Simulation of Simultaneous Confidence Intervals for All Differences of Signal-to-Noise Ratios of Log-Normal Distributions

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Abstract: - A low signal relative to background noise (signal-to-noise ratio: SNR) signifies a challenge in distinguishing the signal from the background noise. Herein, estimates for the simultaneous confidence intervals (SCIs) for the differences between the SNRs of several log-normal (LN) distributions are presented. These are based on the fiducial generalized confidence interval (FGCI), large sample (LS), method of variance estimates recovery (MOVER), and Bayesian (BS) approaches. By using a Monte Carlo simulation study with RStudio programming, all SCIs are compared based on their coverage probabilities and average lengths. The results indicate that LS approach provided shorter average lengths compared to the MOVER approach, the former approach is the most effective for estimating the SCIs for the differences among the SNRs of several LN distributions. Furthermore, these methods were also used to compare the equality of three price-earning ratios: the SET50, SET100, and sSET indexes. In conclusion, the LS approach proved to be the best method and is thus recommended for estimating the SCIs for the differences between the SNRs of multiple LN distributions.

Key-Words: - Bayesian, economics, FGCI, LN distribution, SNR, SCIs.

Received: March 26, 2024. Revised: August 29, 2024. Accepted: December 13, 2024. Published: March 12, 2025.

1 Introduction

The log-normal distribution is well-known to applied to many areas including business and finance [1], such as it has been utilized to model stock prices. The signal-to-noise ratio (SNR) measures the quality of a signal with reference to the background noise, with a higher SNR indicating better signal quality. The SNR is an essential metric in many fields such as telecommunications, finance, economics, business and biology, [2]. For example, in finance, the higher SNR indicates a more reliable trading signal.

Following the investigation of [3] and [4], [5] derived estimates for the simultaneous confidence intervals (SCIs) for the ratios of means of several log-normal distributions. The SCI can be used to assess several parameters concurrently within a predetermined confidence level while the CI concentrates the analysis on a single parameter. The SCI involves specialized methods to consider the joint estimation of multiple parameters and control the overall confidence level, [6]. The SCIs have been used in experimental scenarios involving the comparison of multiple populations. The objective is to enable the drawing of conclusions about differences between the SNRs of more than two LN populations.

The estimation of the SCI for differences between the SNRs of multiple LN distributions is presented herein using four approaches. The first is the fiducial generalized confidence interval (FGCI) approach, in which fiducial intervals are estimated using fiducial distributions. The simultaneous fiducial generalized confidence intervals (SFGCIs) for ratios of means of log-normal distributions using fiducial generalized pivotal quantities (FGPQs) were proposed by [7]. The second is the large sample (LS) approach, for which the Central Limit Theorem (CLT) is key. Several researchers have used it to estimate the CI for various distribution parameters, [8]. The third is the method of variance estimates recovery (MOVER) approach, which involves a two-step process. First, initial estimates of the variance are derived from the data, which provides crucial insights into sample variability. Second, the variance estimates are used to estimate the CI for the parameter of interest, [9]. The fourth is the Bayesian (BS) approach, which is a statistical methodology that centers around probabilistic inference and the interpretation of beliefs regarding hypotheses. Normally, the BS analysis involves a likelihood function and defining a prior distribution which are then used to explore the posterior distribution, [10].

2 Methods

Let $X_i = (X_{i1}, X_{i2}, ..., X_{in_i}) = (\log(Y_{i1}), \log(Y_{i2}), ..., \log(Y_{in_i}))$ be a random sample of size n_i from normal distribution with mean μ_i and variance σ_i^2, where $i = 1,2,...,k$. Let Y_i be the LN distribution with mean $\mu_{Y(i)} = \exp(\mu_i + (\sigma_i^2 / 2))$ and variance $\sigma_{Y(i)}^2 = (\exp(\sigma_i^2) - 1)\exp(2\mu_i + \sigma_i^2)$. The SNR of Y_i is

$$\theta_i = \frac{\mu_{Y(i)}}{\sqrt{\sigma_{Y(i)}^2}} = \frac{1}{\sqrt{\exp(\sigma_i^2) - 1}}. \tag{1}$$

The estimator of θ_i is

$$\hat{\theta}_i = \frac{1}{\sqrt{\exp(S_i^2) - 1}}, \tag{2}$$

where $S_i^2 = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2 / (n_i - 1)$.

The variance of $\hat{\theta}_i$ is

$$\mathrm{Var}(\hat{\theta}_i) = \frac{\sigma_i^4 \exp(2\sigma_i^2)}{2(n_i - 1)(\exp(\sigma_i^2) - 1)^3}. \tag{3}$$

Therefore, the difference between SNRs is

$$\hat{\theta}_{il} = \hat{\theta}_i - \hat{\theta}_l = \frac{1}{\sqrt{\exp(S_i^2) - 1}} - \frac{1}{\sqrt{\exp(S_l^2) - 1}}, \tag{4}$$

where $i, l = 1,2,...,k$ and $i \neq l$.

The variance of $\hat{\theta}_{il} = \hat{\theta}_i - \hat{\theta}_l$ is

$$\mathrm{Var}(\hat{\theta}_{il}) = \frac{\sigma_i^4 \exp(2\sigma_i^2)}{2(n_i - 1)(\exp(\sigma_i^2) - 1)^3} + \frac{\sigma_l^4 \exp(2\sigma_l^2)}{2(n_l - 1)(\exp(\sigma_l^2) - 1)^3}. \tag{5}$$

2.1 Fiducial Generalized Confidence Interval Approach

The concept of generalized pivotal quantities (GPQs) for constructing generalized confidence intervals (GCIs) were proposed by [11]. A subclass of GPQs known as fiducial generalized pivotal quantities (FGPQs) was introduce by [12]. Following [12], the FGPQs were used to estimate fiducial generalized confidence intervals (FGCIs). In a related work, proposed FGPQs to construct simultaneous fiducial generalized confidence intervals (SFGCIs) were proposed by [7]. The FGCI technique provides an advantage by being able to determine confidence intervals for complex parameters. However, its downside stems from its reliance on simulated data.

Following [12], let $X_i = (X_{i1}, X_{i2}, ..., X_{in_i})$ and let $x_i = (x_{i1}, x_{i2}, ..., x_{in_i})$ be the observation of $X_i = (X_{i1}, X_{i2}, ..., X_{in_i})$ with sample size n_i, and $X = (X_1, X_2, ..., X_k)$, $x = (x_1, x_2, ..., x_k)$, $\mu = (\mu_1, \mu_2, ..., \mu_k)$, $\sigma^2 = (\sigma_1^2, \sigma_2^2, ..., \sigma_k^2)$, $\zeta_i = (\mu_i, \sigma_i^2)$, and $\zeta = (\zeta_1, \zeta_2, ..., \zeta_k)$. Let X^* be an independent copy of X. Let $R_i(X, X^*, \zeta)$ be a function of X, X^*, and ζ. Since $R(X, X^*, \zeta) = (R_1(X, X^*, \zeta), R_2(X, X^*, \zeta), ..., R_k(X, X^*, \zeta))$. For real functions $g_1, g_2, ..., g_k$, the random quantities $g_1(R(X, X^*, \zeta)), g_2(R(X, X^*, \zeta)), ..., g_k(R(X, X^*, \zeta))$ are simultaneous fiducial generalized pivotal quantities (SFGPQs) for $g_1(\theta), g_2(\theta), ..., g_k(\theta)$ if the following two properties are satisfied:

FGPQ1: The conditionnal distribution of $R(X, X^*, \zeta)$ is free of ζ.

FGPQ2: For every x, $R(x, x^*, \zeta) = \theta$

Let $\bar{X}_i$ and $\bar{X}_i^*$ be both independent and identically distributed. Let S_i^2 and S_i^{2*} be both independent and identically distributed. Therefore, $\bar{X}_i^*$ and S_i^{2*} are independent. Since $\bar{X}_i^*$ and S_i^{2*} are defined by

$$\bar{X}_i^* \sim N\left(\mu_i, \frac{\sigma_i^2}{n_i}\right) \tag{6}$$

and

$$\frac{(n_i - 1)S_i^{2*}}{\sigma_i^2} \sim \chi_{n_i - 1}^2, \tag{7}$$

where $\chi_{n_i - 1}^2$ is chi-squared distribution with $n_i - 1$ degrees of freedom.

According to [12], the GPQs for μ_i and σ_i^2 are defined as

$$R_{\mu_i} = \bar{X}_i - \frac{S_i}{S_i^*}\left(\bar{X}_i^* - \mu_i\right) \tag{8}$$

and

$$R_{\sigma_i^2} = \frac{S_i^2}{S_i^{2*}} \sigma_i^2. \tag{9}$$

The simultaneous fiducial generalized pivotal quantities (SFGPQs) for $\theta_i - \theta_l$ are

$$R_{\theta_{il}}(X, X^*, \mu, \sigma^2) = R_{\theta_i}(X, X^*, \mu, \sigma^2) - R_{\theta_l}(X, X^*, \mu, \sigma^2)$$

$$= \frac{1}{\sqrt{\exp(R_{\sigma_i^2}) - 1}} - \frac{1}{\sqrt{\exp(R_{\sigma_l^2}) - 1}}, \quad (10)$$

where $R_{\sigma_i^2}$ is defined as in Equation (9).

The variance of $\hat{\theta}_i - \hat{\theta}_l$ is:

$$V_{il} = \frac{S_i^4 \exp(2S_i^2)}{2(n_i - 1)(\exp(S_i^2) - 1)^3} + \frac{S_l^4 \exp(2S_l^2)}{2(n_l - 1)(\exp(S_l^2) - 1)^3}. \quad (11)$$

According to [7] and [12], random quantity is:

$$T = \max_{i \neq l} \left| \frac{\hat{\theta}_{il} - R_{\theta_{il}}(X, X^*, \mu, \sigma^2)}{\sqrt{V_{il}}} \right|, \quad (12)$$

where $\hat{\theta}_{il}$ is defined as in Equation (4), $R_{\theta_{il}}(X, X^*, \mu, \sigma^2)$ is defined as in Equation (10), and V_{il} is defined as in Equation (11).

Therefore, the $100(1-\alpha)\%$ two-sided SCIs using the FGCI approach are

$$SCI_{il(FGCI)} = [L_{il(FGCI)}, U_{il(FGCI)}]$$
$$= \left[\hat{\theta}_{il} - d_{1-\alpha}\sqrt{V_{il}}, \hat{\theta}_{il} + d_{1-\alpha}\sqrt{V_{il}}\right], \quad (13)$$

where $d_{1-\alpha}$ is the $(1-\alpha)$-th quantile of T.

The following algorithm is used to construct SCIs using the FGCI approach.

Algorithm 1
For a given $\bar{x}_i$, $\bar{x}_l$, s_i^2, and s_l^2
For $g =$ 1 to h
Generate $\bar{x}_i^*$, $\bar{x}_l^*$, s_i^{2*}, and s_l^{2*}
Compute $R_{\sigma_i^2}$ and $R_{\sigma_l^2}$
Compute $R_{\theta_{il}}(X, X^*, \mu, \sigma^2)$
Compute V_{il}
Compute T
End g loop
Compute $d_{1-\alpha}$
Compute $L_{il(FGCI)}$ and $U_{il(FGCI)}$

2.2 Large Sample Approach

The LS approach relies on the CLT. In the context of a substantial sample size, the distribution of the sample mean tends to approximate a normal distribution, as predicted by the CLT. In practical terms, this implies that statistical methods based on the normal distribution can be effectively applied to inferential statistics when dealing with large samples, facilitating certain types of analyses and hypothesis testing. The LS approach simplifies the construction of confidence intervals using precise formulas, yet its reliance on the normal distribution makes it particularly applicable to inferential statistics in the context of large sample sizes.

Therefore, the $100(1-\alpha)\%$ two-sided SCIs using the LS approach are

$$SCI_{il(LS)} = [L_{il(LS)}, U_{il(LS)}]$$
$$= \left[\hat{\theta}_{il} - z_{1-\alpha/2}\sqrt{Var(\hat{\theta}_{il})}, \hat{\theta}_{il} + z_{1-\alpha/2}\sqrt{Var(\hat{\theta}_{il})}\right] \quad (14)$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)$-th quantile of the standard normal distribution, and $\hat{\theta}_{il}$ is defined as in Equation (4), and $Var(\hat{\theta}_{il})$ is defined as in Equation (5) with σ_i and σ_l replaced by s_i and s_l, respectively.

2.3 Method of Variance Estimates Recovery Approach

The MOVER CI relies on individual exact CIs, indicating that the MOVER approach constructs CIs by using precise intervals for each parameter separately. In simpler terms, the MOVER approach involves obtaining accurate CIs for individual parameters, and these specific intervals are combined to form the MOVER CI. This collective approach aims to offer a more precise and dependable range for the overall parameter of interest. The MOVER CI for the difference of SNRs is based on the individual exact CIs of two SNRs. Constructing confidence intervals based on the exact formula is straightforward with the MOVER approach. This approach relies on the initial confidence interval for a single parameter.

Let l_i and u_i be the lower limit and the upper limit of the CI for the SNR of LN distribution based on i-th sample is

$$[l_i, u_i] = \left[\hat{\theta}_i - t_{1-\alpha/2}\sqrt{Var(\hat{\theta}_i)}, \hat{\theta}_i + t_{1-\alpha/2}\sqrt{Var(\hat{\theta}_i)}\right], \quad (15)$$

where $t_{1-\alpha/2}$ is the $(1-\alpha/2)$-th quantile of t distribution, $\hat{\theta}_i$ is defined as in Equation (2), and $Var(\hat{\theta}_i)$ is defined as in Equation (3) with σ_i and σ_l replaced by s_i and s_l, respectively.

Following [13], the $100(1-\alpha)\%$ two-sided SCIs using the MOVER approach are:

$$SCI_{il(MOVER)} = [L_{il(MOVER)}, U_{il(MOVER)}]$$
$$= \left[\hat{\theta}_i - \hat{\theta}_l - \sqrt{(\hat{\theta}_i - l_i)^2 + (u_l - \hat{\theta}_l)^2}, \hat{\theta}_i - \hat{\theta}_l + \sqrt{(u_i - \hat{\theta}_i)^2 + (\hat{\theta}_l - l_l)^2}\right], \quad (16)$$

where

$$l_i = \hat{\theta}_i - t_{1-\alpha/2}\sqrt{Var(\hat{\theta}_i)},$$
$$u_i = \hat{\theta}_i + t_{1-\alpha/2}\sqrt{Var(\hat{\theta}_i)},$$
$$l_l = \hat{\theta}_l - t_{1-\alpha/2}\sqrt{Var(\hat{\theta}_l)},$$

and

$$u_l = \hat{\theta}_l + t_{1-\alpha/2}\sqrt{Var(\hat{\theta}_l)}.$$

2.4 Bayesian Approach

The BS approach involves applying Bayes' theorem, wherein the prior distribution is adjusted using observed data to generate the posterior distribution. This posterior distribution serves as the updated probability distribution for the parameters, incorporating the information gleaned from the observed data. The BS approach is applicable for constructing complex parameters. It relies on the posterior probability obtained from both a prior probability and a likelihood function.

The posterior distribution for σ_i^2 is:

$$\sigma_i^2 \mid x_i \sim IG\left(\frac{n_i-1}{2}, \frac{(n_i-1)s_i^2}{2}\right). \quad (17)$$

The conditional posterior distribution for μ_i given σ_i^2 and x_i is

$$\mu_i \mid \sigma_i^2, x_i \sim N\left(\hat{\mu}_i, \frac{\sigma_i^2}{n_i}\right), \quad (18)$$

where σ_i^2 is defined as in Equation (17).

The posterior distribution for θ_i is

$$\theta_{i(BS)} = \frac{1}{\sqrt{\exp(\sigma_i^2)-1}}, \quad (19)$$

where σ_i^2 is defined as in Equation (17).

The posterior distribution for $\theta_i - \theta_l$ is

$$\theta_{il(BS)} = \theta_{i(BS)} - \theta_{l(BS)}, \quad (20)$$

where $\theta_{i(BS)}$ and $\theta_{l(BS)}$ are defined as in Equation (19).

Therefore, the $100(1-\alpha)\%$ two-sided SCIs using the BS approach are

$$SCI_{il(BS)} = [L_{il(BS)}, U_{il(BS)}], \quad (21)$$

where $L_{il(BS)}$ and $U_{il(BS)}$ are the lower limit and the upper limit of the shortest $100(1-\alpha)\%$ highest posterior density interval of $\theta_{il(BS)}$, respectively.

The following algorithm is used to construct the SCIs using the BS approach.

Algorithm 2.
For a given $\bar{x}_i$, $\bar{x}_l$, s_i^2, and s_l^2
For $g = 1$ to m
Generate σ_i^2 and σ_l^2
Compute $\theta_{i(BS)}$ and $\theta_{l(BS)}$
Compute $\theta_{il(BS)}$
End g loop
Compute $L_{il(BS)}$ and $U_{il(BS)}$

3 Results

A simulation study was conducted to assess the coverage probabilities (CPs), average lengths (ALs), and standard errors (SEs) of the SCIs using RStudio programming. The evaluation involved comparing the effectiveness of these SCIs, with a focus on their CPs and ALs. The preferred confidence interval is identified by achieving a CP equal to or exceeding the nominal confidence level of 0.95, while also having the shortest AL.

In the simulation study, two scenarios were considered with sample cased $k = 3$ and $k = 6$. The sample sizes were denoted as $n_1, n_2, ..., n_k$, the population means as $\mu_1 = \mu_2 = ... = \mu_k = 1$, the population SNRs as $\theta_1, \theta_2, ..., \theta_k$, and the population variances as $\sigma_i^2 = \log((1/\theta_i^2)+1)$, where $i = 1, 2, ..., k$. The specific combinations are presented in the following tables. For each parameter steering, 5000 random samples were generated using Algorithm 3. For each of the random samples, 2500 T was simulated using Algorithm 1, and 2500 $\theta_{il(BS)}$ was simulated using Algorithm 2.

Algorithm 3.
For a given n_i, n_l, μ_i, μ_l, θ_i, and θ_l
Compute σ_i^2 and σ_l^2
For $h = 1$ to M
Generate x_{ij} from $N(\mu_i, \sigma_i^2)$ and x_{lj} from $N(\mu_l, \sigma_l^2)$
Compute $\bar{x}_i$, $\bar{x}_l$, s_i^2, and s_l^2
Construct $SCI_{il(FGCI)} = [L_{il(FGCI)}, U_{il(FGCI)}]$
Construct $SCI_{il(LS)} = [L_{il(LS)}, U_{il(LS)}]$
Construct $SCI_{il(MOVER)} = [L_{il(MOVER)}, U_{il(MOVER)}]$
Construct $SCI_{il(BS)} = [L_{il(BS)}, U_{il(BS)}]$
Record whether or not all the values of θ_{il} fall in CIs
Compute $U_{il} - L_{il}$
End h loop
Compute the CPs, ALs, and SEs for each CI

Table 1 and Table 2 in Appendix present the CPs, ALs, and SEs of SCIs for all differences of SNRs of LN distributions for $k = 3$ and $k = 6$, respectively. For $k = 3$, the results indicate that the CPs of the SCIs based on the FGCI approach exceed 0.9700 for all cases. Furthermore, the CPs of the SCIs based on the LS and MOVER approaches surpass 0.9500 for almost all cases. In addition, the CPs of the SCIs based on the BS approach exceed 0.9500 for some cases. The ALs of the SCIs based on the BS approach are shorter than those of the SCIs. For $k = 6$, the CPs of the SCIs based on the FGCI approach are close to 1.0000. The CPs of the SCIs based on the LS and MOVER approaches exceed 0.9500 for some cases. Moreover, the CPs of the SCIs based on the BS approach are less than

0.9500 for almost all cases. Hence, the LS approach is recommended for constructing the SCIs for all differences of SNRs of LN distributions, as it consistently achieves CPs above 0.9500 and shorter ALs.

4 Empirical Application

Monthly index data from January 2023 to November 2023 were provided by [14]. The sample statistics of price-earnings ratios of the SET50 index are $n_1 =$ 11, $\bar{y}_1 =$ 19.57, $s_{Y_1} =$ 1.00, $\bar{x}_1 =$ 2.97, $s_{X_1} =$ 0.05, and $\hat{\theta}_1 =$ 19.98. The sample statistics of price-earnings ratios of the SET100 index are $n_2 =$ 11, $\bar{y}_2 =$ 18.83, $s_{Y_2} =$ 1.29, $\bar{x}_2 =$ 2.93, $s_{X_2} =$ 0.07, and $\hat{\theta}_2 =$ 15.07. The sample statistics of price-earning ratios of the sSET index are $n_3 =$ 11, $\bar{y}_3 =$ 17.21, $s_{Y_3} =$ 1.26, $\bar{x}_3 =$ 2.84, $s_{X_3} =$ 0.07, and $\hat{\theta}_3 =$ 14.09. The SET50, SET100, and sSET indexes follow LN distributions, [14].

The SCIs for all differences of SNRs of price-earnings ratios using the FGCI, LS, MOVER, and BS approaches are given in Table 3 (Appendix). According to Table 3 (Appendix), the findings indicate that the FGCI, LS, MOVER, and BS approaches encompass accurate differences in SNRs. Notably, the BS approach yields shorter intervals compared to the other approaches, demonstrating its superiority in terms of interval lengths.

5 Discussion

The efficacies of the SCI estimates derived using the FGCI, LS, MOVER, and BS approaches for differences among the SNRs of multiple LN distributions were compared via simulation studies. The results indicate that the FGCI approach yielded a conservative SCI estimate as its CPs approached 1.0000. Conversely, the CPs of the BS approach were below 0.9500, thereby indicating that it is unsuitable in this scenario. Meanwhile, those of both the LS and MOVER approaches exceeded 0.9500, and since the LS approach provided shorter ALs compared to the MOVER approach, the former approach is the most effective for estimating the SCI for the differences among the SNRs of several LN distributions.

6 Conclusions

The LS approach is the most effective for estimating the SCI for differences among the SNRs of multiple LN distributions. In the future, we will investigate using the LS approach for estimating the SCI for differences among the SNRs of other distributions.

References:

[1] Jafari, A.A. and Abdollahnezhad, K., Inferences on the Means of Two Log-Normal Distributions: A Computational Approach Test, *Communications in Statistics - Simulation and Computation*, Vol. 44, No. 7, 2015, pp. 1659-1672, DOI: 10.1080/03610918.2013.824094.

[2] Holgersson, H.E.T., Karlsson, P.S. and Mansoor, R., Estimating Mean-Standard Deviation Ratios of Financial Data, *Journal of Applied Statistics,* Vol. 39, No. 3, 2012, pp. 657-671, DOI: 10.1080/02664763.2011.610443.

[3] Chen, Y.H. and Zhou, X.H., Interval Estimates for the Ratio and Difference of Two Lognormal Means, *Statistics in Medicine*, Vol. 25, No. 23, 2006, pp. 4099-4113, DOI: 10.1002/sim.2504.

[4] Schaarschmidt, F., Simultaneous Confidence Intervals for Multiple Comparisons Among Expected Values of Log-Normal Variables, *Computational Statistics and Data Analysis*, Vol. 58, 2013, pp. 265-275, DOI: 10.1016/j.csda.2012.08.011.

[5] Sadooghi-Alvandi, S.M. and Malekzadeh, A., Simultaneous Confidence Intervals for Ratios of Means of Several Lognormal Distributions: A Parametric Bootstrap Approach, *Computational Statistics and Data Analysis*, Vol. 69, 2014, pp. 133-140, DOI: 10.1016/j.csda.2013.07.039.

[6] Kharrati-Kopaei, M., Malekadeh, A. and Sadooghi-Alvandi, M. Simultaneous Fiducial Generalized Confidence Intervals for the Successive Differences of Exponential Location Parameters under Heteroscedasticity, *Statistics and Probability Letters*, Vol. 83, No. 6, 2013, pp. 1547-1552.

[7] Hannig, J., Lidong, E., Abdel-Karimand, A. and Iyer, H., Simultaneous Fiducial Generalized Confidence Intervals for Ratios of Means of Lognormal Distributions, *Austrian Journal of Statistics*, Vol. 35, No. 2&3, 2006, pp.261-269, DOI: 10.17713/ajs.v35i2&3.372.

[8] Tian, L. and Wu, J., Inferences on the Common Mean of Several Log-Normal Populations: the Generalized Variable Approach, *Biometrical Journal*, Vol. 49, No. 6, 2007, pp.944-951, DOI: 10.1002/bimj.200710391.

[9] Donner, A. and Zou, G.Y., Closed-Form Confidence Intervals for Functions of the Normal Mean and Standard Deviation, Statistical Methods in Medical Research, Vol. 21, No. 4, 2012, pp. 347 - 359, DOI: 10.1177/ 0962280210383082.

[10] Casella, G. and Berger, R.L., *Statistical Inference*. Duxbury: California, USA, 2002.

[11] Weerahandi, S., *Exact Statistical Methods for Data Analysis*, Springer-Verlag: New York, 1995.

[12] Hannig, J., Iyer, H. and Patterson, P., Fiducial Generalized Confidence Intervals, *Journal of the American Statistical Association*, Vol. 101, No. 473, 2006, pp. 254-269, DOI: 10.1198/ 016214505000000736.

[13] Zou, G.Y., Taleban, J. and Hao, C.Y., Confidence Interval Estimation for Lognormal Data with Application to Health Economics, *Computational Statistics and Data Analysis*, Vol. 53, No. 11, 2009, pp. 3755-3764, DOI: 10.1016/j.csda.2009.03.016.

[14] Thangjai, W. and Niwitpong, S-A., Confidence Intervals for Functions of Signal-to-Noise Ratio with Application to Economics and Finance, *Asian Journal of Economics and Banking*, Vol. 8, No. 2, 2024, pp. 1-20, DOI: 10.1108/AJEB-12-2023-0129.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

This research was funded by King Mongkut's University of Technology North Bangkok, Contract no. KMUTNB-67-BASIC-22.

Conflict of Interest

The authors have no conflicts of interest to declare.

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APPENDIX

Table 1. The CPs, ALs, and SEs of 95% two-sided SCIs for all differences of SNRs of LN distributions: 3 sample cases

(n_1,n_2,n_3)	(μ_1,μ_2,μ_3)	$(\theta_1,\theta_2,\theta_3)$	CI_{FGCI}			CI_{LS}			CI_{MOVER}			CI_{BS}		
			CP	**AL**	**SE**	**CP**	**AL**	**SE**	**CP**	**AL**	**SE**	**CP**	**AL**	**SE**
(10,10,10)	(1,1,1)	(1,1,1)	0.9811	2.3259	0.1245	0.9513	1.9697	0.1057	0.9793	2.2734	0.1220	**0.9509**	**1.9373**	**0.1083**
		(1,2,5)	0.9813	5.2871	1.1107	0.9540	4.5585	0.9593	0.9808	5.2613	1.1073	**0.9501**	**4.4532**	**0.9296**
(10,20,20)	(1,1,1)	(1,1,1)	0.9808	1.8208	0.1527	**0.9501**	**1.5375**	**0.1290**	0.9713	1.7085	0.1686	0.9481	1.5167	0.1262
		(1,2,5)	0.9795	3.6767	0.6506	0.9535	3.1330	0.5551	0.9691	3.3896	0.5740	**0.9504**	**3.0869**	**0.5460**
(20,20,20)	(1,1,1)	(1,1,1)	0.9811	1.5438	0.0534	0.9495	1.2948	0.0448	**0.9641**	**1.3827**	**0.0479**	0.9487	1.2824	0.0464
		(1,2,5)	0.9791	3.4794	0.7249	**0.9504**	**2.9704**	**0.6193**	0.9634	3.1721	0.6614	0.9475	2.9293	0.6086
(20,30,30)	(1,1,1)	(1,1,1)	0.9814	1.3422	0.0630	0.9523	1.1254	0.0528	0.9622	1.1860	0.0599	**0.9513**	**1.1157**	**0.0532**
		(1,2,5)	0.9821	2.8518	0.5494	0.9539	2.4248	0.4673	0.9643	2.5374	0.4842	**0.9504**	**2.3965**	**0.4603**
(30,30,30)	(1,1,1)	(1,1,1)	0.9813	1.2361	0.0333	0.9497	1.0349	0.0279	**0.9600**	**1.0800**	**0.0291**	0.9477	1.0262	0.0291
		(1,2,5)	0.9799	2.7910	0.5828	**0.9505**	**2.3785**	**0.4970**	0.9609	2.4820	0.5186	0.9473	2.3495	0.4893
(30,50,50)	(1,1,1)	(1,1,1)	0.9823	1.0435	0.0546	**0.9510**	**0.8743**	**0.0457**	0.9583	0.9034	0.0502	0.9488	0.8673	0.0461
		(1,2,5)	0.9801	2.1888	0.4124	**0.9510**	**1.8568**	**0.3499**	0.9579	1.9081	0.3567	0.9492	1.8388	0.3460
(50,50,50)	(1,1,1)	(1,1,1)	0.9818	0.9399	0.0191	**0.9513**	**0.7863**	**0.0159**	0.9573	0.8062	0.0163	0.9498	0.7803	0.0172
		(1,2,5)	0.9781	2.1246	0.4431	0.9473	1.8069	0.3770	**0.9529**	**1.8526**	**0.3865**	0.9445	1.7881	0.3723
(50,100,100)	(1,1,1)	(1,1,1)	0.9806	0.7597	0.0520	**0.9505**	**0.6364**	**0.0436**	0.9554	0.6482	0.0460	0.9488	0.6317	0.0435
		(1,2,5)	0.9773	1.5498	0.2802	0.9475	1.3117	0.2372	**0.9505**	**1.3304**	**0.2390**	0.9443	1.3004	0.2350
(100,100,100)	(1,1,1)	(1,1,1)	0.9811	0.6572	0.0094	**0.9509**	**0.5496**	**0.0078**	0.9535	0.5564	0.0079	0.9485	0.5455	0.0091
		(1,2,5)	0.9796	1.4836	0.3089	**0.9523**	**1.2603**	**0.2625**	0.9547	1.2759	0.2657	0.9497	1.2491	0.2599
(100,200,200)	(1,1,1)	(1,1,1)	0.9819	0.5320	0.0355	**0.9513**	**0.4456**	**0.0297**	0.9529	0.4497	0.0306	0.9487	0.4424	0.0297
		(1,2,5)	0.9809	1.0889	0.1976	0.9487	0.9212	0.1672	**0.9505**	**0.9276**	**0.1678**	0.9463	0.9138	0.1658
(200,200,200)	(1,1,1)	(1,1,1)	**0.9797**	**0.4622**	**0.0046**	0.9475	0.3865	0.0038	0.9486	0.3889	0.0039	0.9456	0.3835	0.0051
		(1,2,5)	0.9815	1.0436	0.2176	**0.9512**	**0.8860**	**0.1847**	0.9525	0.8914	0.1858	0.9489	0.8788	0.1833

Table 2. The CPs, ALs, and SEs of 95% two-sided SCIs for all differences of SNRs of LN distributions: 6 sample cases

$(n_1,n_2,n_3,$ $n_4,n_5,n_6)$	$(\mu_1,\mu_2,\mu_3,$ $\mu_4,\mu_5,\mu_6)$	$(\theta_1,\theta_2,\theta_3,$ $\theta_4,\theta_5,\theta_6)$	CI_{FGCI}			CI_{LS}			CI_{MOVER}			CI_{BS}		
			CP	**AL**	**SE**	**CP**	**AL**	**SE**	**CP**	**AL**	**SE**	**CP**	**AL**	**SE**
(10,10,10, 10,10,10)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9968	2.8092	0.0890	**0.9507**	**1.9690**	**0.0624**	0.9794	2.2726	0.0721	0.9498	1.9367	0.0637
		(1,1,2,2,5,5)	0.9960	6.2552	0.6450	0.9549	4.4715	0.4615	0.9801	5.1610	0.5326	**0.9520**	**4.3719**	**0.4485**
(10,10,10, 20,20,20)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9959	2.3582	0.1059	**0.9516**	**1.6510**	**0.0742**	0.9736	1.8574	0.0933	0.9492	1.6273	0.0735
		(1,1,2,2,5,5)	0.9955	4.6447	0.3471	**0.9505**	**3.2749**	**0.2449**	0.9696	3.5888	0.2530	0.9488	3.2262	0.2416
(20,20,20, 20,20,20)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9968	1.8721	0.0371	**0.9508**	**1.2936**	**0.0256**	0.9648	1.3814	0.0274	0.9495	1.2819	0.0262
		(1,1,2,2,5,5)	0.9951	4.1345	0.4072	**0.9526**	**2.9239**	**0.2881**	0.9661	3.1224	0.3076	0.9494	2.8828	0.2830
(20,20,20, 30,30,30)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9959	1.6895	0.0439	**0.9516**	**1.1676**	**0.0303**	0.9630	1.2352	0.0339	0.9499	1.1571	0.0306
		(1,1,2,2,5,5)	0.9953	3.5047	0.2922	0.9490	2.4632	0.2054	**0.9605**	**2.5863**	**0.2119**	0.9465	2.4343	0.2028
(30,30,30, 30,30,30)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9959	1.4994	0.0237	0.9485	1.0335	0.0163	**0.9581**	**1.0785**	**0.0170**	0.9464	1.0250	0.0169
		(1,1,2,2,5,5)	0.9954	3.3126	0.3217	**0.9521**	**2.3366**	**0.2270**	0.9612	2.4382	0.2368	0.9494	2.3094	0.2239
(30,30,30, 50,50,50)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9956	1.3277	0.0356	0.9487	0.9162	0.0246	**0.9568**	**0.9495**	**0.0267**	0.9471	0.9089	0.0247
		(1,1,2,2,5,5)	0.9952	2.7105	0.2151	0.9496	1.8986	0.1507	**0.9563**	**1.9562**	**0.1531**	0.9475	1.8798	0.1492
(50,50,50, 50,50,50)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9957	1.1447	0.0135	0.9499	0.7879	0.0093	**0.9552**	**0.8078**	**0.0095**	0.9475	0.7818	0.0099
		(1,1,2,2,5,5)	0.9944	2.5239	0.2419	0.9488	1.7766	0.1703	**0.9552**	**1.8215**	**0.1746**	0.9464	1.7594	0.1685
(50,50,50, 100,100,100)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9957	0.9770	0.0309	0.9499	0.6743	0.0213	**0.9542**	**0.6882**	**0.0224**	0.9477	0.6691	0.0213
		(1,1,2,2,5,5)	0.9957	1.9427	0.1413	**0.9501**	**1.3570**	**0.0987**	0.9543	1.3791	0.0992	0.9478	1.3454	0.0979
(100,100,100, 100,100,100)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9958	0.7987	0.0065	**0.9507**	**0.5494**	**0.0045**	0.9536	0.5562	0.0045	0.9486	0.5453	0.0051
		(1,1,2,2,5,5)	0.9949	1.7631	0.1678	0.9498	1.2396	0.1180	**0.9528**	**1.2549**	**0.1195**	0.9466	1.2290	0.1171
(100,100,100, 200,200,200)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9958	0.6846	0.0208	0.9490	0.4725	0.0143	**0.9513**	**0.4772**	**0.0147**	0.9468	0.4690	0.0144
		(1,1,2,2,5,5)	0.9948	1.3629	0.0991	**0.9507**	**0.9511**	**0.0691**	0.9528	0.9587	0.0693	0.9481	0.9435	0.0687
(200,200,200, 200,200,200)	(1,1,1,1,1,1)	(1,1,1,1,1,1)	0.9957	0.5621	0.0032	**0.9509**	**0.3866**	**0.0022**	0.9523	0.3890	0.0022	0.9487	0.3836	0.0028
		(1,1,2,2,5,5)	0.9950	1.2389	0.1173	0.9492	0.8704	0.0824	0.9505	0.8757	0.0830	0.9470	0.8632	0.0818

Table 3. The 95% two-sided SCIs for all differences of SNRs of price-earnings ratios

Comparison	CI_{FGCI}		CI_{LS}		CI_{MOVER}		CI_{BS}	
	Lower	**Upper**	**Lower**	**Upper**	**Lower**	**Upper**	**Lower**	**Upper**
SET50/ SET100	-17.8727	8.0469	-15.8977	6.0718	-17.4007	7.5748	-15.6252	5.1416
SET50/ sSET	-18.5532	6.7713	-16.6235	4.8416	-18.0920	6.3101	-16.3131	4.9186
SET100/ sSET	-11.6691	9.7131	-10.0398	8.0837	-11.2796	9.3236	-10.1369	7.3696