Magneto Thermal Convection in Rotating Viscoelastic Fluid

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Abstract: The aim of the present research was to study the effect of uniform vertical magnetic field on the Rivlin-Ericksen viscoelastic fluid heated from below in the presence of uniform rotation. Following the linearized stability theory, Boussinesq approximation and normal mode analysis, the dispersion relation is obtained. In the scenario of stationary convection, Rivlin-Ericksen viscoelastic fluid behaves like a Newtonian Fluid. Further, it is found that rotation has a stabilizing effect whereas the magnetic field has both stabilizing and destabilizing effects. In addition to this, it has been discovered that the system is reliable for $\frac{g\alpha\chi}{\nu\beta} \le \frac{27\pi^4}{4}$ and under the condition $\frac{g\alpha\chi}{\nu\beta} > \frac{27\pi^4}{4}$ the system goes into an unstable state. Overstability has also been looked at from the perspective of a scenario in which sufficient circumstances are met to rule out the possibility of the phenomenon occurring. It has been discovered that the rotation and the magnetic field both introduce oscillatory modes in the system which were non-existent in their absence.

Key-Words: Thermal Convection; Rivlin-Ericksen Viscoelastic Fluid; Uniform Vertical Magnetic Field; Uniform Rotation

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1 Introduction

A comprehensive account of thermal convection (Be'nard convection) in a fluid layer, in the absence and presence of rotation and magnetic field has been summarized in the celebrated monograph by Chandrasekhar [1]. The use of Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in its association with the external force. The approximation is well justified in the case of incompressible fluids. Abdul-Bari and Al-Rubai [2] have studied the influence of Rayleigh-number in turbulent and laminar region in parallel-plate vertical channels. The influence of radiation on the unsteady free convection flow of a viscous incompressible fluid past a moving vertical plate with Newtonian heating has been investigated theoretically by Narahari and Ishak [3]. Vest and Arpaci [4] have studied the stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below. The nature of instability and some factors may have different effects, on viscoelastic fluids as compared to the Newtonian fluids. For example, Bhatia and Steiner [5] have considered the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that rotation has a destabilizing influence, in contrast to the stabilizing effect on a Newtonian fluid. Bhatia and Steiner [6] have also considered the thermal instability of a Maxwell fluid in hydromagnetics and have found that the magnetic field has stabilizing effect on viscoelastic fluid, just as in case of Newtonian fluid. Sharma [7] has studied the thermal instability of a layer of Oldroydian [8] fluid acted on by a uniform rotation and found that rotation has destabilizing and stabilizing effects under certain conditions, in contrast to a Maxwell fluid where the effect is destabilizing. In another study, Sharma [9] has studied the stability of a layer of an electrically conducting Oldroyd fluid in the presence of a magnetic field and has found that the magnetic field has a stabilizing influence. Many common materials such as paints, polymers, plastics and more exotic one such as silicic magma, saturated soils and the Earth's lithosphere behaves as viscoelastic fluids. Due to the growing use of these viscoelastic materials in various manufacturing and processing industries, in geophysical fluid dynamics, in chemical technology and in petroleum industry, considerable effort has been directed towards understanding their flow.

There are many elastico-viscous fluids that cannot be characterized by Maxwell's

constitutive relations or Oldroyd's [8] constitutive relations. One such class of elastico-viscous fluids is Rivlin-Ericksen fluid [10]. It is well known that the Rivlin-Ericksen fluid [10] is characterized by the constitutive equations

$$S = -pI + \mu A_1 + \mu' A_2 + \mu^{ii} A_1^2 + \mu^{iii} A_2^2 +$$
$$\mu^{iv}(A_1 A_2 + A_2 A_1) + \mu^{v}(A_1^2 A_2 + A_2 A_1^2)$$
$$+ \mu^{vi}(A_1 A_2^2 + A_2^2 + A_1) + \mu^{vii}(A_1^2 A_2^2 + A_2^2 A_1^2)$$

where S is the Cauchy stress tensor, 'p' is an arbitrary hydrostatic pressure, I is the unit tensor and μ′s are polynomial functions of the traces of the various tensors occurring in the representation, matrices 'A_1' and 'A_2' are defined by

$$[A_1]_{ij} = (q_{i,j} + q_{j,i})$$

and

$$[A_2]_{ij} = \frac{\partial [A_1]_{ij}}{\partial t} + q_p [A_1]_{ij,p} + [A_1]_{ip} q_{p,j} + [A_1]_{pj} q_{p,i}$$

'q_p' being velocity vector.

On neglecting the squares and products of 'A_2', we have

$$S = -pI + \mu A_1 + \mu' A_2 + \mu^{ii} A_1^2 \quad ,$$

where μ, μ^{i} *and* μ^{ii} are three material constants. It is customary to call μ , the coefficient of ordinary viscosity, μ' the coefficient of viscoelasticity *and* μ^{ii}, the coefficient of cross-viscosity.

The μ, μ^{i} *and* μ^{ii} are general functions of temperature and material properties. For many fluids such as aqueous solution of polycrylamid and poly-isobutylene, μ, μ^{i} *and* μ^{ii} may be taken as constants. Such and other polymers are used in the manufacture of parts of spacecrafts, aeroplane parts, tyres, belt conveyers, ropes, cushions, seats, foams, plastics, engineering equipments, adhesives, contact lens etc. Recently, polymers are also used in agriculture, communication appliances and in biomedical applications. Srivastava and Singh [11] have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channels of different cross-sections in the presence of a time-dependent pressure gradient. In another study, Garg et al. [12] have studied the rectilinear oscillations of a sphere along its diameter in a conducting dusty Rivlin-Ericksen fluid in the presence of a uniform magnetic field. Sharma and Kumar [13] have studied the effect of rotation on thermal instability in Rivlin-Ericksen elastico-viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Kumar and Singh [14] have studied the stability of two superposed Rivlin-Ericksen viscoelastic fluids in the presence of suspended particles. In another study, Kumar et al. [15] have studied the hydrodynamic and hydromagnetic stability of two stratified Rivlin-Ericksen elastic-viscous superposed fluids. Kumar [16] has studied the effects of uniform vertical magnetic field and uniform rotation on the double-diffusive convection in Rivlin-Ericksen viscoelastic fluid through permeable media. Kumar [17] has also studied the thermosolutal convection in a compressible rotator Rivlin-Ericksen viscoelastic fluid in permeable media. A layer of such fluid heated from below under the action of magnetic field and rotation may find applications in geophysics, interior of the Earth, oceanography and the atmospheric physics.

Keeping in mind the importance of viscoelastic fluids, convection in fluid layer heated from below, magnetic field and rotation; the present paper attempts to study the effect of uniform vertical magnetic field on Rivlin-Ericksen viscoelastic fluid heated from below in the presence of a uniform rotation.

2 Structure of the Problem and Basic Equations

The present problem is studied using methods of linearized stability theory and normal mode analysis. First of all linearized perturbation equations relevant to the problem are obtained and then in section 3 the dispersion relation obtained by using normal analysis method.

Consider an infinite, horizontal, incompressible electrically conducting Rivlin-Ericksen viscoelastic fluid layer of thickness *d*, heated from below so that, the temperatures and densities at the bottom surface $z = 0$ are T_0 and ρ_0 and at the upper surface $z = d$ are T_d and

ρ_d respectively and that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The gravity field $\vec{g}(0,0,-g)$, a uniform vertical magnetic field $\vec{H}(0,0,H)$ and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ act on the system.

The equations of motion, continuity, heat conduction and Maxwell's equations governing the flow of Rivlin-Ericksen viscoelastic fluid in the presence of magnetic field and rotation are

$$\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}=-\nabla\left(\frac{p}{\rho_0}-\frac{1}{2}\left|\vec{\Omega}\times\vec{r}\right|^2\right)+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\nabla^2\vec{v}+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{H})\times\vec{H}+2(\vec{v}\times\vec{\Omega}), \quad (1)$$

$$\nabla\cdot\vec{v}=0, \quad (2)$$

$$\frac{\partial T}{\partial t}+(\vec{v}\cdot\nabla)T=\chi\nabla^2T, \quad (3)$$

$$\nabla\cdot\vec{H}=0, \quad (4)$$

$$\frac{\partial\vec{H}}{\partial t}=(\vec{H}\cdot\nabla)\vec{v}+\eta\nabla^2\vec{H}, \quad (5)$$

where $\vec{v}(u,v,w)$, p, ρ, T, υ and υ' denote the velocity, pressure, density, temperature, kinematic viscosity and kinematic viscoelasticity, respectively and $\vec{r}(x,y,z)$.

The equation of state for the fluid is

$$\rho=\rho_0[1-\alpha(T-T_0)], \quad (6)$$

where ρ_0, T_0 are respectively, the density and temperature of the fluid at the reference level z = 0 and α is the coefficient of thermal expansion. In writing equation (1), use has been made of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equations of motion except the external force term. The magnetic permeability μ_e, thermal diffusivity χ and electrical resistivity η are all assumed to be constant.

The initial state is one in which the velocity, density, pressure and temperature at any point in the fluid are respectively given by

$$\vec{v}=(0,0,0),\ \rho=\rho(z),\ p=p(z), \text{ and } T=T(z). \quad (7)$$

Let $\vec{v}(u,v,w)$, δp, $\delta\rho$, θ and $\vec{h}(h_x,h_y,h_z)$ denote respectively the perturbations in velocity $\vec{v}$ (initially zero), pressure p, density ρ, temperature T and the magnetic field $\vec{H}(0,0,H)$. The change in density $\delta\rho$, caused by the perturbation θ in temperature, is given by

$$\rho+\delta\rho=\rho_0[1-\alpha(T+\theta-T_0)]=\rho-\alpha\rho_0\theta,$$

i.e. $$\delta\rho=-\alpha\rho_0\theta. \quad (8)$$

Then the linearized perturbation equations are

$$\frac{\partial\vec{v}}{\partial t}=-\frac{1}{\rho_0}(\nabla\delta p)-\vec{g}\alpha\theta+\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\nabla^2\vec{v}+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{h})\times\vec{H}+2(\vec{v}\times\vec{\Omega}) \quad (9)$$

$$\nabla\cdot\vec{v}=0, \quad (10)$$

$$\frac{\partial\theta}{\partial t}=\beta w+\chi\nabla^2\theta, \quad (11)$$

$$\nabla\cdot\vec{h}=0, \quad (12)$$

$$\frac{\partial\vec{h}}{\partial t}=(\vec{H}\cdot\nabla)\vec{v}+\eta\nabla^2\vec{h}. \quad (13)$$

Within the framework of Boussinesq approximation, equations (9)-(13), become

$$\frac{\partial}{\partial t}\nabla^2 w=\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\nabla^4 w+\frac{\mu_e H}{4\pi\rho_0}\nabla^2\left(\frac{\partial h_z}{\partial z}\right)+g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-2\Omega\frac{\partial\zeta}{\partial z} \quad (14)$$

$$\frac{\partial\zeta}{\partial t}=\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\nabla^2\zeta+2\Omega\frac{\partial w}{\partial z}-\frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial z}, \quad (15)$$

$$\left[\frac{\partial}{\partial t}-\chi\nabla^2\right]\theta=\beta w, \quad (16)$$

$$\left[\frac{\partial}{\partial t}-\eta\nabla^2\right]h_z=H\frac{\partial w}{\partial z}, \quad (17)$$

$$\left[\frac{\partial}{\partial t}-\eta\nabla^2\right]\xi=H\frac{\partial\zeta}{\partial z}, \quad (18)$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ and $\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$; $\xi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$, stand for the z-components of vorticity and current density, respectively.

3 The Dispersion Relation

We decompose the disturbances into normal modes and assume that the perturbed quantities are of the form

$$[w,\theta,h_z,\zeta,\xi]=[W(z),\Theta(z),K(z),Z(z),X(z)]\exp(ik_x x+ik_y y+nt) \quad (19)$$

where k_x, k_y are the wave numbers along x- and y-directions, respectively, $k=(k_x^2+k_y^2)^{1/2}$ is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using expression (19), equations (14)-(18) in non-dimensional form transform to

$$\left[\sigma(D^2-a^2)W+\left(\frac{g\alpha d^2}{\upsilon}\right)a^2\Theta+\frac{2\Omega d^3}{\upsilon}DZ\right]-\frac{\mu_e Hd}{4\pi\rho_0\upsilon}(D^2-a^2)DK=[1+F\sigma](D^2-a^2)^2W, \quad (20)$$

$$[\{1+F\sigma\}(D^2-a^2)-\sigma]Z=-\left(\frac{2\Omega d}{\upsilon}\right)DW-\left(\frac{\mu_e Hd}{4\pi\rho_0\upsilon}\right)DX, \quad (21)$$

$$[D^2-a^2-p_1\sigma]\Theta=-\left(\frac{\beta d^2}{\chi}\right)W, \quad (22)$$

$$[D^2-a^2-p_2\sigma]K=-\left(\frac{Hd}{\eta}\right)DW, \quad (23)$$

$$[D^2-a^2-p_2\sigma]X=-\left(\frac{Hd}{\eta}\right)DZ, \quad (24)$$

where we have introduced new co-ordinates $(x',y',z')=\left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right)$ in new units of length d and $D=\frac{d}{dz'}$. For convenience, the dashes are dropped hereafter. Also we have put $a = kd$, $\sigma=\frac{nd^2}{\upsilon}$, $F=\frac{\upsilon'}{d^2}$, $p_1=\frac{\upsilon}{\chi}$ is the Prandtl number and $p_2=\frac{\upsilon}{\eta}$ is the magnetic Prandtl number.

We now consider the case where both the boundaries are free as well as perfect conductors of heat, while the adjoining medium is also perfectly conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres [Spiegel [18]] and in certain geophysical situations where it is most appropriate. However, the case of two free boundaries allows us to obtain analytical solution without affecting the essential features of the problem. The appropriate boundary

conditions, with respect to which equations (20)-(24) must be solved, are

$$W = D^2W = 0, DZ = 0, \Theta = 0$$
$$at\ z = 0 \text{ and } z = 1,$$
$$DX = 0,\ K = 0$$

on a perfectly conducting boundary. (25)

Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$, and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \quad (26)$$

where W_0 is a constant.

Eliminating $\Theta, K, Z \text{ and } X$ between equations (20)-(24) and substituting (26) in the resultant equation, we obtain the dispersion relation

$$R_1 = \left(\frac{1+x}{x}\right) \frac{\left[\begin{Bmatrix}\{(1+iF_1\sigma_1\pi^2)(1+x)+i\sigma_1\} \\ \{1+x+i\sigma_1 p_2\}+Q_1\end{Bmatrix}\right][1+x+i\sigma_1 p_1]}{(1+x+i\sigma_1 p_2)}$$
$$+ \frac{T_1(1+x+i\sigma_1 p_2)(1+x+i\sigma_1 p_1)}{\left\{x\left[\begin{Bmatrix}\{(1+iF_1\sigma_1\pi^2)(1+x)+i\sigma_1\} \\ \{1+x+i\sigma_1 p_2\}+Q_1\end{Bmatrix}\right]\right\}}, \quad (27)$$

where

$$R = \frac{g\alpha\beta d^4}{\upsilon\chi}, Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\upsilon\eta}, T_A = \frac{4\Omega^2 d^4}{\upsilon^2},$$

stand for the Rayleigh-number, the Chandrasekhar number, the Taylor number, respectively and we have also put

$$x = \frac{a^2}{\pi^2},\ R_1 = \frac{R}{\pi^4}, i\sigma_1 = \frac{\sigma}{\pi^2},\ F_1 = \pi^2 F,$$
$$T_1 = \frac{T_A}{\pi^4},\ Q_1 = \frac{Q}{\pi^2}$$

and $i = \sqrt{-1}$.

4 Important Theorems and Discussion

Theorem 1: For stationary convection case:

(I) The viscoelasticity parameter F vanishes with σ and Rivlin-Ericksen viscoelastic fluid behaves like an ordinary Newtonian fluid.

(II) Rotation postpones the onset of convection i.e. rotation has a stabilizing effect on the system.

(III) The magnetic field has both stabilizing and destabilizing effects on the system.

Proof: When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (27) reduces to

$$R_1 = \left(\frac{1+x}{x}\right)\left[(1+x)^2 + Q_1\right] + \frac{T_1(1+x)^2}{x\left[(1+x)^2 + Q_1\right]} \quad (28)$$

[a result given by Chandrasekhar [1], equation (59), p. 202] .

To investigate the effects of rotation and magnetic field, we examine the behavior of $\frac{dR_1}{dT_1}$ and $\frac{dR_1}{dQ_1}$ analytically.

(I) It is evident from relation (28) that for the stationary convection, the viscoelasticity parameter F vanishes with σ and Rivlin-Ericksen viscoelastic fluid behaves like an ordinary Newtonian fluid.

(II) Equation (28) yields

$$\frac{dR_1}{dT_1} = \frac{(1+x)^2}{x\left[(1+x)^2 + Q_1\right]}, \quad (29)$$

which is always positive. The rotation, therefore, has a stabilizing effect on the system for the case of stationary convection.

(III) It is also evident from equation (28) that

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{x} - \frac{T_1(1+x)^2}{x\left[(1+x)^2 + Q_1\right]^2}. \qquad (30)$$

It is clear from equation (30) that, for stationary convection, $\frac{dR_1}{dQ_1}$ may be positive as well as negative, thus, the magnetic field has both stabilizing and destabilizing effects on the system.

Theorem 2: The system is stable or unstable.

Proof: Multiplying equation (20) by W^*, the complex conjugate of W, integrating the resulting equation over the range of z and using equations (21)-(24) together with the boundary conditions (25), we obtain

$$\begin{aligned} &-\sigma I_1 + \frac{g\alpha\chi a^2}{\upsilon\beta}\left(I_2 + p_1\sigma^* I_3\right) - d^2\left(1 + F\sigma^*\right)I_4 \\ &- d^2\sigma^* I_5 - \\ &-\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}\left(I_6 + p_2\sigma I_7\right) - \frac{\mu_e\eta}{4\pi\rho_0\upsilon}\left(I_8 + p_2\sigma^* I_9\right) \\ &= (1+F\sigma)I_{10} \end{aligned} \qquad (31)$$

where

$$I_1 = \int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz,$$

$$I_2 = \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz,$$

$$I_3 = \int_0^1 \left(|\Theta|^2\right)dz, \quad I_4 = \int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz,$$

$$I_5 = \int_0^1 \left(|Z|^2\right)dz, \quad I_6 = \int_0^1 \left(|DX|^2 + a^2|X|^2\right)dz,$$

$$I_7 = \int_0^1 \left(|X|^2\right)dz,$$

$$I_8 = \int_0^1 \left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz,$$

$$I_9 = \int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz,$$

$$I_{10} = \int_0^1 \left(|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right)dz, \qquad (32)$$

and σ^* is the complex conjugate of σ. The integrals $I_1, \ldots\ldots\ldots, I_{10}$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$, where σ_r, σ_i are real and equating the real and imaginary parts of equation (31), we obtain

$$\begin{aligned} &\sigma_r\begin{bmatrix} -I_1 + \frac{g\alpha\chi a^2}{\upsilon\beta}p_1 I_3 - d^2 F I_4 - d^2 I_5 \\ -\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}p_2 I_7 - \frac{\mu_e\eta}{4\pi\rho_0\upsilon}p_2 I_9 - F I_{10} \end{bmatrix} \\ &= -\frac{g\alpha\chi a^2}{\upsilon\beta}I_2 + d^2 I_4 + \frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}I_6 \\ &+ \frac{\mu_e\eta}{4\pi\rho_0\upsilon}I_8 + I_{10} \end{aligned}, \qquad (33)$$

$$\sigma_i\begin{bmatrix} I_1 + \frac{g\alpha\chi a^2}{\upsilon\beta}p_1 I_3 - d^2 F I_4 - d^2 I_5 \\ +\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}p_2 I_7 - \frac{\mu_e\eta}{4\pi\rho_0\upsilon}p_2 I_9 + F I_{10} \end{bmatrix} = 0. \qquad (34)$$

It is evident from equation (33) that σ_r is positive or negative. The system is, therefore, stable or unstable.

Theorem 3: The modes may be either oscillatory or non-oscillatory in contrast to the non-magneto-rotatory case.

Proof: Equation (34) yields that σ_i may be either zero or non-zero, which means that the modes may be either non-oscillatory or

oscillatory. In the absence of rotation and magnetic field, equation (34) reduces to

$$\sigma_i\left[I_1+\frac{g\alpha\chi a^2}{\upsilon\beta}p_1I_3+FI_{10}\right]=0,$$

and the terms in brackets are positive definite. Thus, $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for Rivlin-Ericksen viscoelastic fluid heated from below in the absence of rotation and magnetic field. It is clear from equation (34) that the oscillatory modes are introduced due to the presence of rotation and the magnetic field, which were non-existent in their absence.

Theorem 4: The system is stable for $\frac{g\alpha\chi}{\nu\beta}\le\frac{27\pi^4}{4}$ and under the condition $\frac{g\alpha\chi}{\nu\beta}>\frac{27\pi^4}{4}$, the system becomes unstable.

Proof: From equation (34), it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If $\sigma_i\neq 0$, then equation (34) gives

$$\frac{g\alpha\chi a^2}{\upsilon\beta}p_1I_3-d^2FI_4-d^2I_5-\frac{\mu_e\eta}{4\pi\rho_0\upsilon}p_2I_9$$
$$=-I_1-\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}p_2I_7-FI_{10}. \quad (35)$$

Substituting in equation (33), we have

$$I_{10}+\frac{\mu_e\eta}{4\pi\rho_0\upsilon}I_8+\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}I_6+d^2I_4+$$
$$2\sigma_r\left[I_1+\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}p_2I_7+FI_{10}\right]=\frac{g\alpha\chi a^2}{\upsilon\beta}I_2, \quad (36)$$

Equation (36) on using Rayleigh-Ritz inequality, gives

$$\frac{\left(\pi^2+a^2\right)^3}{a^2}\int_0^1|W|dz+$$
$$\frac{\left(\pi^2+a^2\right)}{a^2}\left\{\begin{array}{l}\frac{\mu_e\eta}{4\pi\rho_0\upsilon}I_8+\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}I_6+d^2I_4+\\ 2\sigma_r\left[I_1+\frac{\mu_e d^2\eta}{4\pi\rho_0\nu}p_2I_7+FI_{10}\right]\end{array}\right\}$$
$$\le\frac{g\alpha\chi}{\nu\beta}\int_0^1|W|\,dz. \quad (37)$$

Therefore, it follows from equation (37) that

$$\left[\frac{27\pi^4}{4}-\frac{g\alpha\chi}{\nu\beta}\right]\int_0^1|W|\,dz+$$
$$\frac{\left(\pi^2+a^2\right)}{a^2}\left\{\begin{array}{l}\frac{\mu_e\eta}{4\pi\rho_0\upsilon}I_8+\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}I_6+d^2I_4+\\ 2\sigma_r\left[I_1+\frac{\mu_e d^2\eta}{4\pi\rho_0\nu}p_2I_7+FI_{10}\right]\end{array}\right\}$$
$$\le 0, \quad (38)$$

since minimum value of $\frac{\left(\pi^2+a^2\right)^3}{a^2}$ w.r.t. a^2 is $\frac{27\pi^4}{4}$.

Now, let $\sigma_r\ge 0$, we necessarily have from equation (38) that

$$\frac{g\alpha\chi}{\nu\beta}>\frac{27\pi^4}{4}. \quad (39)$$

Hence, if $\frac{g\alpha\chi}{\nu\beta}\le\frac{27\pi^4}{4}$, (40)

then $\sigma_r<0$. Therefore, the system is stable. Therefore, under condition (40), the system is stable and under condition (39) the system becomes unstable.

5 Conclusions

A layer of Newtonian fluid heated from below under varying assumptions of hydrodynamics in the presence and absence of rotation and magnetic field has been studied by Chandrasekhar [1]. With the growing importance of viscoelastic fluids in chemical engineering, modern technology and industry, the investigations on such fluids are desirable. The Rivlin-Ericksen fluid is one such important

viscoelastic fluid. Keeping in mind the importance of viscoelastic fluids, the present paper considering the effect of rotation on the Rivlin-Ericksen viscoelastic fluid heated from below in the presence of a uniform vertical magnetic field.
The main conclusions from the analysis of this paper are as follows:

1. For the case of stationary convection the following observations are made:

   a. The viscoelasticity parameter F vanishes with σ and Rivlin-Ericksen viscoelastic fluid behaves like an ordinary Newtonian fluid.

   b. The rotation postpones the onset of convection i.e. rotation has a stabilizing effect on the system.

   c. The magnetic field has both stabilizing and destabilizing effects on the system.

2. It is found that rotation and magnetic field introduce oscillatory modes in the system which was non-existent in their absence.

3. It is observed that the system is stable for $\frac{g\alpha\chi}{\nu\beta} \le \frac{27\pi^4}{4}$ and under the condition $\frac{g\alpha\chi}{\nu\beta} > \frac{27\pi^4}{4}$, the system becomes unstable.

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