Residual-Based RKF with Recursive Measurement Noise Covariance Matching

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Abstract: - A new residual-based recursive measurement noise covariance estimation method is proposed. The presented algorithm is used for Kalman filter tuning, as a result, the robust Kalman filter (RKF) against measurement malfunctions is derived. The proposed residual-based RKF with recursive estimation of measurement noise covariance is applied to the model of Unmanned Aerial Vehicle (UAV) dynamics. Algorithms are examined for two types of measurement fault scenarios; constant bias at measurements (additive sensor faults) and measurement noise increments (multiplicative sensor faults). The simulation results show that the proposed RKF can accurately estimate UAV dynamics in real-time in the presence of various types of sensor faults. Estimation accuracies of the proposed RKF and conventional KF are investigated and compared.

Key-Words: - Kalman filter, residual, robust estimation, unmanned aerial vehicle, sensor faults, covariance matching.

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1 Introduction

The Kalman Filter can be used to estimate the states of an Unmanned Aerial Vehicle (UAV). That is the preferred method because it is crucial to exactly know the characteristics, such as velocity, altitude, attitude, etc. Successful aircraft control can be attained when these UAV states are attained without any issues. However, that procedure is contingent on how accurate the measurements are. The filter produces erroneous findings and diverges over time if the measurements are unreliable due to any type of sensor fault. Due to the significance of obtaining fault tolerance in the design of a UAV flight control system, filters should be constructed robustly to overcome such issues.

The Kalman filter method of state estimation is extremely sensitive to defects in the measurement system. Changes in the measurement channels significantly degrade the performance of the estimating systems if the measurement system's state of operation differs from the models used in the filter's synthesis. The possible errors can be recovered with adaptive Kalman filters.

A variety of alternative strategies can be used to make the Kalman filter flexible and hence insensitive to a priori measurements or system uncertainties. Multiple-model adaptive estimation (MMAE) [1], [2], innovation-based adaptive estimation (IAE) [3], [4], [5], and residual-based adaptive estimation (RAE) [5], [6] are all essential techniques to addressing the adaptive Kalman filtering problem. Changes in the innovation or residual sequences cause rapid adjustments to the measurement and/or process noise covariance matrices.

The MMAE approach can only be utilized in specific situations because it calls for a number of parallel Kalman filters, and the faults should be known. IAE and RAE methods must use the innovation vectors or residual vectors of *m* epochs in the moving window to estimate the covariance matrices. The number, kind, and distribution of the measurements for all epochs inside a window must be consistent for IAE and RAE estimators. If not, neither the innovation nor the residual vectors can be used to estimate the covariance matrices of the measurement noises.

Another idea is to multiply the noise covariance matrix by a time-dependent variable to scale it. This algorithm is called adaptive fading Kalman filter (AFKF). One approach to creating such an algorithm is to multiply the process or measurement noise covariance matrices by a single adaptive factor [5], [7], [8]. The AFKF technique can be used if there is a fault in the measurement system, and the filter's insensitivity to present measurement faults can be ensured by multiplying the measurement noise scale factor by the measurement noise covariance matrix. As a result, by applying an adjustment to the filter gain, the filter's accurate estimating behavior will be protected from being affected by inaccurate measurements.

An adaptation method based on the multiple fading factor is provided in [5], and [9]. The variation in the impacts of measurement noise covariance change on estimating the performance of each estimated state is the primary reason for adopting several fading factors. It is critical to carefully evaluate how modifying the measurement noise covariance would affect each state, particularly for complicated multivariable systems, and to employ a matrix with many fading factors rather than a single factor (so that the adaptation is weighted differently for each state).

The measurement covariance matrix can be modified with the help of the fuzzy inference system, [10]. The results showed that the proposed adaptive fuzzy extended Kalman filter is robust against disturbances and outliers. Although adaptive Kalman filter algorithms based on fuzzy logic work well in some situations, they are knowledge-based systems that function with linguistic variables and cannot be widely applied to critical systems like aircraft flight control systems since they are human experience-based.

Because the noise estimator cannot be expressed in a recursive form and each previous state vector must be smoothed by the most recent measurements at each point in time, the algorithm in the studies mentioned cannot be used to directly estimate the measurement noise covariance in practical operations.

In this study, a residual-based robust Kalman filter with a recursive measurement noise covariance estimator is proposed and applied for the state estimation process of a UAV platform. The results of the proposed robust and conventional Kalman filter algorithms are compared for different types of measurement faults and recommendations about their utilization are given.

The article is presented as follows. The UAV's flight dynamics model is presented first. The optimal Kalman filter for estimating UAV state is then described in the following section. After that, a novel recursive measurement noise covariance estimation method for Kalman filter tuning is proposed. Based on the introduced residual-based recursive measurement noise covariance estimation approach the robust Kalman filter (RKF) against sensor faults is derived. The proposed RKF with recursive measurement noise covariance estimation algorithm is applied for the model of UAV dynamics and the performance of the proposed filter is tested via simulations for the state estimation process of a UAV platform. The conclusion and the results are briefly summarized in the final section.

2 Preliminaries

Consider the linear dynamic system represented by the state equation:

$$x(j+1) = Ax(j) + Bu(j) + Gw(j) \tag{1}$$

and measurement equation:

$$z(j) = H(j)x(j) + V(j), \tag{2}$$

where $x(j)$ is the system state; A is the system transition matrix; B is the control distribution matrix; $u(j)$ is the control input; $w(j)$ is the random system noise; G is the system noise transition matrix; $z(j)$ is the measurement vector; $H(j)$ is the measurement matrix; $V(j)$ is a random measurement noise.

Assume that $w(j)$ and $V(j)$ are Gaussian white noise random vectors with zero mean and covariances:

$$\begin{aligned} &E\left[w(j)w^T(k)\right] = Q(j)\delta(jk); \\ &E\left[V(j)V^T(k)\right] = R(j)\delta(jk). \end{aligned} \tag{3}$$

where $\delta(jk)$ *is* the Kronecker delta symbol. Note that $\{w(j)\}$ and $\{V(j)\}$ are assumed mutually uncorrelated.

The state vector (1) can be estimated via the optimal linear Kalman filter (LKF), [11]. Equations for the estimation value and gain matrix of the LKF respectively are:

$$\hat{x}(j/j) = \hat{x}(j/j-1) + K(j)\Delta(j) \quad (4)$$

$$K(j) = P(j/j-1)H^T(j)P_\Delta^{-1} \quad (5)$$

where $\hat{x}(j/j-1) = A\hat{x}(j-1/j-1) + Bu(j-1)$ is the extrapolation value, $\Delta(j)$ and $P_\Delta(j)$ are the innovation and innovation covariance respectively.

The expressions for the $\Delta(j)$ and $P_\Delta(j)$ are:

$$\Delta(j) = z(j) - H(j)\hat{x}(j/j-1) \quad (6)$$

$$P_\Delta(j) = H(j)P(j/j-1)H(j) + R(j) \quad (7)$$

Here $P(j/j-1)$ is the covariance matrix of the extrapolation error.

The residual of Kalman filter is defined as:

$$\nu(j) = z(j) - H(j)\hat{x}(j/j) \quad (8)$$

The residual covariance is [6]

$$P_\nu(j) = R(j) - H(j)P(j/j)H(j) \quad (9)$$

The residual sequence (8) will be white Gaussian noise with zero-mean and covariance (9) if the system is functioning normally [3], i.e. $\nu(j) \sim N(0, P_\nu(j))$. On the other hand, when there are abnormal changes occurring in the system or measurement channels, it can be assumed that $\nu_f(j) \sim N\left(\mu(j), P_{\nu_f}(j)\right)$, where either $\mu(j) \neq 0$ or $P_{\nu_f}(j) \neq P_\nu(j)$ or both. Note that faults that only result in $\mu(j) \neq 0$ are generally called additive or bias type faults. They can be denoted as $\nu_f(j) = \nu(j) + f(j)$ and satisfy $\nu_f(j) \sim N(\mu(j), P_\nu(j))$, where $E[f(j)] = \mu(j)$. Those faults that lead to changes in innovation covariance $P_\nu(j)$ are called multiplicative or noise increment type faults, which can be denoted as $\nu_f(j) = F(j)\nu(j)$ with $\nu_f(j) \sim N\left(0, F(j)P_\nu(j)F^T(j)\right)$.

3 The Influence of Sensor Faults on Kalman Fılter Residual

The statistical properties of the Kalman filter residual will alter as a result of measurement bias and sensor noise increase type sensor faults. This section examines the impact of these types of sensor faults on the Kalman filter's residual sequence.

3.1 Influence of Sensor Biases on the Kalman Filter Residual

Theorem 1: In the event that measurements are processed using LKF (4)–(7) and a measurement bias arises at an iteration step $j = \tau$, then at all $j > \tau$ steps the residual bias will be equal to the difference between the measurement bias and the estimated observation bias.

Proof: At the first step following the bias occurring at iteration $j = \tau$, the extrapolation value can be expressed as

$$\begin{aligned} &\hat{x}_b(j+1/j) = A\hat{x}_b(j/j) + Bu(j) + Gw(j) = \\ &A\hat{x}(j/j) + A\Delta\hat{x}(j/j) + Bu(j) + Gw(j) \\ &= \hat{x}(j+1/j) + \Delta\hat{x}(j+1/j) \end{aligned} \quad (10)$$

where $\Delta\hat{x}(j+1/j) = A\Delta\hat{x}(j/j)$ is the extrapolation value bias.

Residual of Kalman filter is:

$$\begin{aligned} &\nu_b(j+1) = z(j+1) + b_z(j+1) - H(j+1)\hat{x}_b(j+1/j+1) \\ &= z(j+1) - H(j+1)\hat{x}(j+1/j+1) + b_z(j+1) - \\ &H(j+1)\Delta\hat{x}(j+1/j+1) = \nu(j+1) + \mu(j+1) \end{aligned} \quad (11)$$

where

$$\mu(j+1) = b_z(j+1) - H(j+1)\Delta\hat{x}(j+1/j+1) \quad (12)$$

is the residual bias.

The residual bias is equal to the difference between the measurement bias and estimated observation bias, as may be observed from expression (12), as shown. For all $j > \tau$ steps, this situation applies. As a result, Theorem 1 is proven. Consequently, measurement bias type sensor faults will cause a bias in the residual of the Kalman filter.

3.2 Influence of Measurement Noise Increment to the Residual

Let the measurements are processed by the LKF (4)-(7) and a measurement noise increment occurs at the iteration step $j = \tau$. Measurement noise increment can be simulated by multiplying the measurement noise vector with the diagonal matrix $F(j)$, which diagonal elements meet the following condition: $\sigma_{ii}(j) \geq 1$, $(i = \overline{1,n})$ for $\forall j \geq \tau$. Here n is the dimension of the measurement vector. As it is clear, for the noise increment type sensor fault in the i th measurement channel, the appropriate diagonal element of $F(j)$ will be larger than 1, i.e. $\sigma_{ii}(j) > 1$ for $\forall j \geq \tau$ and rest of the measurement channels become 1. Consequently, the diagonal elements of $F(j)$ can be presented in the following form:

$$\sigma_{ii} = \begin{cases} 1: \text{ no measurement fault} \\ >1: \text{measurement fault} \end{cases}$$

The measurement model in this case can be written in the form:

$$z(j) = H(j)x(j) + F(j)V(j), \tag{13}$$

where

$$diag\left(F(j)\right) = \begin{cases} \begin{pmatrix} 1 & 1 & . & . & . & 1 \end{pmatrix}, \quad \text{for } j < \tau \\ \begin{pmatrix} \sigma_{11} & \sigma_{22} & . & . & . & \sigma_{nn} \end{pmatrix} \text{ if } \exists i \in \left(\overline{1,n}\right), \\ \text{where } \sigma_{ii} > 1 \text{ for } j \geq \tau \end{cases} \tag{14}$$

Theorem 2: In the event that measurements are processed using LKF (4)–(7) and a measurement noise increment occurs at an iteration $j = \tau$, then at all $j \geq \tau$ steps the measurement noise increment leads to increment in the residual covariance (7).

Proof. The innovation covariance at the iteration steps $j \geq \tau$ can be expressed as

$$P_{\nu_{ni}}(j) = F(j)R(j)F^T(j) - H(j)P(j/j)H^T(j) \tag{15}$$

The residual covariance increment is:

$$\Delta P_{\nu_{ni}}(j) = F(j)R(j)F^T(j) - R(j) \tag{16}$$

Since the matrices $F(j)$ and $R(j)$ are assumed to be diagonal, the expression (16) can be rewritten in the following form:

$$\Delta P_{\nu_{ni}}(j) = \left[F(j)\right]^2 R(j) - R(j) = \left\{\left[F(j)\right]^2 - I\right\} R(j) \tag{17}$$

where I is the $n \times n$ identity matrix. Because $R(j)$ and $F(j)$ are positive definite diagonal matrices and $F(j)$ has diagonal elements $\sigma_{ii}(j) \geq 1$, ($i = \overline{1,s}$) for $\forall\ j \geq \tau$, then the matrix $\left\{\left[F(j)\right]^2 - I\right\} R(j)$ is also positive definite. Since the innovation covariance increment is a positive definite matrix, the Theorem 2 is valid, therefore, the noise increment type sensor fault leads to an increment of residual covariance (7).

4 Resudual-Based Recursive Measurement Noise Covariance Estimator

The statistical properties of the Kalman filter residual will change as a result of measurement bias and measurement noise increment. For the compensation of measurement bias or measurement noise increment, the real and theoretical values of the residual covariance matrices must be compared.

In the absence of measurement fault in the estimation system, the real innovation covariance $C(j)$ is equal to the theoretical one, [6]

$$C(j) = R(j) - H(j)P(j/j)H^T(j) \tag{18}$$

The real covariance matrix of $\nu(j)$ is an average of $\nu(k)\nu^T(k)$ within a moving window M :

$$C(j) = \frac{1}{M} \sum_{k=j-M+1}^{j} \nu(k)\nu^T(k) \tag{19}$$

Substituting Eq. (19) into (18) we have

$$\frac{1}{M} \sum_{k=j-M+1}^{j} \nu(k)\nu^T(k) = R(j) - H(j)P(j/j)H^T(j) \tag{20}$$

The real residual covariance matrix $C(j)$ can be estimated by $\nu(j)\nu^T(j)$ at the current epoch in order to avoid the smoothness of the average of $\nu(j)\nu^T(j)$ within M epochs, which does not adequately reflect the uncertainty of the model errors at the current step

$$C(j) = \nu(j)\nu^T(j) \tag{21}$$

Taking into account (18) and (21), the expressions for the measurement noise covariances for $j+1$ and j iterations can be written in the following form:

$$R(j+1) = \nu(j+1)\nu^T(j+1) + H(j+1)P(j+1/j+1)H^T(j+1) \tag{22}$$

$$R(j) = \nu(j)\nu^T(j) + H(j)P(j/j)H^T(j) \tag{23}$$

Therefore $R(j+1)$ minus $R(j)$ equals:

$$\begin{aligned} R(j+1) - R(j) &= \nu(j+1)\nu^T(j+1) - \\ &\nu(j)\nu^T(j) + H(j+1)P(j+1/j+1)H^T(j+1) \\ &- H(j)P(j/j)H^T(j) \end{aligned} \tag{24}$$

The equation (24) can be written as:

$$R(j+1) = R(j) + \nu(j+1)\nu^T(j+1) - \nu(j)\nu^T(j) + H(j+1)P(j+1/j+1)H^T(j+1) - H(j)P(j/j)H^T(j) \quad (25)$$

If measurements are linear, than $H(j+1) = H(j)$ and the expression (25) can be written in simple form as:

$$R(j+1) = R(j) + \nu(j+1)\nu^T(j+1) - \nu(j)\nu^T(j) - H\left[P(j/j) - P(j+1/j+1)\right]H^T \quad (26)$$

The resulting expression (26) makes it possible to recursively estimate the measurement noise covariance for the Kalman filter tuning. Below the RKF with recursive estimation of measurement noise covariance is applied for the UAV dynamics model.

If a measurement bias occurs at the iteration step $j = \tau$, and the biased residual sequence is denoted by $\nu_b(j)$, then the biased residual is defined as:

$$\nu_b(j) = \nu(j) \qquad j=1,2,\dots\ \tau-1 \quad (27)$$

$$\nu_b(j+1) = \nu(j+1) + \mu(j+1) \quad j=\tau+1,\ \tau+2,\dots \quad (28)$$

When $j<\tau$, the mathematical expectation of the real residual covariance matrix (28) can be determined by the following equation:

$$E\left[C(j)\right] = P_\nu(k) = R(j) - H(j)P(j/j)H^T(j) \quad (29)$$

In the case of $j>\tau$, in the real residual covariance, a biased values $\nu_b(j+1) = \nu(j+1) + \mu(j+1)$ is used instead of an unbiased value $\nu(j+1)$, where $\mu(j+1)$ is the residual bias

$$C_b(j) = \nu_b(j)\nu_b^T(j) \quad (30)$$

Remark. Note that the expected value of the residual $\nu_b(j)$ in this case is not zero, therefore the formula (30) is not a real covariance. This is the square of the residual. Bias type measurement fault may be converted to the square of residual and such types of faults can be compensated using covariance matching techniques.

Statement. For iteration steps $j>\tau$, measurement bias leads to an increase in the mathematical expectation of the square of residual.

Proof. It is proven in *Theorem 1* that the measurement bias will cause bias in the residual of the Kalman filter.

The mathematical expectation of the square of innovation (30) for $j>\tau$ can be written as:

$$E\left[C_b(j)\right] = E\left[\nu_b(j)\nu_b^T(j)\right] = E\left\{\left[\nu(j)+\mu(j)\right]\left[\nu(j)+\mu(j)\right]^T\right\}$$
$$= E\left[\nu(j)\nu^T(j) + \nu(j)\mu^T(j) + \mu(j)\nu^T(j) + \mu(j)\mu^T(j)\right] \quad (31)$$

Taking into account $E\left[\nu(j)\right] = 0$, and the absence of correlation between the parameters $\nu(j)$ and $\mu(j)$, we have

$$E\left[C_b(j)\right] = E\left[C(j)\right] + E\left[\mu(j)\mu^T(j)\right] \quad (32)$$

Expressions (11) and (32) prove the *Statement 1*. Consequently, the measurement bias will increase the mathematical expectation of the square of residual. It can be seen from the *Theorem 1* and the *Statement* above that the measurement bias is transferred to the residual bias and changes the mathematical expectation of the square of residual (21). As a result, the measurement bias is transferred to the mathematical expectation of the square of residual. Thus, the square of residual can be used to compensate of measurement bias.

Therefore, the measurement bias will increase the mathematical expectation of the square of residual (23). As a result, according to formulas (23) and (26), the measurement noise covariance matrix R will increase, resulting in a smaller Kalman gain, which will reduce the influence of measurements on the state update process and increase the influence of the mathematical model of the system. As a result, the robustness of the filter against the measurement bias fault is ensured and the deterioration of the estimation procedure caused by the measurement bias fault is prevented.

5 Results of Simulation

The proposed innovation-based adaptive KF algorithm is applied to the UAV platform dynamics model. As the experimental platform, the ZAGI UAV was selected, and Kalman filter applications were carried out while taking into consideration its dynamics and characteristics, [12]. In order to estimate the UAV state vector

$$x(k) = \left[\Delta u \quad \Delta w \quad \Delta q \quad \Delta\theta \quad \Delta h \quad \Delta\beta \quad \Delta p \quad \Delta r \quad \Delta\varphi\right]_k^T$$

the proposed residual-based RKF with R-adaptation and conventional LKF (4)-(7) is used. Here, Δu, Δv, Δw are the velocities along x, y and z directions in the body frame, Δp, Δq, Δr are the angular rates around x, y and z axes, $\Delta\theta$ is the pitch angle, $\Delta\phi$ is the roll angle, $\Delta\beta$ is the sideslip angle, Δh is the height.

Simulations are carried out in 1000 steps over a time frame of 100 seconds with a sampling time of 0.1 seconds. Two different types of measurement fault scenarios—constant bias in measurements and measurement noise increment—are taken into consideration during simulations to test the proposed residual-based RKF with recursive estimate of measurement noise covariance.

5.1 Constant Bias in Measurements

A constant bias term is added to the measurements of the pitch angle gyro after the 30th second of the simulation

$$z_\theta(j) = z_\theta(j) + v_\theta(j) + 0.5\,,\ (j \geq 300) \qquad (33)$$

The residual-based RKF with recursive R-adaptation simulation results for the pitch angle in the presence of pitch angle gyro bias are presented in Figure 1. The findings of the RKF's state estimation are compared to the actual values in the first section of the figure. The estimation error based on the actual values of the UAV states is depicted in the second portion of the picture. The estimation error variance is shown in the final section.

Figure 1 shows that the proposed residual-based RKF with recursive estimation of measurement noise covariance achieves estimation of the states accurately in the presence of bias at the pitch angle gyro. In this case, RKF gives sufficiently good estimation results by totally eliminating the estimation error caused by the bias in the pitch angle gyro.

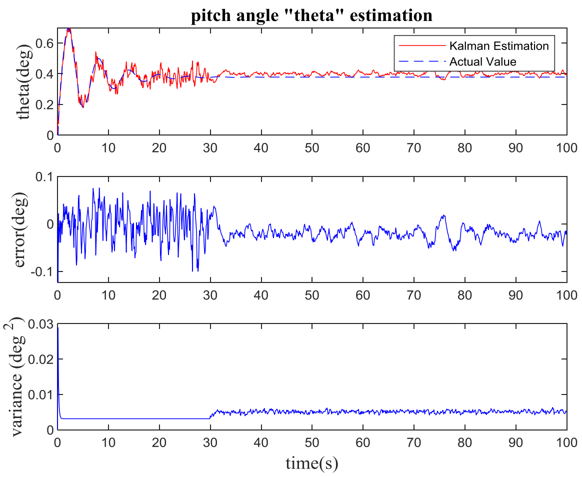

Fig. 1: Pitch angle estimation results using residual-based RKF with recursive R-adaptation in the presence of bias at the pitch angle gyro

Figure 2 displays the results of the conventional KF estimation for the pitch angle in the presence of pitch gyro bias. As can be seen, the conventional KF estimates shift after the 30th second of simulation (after the pitch angle gyroscope fails), and the estimation results are erroneous.

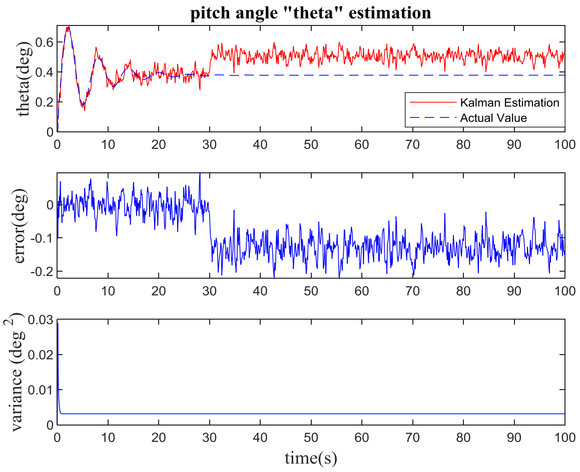

Fig. 2: Pitch angle estimation results using conventional KF in the presence of bias at the pitch angle gyro

5.2 Measurement Noise Increment

In the second measurement malfunction scenario, the measurement fault is defined as the pitch angle gyro measurement noise's standard deviation multiplied by a constant term after the 30th second:

$$z_\theta(j) = z_\theta(j) + v_\theta(j) \times 3\,,\ (\ j \geq 300)\,. \qquad (34)$$

The proposed residual-based RKF with recursive R-adaptation estimation results for the pitch angle in the presence of measurement noise increment at the pitch angle gyro are presented in Figure 3. As seen from the graphs presented in Figure 3, the proposed residual-based RKF with recursive estimation of measurement noise covariance gives sufficiently good estimation results in the presence of measurement noise increment at the pitch angle gyro.

Figure 4 displays the results of the conventional KF estimation for the pitch angle and roll rate in the presence of pitch gyro bias. As seen, the accuracy of conventional KF estimates deteriorates after the 30th second of simulation (after the pitch angle gyroscope fails).

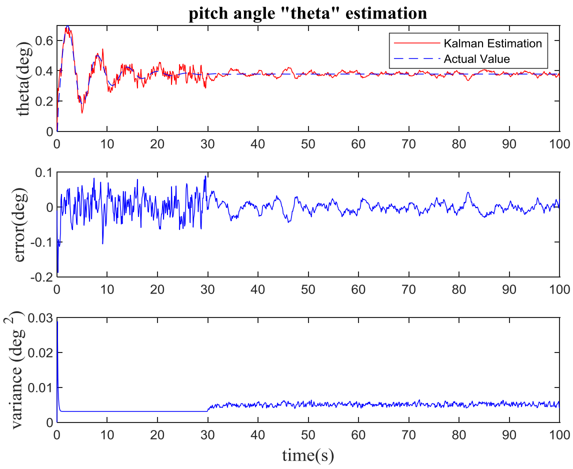

Fig. 3: Pitch angle estimation results using residual-based RKF with recursive R-adaptation in the presence of measurement noise increment at the pitch angle gyro

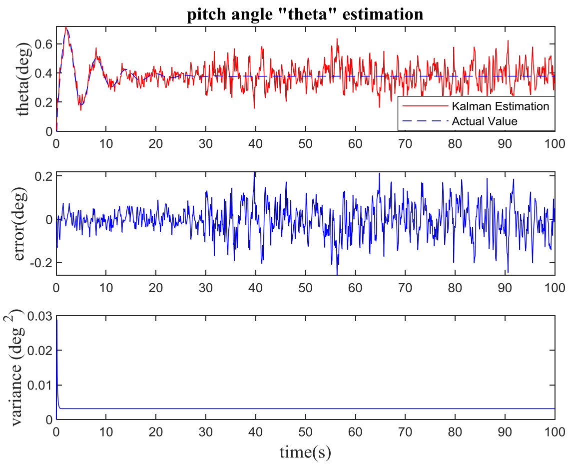

Fig. 4: Pitch angle estimation results using conventional KF in the presence of measurement noise increment at the pitch angle gyro

5.3 RMS Errors of the Innovation-based RKF

RMS errors of the residual-based RKF with recursive estimation of measurement noise covariance and conventional KF estimates in the presence of pitch angle gyro bias are presented in Table 1. As can be seen from the results presented in Table 1, the proposed RKF is superior for both longitudinal and lateral parameters in the presence of pitch angle gyro bias. RMS errors of conventional KF are sufficiently greater than the RMS errors of the proposed robust filter.

RMS errors of the proposed residual-based RKF and conventional KF estimates in the presence of measurement noise increment at the pitch angle gyro are presented in Table 2.

Table 1. RMS errors of the proposed RKF and conventional KF estimates in the presence of pitch angle gyro bias

Method	RKF	Conv.KF
Δu	0.1134	0.5309
Δw	0.0300	0.1738
Δq	0.0156	0.0651
$\Delta \theta$	0.0208	0.1357
Δh	0.8210	0.3068
$\Delta \beta$	0.0164	0.0286
Δp	0.0124	0.0386
Δr	0.0124	0.0285
$\Delta \phi$	0.0434	0.0475

Table 2. RMS errors of the proposed RKF and conventional KF estimates in the presence of measurement noise increment at the pitch angle gyro

Method	RKF	Conv.KF
Δu	0.1505	0.1782
Δw	0.0285	0.0847
Δq	0.0222	0.0648
$\Delta \theta$	0.0158	0.0749
Δh	1.4480	0.1523
$\Delta \beta$	0.0148	0.0289
Δp	0.0091	0.0413
Δr	0.0135	0.0284
$\Delta \phi$	0.0277	0.0497

The presented in Table 2 results show that the residual-based RKF with recursive R-adaptation gives better results for both longitudinal and lateral parameters in the presence of measurement noise increment at the pitch angle gyro. The RMSE results of conventional KF are worse compared to the robust filter.

In all investigated sensor fault sceneries, the proposed residual-based RKF gives better estimation results than the conventional KF.

6 Conclusion

This study proposes a novel recursive method for estimating measurement noise covariance for Kalman filter tuning. Based on the covariance difference approach to recursively estimate the measurement noise covariance, a residual-based robust Kalman filter against sensor faults is presented. The sensor fault compensation in this filter is accomplished with a simple change to the conventional KF.

The proposed residual-based RKF with recursive estimation of measurement noise covariance is applied to the UAV dynamics model. Two alternative scenarios of measurement error are evaluated on algorithms; constant bias at measurements (additive sensor faults) and measurement noise increments (multiplicative sensor faults). The simulation results show that the proposed residual-based RKF with recursive R-adaptation can accurately estimate the UAV dynamics in real-time in the presence of various types of sensor faults.

Estimation accuracies of the proposed residual-based RKF and conventional KF are compared. In all investigated sensor fault sceneries, the results of the proposed RKF are superior. The conventional KF gives the worst estimation results in the presence of sensor faults.

The residual-based RKF with recursive estimation of measurement noise covariance can be recommended as the reliable UAV state estimator in the flight control system in the presence of sensor faults.

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- Ulviye Hacizade has organized and executed the investigation, validation, simulation, data curation, formal analysis.

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