsilon_t ; \varepsilon_t \sim Exp(3.6471) \qquad (18)$$

Subsequently, the NIE technique to solve the integral equation in Equation (14) for the ARL of the process running on an EWMA control chart becomes

$$\hat{\Upsilon}(\varphi) \approx 1 + \frac{1}{\lambda} + \sum_{j=1}^{m} w_j \hat{\Upsilon}(\nu_j) f\left(\frac{\nu_j - (1-\lambda)\varphi}{\lambda} - \begin{pmatrix} 1.136Y_{t-1} - 0.0949Y_{t-2} - 0.0265Y_{t-3} \\ -0.0418Y_{t-4} - 3.4336X_1 + \varepsilon_t \end{pmatrix} \right) \qquad (19)$$

with a set of constant weights $w_j = b/m,$ and $\nu_j = b/m(j - 1/2)$; $j = 1, 2, \ldots, m.$

The results of the calculations and a comparison with those using the explicit formulas are provided in Table 6 (Appendix). They reveal that there was no difference in the ARL values obtained by using the two techniques when ARL_0 = 370 with various smoothing parameter values (0.05, 0.10, or 0.20) for the ARFIX(1, 0.1632,1) process running on an EWMA control chart. As the mean change magnitude was increased, the ARL calculated via both methods decreased, yielding the same findings as those in Table 2 (Appendix) and Table 3 (Appendix). Moreover, the percentage accuracy was 100% in all cases. This indicates high consistency between the two techniques. Moreover, for the same mean change magnitude, the out-of-control ARL increased as the value of the smoothing parameter was increased from 0.01 to 0.05. These results are in agreement with the numerical results in Section 4. The efficacy of both techniques concerning out-of-control processes were assessed by comparing the EWMA control chart to the CUSUM control chart. Calculation of the UCL (b) parameter when the reference value (a) is set to 6 on the CUSUM control chart and ARL_0 is set according to the EWMA control chart. For detecting small changes in process parameters on both control charts, consistent results are obtained. It was later found that the presented results were consistent with the previous. Overall, the NIE technique is performed as a accomplishing choice.

6 Conclusions and Recommendations

The research presented here is an innovative approach for detecting mean changes on EWMA control charts of the ARFIX time series. The Gauss-Legend method is applied to an approximation of ARL with IE interpolation. The results of a numerical study comprising the proposed technique and the ARL derived using explicit formulas showed excellent agreement between the two methods (accuracy percentage > 95%), and it was found that the out-of-control ARL results decreased rapidly and in the same direction for both techniques. Therefore, the NIE technique is a suitable choice for determining the ARL for this specific situation. Moreover, the method could be modified for other control charts and used in practical scenarios that include other time series models.

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APPENDIX

Table 1. Values of the upper control limit (b) with optimal values of λ for combination long-memory ARFIX$(p, 0.1, 1)$ models at $ARL_0 = 370$.

Long-memory ARFIX$(p, 0.1, 1)$ process	Coefficient parameters				λ		
	ϕ_1	ϕ_2	ϕ_3	β_1	0.03	0.05	0.10
$p = 1$	0.1	-	-	0.1	2.5881E-14	2.66337E-08	0.0011283
	-0.1	-	-	0.1	3.0534E-14	3.14211E-08	0.0013324
$p = 2$	0.1	0.2	-	0.1	2.1940E-14	2.25758E-08	0.0009556
	-0.1	0.2	-	0.1	2.5881E-14	2.66337E-08	0.0011283
$p = 3$	0.1	0.2	0.3	0.1	1.7125E-14	1.76181E-08	0.000745
	-0.1	0.2	0.3	0.1	2.0200E-14	2.07849E-08	0.0008795

Table 2. Comparison of out-of-control ARL results between the NIE technique and explicit formulas for the long-memory ARFIX processes when $\phi_1 = 0.1$ running on an EWMA control chart

Long-memory ARFIX$(p,0.1,1)$	λ	Technique	α_1									
			1.00	1.025	1.05	1.075	1.100	1.125	1.15	1.20	1.30	1.50
$p = 1$	0.03	NIE	370.000	159.284	71.648	33.721	16.686	8.766	4.962	2.120	1.119	1.003
		Explicit	370.000	159.284	71.644	33.720	16.686	8.766	4.962	2.120	1.119	1.003
		%Accuracy	100.000	100.000	99.997	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.05	NIE	370.000	220.118	134.308	83.953	53.716	35.165	23.553	11.338	3.588	1.278
		Explicit	370.000	220.118	134.308	83.953	53.716	35.165	23.553	11.338	3.588	1.278
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.10	NIE	370.000	279.965	214.627	166.595	130.747	103.744	83.156	54.956	26.535	8.608
		Explicit	370.000	279.965	214.627	166.595	130.747	103.744	83.156	54.956	26.535	8.608
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
$p = 2$	0.03	NIE	370.000	158.663	71.102	33.350	16.453	8.625	4.878	2.089	1.115	1.003
		Explicit	370.000	158.672	71.107	33.351	16.453	8.625	4.878	2.089	1.115	1.003
		%Accuracy	100.000	99.994	99.993	99.997	100.000	100.000	100.000	100.000	100.000	100.000
	0.05	NIE	370.000	219.237	133.263	83.002	52.930	34.543	23.072	11.056	3.491	1.263
		Explicit	370.000	219.237	133.263	83.002	52.930	34.543	23.072	11.056	3.491	1.263
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.10	NIE	370.000	278.789	212.877	164.581	128.733	101.802	81.340	53.440	25.551	8.191
		Explicit	370.000	278.789	212.877	164.581	128.733	101.802	81.340	53.440	25.551	8.191
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
$p = 3$	0.03	NIE	370.000	157.739	70.292	32.800	16.112	8.419	4.755	2.046	1.108	1.003
		Explicit	370.000	157.733	70.295	32.801	16.112	8.419	4.755	2.046	1.108	1.003
		%Accuracy	100.000	99.996	99.996	99.997	100.000	100.000	100.000	100.000	100.000	100.000
	0.05	NIE	370.000	217.921	131.711	81.596	51.773	33.632	22.370	10.650	3.352	1.242
		Explicit	370.000	217.921	131.711	81.596	51.773	33.632	22.370	10.650	3.352	1.242
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.10	NIE	370.000	277.037	210.281	161.659	125.776	98.962	78.693	51.250	24.149	7.609
		Explicit	370.000	277.037	210.281	161.659	125.776	98.962	78.693	51.250	24.149	7.609
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000

Table 3. Comparison of out-of-control ARL results between the NIE technique and explicit formulas for the long-memory ARFIX processes when $\phi_1 = -0.1$ running on an EWMA control chart

Long-memory ARFIX(p,0.1,1)	λ	Technique	α_1									
			1.00	1.025	1.05	1.075	1.100	1.125	1.15	1.20	1.30	1.50
p = 1	0.03	NIE	370.000	159.928	72.209	34.102	16.924	8.910	5.048	2.151	1.124	1.003
		Explicit	370.000	159.929	72.209	34.102	16.924	8.910	5.049	2.152	1.124	1.003
		%Accuracy	100.000	99.999	100.000	100.000	100.000	100.000	99.980	99.954	100.000	100.000
	0.05	NIE	370.000	221.004	135.362	84.916	54.515	35.799	24.045	11.626	3.689	1.293
		Explicit	370.000	221.004	135.362	84.916	54.515	35.799	24.045	11.626	3.689	1.293
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.10	NIE	370.000	281.147	216.394	168.566	132.795	105.726	85.017	56.518	27.559	9.050
		Explicit	370.000	281.147	216.394	168.566	132.795	105.726	85.017	56.518	27.559	9.050
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
p = 2	0.03	NIE	370.000	159.285	71.650	33.721	16.686	8.765	4.962	2.120	1.119	1.003
		Explicit	370.000	159.284	71.654	33.720	16.686	8.765	4.962	2.120	1.119	1.003
		%Accuracy	100.000	99.999	99.994	99.997	100.000	100.000	100.000	100.000	100.000	100.000
	0.05	NIE	370.000	220.118	134.308	83.953	53.716	35.165	23.553	11.338	3.588	1.278
		Explicit	370.000	220.118	134.308	83.953	53.716	35.165	23.553	11.338	3.588	1.278
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.10	NIE	370.000	279.965	214.627	166.559	130.747	103.744	83.156	54.956	26.535	8.608
		Explicit	370.000	279.965	214.627	166.559	130.747	103.744	83.156	54.956	26.535	8.608
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
p = 3	0.03	NIE	370.000	158.350	70.826	33.166	16.337	8.556	4.836	2.075	1.113	1.003
		Explicit	370.000	158.349	70.828	33.166	16.337	8.556	4.836	2.075	1.113	1.003
		%Accuracy	100.000	99.999	99.997	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.05	NIE	370.000	218.797	132.744	82.531	52.541	34.237	22.835	10.919	3.444	1.256
		Explicit	370.000	218.797	132.744	82.531	52.541	34.237	22.835	10.919	3.444	1.256
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.10	NIE	370.000	278.204	212.008	163.601	127.739	100.846	80.446	52.699	25.074	7.991
		Explicit	370.000	278.204	212.008	163.601	127.739	100.846	80.446	52.699	25.074	7.991
		%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000

Table 6. Comparison of out-of-control ARL results between the NIE technique and explicit formulas for the TIOc1 dataset-based long-memory ARFIX process running on an EWMA and CUSUM control chart

Control chart	λ	b	ARL techniques	α_1								
				1.025	1.05	1.075	1.100	1.125	1.15	1.20	1.30	1.50
EWMA	0.01	3.2422×10^{-11}	NIE	188.471	99.309	54.095	30.474	17.787	10.793	4.56032	1.5915	1.0330
			Explicit	188.471	99.309	54.095	30.474	17.787	10.793	4.56032	1.5915	1.0330
			%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.03	0.008320382	NIE	293.823	235.804	191.116	156.333	128.992	107.302	75.945	41.184	15.598
			Explicit	293.823	235.804	191.116	156.333	128.992	107.302	75.945	41.184	15.598
			%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
	0.05	0.213871	NIE	310.138	262.247	223.55	191.994	166.039	144.523	111.442	70.408	33.818
			Explicit	310.138	262.247	223.55	191.994	166.039	144.523	111.442	70.408	33.818
			%Accuracy	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
Control chart	a	b	ARL techniques	1.025	1.05	1.075	1.100	1.125	1.15	1.20	1.30	1.50
CUSUM	6	16.35156	NIE	310.012	262.582	224.211	192.904	167.156	145.820	113.057	72.541	36.565
			Explicit	310.674	263.115	224.643	193.258	167.448	146.063	113.228	72.631	36.597
			%Accuracy	99.787	99.797	99.808	99.817	99.826	99.834	99.849	99.876	99.913

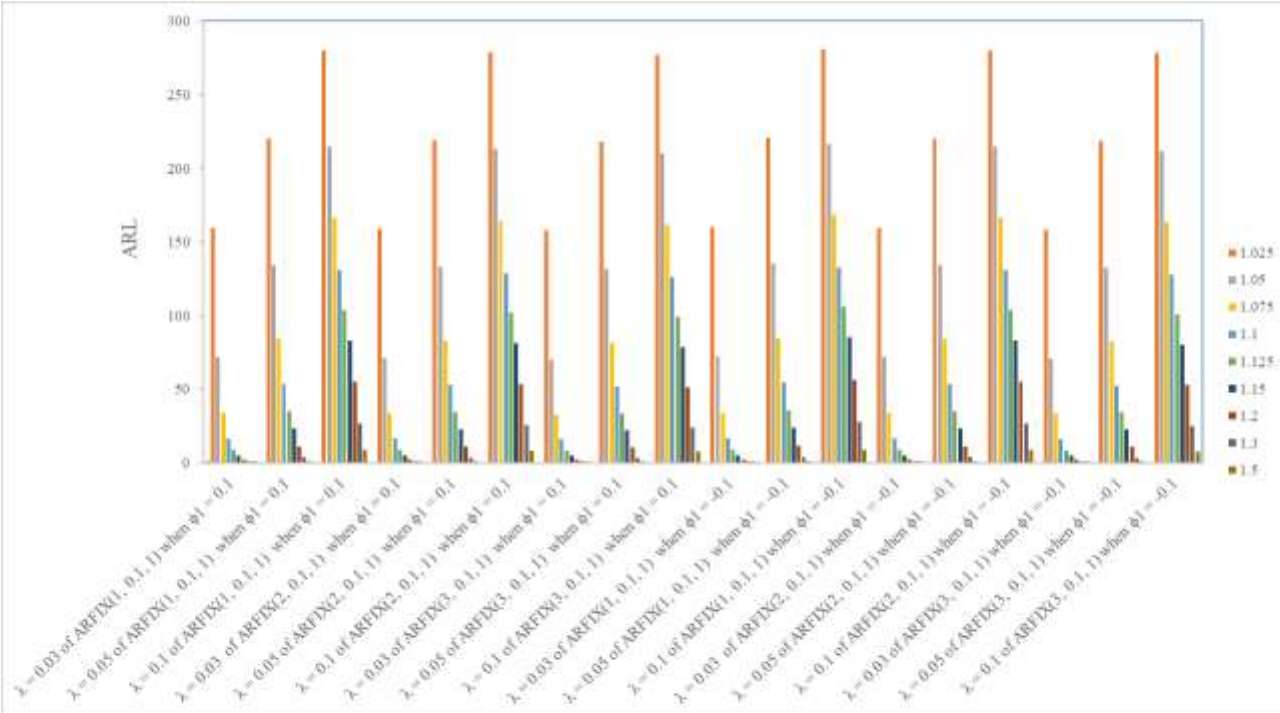

Fig. 1: Graphical displays of ARL_1 results using NIE method running on an EWMA control chart for the long-memory ARFIX processes with coefficient value

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Conflict of Interest

The authors have no conflict of interest to declare.