A Novel Semi - analytical Regular and Singular Solution for Calculating Mutual Inductance between Inclined Solenoids in Crossing or Contact Positions

SLOBODAN BABIC[1], NIKOS E. MASTORAKIS[2]
[1]Independent Consultant,
53 Berlioz 101, H3E 1N2, Montréal, Québec,
CANADA

[2]Technical University of Sofia,
English Language Faculty of Engineering
Sofia
BULGARIA
and
Hellenic Naval Academy,
Sector of Electrical Engineering and Computer Science, Piraeus
GREECE

Abstract: - In this paper, we introduce new formulas for calculating the mutual inductance between two inclined solenoids positioned in free space. We employ the filament method to derive these calculations and explore the optimal number of subdivisions for coil representations, balancing computational efficiency and accuracy. Notably, we present the first calculations in the literature addressing singular cases where solenoids intersect, acknowledging the theoretical implications of these intersections despite their physical improbability. Our results appear reasonable and may challenge engineers and physicists to evaluate their analytical and semi-analytical methods in this context. We provide examples illustrating both regular and singular cases, demonstrating the applicability of our findings.

Key-Words: - Inclined solenoids, mutual inductance, radial subdivisions of coils, axial subdivisions of coils, singular cases, Gaussian numerical integration, complete elliptical integrals of the first and second kind.

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1 Introduction

A new method has been developed for calculating the mutual inductance between two inclined solenoids in any relative position and angular orientation. This method covers all scenarios, whether regular or singular, including cases where the solenoids intersect or are in contact. There are relatively few papers on this topic in the literatures [1], [2], [3], [4], [5], [6] and [7] and the existing ones primarily rely on solutions involving special functions such as Bessel, Struve, and Hypergeometric series, which often require numerical integration. An excellent contribution in this domain was made in [3].

However, engineers, physicists, and others working in this field may not be familiar with these special functions and often prefer easier, faster, and more precise methods for obtaining practical solutions. This new method offers a valuable contribution to the treatment of inclined solenoids in any desired orientation, enabling the calculation of key electromagnetic quantities such as mutual inductance, magnetic force, torque, and stiffness. The method leverages the filament technique, which is known for its accuracy and efficiency, minimizing computational time. A particularly significant contribution of this method is its handling of singular cases where solenoids intersect, even though such cases may be physically controversial. This is critical from an engineering perspective, where solutions must be both highly accurate and computationally efficient.

The filament method used in this approach replaces each solenoid with a set of filamentary Maxwell loops, distributed in a way that respects the uniform current distribution over the cross-section and coil dimensions. The coils are divided axially, with each segment representing a loop carrying the corresponding current. The number of segments depends strictly on the coil dimensions, which is crucial for optimizing both accuracy and computation time. The optimal balance between the number of axial divisions and computational efficiency is explored in detail in [1] and [2]. All calculations are implemented using MATLAB, and the provided examples can serve as benchmark problems for testing other methods that address similar challenges.

Additionally, this approach provides a comprehensive method to calculate all combinations of inclined coils (e.g., coil-solenoid, two disk coils), ensuring consideration of both conventional and non-conventional coils.

2 Problem Formulation

To calculate the mutual inductance between two inclined solenoids positioned in any desired configuration (Figure 1 and Figure 2), we use the fundamental general formulas provided in [1] and [2]: Which calculates the mutual inductance between two loops in arbitrary positions. In [1] and [2], we developed highly effective formulas for calculating the mutual inductance between two inclined coils with rectangular cross-sections in any position, as well as between circular coils with rectangular cross-sections and parallel axes.

By applying certain modifications to these formulas—specifically by neglecting the coil thickness in the case of rectangular cross-sections—we derive the mutual inductance between two inclined solenoids in any desired orientation as follows:

$$M = \frac{N_1 N_2}{(2K+1)(2m+1)} \sum_{g=-K}^{g=K} \sum_{p=-m}^{p=m} M(g,p) \qquad (1)$$

$$M(g,p) = \frac{\mu_0 R_S}{\pi} \sqrt{Ll}\ I \qquad (2)$$

where,

$$I = \int_0^{2\pi} \frac{p_1 \cos(t) + p_2 \sin(t) + p_3}{\sqrt{kV_1^3}} \phi(k) dt$$

$$x = x_C + \frac{ba_1}{(2m+1)} p\,,$$

$$y = y_C + \frac{bb_1}{(2m+1)} p,$$

$$z = z_C - \frac{a}{(2K+1)} g + \frac{bc_1}{(2m+1)} p$$

$$(g = -K, \dots, 0, \dots, K;\ p = -m, \dots, 0, \dots, m)$$

$$\vec{N} = \left\{\frac{a_1}{L}, \frac{b_1}{L}, \frac{c_1}{L}\right\} = \{n_x, n_y, n_z\}$$

$$L = \sqrt{a_1^2 + b_1^2 + c_1^2}\ ,\ l = \sqrt{a_1^2 + c_1^2}$$

$$\alpha = \frac{R_S}{R_P}, \qquad \beta = \frac{x}{R_P}\ , \qquad \gamma = \frac{y}{R_P}\ , \qquad \delta = \frac{z}{R_P}$$

$$p_1 = L\gamma c_1,\ \ p_2 = -(\beta l^2 + \gamma a_1 b_1),\ \ p_3 = l\alpha c_1$$

$$p_4 = -(\beta a_1 b_1 - \gamma l^2 + \delta b_1 c_1)$$

$$p_5 = -L(\beta c_1 - \delta a_1)$$

$$V_1 = L^2 l^2 (\beta^2 + \gamma^2) + \alpha^2 (L^2 l^2 - b_1^2 c_1^2) cos^2(t) +$$

$$\alpha^2 L^2 c_1^2 sin^2(t) + \alpha^2 L a_1 b_1 c_1 \sin(2t) -$$

$$2Ll\alpha(\beta a_1 b_1 - \gamma l^2)\cos(t) - 2L^2 l\ \alpha\beta c_1 ain(t)$$

$$A = Ll(1 + \alpha^2 + \beta^2 + \gamma^2 + \delta^2) +$$

$$2\alpha[p_4 \cos(t) + p_5 \sin(t)]$$

$$k^2 = 4V_1/(2V_1 + A)$$

$$\phi(k) = \left(1 - \frac{k^2}{2}\right) K(k) - E(k)$$

$K(k)$ and E(k) are the complete elliptical integrals of the first and second kind, [8] and [9].

Special case:

$$a_1 = c_1 = 0 \rightarrow l = 0$$

$$u_x = 0, \qquad u_y = 0, \qquad u_z = 1$$

$$v_x = 1, \qquad v_y = 0, \qquad v_z = 0$$

The parameters used in this method are defined as follows:

- R_P — inner radius of the primary solenoid, which is positioned parallel to the Z-axis with its center at the origin.
- R_S — outer radius of the secondary inclined solenoid, whose center is located at point C, the origin of the system x′, y', and z'.
- N_1 — number of turns in the primary solenoid.
- N_2 — number of turns in the secondary solenoid.
- a and b — axial heights of the primary and secondary solenoids, respectively.
- d — perpendicular displacement between the axes of the solenoids.
- **c** — displacement between the planes of the centers of the solenoids.
- K and m — number of axial subdivisions for primary and secondary solenoids,
- a_1, b_1 and c_1— the component of the unit vector $\vec{N}$ positioned at point C following the axis z'.

In this paper, we employed Gaussian numerical integration for the evaluation of the single integrals [8] and [9].

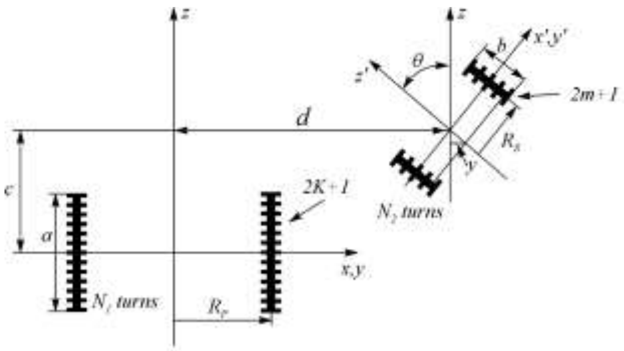

Fig. 1: Configuration of mesh matrix: Two inclined solenoids (axes intersect but not at the center of either)

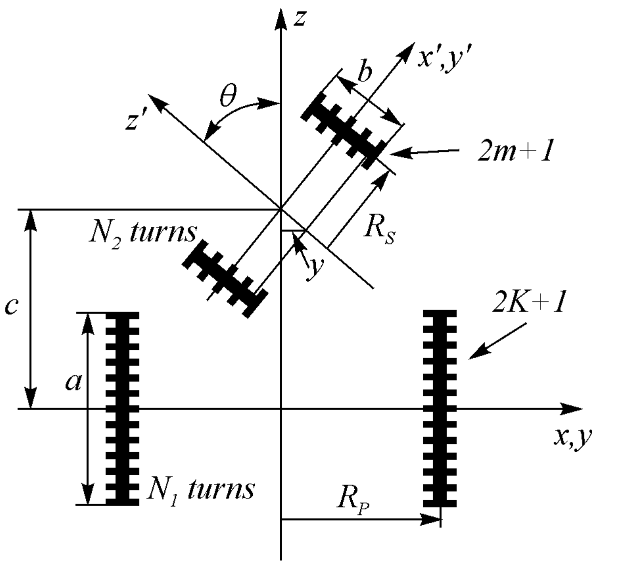

Fig. 2: Configuration of mesh matrix: Two inclined solenoids (axes intersect at the center of either)

3 The Optimal Choice of the Number of the Subdivisions

In [1] and [2], we discussed and presented the relationship between the radial and axial divisions as a function of the number of subdivisions for coils with rectangular cross-sections, whether inclined or with parallel axes.

Let us define the following radial distances:

- L_N— corresponding to the radial subdivision N,
- Ln — corresponding to the radial subdivision n.

$$L_N = R_2 = R_1 \rightarrow N = 0 \quad (3)$$

$$L_n = R_4 = R_3 \rightarrow n = 0 \quad (4)$$

Let us also define the following axial distances:
La — corresponding to the axial subdivision K,
Lb — corresponding to the axial subdivision m.

$$L_a = a \rightarrow K \quad (5)$$

$$L_b = b \rightarrow m \quad (6)$$

$L_a/L_b = a/b = K/m = [t\,] \leq 1 \; or \; [t] \geq 1$

We have only two dependable variables in the axial subdivisions of the solenoids.

Thus, the number of subdivisions for the solenoids are as follows:

If $[t] < 1$
$N = 0$, $K = K$, $n = 0$ and $m = 1/[t]\, K$

If $[t] > 1$
$N = 0$, $K = [t]\, m$, $n = 0$ and $m = m$

If $t = 1$
$N = 0$, $K = m$, $n = 0$ and $m = m$

In the case of solenoids, whether inclined or with parallel axes, the problem becomes significantly simpler because the two variables N and n are zero. This reduces the problem to two variables, K and m, which are linearly dependent. The selection of these variables, as well as the iterative process used to determine the optimal balance between minimal computational time and maximum accuracy, is detailed in [1] and [2].

4 Singular Treatment

The singular cases will be solved by ansatz. Singular cases often arise in electromagnetic systems when there are specific configurations (e.g., coils in direct contact, overlapping fields) that lead to undefined or infinite values in the governing equations.

An ansatz is an educated guess or a proposed form for the solution to a mathematical problem, which is then verified through substitution into the governing equations. In this case, it allows you to find solutions for complex configurations.

Using an ansatz for singular cases allows for obtaining meaningful solutions where direct analytical methods might fail, facilitating the understanding of complex electromagnetic interactions.

We are discussing the orientation and positioning of a secondary coil relative to a primary coil using both cylindrical and Cartesian coordinate systems, [3].

The key points are:

1.Primary Coil (Axisymmetric): The primary coil is symmetric around its axis (like the *z*-axis in cylindrical coordinates.

2. Secondary Coil (Position and Orientation):
The secondary coil's position is described in the Cartesian coordinate system by a position vector, $rc=(x_c,y_c,z_c)$ where x_c,y_c and z_c are its coordinates relative to the origin.
The vector $\vec{N}$ likely represents the orientation of the secondary coil's axis.

3. Rotation to Simplify Calculations:
Since the primary coil is axisymmetric, you can rotate the Cartesian coordinate system in such a way that the *Y*-component of the secondary coil's orientation vector n_y becomes zero. This simplifies the problem, reducing the 3*D* orientation to a plane with only two components, likely n_x and nz.

This setup likely helps simplify the analytical or numerical evaluation of mutual inductance or magnetic interactions between the two coils, especially when solving for cases where the coils have a general orientation. The symmetry of the primary coil means that one degree of freedom in orientation can be removed through this rotation, [3].

Let's define the unit vector at point *C* as follows:

$$\vec{N} = \{n_x, n_y, n_z\} = \{n_x, 0, n_z\} \tag{7}$$

Let's consider $cos(\theta) = a$ as a singular case for the filament method. To overcome this problem let us find the mutual inductance when $cos(\theta)$ is near a.

Let's take a very small positive number ε. By introducing $\cos(\theta) = a - \varepsilon$ and $\sin(\theta) = \sqrt{1-(a-\varepsilon)^2}$ we calculate the mutual inductance $M(a-\varepsilon)$.

Next, we introduce $\cos(\theta) = a + \varepsilon$ and $\sin(\theta) = \sqrt{1-(a+\varepsilon)^2}$ to calculate the mutual inductance $M(\mathrm{a}+\varepsilon)$.

For different values of $\varepsilon_i = 0.1; 0.01; 0.001; ...$
we calculate the corresponding mutual inductance as

$$M\varepsilon_i(a) = \frac{[M(a-\varepsilon_i)+M(a+\varepsilon_i)]}{2},\ i = 1,2,3, \dots, \tag{8}$$

We terminate the process when

$$M\varepsilon_{i-1} = M\varepsilon_i \tag{9}$$

or when the calculation diverges to infinity.
In the case of two solenoids with parallel axes $rc = (x_c = d,\ y_c = 0,\ z_c = c)$ and $\vec{N} = \{n_x, n_y, n_z\} = \{0,0,1\}$.

5 Examples

5.1 Example 1

Calculate the mutual inductance between two inclined solenoids for which we have:

$R_1 = 1$ m, $a = l_1 = 1$ m, $N_1 = 100$ turns
$R_2 = 0.25$ m, $b = l_2 = 0.5$ m, $N_2 = 50$ turns.

The center of the inclined solenoid is *C* (0.2 m;0 cm;0.3m).

$a/b = K/m = 1/0.5 = 2$, $(N = n = 0)$

From the coils 'dimensions the number of axial subdivisions can be taken as follows, [1] and [2]:
$N = 0$, $K = 2m$, $n = 0$, $m = m$

Let's examine the computational time and accuracy in the function of the number of the subdivisions.

We tested the case where $n_x = sin(\theta) = 0.51^{0.5}$, $n_y = 0$ and $n_z = cos(\theta) = 0.7$. From Table 1 we can see that this case is not the singular.

Table 1. Test of the computational time and accuracy for the mutual inductance

N/ K/ m/ n	*M* (μH)	*Time*(s)	*ARE* (%)
0/50/25/0	381.2429704988	1.861	----
0/100/50/0	381.2408304499	6.039	0.00056
0/150/75/0	381.2404299999	13.093	0.00011
0/200/100/0	381.2402893125	23.532	0.00004
0/ 250/125/0	381.2402240645	35.825	0.00002
0/300/150/0	381.2401885773	50.334	0.00001
0/350/175/0	381.2401671615	68.221	0.00001
0/400/200/0	381.2401532533	89.042	0.000004

All results are in very good agreement, with differences appearing only after the sixth decimal place. All cases demonstrate excellent accuracy, and the first four cases can be considered without any reservations. We focus on the case where *K*=200 and *m*=100, for which the computational time is approximately 23.532 seconds. In Table 2, all calculations for the different angles of inclination are provided.

Table 2. Computation of Mutual Inductance between Two Inclined Solenoids.

$n_x = sin(\theta)$	$n_z = cos(\theta)$	M(μH) This work
0.0	1.0	511.8057225358491
$\sqrt{0.19}$	0.9	474.1189590889916
0.6	0.8	NaN
$\sqrt{0.51}$	0.7	381.2402893124998
0.8	0.6	332.8678266511847
$\sqrt{0.75}$	0.5	283.7002109910534
$\sqrt{0.84}$	0.4	233.8997743391140
$\sqrt{0.91}$	0.3	183.5735208228110
$\sqrt{0.96}$	0.2	132.7964260156655
$\sqrt{0.99}$	0.1	8.162237991774454
1.0	0.0	30.09018080816064

Obviously, the singularity appears for $n_x = sin(\theta)$= 0.6, n_y =0 and $n_z = cos(\theta)$ = 0.8.

For different values of epsilon (Table 3), using equation (8), we test the mutual inductance near the singularity.

Table 3. Test of the mutual inductance close to the singularity

ε	M(μH)
0.001	428.619819230
0.0001	428.537735536
0.00001	428.537736336
0.000001	428.537736344
0.0000001	428.537736344
0.00000001	NaN

Thus, we stop here before the mutual inductance begins infinity. Let us verify this reasoning with condition (9) with a =0.8 and ε_i = 0.0000001.

$$M(a - \varepsilon_i) = 428.5376897385664 \mu\text{H}$$

and

$$M(a + \varepsilon_i) = 428.537782949496 \mu\text{H}$$

Finally,

$$M(a) = (M(a - \varepsilon_i) + M(a + \varepsilon_i))/2$$

The condition in equation (9) is satisfied, indicating that the mutual inductance at this point can be considered extremely precise, and it is

M = 428.537736344 (μH).

It should be noted that this type of singularity is referred to as a *smooth singularity*. This means that the intersection of the solenoids occurs at a single point, and it is possible to calculate this singularity from both sides of that point.

5.2 Example 2

Calculate the mutual inductance between two inclined solenoids for which we have:

R_1 = 2 cm, $a = l_1$ = 2 cm, N_1 = 100 turns

R_2 = 1 cm, $b = l_2$ = 2 cm, N_2 = 100 turns.

The center of the inclined solenoid is C (1.1 cm;0 cm; 0.5 cm).

$a/b = K/m = 2/2 = 1$, ($N = n = 0$)

From the coils' dimensions the number of axial subdivisions can be taken as follows, [1], [2]:

$N = 0$, $K = m$, $n = 0$, $m = m$

Let's examine the computational time and accuracy in the function of the number of subdivisions.

We test the case where $n_x = sin(\theta)$= 0.8, n_y =0 and $n_z = cos(\theta)$ = 0.6. From Table 4 we can see that this case is not singular.

Table 4. Test of the computational time and accuracy for the mutual inductance

$N/ K/ m/ n$	M (mH)	*Time*(s)	*ARE* (%)
0/40/40/0	6.0083378010924	2.195	----
0/80/80/0	6.008246454441	7.733	0.00220
0/120/120/0	6.008229897189	16.659	0.00028
0/160/160/0	6.008218245204	28.811	0.00019
0/ 200/200/0	6.008216146003	70.465	0.00004
0/ 240/240/0	6.008214413430	104.209	0.00003

All results are in very good agreement, with differences appearing only after the five or the sixth decimal place. All cases demonstrate excellent accuracy, and the first four cases can be considered without any reservations. We focus on the case where $K = m$ =160, for which the computational time is approximately 28.811185 seconds. In Table 5, all calculations for the different angles of inclination are provided.

Obviously, all cases are regular, so the presented method is applicable to both regular and singular cases.

Table 5. Computation of Mutual Inductance between Two Inclined Solenoids.

$n_x = sin(\theta)$	$n_z = cos(\theta)$	M(mH) This work
0.0	1.0	8.74011321164347
$\sqrt{0.19}$	0.9	8.238882046221992
0.6	0.8	7.487950826302638
$\sqrt{0.51}$	0.7	6.745801670369136
0.8	0.6	6.008218245204718
$\sqrt{0.75}$	0.5	5.270331596007753
$\sqrt{0.84}$	0.4	4.527506873290331
$\sqrt{0.91}$	0.3	3.775026595666628
$\sqrt{0.96}$	0.2	3.007712605621282
$\sqrt{0.99}$	0.1	2.219583638360655
1.0	0.0	1.403399378413527

5.3 Example 3

Calculate the mutual inductance between two solenoids with parallel axes for which we have:

R_1 = 1 m, $a = l_1$ =1 m, N_1 = 1000 turns

R_2 = 1 m, $b = l_2$ = 1 m, N_2 = 1000 turns.

The center of the displaced solenoid is $C(x_C = d$; y_C = 0; $z_C = c$ =1), and the component of the unit vector at this point $\vec{N} = \{n_{xC}, n_{yC}, n_{zC}\} = \{0,0,1\}$.

From the coils' dimensions the number of axial subdivisions can be taken as follows, [5], [6]:

$N = 0$, $K = m$, $n = 0$, $m = m$

The results in fived column are for $K = m$ = 100, $N = n = 0$.

Let's take as in the previous case $\varepsilon = 0.0000001$ and find for d = 1-0.0000001and d = 1+0.0000001 respectively.

M_-=509.5184217293739 mH.

The elapsed time is 6.933516 seconds.

M_+=509.5164125027133mH.

The elapsed time is 8.352631 seconds.

Now, the supposed mutual inductance for this case could be:

$\mathbf{M = (M_- + M_+)/2 = 509.5174171160436}$ **mH.**

Thus, the problem of the singularity is overcome by ansatz, Table 6.

Table 6. Computation of Mutual Inductance between Two Inclined Solenoids.

d(m)	$M_{This\ Work}$ (mH)	$Time$(s)
0.0	1870.827527140828	4.658099
0.25	1442.641111354813	4.580331
0.5	1084.060166389914	8.621715
0.75	777.3078274413109	8.703822
0.999999	509.5184217293739	7.699235
1.0	NaN	
1.000001	509.5164125027133	8.674804
1.25	274.5559991375439	8.315158
1.5	72.59962217729628	8.410469
1.75	-86.60801122260487	8.799677
2.0	-159.1494780152672	6.926220
2.25	-107.5928368505295	7.882158
2.5	-75.74174178415984	8.464918
2.75	-55.3147627583018	8.337810
3.0	-41.64955582522267	7.441429

It should be noted that this type of singularity is referred to as a *smooth singularity*. This means that the intersection of the solenoids occurs at a single point, and it is possible to calculate this singularity from both sides of that point.

5.4 Example 4

Calculate the mutual inductance between two inclined solenoids for which we have:

R_1 = 3 m, a = 4 m, N_1 = 1000 turns

R_2 = 1 m, b = 4 m, N_2 = 1000 turns.

The center of the inclined solenoid is $C(x_C$ = 1 m; y_C = 0 m; z_C =2m)

From the coils' dimensions the number of axial subdivisions can be taken as follows, [5], [6]:

$N = 0$, $K = m$, $n = 0$, $m = m$ =300

Table 7. Computation of Mutual Inductance between Two Inclined Solenoids

$n_x = sin(\theta)$	$n_z = cos(\theta)$	$M_{This\ Work}$ (mH)
0.0	1.0	NaN
$\sqrt{0.19}$	0.9	367.9139516949478
0.6	0.8	340.3142988590226
$\sqrt{0.51}$	0.7	312.0164553561316
0.8	0.6	283.5531178651556
$\sqrt{0.75}$	0.5	255.2917023555278
$\sqrt{0.84}$	0.4	226.4840684770490
$\sqrt{0.91}$	0.3	195.2347084483120
$\sqrt{0.96}$	0.2	161.4129841218354
$\sqrt{0.99}$	0.1	125.0515637025127
1.0	0.0	85.98299013180394

Table 8. Test of the mutual inductance close to the singularity

ε	M(mH)
0.001	390.4285350889643
0.00001	390.2334580148851
0.000001	390.1650923006923
0.0000001	390.1404249985815
0.00000001	**NaN**

Thus, for the case where $cos(\theta)$ = 1, the mutual inductance is calculated as

$$\boldsymbol{M} = \mathbf{390.1404249985815\ mH}$$

which can be considered extremely close to the exact value.

This scenario represents a singular case that can only be addressed at a specific point at the end of the calculation process.

The mutual inductance for this singular case was determined using an ansatz, as it was intuitively anticipated.

It should be noted that in all previous examples where singularities appear, we are referring to smooth singularities. This means the intersection of the solenoids occurs at a single point, Table 7 and Table 8.

Hard singularities, on the other hand, arise when the solenoids are completely overlapped, such as in the case when calculating the self-inductance of a solenoid.

5.5 Example 5

Calculate the mutual (self) inductance between two inclined solenoids for which we have:
R_1 =1 m, a = 2 m, N_1 = 1000 turns
R_2 = 1 m, b = 2 m, N_2 = 1000 turns.

This is the problem where both solenoids are completely overlapped. It means we have the double singularity or hard singularity (axial and radial), so this problem can not be solved by the presented method. Helpfully, we can use the well-known formula [10], from which the self-inductance is:

$$L = 1.35889175900372\ \mathrm{H}$$

6 Conclusion

In this paper, a highly efficient and straightforward method is presented for calculating the mutual inductance between two inclined solenoids positioned in any desired spatial configuration. The key aspect of this calculation is the treatment of singular cases when the solenoids intersect or come into contact. This can raise controversy, as one might question whether it is physically possible for coils to intersect and what the practical use of such calculations would be. For this reason, we aim to open a discussion on this approach, inviting experts in the field to share their opinions on the validity and regularity of these calculations.

From an engineering perspective, the method ensures optimal accuracy and minimal computation time. Additionally, we provide an incredibly effective approach to determine the most optimal relationship between the coil dimensions and the corresponding number of subdivisions, leading to high accuracy with minimal computational effort. Furthermore, we offer a solution for resolving potential singular cases using an ansatz.

The filament method is employed, which simplifies all procedures for engineers, physicists, and others working in the field of electromagnetics, without requiring familiarity with complex special functions that often pose similar challenges. Some representative examples are provided to confirm the validity of the present method, and these examples can serve as benchmark problems for testing other potential methods. All programming is implemented in MATLAB, making the method highly accessible for potential users.

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- Slobodan Babic carried out Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Writing original draft preparation, Visualization.
- Nikos E. Mastorakis carried out Methodology, Validation, Formal analysis, Investigation, Visualization.

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