Differential Games: A New Perspective with Artificial Intelligence

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Abstract: Differential Game Theory has seen significant advancements in recent years, driven by numerous scholars exploring theoretical and practical aspects of the field. In this paper, we summarize the principal findings in pursuit-evasion games, focusing on Cop-Win and Robber-Win games on graphs. A dedicated section explores the demonstration technique introduced by G. Ibragimov, showcasing how evasion or capture can be achieved in a chase game by defining time intervals. The paper concludes by presenting key open problems in this area, with a special emphasis on applying artificial intelligence to trajectory prediction.

Key-Words: Differential games; Pursuit Evasion games; Cop-win games; robber-win game; Artificial Intelligence.

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1 Introduction

Differential games are a class of mathematical problems that model dynamic competitive or cooperative situations between two or more agents. They involve differential equations to describe the temporal evolution of the agents' strategies and choices, considering either conflicting or shared objectives. They find applications in economics, biology, robotics, and optimal control. Differential games are closely related to pursuit-evasion games, a specific subset where one or more agents (pursuers) aim to catch or track others (evaders) over time. This dynamic can be likened to the primal struggle between a lion chasing a gazelle, as seen in documentaries. The essence of these games lies in modeling such interactions mathematically. The differential games generalize finite extensive-form games, particularly those characterized as pursuit-evasion scenarios. These games, in fact, use differential equations to model the dynamics of motion and strategy, capturing the interplay between the pursuers' efforts to minimize the distance and the evaders' attempts to maximize it. Here, the evader's goal is to avoid capture, while the pursuer's objective is to achieve it. Then a pursuit-evasion game focuses on devising strategies for one or more pursuers to intercept or capture one or more evaders in motion. In a broader sense, such games typically involve N players with opposing objectives, where each player's goal is in direct conflict with the others. Every player aims to achieve their objectives, operating under the assumption that all participants act optimally to meet their goals. These objectives are mathematically defined by minimizing or maximizing a payoff function. In these games, the players' strategies are represented by the control function u_i, which is selected from a predefined set of possible options U_i. The choice of u_i is determined by the player's discretion, influenced by their understanding of the strategies employed by the other participants. Differential games defined in continuous time are analyzed using differential equations and share a strong connection with optimal control theory. PE games are often represented as endless differential games, where differential equations provide a framework to control and analyze a variety of real-world scenarios. Pursuit-evasion games are widely studied in fields like robotics, military strategy, and autonomous systems. Examples include a guided missile chasing an aircraft or a swarm of mobile robots surveilling and attempting to apprehend a target. Modeling player behaviors abstractly enables the formulation of strategies, defined as sequences of spatial positions that achieve the game's objectives. In this paper, we aim to introduce a possible scenario involving the application of AI to pursuit-evasion games. There are examples of applying AI to chase games such as [1] where authors used multi-agent reinforcement learning to develop strategies in two-player pursuit games with complex dynamics and sensory limitations. The results of this research were tested on real robots, demonstrating the effectiveness of the learned strategies. In [2], the authors established a model for attack and defense scenarios, utilizing twin delayed deep deterministic gradient algorithms to enhance decision-making in pursuit-evasion contexts. In [3], the authors apply generative machine learning models to optimize policies in pursuit-evasion games, highlighting the effectiveness of data-driven approaches in sequential decision-making problems. In [4] the authors addressed the coordination of agents in pursuit-evasion scenarios on graphs, utilizing pre-trained models to enhance decision-making strategies. In pursuit-evasion games, in fact, the analysis begins with defining the strategies adopted by evaders and pursuers during the game's evolution. These strategies represent predetermined paths. What would the scenario look like if the principles of AI could be used to predict the positions occupied by the players? What results can be achieved, and under what assumptions can a model of such nature be defined? Is it possible, starting from the models introduced in [5],[6],[7],[8], to apply deep learning principles to predict the opponent's moves and avoid them? Does the use of AI allow constants to be defined precisely?

2 History and Historical Development of PE Games

Pursuit-evasion problems date back to at least the 1700s when Bouguer considered scenarios involving pirates chasing merchant ships. In the 20th century, game theory emerged as a powerful mathematical tool to formalize and solve these problems. For example, the homicidal chauffeur problem features a pursuer and evader moving at finite speeds in an obstacle-free environment. Research in the 21st century includes the study of whether a pursuer can maintain the visibility of an evader in environments with polygonal obstacles. While a complete answer remains elusive, significant progress has been made, revealing connections to related pursuit-evasion problems. The field of differential games is generally considered to have been founded in [9]. His groundbreaking work established the theoretical groundwork for the investigation of strategic decision-making in differential equation-described dynamic systems. This field includes a wide range of tasks, including guarding problems, in which pursuers defend objectives from invading evaders, and search problems, in which the pursuer must find the evader. Every variant adds distinct dynamics that enhance the pursuit-evasion architecture as a whole. In [10] the author examined, in R^n, a basic motion differential game with numerous pursuers and a single evader, in which each player has the same dynamic possibilities. In particular, it was demonstrated that evasion is achievable if the evader's starting state falls within the convex hull of the pursuers' initial states, and that pursuit is possible otherwise. The technique of resolving functions for linear chase problems with several pursuers was created in [11]. In [12], the author developed the approach of resolving functions in the case of integral restrictions.

In [13] a linear differential game involving many pursuers and one evader was studied for the first time, where the control functions of players are subject to integral constraints. In particular, they focus on the possibility of escape or capture in terms of the players' energy. In particular, a differential pursuit-evasion game, with K pursuers and M and integral constraints, can be modeled through a series of differential equations as in the following system:

$$\dot{x}_i = u_i, \;\; x_i(0) = x_{i0}, \;\; i = 1, \ldots, K, \quad (1)$$
$$\dot{y}_j = v_j, \;\; y_j(0) = y_{j0}, \;\; j = 1, \ldots, M, \quad (2)$$

with integral constraints:

$$\int_0^\infty |u_i(s)|^2 ds \leq \rho_i^2, \; i = 1, \ldots, K, \quad (3)$$
$$\int_0^\infty |v_j(s)|^2 ds \leq \sigma_j^2, \; j = 1, \ldots, M, \quad (4)$$

where $x_i, y_j, u_i, v_j \in R^n$, $n \geq 2$, and ρ_i, σ_j are given positive numbers, $x_{i0} \neq y_{j0}$ for all $i = 1, \ldots, K$, and $j = 1, \ldots, M$.

The formulation of the problem solved in [13], with K pursuers and one evader, is the follows:

Theorem 1. *For the game (1)-(4), if $\rho_1^2 + \ldots + \rho_K^2 > \sigma_1^2$, then pursuit can be completed.*

In a similar way, in [14], it was shown that: if

$$\rho_1^2 + \ldots + \rho_K^2 \leq \sigma_1^2 \quad (5)$$

then the evasion is possible from some initial positions of players.

In general, an evasion game of K pursuers and $M = 1$ evader was studied in [15] and it was proven that if (5) holds, then for any initial positions of players evasion is possible.

In [15] the autors studied a pursuit game problem of K pursuers and M evaders described by equations (1)-(4) in a closed convex subset of R^n. It was established that if

$$\rho_1^2 + \ldots + \rho_K^2 > \sigma_1^2 + \ldots + \sigma_M^2, \tag{6}$$

then pursuit can be completed.

In [8], the authors studied the game (1-(4) in R^n in the case where

$$\rho_1^2 + \ldots + \rho_K^2 \leq \sigma_1^2 + \ldots + \sigma_M^2. \tag{7}$$

If this is the case, we show that evasion is possible from any initial positions of players.

This brief excursus aims to focus on mathematical problems and on some possible strategies aimed at resolving conflicting problems. The evolution of chase games has seen an ever-increasing activity from many researchers scattered around the world.

The result of this work was a different vision of classical problems through the use of theoretical mathematics, which gives these issues the rigor of an optimal solution. The impulse that technology is providing in terms of machine learning processes has revolutionized the way of tackling abstract problems by simulating physical realities and therefore declining and adapting abstract results to the complexity of the real world.

We have reached the point where moving objects can learn to deviate from their trajectory in the face of obstacles, even unpredictable ones. This is the case of ”driverless” driving, for example. A series of problems regarding the evolution of pursuit-evasion games in the age of AI therefore remain open. This paper aims to raise awareness in the academic world on these issues to create a new line of research that combines various aspects of scientific research in order to apply these results for military defense actions, for the defense of cyber attacks, as we will see in the case of PE games on the graphs.

3 The Ibragimov's Demonstration Technique

An excellent starting point for expanding pursuit-evasion games is to know how an approach to solving an escape or capture problem can become a technique to adapt to different cases, such as the simple, linear game. G. Ibragimov's seminal contributions [5] introduced a new demonstration technique, particularly in two-dimensional differential escape games involving multiple pursuers and an evader.

It can be observed, in fact, that there are similarities between the construction of evasion strategies and the main results obtained by the same author between 2012 and 2018. These similarities are:

1. the definition of time intervals;
2. the construction of a strategy for the evader which allows the evader(s) to use a manoeuvre on against the i-th pursuer;
3. estimating the distances between the evader and pursuers, and establishing that evasion is possible.

The proof is divided into a reduction part, followed by construction of the strategies and possibility of the evasion, checking the admissibility, and estimating distances between the evader and pursuers [7].

For the first time in [7] a two-dimensional pursuit-evasion game is studied, with several pursuers and an evader, with integral constraints on the control functions of the players, assuming that if the total resource of the pursuers does not exceed that of the evader then the escape it is possible.

The game that was solved in [7] is the following: we consider in R^2, n pursuers P_i, $i = 1, \ldots, n$, and one evader E, whose movements are described by the following differential equations:

$$P_i : \dot{x}_i = u_i, x_i(0) = x_{i0}, \tag{8}$$

$$E : \dot{y} = v, y(0) = y_0, \tag{9}$$

where $x_i, x_{i0}, u_i, y, y_0, v \in R^2$, u_i is the control parameter of the pursuer P_i and v is that of the evader E.

In the same paper, the authors introduce the definitions of admissible control of the pursuers and the evader, the evader's strategy through the following definitions:

Definition 1. *[7] A Borel measurable function $u_i(\cdot)$, $u_i : [0, \infty) \to R^2$ (respectively, $v(\cdot)$, $v : [0, \infty) \to R^2$) such that*

$$\left(\int_0^\infty |u_i(s)|^2 ds \right)^{1/2} \leq \rho_i, \quad \left([\left(\int_0^\infty |v(s)|^2 ds \right)^{1/2} \leq \sigma \right)$$

is called an admissible control of the pursuer P_i (evader E). Here ρ_i and σ are given positive numbers called the resources of the pursuer P_i and evader E, respectively.

Definition 2. *[7] The strategy of the evader E is a function $V = V(y, x_1, \ldots, x_k, u_1, \ldots, u_k)$, $V : R^{2k+1} \to R^2$, for which the system*

$$\dot{x}_i = u_i, \;\; x_i(0) = x_{i0}, \;\; i = 1, \ldots, K, \tag{10}$$

$$\dot{y} = V(y, x_1, \ldots, x_k, u_1, \ldots, u_k), \;\; y(0) = y_0, \tag{11}$$

has a unique absolutely continuous solution $(x_1(t), \ldots, x_k(t), y(t))$, for arbitrary admissible controls $u_i(\cdot)$ of the pursuers P_i. The strategy V is called admissible if each control generated by V is admissible.

Definition 3. *[7] Evasion for evader E from pursuers P_i is possible if there exists a strategy V of the evader E such that $x_i(t) \neq y(t), t > 0$ for any admissible controls $u_i(\cdot)$ of the pursuers P_i.*

It is important to underline how these introductions are the basis of the demonstrations of the papers that followed it and which that are remodulated these concepts following the steps of the demonstration technique just exposed.

Following this scheme, the main result was demonstrated in terms of the resources of the pursuers and evaders.

Theorem 2. *[7] If*

$$\sigma \geq \left(\sum_{i=1}^{k} \rho_i^2\right)^{1/2} := \rho$$

then evasion is possible in the game 8.

Using this result, in 2018 in [6] the game was extended to a number of evaders greater than 1.

In fact, in [6] a simple differential game of evasion of the movement of many pursuers and evaders has been studied, where the control functions of the players are subject to integral constraints. The game under consideration in [6] is described through the following different equations:

$$\dot{x}_i = u_i, \;\; x_i(0) = x_{i0}, \;\; i = 1, \ldots, K, \qquad (12)$$
$$\dot{y_j} = v_j, \;\; yJ(0) = y_{j0}, \;\; j = 1, \ldots, M, \qquad (13)$$

where $x_i, x_{i0}, u_i, y_j, y_j, v_j \in R^n, n \geq 2$, u_i is the control parameter of the pursuer P_i and v_j is that of the evader E_j.

Control systems with integral constraints on control functions are as follows:

$$\int_0^\infty |u_i|^2 \leq \rho_i^2, \;\; i = 1, \ldots, K \qquad (14)$$
$$\int_0^\infty |v_j|^2 \leq \sigma_j^2, \;\; j = 1, \ldots, M. \qquad (15)$$

The control resource is exhausted by consumption, such as energy, finance, and food.

The main result of [6] is:

Theorem 3. [6] *If $\rho = (\rho_1^2 + \ldots + \rho_K^2)^{1/2}$ and $\sigma = (\sigma_1^2 + \ldots + \sigma_M^2)^{1/2}$ and $\sigma \geq \rho$, then for any initial position of players, evasion is possible in the game 12.*

4 The Cop-win and the Robber-win Games

Let G be a finite, connected, and undirected graph. Two players, the cop (C) and the robber (R), take turns occupying vertices of G and moving along its edges. The cop wins by reaching the same vertex as the robber, thereby capturing them; otherwise, the robber wins. This scenario represents a classic pursuit-evasion game. In [16] the authors identified the exact class of graphs, known as cop-win graphs, where the cop is guaranteed to win.

The cop selects their initial vertex first, followed by the robber. The game then progresses in alternating turns, with the cop moving first. On a turn, a player may either move to an adjacent vertex or remain stationary. The game concludes with a win for the cop if they manage to occupy the same vertex as the robber. Conversely, the robber wins by evading capture indefinitely. A graph is classified as a cop-win graph if the cop can always guarantee a win, regardless of the players' starting positions. Graphs that are not cop-win are referred to as robber-win graphs.

Definition 4. *A strategy for the cop is given by the function $s_p : V^2 \times T^{even} \to V$, where T^{even} is the set of all even numbers, whereas a strategy for the robber is given by the function $s_e : V^2 \times T^{odd} \to V$, where T^{odd} is the set of all odd numbers. Let S_p denote the set of all possible strategies of the cop and S_e denote the set of all possible strategies of the robber.*

It is clear that for each graph G one of the two players wins. For instance, considered a winning strategy for C, C should capture R at most after $n(n-1)+1$ moves, where we denote by n the number of vertices in G.

In light of the above, it seems natural to ask what the minimum number of cops can be that can be added to the game so that the graph becomes cop-win (see,[17]).

In [18] the authors introduced different notions of cop-wins and thief-wins games based on decreasing order of strength.

Definition 5. *We say that graph $G = (V, E)$ is*

1. *strongly cop-win iff $\exists s_p \in S_p$ such that $\forall (v_{p_0}, v_{e_0} \in) V^2$ and $\forall s_e \in S_e$, we have $v_{p_t} = v_{e_t}$ for some t.*

2. *strongly robber-win iff* $\exists s_e \in S_e$ *such that* $\forall (v_{p_0}, v_{e_0}) \in V^2$ *and* $\forall s_p \in S_p$*, we have* $v_{p_t} = v_{e_t}$ *for all* t*.*

3. *is cop-win iff* $\exists s_p \in S_p$ *and* $\exists v_{p_0} \in V$ *such that* $\forall v_{e_0} \in V$ *and* $\forall s_e \in S_e$*,weha* $v_{p_t} = v_{e_t}$ *for some* t*.*

4. *is robber-win iff* $\exists s_e \in S_e$ *and* $\exists v_{e_0} \in V$*, such that* $\forall v_{p_0} \in V$ *and* $\forall s_p \in S_p$*, we have* $v_{p_t} = v_{e_t}$*, for all* t*.*

5. *is weakly cop-win iff* $\exists v_{p_0} \in V$ *and* $\exists s_p \in S_p$ *such that* $\forall s_e \in S_e$*,we have* $v_{p_t} = v_{e_t}$ *for some* t *and for some* $v_{e_0} \in V$*.*

6. *is weakly robber-win iff* $\exists v_{e_0} \in V$ *and* $\exists s_e \in S_e$*, such that* $\forall s_p \in S_p$*,we have* $v_{p_t} = v_{e_t}$ *for all* t *and for some* $v_{p_0} \in V$*.*

One of the most important theorems in this class of games is the one in [16] which involves the notion of dismantlable; meaning that graphs can be reduced a single vertex through a sequence of retracts.

In the context of graphs, a vertex v is said to be *dominated* by another vertex w if $N[v] \subseteq N[w]$, meaning v and w are adjacent, and every neighbor of v is also a neighbor of w. We denote by $N[v]$ the closed neighborhood of a vertex v, i.e. the vertex v and all vertices adjacent to it.

A vertex dominated by another is called an *irreducible vertex*, as defined in [16]. A *dismantling order* of a graph is an arrangement of its vertices such that, during sequential removal of vertices in this order, each removed vertex (except the final one) is dominated at the time of removal. A graph is considered dismantlable if and only if it admits such an ordering.

A representation of a dismantlable graph introduced in [18] is the following Figure 1.

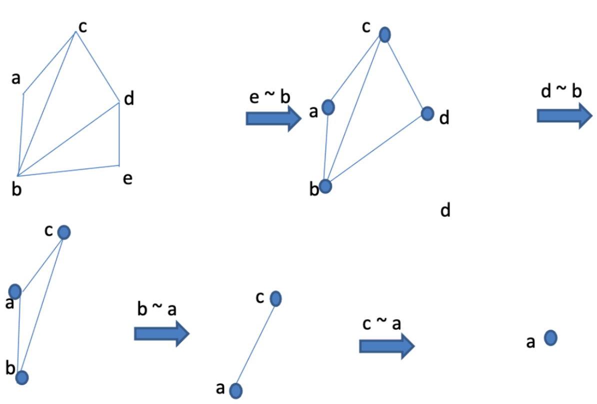

Figure 1: Dismantlable graph

Theorem 4. *[16] G is cop-win iff it is dismantlable.*

This theory has numerous applications, including artificial intelligence and robotics path planning. For instance, in a competitive setting, a robot may employ tactics based on cop-win graphs to intercept a target or evade capture.Cop-win tactics can be applied to AI-based surveillance systems to optimize the movement and positioning of patrol agents (such as security robots or drones) in order to monitor regions and apprehend intruders. AI can employ dismantlability features to discover the best patrol routes and guarantee coverage by representing the environment as a graph. An essential component of game theory and multi-agent systems, adversarial reasoning is shown by op-win games. These concepts can be used by AI systems to model and forecast adversary behavior in competitive situations, such as financial trading or cybersecurity (e.g., by simulating attacker-defender interactions).

5 Further Developments and Open Problems

Numerous challenges remain in pursuit-evasion games. For instance, studying capture and evasion strategies using second-order differential equations offers fertile ground for exploration. One promising avenue involves applying artificial intelligence (AI) to these games. AI could help predict player positions by leveraging strategies defined through functions that satisfy imposed constraints.

Graph theory plays a pivotal role here, modeling and analyzing relationships in environments such as social networks or cybersecurity. For example, a graph-based chase game could model a cyberattack, where a virus (robber) and antivirus (cop) compete. Using AI, one could predict attack patterns and mitigate risks.

In the realm of social media, pursuit-evasion games could optimize user engagement by modeling interactions and data exchange online. These possibilities highlight the potential of integrating AI with pursuit-evasion frameworks.

In this framework, we are going to develop in a next paper new modeling—starting from the techniques and mathematical tools that have been presented by this note—which combines three selected machine learning techniques: genetic algorithms, k-nearest neighbor learners, and reinforcement learning.

Here is a sketch of our approach and the essential line of research that it will be developed:

5.1 κ-nearest Neighbor Learner

Our idea is to use a 1-nearest neighbor algorithm. The essential principle is that the agent maintains a database of past circumstances, or ”cases.” The database entry that most closely resembles the current circumstance is obtained whenever the agent is required to make a decision, and the action recommended by this case is carried out. The same range, bearing, and heading criteria that were applied to the genetic algorithm learner are utilized to determine each state. The total of the normalized feature (F) distances (measured by a norm) is the similarity metric, denoted by Similiarity$(.,.)$ as below:

$$\text{Similarity}(F_i, F_j) = \sum_{k=1}^{3} \frac{|F_{ik} - F_{jk}|}{max||x||_k}$$

The agent would perform random acts while learning. Every circumstance that arose during this exploration stage was documented. It is regarded as a potential case candidate to be added to the database if the game was successful, meaning the evader was either caught or not caught. The majority of case-based reasoning systems have issues with case-based maintenance. The database's size is constrained by memory and processing limitations. If it hasn't already been absorbed by an existing case in the database, a new case will be added following a learning session. If adding this case exceeds the case base's maximum size for a particular action, a randomly selected case.

5.2 Genetic Algorithms and Reinforcement Learning. The “Learner” Case

We are also implementing a standard “L” learner. Usually learners used the standard L learning update rule shortly presented below. However, since the main aim of the Pursuer and Evader are opposite, different “reward” algorithms can be used for the two potential players.

Update: $R(x) = R(x) + \rho \cdot (R(i) + \delta(L(y) - R(x)))$

Reward Evader: $x \cdot t$ iff E is caught at time t

Reward Pursuer: $R(E) = x \cdot (T - t)$ iff P catches E at time t

6 Conclusion

In this article we have retraced some fundamental steps of differential game theory with a specific focus on pursuit-evasion games. The elements under consideration represent a very robust mathematical framework for addressing this class of games, which has many real-world applications. From robotics to military defense, from cyber attacks to the use of driverless cars, in fact, We can observe how the theory of differential games defines the essential mathematical framework for their description.

For this reason we would like to introduce into this research some ideas that lead to the application of AI to better manage this class of problems. This new model involves the combination of three selected machine learning techniques: genetic algorithms, k nearest neighbor learners, and reinforcement learning. The intent is to arouse a new fervor in the world of research in this sector to define an implementation of this classic and current theme.

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1. the choice of main articles to describe the literature review (see introduction and paragraph 1,2)
2. for the final revision of the document and Conclusions.

The article, after the application of this software, was further revised by the authors, who added new elements useful for improving the article. The authors assume full responsibility for the content of the publication.

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