Two isomorphic forms of quantum e.-m. field existence and Lamb and Casimir effects

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Abstract: - It is shown that the existence of Lamb and Casimir experiment results indebted to quantum e.-m. field of a photon with «deinterlaced» spin components existing at the same time with usual quantum e.-m. field and attributed as e.-m. field that exist outside the light cone.

Key-Words: - Casimir force, Feynman paths, generalized quantum mechanics, generalized path integral, Lamb shift, quantum field of a photon.

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1 Introduction

The central point of the article, as well as, [1], is the study of the quantum nature of a free e.-m. Majorana–Maxwell field existing only for non-zero Planck constant $\hbar$, although this field itself does not depend on $\hbar$ explicitly. This statement leads to the study of the quantum e.-m. field of the photon with «deinterlaced» spin components (the generalized quantum Cauchy field), which was attributed in the work, [1], as quantum field of a photon existing in exterior of the light cone. Remark, that the reality of the existence of such physical object with paradoxical properties is confirmed by experimental work, [2].

Note, that the presence of such quantum field of a photon leads to discovery of mathematically correct «Feynman path integral» in electrodynamics, which should now be understood as a quantum theory.

In present work «Lamb shift» i. e. a small shift of energy levels of ground state an electron in the Hydrogen atom [3], [4], [5], calculated by Dirac, is considered and calculated as taking into account action on electron by quantum e.-m. filed of «deinterlaced» photon of the Hydrogen nucleus charge.

The article uses the notation and terminology accepted in first book of the monographs «Generalized functions» [6]. The light velocity c and Planck constant $\hbar$ are assumed equaled 1.

Before reading present article it is desirable to acquainting with article, [1].

2 Spherically symmetric generalized quantum Cauchy e.-m. field

Consider arbitrary generalized quantum e.-m. field in Majorana variables at first [7]

$$M_t(x) = E_t(x)+iH_t(x), \quad \bar{M}_t(x) = E_t(x)-iH_t(x).$$

In momentum representation, its components satisfy the equations

$$i\frac{\partial}{\partial t}\tilde{M}_t(p) = (S,p)\tilde{M}_t(p),$$
$$i\frac{\partial}{\partial t}\tilde{\bar{M}}_t(p) = -(S,p)\tilde{\bar{M}}_t(p),$$

where

$$(S,p) = \sum_{j=1}^{3} s^j p_j,$$

$$s^1 = \begin{pmatrix} 0, & 0, & 0 \\ 0, & 0, & -i \\ 0, & i, & 0 \end{pmatrix}, \quad s^2 = \begin{pmatrix} 0, & 0, & i \\ 0, & 0, & 0 \\ -i, & 0, & 0 \end{pmatrix},$$
$$s^3 = \begin{pmatrix} 0, & -i, & 0 \\ i, & 0, & 0 \\ 0, & 0, & 0 \end{pmatrix}.$$

Here s^1, s^2, s^3 is infinitesimal spin operators of a photon [8].

Recall, that despite the fact that the photon spin is $\hbar$, the Maxwell–Majorana equations clearly do not depend on the quantity $\hbar$, but only if $\hbar \neq 0$. That is why an e.-m. field always is quantum (at $\hbar = 0$ the Maxwell equations just disappear).

Remark, that therefore, for a photon that has no classic limit, the energy density can be interpreted as the probability density of such state occurring, cf, [7].

Also take into account, that when ground electronic state forming in hydrogen atom only a spherically symmetric quantum e.-m. field is involved.

The system of Maxwell–Majorana equations is separated on equations for a photon $\tilde{M}_t(p)$ and antiphoton $\tilde{\bar{M}}_t(p)$ in momentum representation [7]; consider the momentum representation of the retarded Green function for a photon

$$\tilde{M}_t(p) = \exp(-it(S,p)).$$

Since (S,p) is Hermitian matrix, it can be reduced to diagonal form by certain unitary transform $\tilde{Q}(p)$. It is easy to see that the momentum representation of the Green function

$$\tilde{\mu}_t(p) = \exp\left(-it\begin{pmatrix}|p|, & 0, & 0\\ 0, & -|p|, & 0\\ 0, & 0, & 0\end{pmatrix}\right)$$

for the new quantum e.-m. field ($|p| = \sqrt{{p_1}^2+{p_2}^2+{p_3}^2}$) characterized by no spin interaction of the photon components (photon with «deinterlaced» spins) has arisen at this.

Remark, that the existence of such quantum e.-m. field is proved by the last experiments, [2], (where such photons are called as «entangled photons»).

Bearing in mind the constructing of the coordinate representation $\mu_t(x)$, we will consider its Fourier image $\tilde{\mu}_t(p)$ as an analytic functional on testing functions $\psi(p) \in Z$; then its preimage $\mu_t(x)$ will be a functional on finite (bump) functions (the Paley–Wiener theorem, [6]).

Therefore, the Green function of a «deinterlaced» photon arises as matrix-valued generalized function

$$\int \bar{\mu}_t(x)\begin{pmatrix}\varphi_1(x)\\ \varphi_2(x)\end{pmatrix} dx$$

on a columns of bump functions.

To simplify the record it will be considered only scalar form $C_{it}(x)$.
It is known that Fourier preimage of the analytic functional $\exp(-it|p|)$ is the space quantum Cauchy functional

$$C_{it}(x) = \frac{\delta^{S_t}(x)}{4\pi t^2} + \frac{2i}{\pi^2}\cdot\frac{t}{(t^2-|x|^2)^2}$$

(here δ^{S_t} is δ-function on a sphere of radius t) on bump functions $\varphi(x) \in K$.

Remark, that the functional

$$\int \bar{C}_{it}(x-\alpha)\psi_0(\alpha)\,d\alpha = \psi_t(x)$$

defines physically interpreted a photon quantum state as the solution of a Cauchy problem, if consider complex-valued bump functions

$$\psi_0(x) = \varphi(x) + i\phi(x) \qquad (\varphi(x), \phi(x) \in K)$$

as initial state of the quantum field [1].

Thus, along with the generalized quantum process $M_t(x)$ its the quantum-theoretic unitarily equivalent double $\mu_t(x)$ has arisen, and also the existence of this double helped to discover the continuation of process $M_t(x)$ to σ-additive generalized quantum measure in the space dual to the space of instantaneous Hilbert velocities of a photon.

Remark, that the Green function of «classic» e.-m. Maxwell field, the evolution process of which by the constructing Huygens–Fresnel envelope is given, turns out to be represented by the generalized state superposition of a photon on «Feynman paths» after indicated continuation. Here the «classic» Maxwell field is interpreted as average value of isochronal variation operator (Euler derivative) vanishing in the field state that representing this Green function expansion by «Feynman paths».

It is easy to see, that the resulting average value of operator on such quantum e.-m. field is the same as «classic» e.-m. field.

Returning to the problem on a relativistic electron in the field of the hydrogen atom nucleus considered by Dirac [3], [7], recall that here the nucleus e.-m. field supposed to be Coulomb, i. e. it is used the «classic» description variant of e.-m. field. Note, that here left over unclarified the cause of small shift of the ground state components of an electron observing experimentally, i. e. «Lamb shift».

Pass on to consider this phenomenon.

3 Attribution of quantum e.-m. field double of Hydrogen nucleus («deinterlaced» photon field) and Lamb effect

Up to this point the quantum field $C_{it}(x)$ of a «deinterlaced» photon was used only as a mathematic construction, which made it possible to construct a correct «path integral» in electrodynamics. However, as the consideration of the problem on Casimir forces is show that a quantum field exists in actuality as double of e.-m. field, existing in exterior of light cone simultaneously with usual e.-m. field, initiate «Lamb shift», cf. [9], [10].

Indeed, consider a physically interpreted state of the quantum photon field with «deinterlaced» spins [1], considering to simplify the record only one element of diagonal matrix

$$\psi_t(x) = \int \bar{C}_{it}(x-\alpha)\psi_0(\alpha)\,d\alpha = \\ = -\int \bar{C}_{it}(\alpha)\psi_0(x-\alpha)\,d\alpha\,.$$

Then, it is easy to see that a spherically symmetric state of this field with generalized Hamiltonian

$$\mathrm{H}(x) = 2\pi^{-1}|x|^{-4}$$

satisfy the Shrödinger equation

$$i\frac{\partial}{\partial t}\psi_t(x) = \frac{2}{\pi}\int \frac{\psi_t(x-\alpha)}{|\alpha|^4}\,d\alpha\,,$$

which at small Δt has a solution

$$\psi_{\Delta t}(x) = \psi_0(x) - \frac{2i\Delta t}{\pi}\int \frac{\psi_0(x-\alpha)}{|\alpha|^4}\,d\alpha\,.$$

Consider the generalized Hamiltonian

$$2\pi^{-1}\int |\alpha|^{-4}\psi_0(x-\alpha)\,d\alpha\,.$$

Passing to spheric coordinates, we have

$$\int |\alpha|^{-4}\psi_0(x-\alpha)\,d\alpha = \\ = 4\pi\int\limits_0^\infty |\alpha|^{-2}\overline{\psi_0(x-\alpha)}^{S_{|\alpha|}}\,d|\alpha|\,,$$

where $\overline{\psi_0(x-\alpha)}^{S_{|\alpha|}}$ is average over the sphere $S_{|\alpha|}$ of radius $|\alpha|$. Therefore, using the mean value theorem, we have

$$\int |\alpha|^{-4}\psi_0(x-\alpha)\,d\alpha = \\ = 4\pi\int\limits_0^\infty |\alpha'|^{-2}\psi_0(x-\alpha')\,d\alpha'\,,$$

where α' is certain fixed point on the sphere $S_{|\alpha'|}$. Therefore, it is easy to see that at x in $\psi_0(x-\alpha')$ it should consider only a component colineared α'.

Take advantage of result obtained in [6], according to which $\psi_0(x-\alpha')$ is even bump function of $|\alpha'|$. Therefore, we have

$$\int\limits_0^\infty |\alpha'|^{-2}\psi_0(x-|\alpha'|)\,d|\alpha'| = \frac{1}{2}\int \alpha^{-2}\psi_0(x-\alpha)\,d\alpha\,.$$

According to [6, p. 74, formula 7], the latter integral should be understandable as

$$\left(\int_{-\infty}^{-\varepsilon} + \int_{\varepsilon}^{\infty}\right)\alpha^{-2}\psi_0(x-\alpha)\,d\alpha$$

when $\varepsilon > 0$. Thus, the Hamiltonian of considered quantum e.-m. field is defined only when $|x| \geqslant \varepsilon$, which allows to attribute this generalized e.-m. field as existing only outside the light cone.

Therefore, returning to original formulation of the problem, we have

$$\psi_{\Delta t}(x) - \psi_0(x) = \Delta_t\psi_0(x) = \\ = -4i\Delta t\left(\int_{-\infty}^{-\varepsilon} + \int_{\varepsilon}^{\infty}\right)\alpha^{-2}\psi_0(x-\alpha)\,d\alpha\,.$$

Recall [1], that when solving the problem on calculation of attraction force between mirrors (Casimir forces) a similar formula

$$\Delta_t\psi_0(x) = \frac{-i\Delta t}{\pi}\left(\int_{-\infty}^{-\varepsilon} + \int_{\varepsilon}^{\infty}\right)\alpha^{-2}\psi_0(x-\alpha)\,d\alpha$$

led to origin of Casimir forces interpreting as the square of the average of «Casimir operator» $\frac{\partial}{\partial\varepsilon}$ in the field state ψ_ε (the distance between mirrors is 2ε):

$$\overline{\frac{\partial}{\partial\varepsilon}} = \frac{\int \bar{\psi_\varepsilon}(z)\frac{\partial}{\partial\varepsilon}\psi_\varepsilon(z)\,dz}{\int \bar{\psi_\varepsilon}(z)\psi_\varepsilon(z)\,dz} = \\ = \frac{\int \bar{\psi_\varepsilon}(z)(\overleftarrow{\varepsilon}^{-2} + \overrightarrow{\varepsilon}^{-2})\psi_0(z-\varepsilon)\,dz}{\int \bar{\psi_\varepsilon}(z)\psi_\varepsilon(z)\,dz};$$

acting between each area unit of mirrors. In considered problem on calculation «Lamb shift» the atomic envelope of the hydrogen in ground state plays role of mirrors. Thus, the Casimir forces between area elements of hydrogen atomic envelope, that are lying on endpoints of each diameters, resulting in a force contracting the atomic envelope (pressure force). Remark, that at here this pressure force does not depend on Hydrogen nucleus charge — cf. with force in Casimir effect. The potential macroscopic energy

$$V(r) = \begin{cases} \dfrac{4}{3}\dfrac{e}{r^3}, & a_0 \leqslant r < \infty \\ 0, & r < a_0 \end{cases}$$

of the double of ground Coulomb field of the hydrogen nucleus corresponding to this force (here a_0 is Bohr radius), that leads to small energy level shift of first excited and ground energy levels of hydrogen atom, i. e. to «Lamb shift».

4 Conclusion

Thus, both the «Casimir force» and «Lamb shift» is due to existence of the generalized quantum e.-m. field of a photon with «deinterlaced» spin components (to generalized quantum e.-m. Cauchy field), which is attributed as dual form of quantum e.-m. field existing in exterior of the light cone simultaneously with usual e.-m. field of hydrogen nucleus and leading to «Lamb shift».

Therefore, both the Casimir mirrors and hydrogen atom are turned to be a detectors of the quantum e.-m. field double.

5 Addition

Remark, that the quantum e.-m. field of a «deinterlaced» photon in nature (in experiment) arises and exists only simultaneously with Maxwellian Lorentz covariant field. Therefore, this pair of quantum fields is written in the same way in all Lorentz reference system.

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